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On Waves in War and Elections*

Wavelet Analysis of Political Time-Series

Luís Aguiar-Conraria† Pedro C. Magalhães‡ Maria Joana Soares§

January 14, 2010

Abstract

Spectral analysis and ARMA models have been the most established weapons of choice for the detection of cycles in time series data. However, such techniques are only appropriate when periodic components are time invariant. This has led some scholars to disregard the possibility of irregular cycles. We aim to contribute to further reconsideration of the possibility of cycles in political phenomena by introducing wavelet analysis. Its main contribution comes from the ability to estimate the spectral characteristics of a time series as a function of time, thus revealing how its different periodic components change over time, while preserving rigorous standards of statistical inference. We demonstrate the usefulness of wavelet analysis with two applications. The first concerns the possibility of long-cycles in wars, one of the central puzzles in the international relations sub-field. The second concerns election returns in presidential and congressional elections in the United States, where the existence of cyclical patterns has been equally controversial. Contrary to the prevailing wisdom, wavelet analysis allows the detection of transient and coexisting cycles, shedding some light over phenomena that have remained unaddressed so far.

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1 Introduction

The notion that cultural, social, economic, and political phenomena display cyclical patterns throughout history has always exerted considerable allure. Cyclical thinking was arguably the dominant conception of time in primitive societies (Eliade 1954), remained central in Greek and Graeco-Roman approaches to history (Nisbet 1980), and was revived by philosophers such as Nietzsche and Spengler, who decried the "linear"conceptions of change inherent to modernity (Mohler 2001). Inevitably, cyclical theories have also found their way into the modern social sciences, especially in economics (Kitchin 1923; Burns and Mitchell 1946; Kuznets 1930 and 1958; Kondratieff 1935; Schumpeter 1939) but also, to a lesser extent, in other disciplines (see, for example, Namenwirth and Weber 1987; Wallerstein 1974; Arrighi 1994).[1] The attraction is not entirely surprising. On the one hand, cyclical theories, particularly when assuming some sort of fixed periodicity, seem to contain the promise that"change"can be predicted to occur at a particular point in time. On the other hand, the detection of cycles, particularly long-term ones, seems to provide a potentially interesting contrast to the (arguably prevalent) views of social reality that focus solely on the strategic actions of individuals and use short-term horizons for the detection of causes and outcomes (Pierson 2003).

However, as van der Eijk and Weber elegantly put it, cyclical thinking has also been a sort of "Lorelei", one of the Rhine nymphs whose singing led boatmen to the rocks where they inevitably perished: "researchers are lured to it because of its theoretical promise, only to become entangled in (if not wrecked by) messy problems of empirical inference"(Eijk and Weber 1987, 271). Political science has been immune neither to the attraction of cycles nor to ensuing wreckage. In the subfield of international relations, for example, although the question of whether there are long cycles of war in the international system has intrigued scholars for a long time, it still remains one of the fundamental sources of conflicting findings in the literature (Mintz 2005)."Long-wave"theories of war have been criticized not only theoretically and conceptually but also on methodological and empirical grounds, including the scarcity of the data available to uncover any alleged cyclicality, the inadequacy of the statistical techniques that have been used for that purpose, and the inability to find the hypothesized empirical patterns once accepted methodological standards are employed. Similarly, in the study of American politics, the existence of cycles in election returns has played a central role in the debates around the most important – but also the most contested – theory about electoral change in the United States: the theory of electoral realignment. The periodicity of those cycles, the kind of events and outcomes supposedly exhibiting cyclical periodicity and even the very existence of any

[1] For reviews of cyclical theories in the social sciences, see Bürklin (1987) or Mohler (2001).

sort of cyclicality have remained contentious throughout the history of American electoral studies (see Rosenof 2003 for a review of these debates since the early 20th century). And although the evidence in favor of some sort of cyclicality is mounting, different strands of scholarship and different statistical techniques have yielded different answers concerning the length and periodicity of such cycles, as well as to the question of whether cyclicality is indeed a feature of the entire history of American elections or just a part of it.

Our aim in this paper is to contribute to further reconsideration of the possibility of cycles in political phenomena. We do that by introducing a new set of statistical tools provided by wavelet analysis. So far, for empirically and statistically minded social researchers, spectral analysis and ARMA models have been the most established weapons of choice for the detection of cycles in time series data. Although the analysis of stationary processes in the frequency domain (spectral analysis) is equivalent to the analysis in the time domain based on the auto-covariance function (ARMA), the former has the advantage of addressing the issue of cycle detection in a direct fashion. Thus, political scientists have resorted to Fourier spectral analysis in attempts to uncover cycles in a variety of phenomena, including terrorist attacks (Enders and Sandler 2000, 2002 and 2006), military expenditures (Williams and McGinnis 1992; Gerace 2002), policy agendas (Howlett 1998), government popularity (Goodhart and Bhansali 1970; Miller and Mackie 1973; Richards 1992), and, closer to our substantive concerns in this paper, the severity of wars (Beck 1991) and election returns (Lin and Guillén 1998; Merrill, Grofman, and Brunell 2008). There are, however, two main problems with spectral analysis. First, it is only appropriate for strictly stationary time series, while many, if not all, economic and political time series are, in fact, noisy, complex, and strongly non-stationary. The second problem goes to the heart of what tends to make debates about cycles often particularly acrimonious (Eijk and Weber 1987, 276): the possibility of irregular cycles without fixed periodicity throughout the entire time series. This has led some proponents of cyclical theories to dismiss spectral analysis on grounds of its inability to deal with the possibility that cycles have evolving periodicity (Goldstein 1988),[2] albeit the alternatives they provide often fall short of meeting rigorous standards of statistical inference. Conversely, it has been argued that"if spectral analysis doesn't find long cycles, then no other technique with known statistical properties will work any better"(Beck 1991, 464). However, this assessment of established techniques risks taking the additional step of turning the basic assumptions of Fourier spectral analysis into questionable assumptions about the nature of the social and political cycles themselves ("only fixed period models are meaningful models of cyclic phenomena"- Beck 1991, 456).

Wavelet analysis provides an alternative to these equally unsatisfactory approaches

[2]Indeed, strict stationarity implies a time invariant data generating process. Therefore cyclicality needs to be assumed as time invariant.

to the conceptualization and analysis of cycles in social and political phenomena. As a coherent mathematical body, wavelet theory was born in the mid-1980s (Grossman and Morlet 1984; Goupillaud *et al.* 1984).[3] It is now used in physics, geophysics, astronomy, epidemiology, signal processing, oceanography, and, more recently, in economics, with the pioneering works of Ramsey and Lampart (1998), and Gençay, Selçuk and Witcher (2001).[4] Its main contribution comes from estimating the spectral characteristics of a time series as a function of time, thus revealing how its different periodic components may change over time. As a result, as we shall see in more detail in the following sections, wavelet analysis addresses both the problems resulting from the analysis of non-stationary time series and the possibility that the different periodic components of those series change over time, allowing for the detection of changes in the periodicity of cycles and, at the same time, rigorous statistical inference. Nevertheless, its use in the field of economics remains infrequent[5] and, to our knowledge, it has never been used in other social sciences. In this paper, we show that the potential of wavelet analysis also extends to the study of political time series, by demonstrating its applicability to the study of two major puzzles in the political science literature: the existence of cycles in the severity of wars and in election outcomes.

The paper proceeds as follows. The next section presents three basic sets of tools in wavelet analysis. The first leads to the estimation of the *wavelet power spectrum*, which describes the evolution of the volatility of a single time series at the different frequencies. The second and third sets of tools move us from univariate to multivariate analysis of time series: *cross-wavelets* and *phase-differences* give us, respectively, quantified indications of the similarity of power between two time series and a measure of the coordination and delays of the oscillations between them. Wavelet analysis comprises a large number of complex technicalities. Thus, throughout the following section, we will illustrate the main concepts with some controlled data experiments. Section three of the paper applies the first two of the aforementioned tools to the analysis of the severity of wars from 1495 to 1975, using well-established data in the literature (Levy 1983; Goldstein 1988). We identity a first statistically significant cycle in the severity of wars but are also able to conclude, thanks to the flexibility of wavelet analysis, that such a cycle is not prevalent throughout the entire time series, being replaced by shorter-length cycle by its end. Furthermore, using the tools of cross-wavelet analysis, we examine the hypothesis that cycles in war severity were correlated to cycles in inflation. Section four applies the same tools, as well as the phase and phase-difference analyses, to presidential and congressional election

[3] See Hubbard (1996) for a very nice non-technical historic account of wavelet analysis.

[4] For a detailed review of wavelet applications to economic data, the reader is referred to Crowley (2007).

[5] In particular, the continuous cross-wavelet transform that we use in this paper has only been applied to economics by Crowley and Mayes (2008) and Aguiar-Conraria, Azevedo and Soares (2008).

returns in the United States. We show that, although the evidence from wavelet analysis of those data does lend additional support to recent attempts to uncover cycles in election returns (Lin and Guillén 1998; Merrill, Grofman, and Brunell 2008), such cyclicality can be shown, unlike what could ever emerge from spectral analysis, as temporally localized. Moreover, we show that the dynamics ruling the presidential elections is distinct from the congressional elections dynamics. Finally, we use the tools of cross-wavelets and phase differences to uncover the relationship between cycles in presidential and congressional elections. Section five concludes.

2 Wavelets: Frequency Analysis Across Time

Fourier spectral analysis can be, and has been, used to determine whether there are frequencies that play predominant roles in explaining the overall variance of a time series, by decomposing the observed pattern over time into a spectrum of cycles of different lengths, just as a prism decomposes white light into a spectrum of colors of different frequencies. In spite of its utility, however, under the Fourier transform, the time information of a time-series is completely lost. Because of this loss of information, it is an almost impossible task to distinguish transient relations or to identify structural changes. Moreover, as we have argued, these techniques are only appropriate for time-series with stable statistical properties. Unfortunately, typical political time-series are noisy, complex and strongly non-stationary.

To overcome the problems associated with the classical Fourier analysis, Gabor (1946) introduced the Short Time Fourier Transform. The basic idea is to break a time-series into smaller sub-samples and apply the Fourier transform to each sub-sample. However, this approach remains inefficient because the frequency resolution is the same across all different frequencies. In order to capture abrupt changes, for example, one would like to have very short functions (narrow windows). At the same time, to isolate slow and persistent movements, one would like to have very long functions (wide windows). This is exactly what can be achieved with the wavelet transform. While the Fourier transform breaks down a time-series into constituent sinusoids of different frequencies and infinite duration in time, the wavelet transform expands the time-series into shifted and scaled versions of a function that has limited spectral band and limited duration in time. This leads to one major advantage afforded by the wavelet transform, i.e., the ability to perform natural local analysis of a time-series in the sense that the length of wavelets varies endogenously: it stretches into a long wavelet function to measure the low frequency movements; and it compresses into a short wavelet function to measure the high frequency movements.

It is true that the Heisenberg uncertainty principle proves that there will always be a trade-off between localization in time and localization in frequency; in particular, we cannot ask for a function to be, simultaneously, band and time limited. However, a mother wavelet can be chosen with a fast decay in time and frequency which, for all practical purposes, corresponds to an effective band and time limiting (see Daubechies 1992).

In this section, we introduce three wavelet tools: (1) the wavelet power spectrum, which describes the evolution of the variance of a time-series at the different frequencies, with periods of large variance associated with periods of large power at the different scales, (2) the cross-wavelet power of two time-series, which describes the local covariance between the time-series, and the wavelet coherency, which can be interpreted as a localized correlation

coefficient in the time frequency space, and (3) the phase, which can be viewed as the position in the cycle of the time-series as a function of frequency, and the phase-difference, which gives us information on the delay, or synchronization, between oscillations of the two time-series.

2.1 The Wavelet

In what follows, $L^2(\mathbb{R})$ denotes the set of square integrable functions, i.e. the set of functions defined on the real line such that $\|x\| := \{\int_{-\infty}^{\infty} |x(t)|^2 dt\}^{1/2} < \infty$, with the usual inner product, $\langle x, y\rangle := \int_{-\infty}^{\infty} x(t)y^*(t)dt$. The asterisk superscript denotes complex conjugation. Given a function $x(t) \in L^2(\mathbb{R})$, $X(f) := \int_{-\infty}^{\infty} x(t)\, e^{-i2\pi ft}dt$ will denote its Fourier transform. We recall the well-known Parseval relation, valid for all $x(t), y(t) \in L^2(\mathbb{R})$, $\langle x(t), y(t)\rangle = \langle X(f), Y(f)\rangle$, from which the Plancherel identity immediately follows: $\|x(t)\| = \|X(f)\|$. The minimum requirements imposed on a function $\psi(t)$ to qualify for being a *mother (admissible or analyzing) wavelet* are that $\psi \in L^2(\mathbb{R})$ and also fulfills a technical condition, usually referred to as the *admissibility condition*, which reads as follows:

$$0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(f)|}{|f|} df < \infty, \tag{1}$$

where $\Psi(f)$ is the Fourier transform of $\psi(t)$,(see Daubechies 1992, 24).

The wavelet ψ is usually normalized to have unit energy: $\|\psi\|^2 = \int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$. The square integrability of ψ is a very mild decay condition; the wavelets used in practice have much faster decay; typical behavior will be exponential decay or even compact support. For functions with sufficient decay it turns out that the admissibility condition (1) is equivalent to requiring $\Psi(0) = \int_{-\infty}^{\infty} \psi(t)\, dt = 0$. This means that the function ψ has to wiggle up and down the $t-$axis, i.e. it must behave like a wave; this, together with the decaying property, justifies the choice of the term wavelet (originally, in French, *ondelette*) to designate ψ.

2.2 The Continuous Wavelet Transform

Starting with a mother wavelet ψ, a family $\psi_{s,\tau}$ of "wavelet daughters"can be obtained by simply scaling ψ by s and translating it by τ

$$\psi_{s,\tau}(t) := \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \quad s, \tau \in \mathbb{R}, s \neq 0. \tag{2}$$

The parameter s is a scaling or dilation factor that controls the length of the wavelet (the factor $1/\sqrt{|s|}$ being introduced to guarantee preservation of the unit energy, $\left\|\psi_{s,\tau}\right\| = 1$)

and τ is a location parameter that indicates where the wavelet is centered. Scaling a wavelet simply means stretching it (if $|s| > 1$), or compressing it (if $|s| < 1$).[6]

Given a function $x(t) \in L^2(\mathbb{R})$ (a time-series), its continuous wavelet transform (CWT) with respect to the wavelet ψ is a function $W_x(s,\tau)$ obtained by projecting $x(t)$, in the L^2 sense, onto the over-complete family $\{\psi_{s,\tau}\}$:

$$W_x(s,\tau) = \langle x, \psi_{s,\tau} \rangle = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^* \left(\frac{t-\tau}{s} \right) dt. \tag{3}$$

When the wavelet $\psi(t)$ is chosen as a complex-valued function, the wavelet transform $W_x(\tau, s)$ is also complex-valued. In this case, the transform can be separated into its real part, $\mathcal{R}(W_x)$, and imaginary part, $\mathcal{I}(W_x)$, or in its amplitude, $|W_x(\tau, s)|$, and phase, $\phi_x(\tau, s) : W_x(\tau, s) = |W_x(\tau, s)| e^{i\phi_x(\tau,s)}$. Recall that the phase-angle $\phi_x(\tau, s)$ of the complex number $W_x(\tau, s)$ can be obtained from the formula: $\tan(\phi_x(\tau, s)) = \frac{\mathcal{I}(W_x(\tau,s))}{\mathcal{R}(W_x(\tau,s))}$, using the information on the signs of $\mathcal{R}(W_x)$ and $\mathcal{I}(W_x)$ to determine to which quadrant the angle belongs to.

For real-valued wavelet functions, the imaginary part is constantly zero and the phase is, therefore, undefined. Hence, in order to separate the phase and amplitude information of a time-series, it is important to make use of complex wavelets. As Lilly and Olhede (2009) explain, analytic wavelets, i.e. wavelets $\psi(t)$ satisfying $\Psi(f) = 0$, for $f < 0$, are ideal for the analysis of oscillatory signals, since the continuous analytic wavelet transform provides an estimate of the instantaneous amplitude and instantaneous phase of the signal in the vicinity of each time/scale location (τ, s).[7] The importance of the admissibility condition (1) comes from the fact that it guarantees that it is possible to recover $x(t)$ from its wavelet transform. When ψ is analytic and $x(t)$ is real, a reconstruction formula is given by

$$x(t) = \frac{2}{C_\psi} \int_0^{\infty} \left[\int_{-\infty}^{\infty} \mathcal{R}\left(W_x(s,\tau) \psi_{s,\tau}(t) \right) d\tau \right] \frac{ds}{s^2}. \tag{4}$$

Therefore, we can easily go from $x(t)$ to its wavelet transform, and from the wavelet transform back to $x(t)$. Note that one can limit the integration over a range of scales, performing a band-pass filtering of the original series. See Daubechies (1992, 27-28) or Kaiser (1994, 70-73) for more details about analytic wavelets.

[6] Note that for negative s, the function is also reflected.

[7] Note that an analytic function is necessarily complex.

2.3 Localization Properties

Let the wavelet ψ be normalized so that $\|\psi\| = 1$. Also, assume that ψ and its Fourier transform Ψ have sufficient decay to guarantee that the quantities defined below are all finite. We define the center μ_t of ψ by

$$\mu_t = \int_{-\infty}^{\infty} t \left|\psi\left(t\right)\right|^2 dt. \tag{5}$$

In other words, the center of the wavelet is simply the mean of the probability distribution obtained from $\left|\psi\left(t\right)\right|^2$. As a measure of concentration of ψ around its center one usually takes the standard deviation σ_t:

$$\sigma_t = \left\{\int_{-\infty}^{\infty} \left(t-\mu_t\right)^2 \left|\psi\left(t\right)\right|^2 dt\right\}^{\frac{1}{2}}. \tag{6}$$

In a total similar manner, one can also define the center μ_f and standard deviation σ_f of the Fourier transform $\Psi(f)$ of ψ.

The interval $[\mu_t - \sigma_t, \mu_t + \sigma_t]$ is the set where ψ attains its "most significant"values, whilst $\left[\mu_f - \sigma_f, \mu_f + \sigma_f\right]$ plays the same role for $\Psi\left(f\right)$. The rectangle $[\mu_t - \sigma_t, \mu_t + \sigma_t] \times \left[\mu_f - \sigma_f, \mu_f + \sigma_f\right]$ in the (t, f) $-$plane is called the Heisenberg box or window in the time-frequency plane. We then say that ψ is localized around the point $\left(\mu_t, \mu_f\right)$ of the time-frequency plane with uncertainty given by $\sigma_t\sigma_f$.

The uncertainty principle, first established by Heisenberg in the context of quantum mechanics, gives a lower bound on the product of the standard deviations of position and momentum for a system, implying that it is impossible to have a particle that has an arbitrarily well-defined position and momentum simultaneously. In our context, the Heisenberg uncertainty principle tells us that there is always a trade-off between localization in time and localization in frequency; in particular, we cannot ask for a function to be, simultaneously, band and time limited. To be more precise, the Heisenberg uncertainty principle establishes that the uncertainty is bounded from below by the quantity $1/4\pi$:

$$\sigma_t\sigma_f \geq \frac{1}{4\pi}. \tag{7}$$

If the mother wavelet ψ is centered at μ_t, has standard deviation σ_t and its wavelet transform $\Psi\left(f\right)$ is centered at μ_f with a standard deviation σ_f, then one can easily show that the daughter wavelet $\psi_{\tau,s}$ will be centered at $\tau + s\mu_t$ with standard deviation $s\sigma_t$, whilst its Fourier transform $\Psi_{s,\tau}$ will have center $\frac{\mu_f}{s}$ and standard deviation $\frac{\sigma_f}{s}$.

From the Parseval relation, we know that $W_x\left(s,\tau\right) = \left\langle x\left(t\right), \psi_{s,\tau}\left(t\right)\right\rangle = \left\langle X\left(f\right), \Psi_{s,\tau}\left(f\right)\right\rangle$.

Therefore, the continuous wavelet transform $W_x(s,\tau)$ gives us local information within a time-frequency window $[\tau + s\mu_t - s\sigma_t, \tau + s\mu_t + s\sigma_t] \times \left[\frac{\mu_f}{s} - \frac{\sigma_f}{s}, \frac{\mu_f}{s} + \frac{\sigma_f}{s}\right]$. In particular, if ψ is chosen so that $\mu_t = 0$ and $\mu_f = 1$, then the window associated with $\psi_{\tau,s}$ becomes

$$[\tau - s\sigma_t, \tau + s\sigma_t] \times \left[\frac{1}{s} - \frac{\sigma_f}{s}, \frac{1}{s} + \frac{\sigma_f}{s}\right] \tag{8}$$

In this case, the wavelet transform $W_x(s,\tau)$ will give us information on $x(t)$ for t near the instant $t = \tau$, with precision $s\sigma_t$, and information about $X(f)$ for frequency values near the frequency $f = \frac{1}{s}$, with precision $\frac{\sigma_f}{s}$. Therefore, small/large values of s correspond to information about $x(t)$ in a fine/broad scale and, even with a constant area of the windows, $A = 4\sigma_t\sigma_f$, their dimensions change according to the scale; the windows stretch for large values of s (broad scales s – low frequencies $f = 1/s$) and compress for small values of s (fine scale s – high frequencies $f = 1/s$).

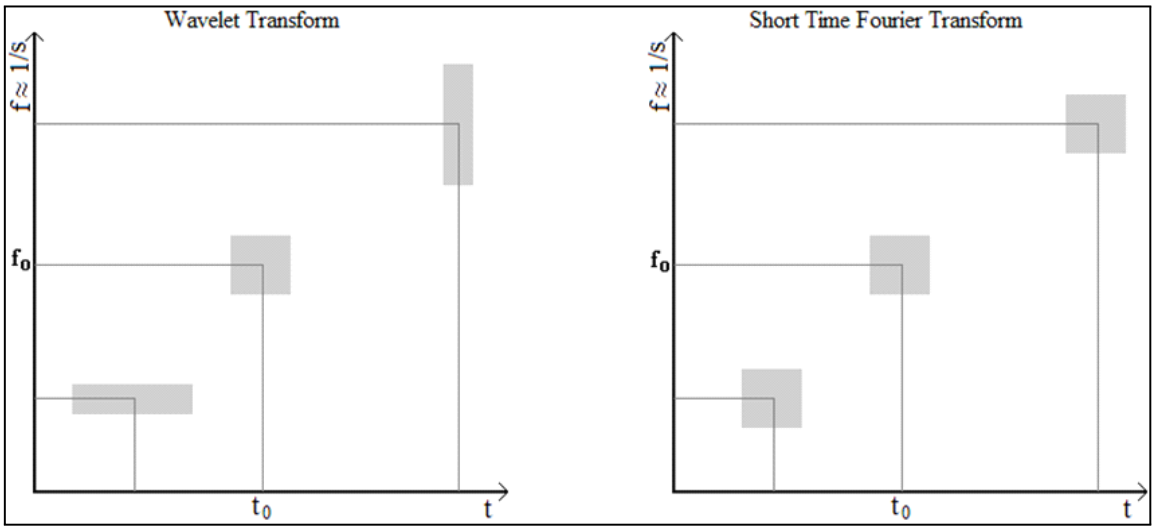


Figure 1: Time-frequency resolution

Figure 1 illustrates this major advantage afforded by the wavelet transform, when compared to the Short Time Fourier Transform: its ability to perform natural local analysis of a time-series in the sense that the length of wavelets varies endogenously. It stretches into a long wavelet function to measure the low frequency movements; and it compresses into a short wavelet function to measure the high frequency movements.

2.4 The Morlet Wavelet: Optimal Joint Time-Frequency Concentration

There are several types of wavelet functions available with different characteristics, such as, Morlet, Mexican hat, Haar, Daubechies, etc. Since the wavelet coefficients $W_x(s,\tau)$ contain combined information on both the function $x(t)$ and the analyzing wavelet $\psi(t)$, the choice of the wavelet is an important aspect to be taken into account, which will depend on the particular application one has in mind. To study cycles, it is important to select a wavelet whose corresponding transform will contain information on both amplitude and phase, and hence, a progressive complex-valued wavelet is a natural choice (the advantage of using a progressive wavelet has already been referred).

We will use the Morlet wavelet, proposed by Goupillaud, Grossman and Morlet (1984):

$$\psi_\eta(t) = \pi^{-\frac{1}{4}}\left(e^{i\eta t} - e^{-\frac{\eta^2}{2}}\right)e^{-\frac{t^2}{2}}. \tag{9}$$

The term $e^{-\frac{\eta^2}{2}}$ is introduced to guarantee the fulfillment of the admissibility condition; however, for $\eta \geq 5$ this term becomes negligible. The simplified version

$$\psi_\eta(t) = \pi^{-\frac{1}{4}}e^{i\eta t}e^{-\frac{t^2}{2}} \tag{10}$$

of (9) is normally used (and still referred to as a Morlet wavelet).

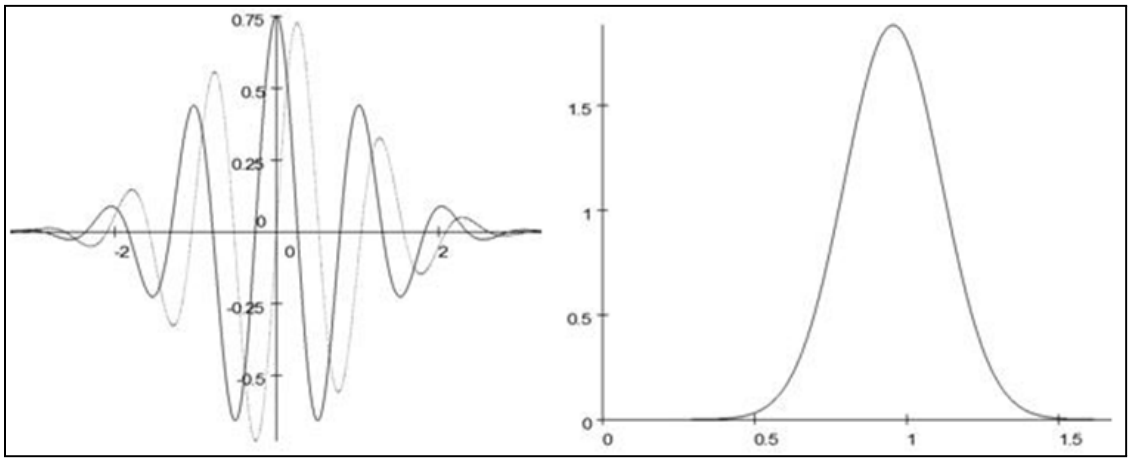


Figure 2: On the left: the Morlet wavelet $\psi_6(t)$ — real part (solid line) and imaginary part (dashed line). On the right: its Fourier transform.

This wavelet has interesting characteristics. For $\eta > 5$, for all practical purposes, the

wavelet can be considered as analytic; see Foufoula-Georgiou and Kumar (1994).[8] The wavelet (10) is centered at the point $\left(0, \frac{\eta}{2\pi}\right)$ of the time-frequency plane; hence, for the particular choice $\eta = 6$, one has that the frequency center is $\mu_f = \frac{6}{2\pi}$and the relationship between the scale and frequency is simply $f = \frac{\mu_f}{s} \approx \frac{1}{s}$. Thanks to the the clear inverse relation between scale and Fourier frequency there is a one-to-one relation between scale and frequency and we will use both terms interchangeably.[9]

It is simple to verify that the time standard deviation is $\sigma_t = 1/\sqrt{2}$ and the frequency standard deviation is $\sigma_f = 1/\left(2\pi\sqrt{2}\right)$. Therefore, the uncertainty of the corresponding Heisenberg box attains the minimum possible value $\sigma_t\sigma_f = \frac{1}{4\pi}$. In this sense, the Morlet wavelet has optimal joint time-frequency concentration.

2.5 Transform of Finite Discrete Data

If one is dealing with a discrete time-series $x = \{x_n,\ n = 0, \ldots, T-1\}$ of T observations with a uniform time step δt, which we can take as the unity ($\delta t = 1$), the integral in (3) has to be discretized and is, therefore, replaced by a summation over the T time steps; also, it is convenient, for computational efficiency, to compute the transform for T values of the parameter τ, $\tau = m\delta t;\ m = 0, \ldots, T-1$. In practice, naturally, the wavelet transform is computed only for a selected set of scale values $s \in \{s_k, k = 0, \ldots, F-1\}$ (corresponding to a certain choice of frequencies f_k). Hence, our computed wavelet spectrum of the discrete-time series x will simply be a $F \times T$ matrix W_x whose (k, m) element is given by

$$W_x(k,m) = \frac{1}{\sqrt{s_k}} \sum_{n=0}^{T-1} x_n \psi^* \left((n-m) \frac{1}{s_k} \right) \quad k = 0, \ldots, F-1, \quad m = 0, \ldots, T-1. \tag{11}$$

Although it is possible to calculate the wavelet transform using the above formula for each value of k and m, one can also identify the computation for all the values of m simultaneously as a simple convolution of two sequences; in this case, one can follow the standard procedure and calculate this convolution as a simple product in the Fourier domain, using the Fast Fourier Transform algorithm to go forth and back from time to spectral domain; this is the technique prescribed by Torrence and Compo (1998).[10]

As with other types of transforms, the CWT applied to a finite length time-series

[8] Our results in the next section, were obtained with the particular choice $\eta = 6$.

[9] As Meyers, Kelly and O'Brien (1993) say,"for a general wavelet, the relation between scale and the more common Fourier wavelength is not necessarily straightforward; for example, some wavelets are highly irregular without any dominant periodic components. In those cases it is probably a meaningless exercise to find a relation between the two disparate measures of distance. However, in the case of the Morlet wavelet, which is a periodic wavelet enveloped by a Gaussian, it seems more reasonable."

[10] A program code based on the above procedure is available at the site http://paos.colorado.edu/research/wavelets/.

inevitably suffers from border distortions; this is due to the fact that the values of the transform at the beginning and the end of the time-series are always incorrectly computed, in the sense that they involve "missing"values of the series which are then artificially prescribed; the most common choices are zero padding – extension of the time-series by zeros – or periodization. Since the "effective support"of the wavelet at scale s is proportional to s, these edge-effects also increase with s. The region in which the transform suffers from these edge effects is called the cone of influence. In this area of the time-frequency plane the results are unreliable and have to be interpreted carefully. In this paper, the cone of influence is defined, following Torrence and Compo (1998), as the $e-$folding time of the wavelet at the scale s, that is, so that the wavelet power of a Dirac δ at the edges decreases by a factor of e^{-2}. In the case of the Morlet wavelet this is given by $\sqrt{2}s$.

2.6 Wavelet Power Spectrum

In view of the energy preservation formula, and in analogy with the terminology used in the Fourier case, we simply define the (local) wavelet power spectrum as $S_x(s,\tau) = |W_x(s,\tau)|^2$, which gives us a measure of the local variance.

The seminal paper by Torrence and Compo (1998) is one of the first to give guidance for conducting significance tests for the wavelet power. By using a large number of Monte-Carlo simulations, they derived empirical distributions for the wavelet power corresponding to an AR(0) or a stationary AR(1) process with a certain background power spectrum (P_f); the test is then performed by comparing the values obtained for the original time-series with the empirical distribution:

$$D\left(\frac{|W_x(s,\tau)|^2}{\sigma_x^2} < p\right) = \frac{1}{2} P_f \chi_v^2(p), \tag{12}$$

at each time τ and scale s. The value of P_f is the mean spectrum at the Fourier frequency f that corresponds to the wavelet scale s — in our case $s \approx \frac{1}{f}$ — and v is equal to 1 or 2, for real or complex wavelets respectively. For more general processes, one has to rely on Monte Carlo Simulations.

Sometimes the wavelet power spectrum is averaged over time for comparison with classical spectral methods. When the average is taken over all times, we obtain the so-called global wavelet power spectrum:

$$GWPS_x(s) = \int |W_x(\tau,s)|^2 \, d\tau. \tag{13}$$

2.6.1 Example

As we have argued before, the main advantage of wavelet analysis over spectral analysis is the possibility of tracing transitional changes across time. To illustrate this, consider the following experiment with simulated data. We generate 50 years of monthly data according to the following data generating process:

$$y_t = \cos\left(\frac{2\pi}{p_1}t\right) + \cos\left(\frac{2\pi}{p_2}t\right) + \varepsilon_t, \qquad t = \frac{1}{12}, \frac{2}{12}, \cdots, 50 \tag{14}$$

$$\text{with} \quad \begin{cases} p_1 = 5 & \text{if } 20 \leq t < 30 \\ p_1 = 3 & \text{otherwise} \end{cases} \quad \text{and} \quad p_2 = 10$$

Formula (14) tells us that the time series y_t is the sum of two periodic components and a white noise.[11] The second period component represents a 10 year cycle while the first periodic component shows some transient dynamics. In the beginning, it represents a 3 year cycle that, temporarily, changes to a 5 year cycle between the second and the third decades.

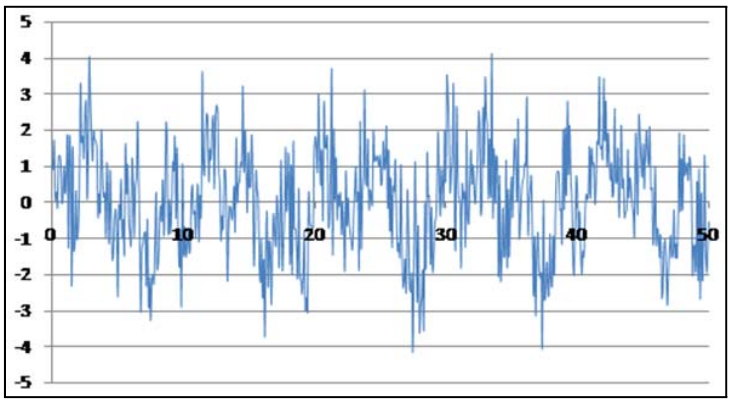


Figure 3: $y_t = \cos\left(\frac{2\pi}{p_1}t\right) + \cos\left(\frac{2\pi}{p_2}t\right) + \varepsilon_t$

This change in the dynamics is nearly impossible to spot in Figure 3. Furthermore, if we use the traditional spectral analysis, the information on the transient dynamics is completely lost, as we can see in Figure 4.c. The power spectral density estimate is able to capture both the 3-year and the 10-year cycles but it completely fails to capture the 5-year cycle that occurred between the second and the third decades. Comparing with Figure 4.b, we observe that spectral analysis gives us essentially the same information as

[11] This formulation is not as restrictive as it may seem. An autoregressive process of order 2, or higher, with an oscillatory behavior, will have a solution that involves sines and cosines. We, therefore, could have generated similar time series using a more common auto regressive process. We chose to explicitly have a cosine because the period of the oscillation is observed directly.

the global wavelet power spectrum, which is an average, across time, of the wavelet power spectrum.

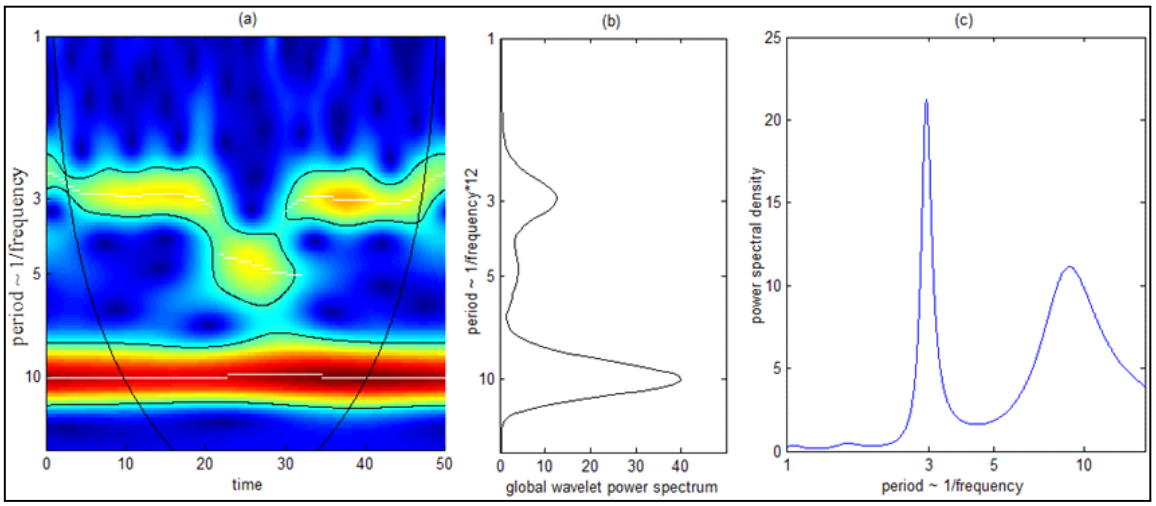


Figure 4: (a) Wavelet Power Spectrum — The black contour designates the 5% significance level based on a AR(1) null. The cone of influence, which indicates the region affected by edge effects, is shown with a thin black line. The color code for power ranges from blue (low power) to red (high power). The white lines show the maxima of the undulations of the wavelet power spectrum. (b) Global Wavelet Power Spectrum — average wavelet power for each frequency. (c) Power Spectral Density — The power spectral density is calculated using a parametric spectral estimation method, which implements the Yule-Walker algorithm.

On the other hand, Figure 4.a shows the wavelet power spectrum itself. In the horizontal axis, we have the time dimension. The vertical axis, gives us the scale, or frequency dimension (recall that $s \approx 1/f$). The power is given by the color. The color code for power ranges from blue (low power) to red (high power). Regions with warm colors represent areas of high power. The white lines show the maxima of the undulations of the wavelet power spectrum, therefore giving us a more precise estimate of the cycle period. The black contour designates the regions where the wavelet power is statistically significant at 5%. We observe a white line on period 10 across all time, meaning that there is a permanent cycle of this period. Both the red color and the black contour tell us that this cycle is strong and statistically significant. We are also able to spot the three year period cycle that occur between time zero and 20 and, again, between time 30 and 50. Finally, we are also able to spot a yellow/orange region between time 20 and 30, with the white stripes identifying the cycle of period five. This means that a cycle of roughly 5-year periodicity, relatively important in explaining the total variance of the time-series and taking place between time 20 and 30, was hidden by the Fourier Power Spectrum estimate.

Figure 4 clearly illustrates the biggest advantage of wavelet analysis over spectral analysis. While the Fourier transform is silent about changes that happen across time, with wavelets we are able to estimate the power spectrum as a function of time and, therefore, we do not loose the time dimension. The wavelet power spectrum is able to capture not only the 3-year and 10-year cycles, but also to capture the change that occurred between years 20 and 30.

2.7 Cross-Wavelets and Phase-Differences

2.7.1 Cross-Wavelet Power

The cross-wavelet transform of two time-series, $x(t)$ and $y(t)$, first introduced by Hudgins, Friehe and Mayer (1993), is simply defined as

$$W_{xy}(s,\tau) = W_x(s,\tau)\, W_y^*(s,\tau), \tag{15}$$

where W_x and W_y are the wavelet transforms of x and y, respectively. The cross-wavelet power is given by $|W_{xy}|$. While we can interpret the wavelet power spectrum as depicting the local variance of a time-series, the cross-wavelet power of two time-series depicts the local covariance between these time-series at each scale and frequency. Therefore, cross-wavelet power gives us a quantified indication of the similarity of power between two time-series. Torrence and Compo (1998) also derived the cross-wavelet distribution assuming that the two time-series have Fourier Spectra P_f^x and P_f^y.Under the null, the cross-wavelet distribution is given by

$$D\left(\frac{|W_x W_y^*|}{\sigma_x \sigma_y} < p\right) = \frac{Z_v(p)}{v}\sqrt{P_f^x\ P_f^y}, \tag{16}$$

where $Z_v(p)$ is the confidence level associated with the probability p for a pdf defined by the square root of the product of two χ^2 distributions. For more general data generating processes one has to rely on Monte Carlo simulations.

2.7.2 Wavelet Coherency

As in the Fourier spectral approaches, wavelet coherency can be defined as the ratio of the cross-spectrum to the product of the spectra of both series, and can be thought of as the local correlation, both in time and frequency, between two time-series. The wavelet

coherency between two time-series, $x(t)$ and $y(t)$, is defined as follows:

$$R_{xy}(s,\tau) = \frac{|S(W_{xy}(s,\tau))|}{|S(W_{xx}(s,\tau))|^{\frac{1}{2}} |S(W_{yy}(s,\tau))|^{\frac{1}{2}}}, \tag{17}$$

where S denotes a smoothing operator in both time and scale. Smoothing is necessary. Without that step, coherency is identically one at all scales and times. Smoothing is achieved by a convolution in time and scale. The time convolution is done with a Gaussian and the scale convolution is performed by a rectangular window (see Cazelles *et al.* 2007 for details).

Theoretical distributions for wavelet coherency have recently been derived, by Ge (2008), but only when two stationary Gaussian white noises processes are assumed and we use the Morlet wavelet; for more general processes, one again has to rely on Monte Carlo simulation methods. We follow Rouyer *et al.* (2008) and, for each time series, construct the beta surrogate series. The beta surrogates display a similar variance and autocorrelation structure as the original time series and, additionally, display the same relative distribution of frequencies, i.e. the same slope of the Fourier spectrum, as the original time series. With this approach, the surrogate series approximately keep the mean, standard deviation, auto-correlation and partial auto-correlation functions and the power spectrum of the original data. We do this 5000 times and compute the wavelet power spectrum for each trial. Then we extract the critical values at 5 and 10%.

2.7.3 Phase Difference

The phase-difference gives us information about the delays of the oscillations between two time-series, $x(t)$ and $y(t)$, as a function of time and frequency. As we said before, the phase of a given time-series, ϕ_x, can be viewed as the position in the pseudo-cycle of the series. The phase-difference, $\phi_{x,y}$, characterizes phase relationships between the two time-series, i.e. their relative position in the pseudo-cycle. The phase-difference is defined as

$$\phi_{x,y}(s,\tau) = \tan^{-1}\left(\frac{\mathcal{I}\{W_{xy}(s,\tau)\}}{\mathcal{R}\{W_{xy}(s,\tau)\}}\right), \qquad \text{with } \phi_{x,y} \in [-\pi,\pi]. \tag{18}$$

A phase-difference of zero indicates that the time-series move together at the specified frequency. If $\phi_{x,y} \in \left(0,\frac{\pi}{2}\right)$ then the series move in phase, but the time-series y leads x. If $\phi_{x,y} \in \left(-\frac{\pi}{2},0\right)$ then it is x that is leading. A phase-difference of π (or $-\pi$) indicates an anti-phase relation. If $\phi_{x,y} \in \left(\frac{\pi}{2},\pi\right)$ then x is leading. Time-series y is leading if $\phi_{x,y} \in \left(-\pi,-\frac{\pi}{2}\right)$.

2.7.4 Example

Again, we illustrate these concepts with the aid of a numerical example. As before, we also show how wavelet analysis is suitable to identify changes in the dynamic relations of two time series. Consider two time series that share two common cycles, with some delays (again, this process can be seen as the solution to some bivariate VAR):

$$x_t = \sin\left(\frac{2\pi}{3}t\right) + 3\sin\left(\frac{2\pi}{6}t\right) + \varepsilon_{x,t}, \qquad t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, 50, \tag{19}$$

$$y_t = \begin{cases} 4\sin\left(\frac{2\pi}{3}\left(t+\frac{4.5}{12}\right)\right) - 3\sin\left(\frac{2\pi}{6}\left(t-\frac{9}{12}\right)\right) + \varepsilon_{y,t}, \\ \qquad\qquad \text{for } t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, 25, \\ 4\sin\left(\frac{2\pi}{3}\left(t-\frac{4.5}{12}\right)\right) - 3\sin\left(\frac{2\pi}{6}\left(t+\frac{9}{12}\right)\right) + \varepsilon_{y,t}, \\ \qquad\qquad \text{for } t = 25+\frac{1}{12}, 25+\frac{2}{12}, \ldots, 50. \end{cases} \tag{20}$$

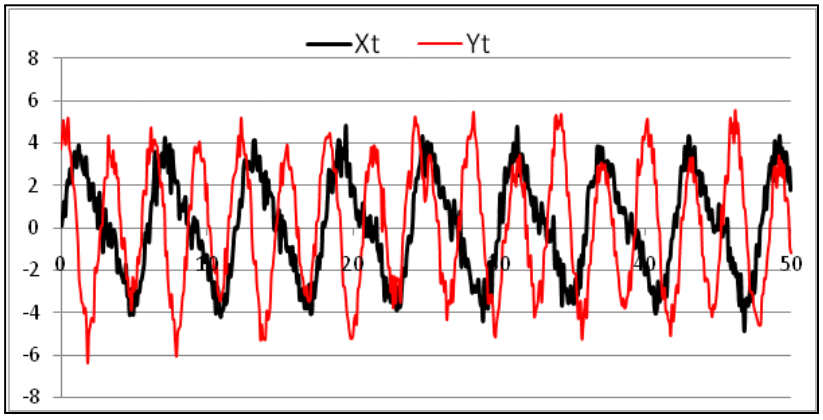


Figure 5: x_t and y_t

Looking at the formulas, it is clear that x_t and y_t share 3-year and 6-year cycles. However, the way their cycles relate to each other evolves with time and is different across frequencies. Consider the shorter period cycle, the 3-year cycle. The cycles are positively correlated. However, while for the first half of the sample the y_t cycle precedes the x_t cycle by four and a half months in the second half of the sample the y_t cycle lags the x_t cycle.

These features are captured in Figure 6. On the left, we have the wavelet coherency. On the right we have the phase and phase-difference computed for two different frequency bands. On the top, we compute the phases for the 2.5∼3.5 year frequency band. In the bottom, we consider the 5∼7 year frequency band. The green line represents the y_t phase and the blue represents x_t's phase. The red represents the phase difference.

That both series have common and highly correlated 3-year and 6-year cycles is revealed by the regions of strong coherency around those frequencies. That the 3-year cycles are in phase (positively correlated) is revealed by the phase difference (red line in the upper right graph),which is consistently situated between $-\pi/2$ and $\pi/2$. Finally, we can see that the 3-year y_t cycle was leading for the first half of the time and lagging in the second half by noting that in the first half of the sample the phase-difference is between zero and $\frac{\pi}{2}$, while in the second half it is between $-\frac{\pi}{2}$ and zero.

Looking at the 6-year cycle, we observe that the series are out of phase (negatively correlated) with x_t leading in the first half and y_t leading in the second half of the sample.

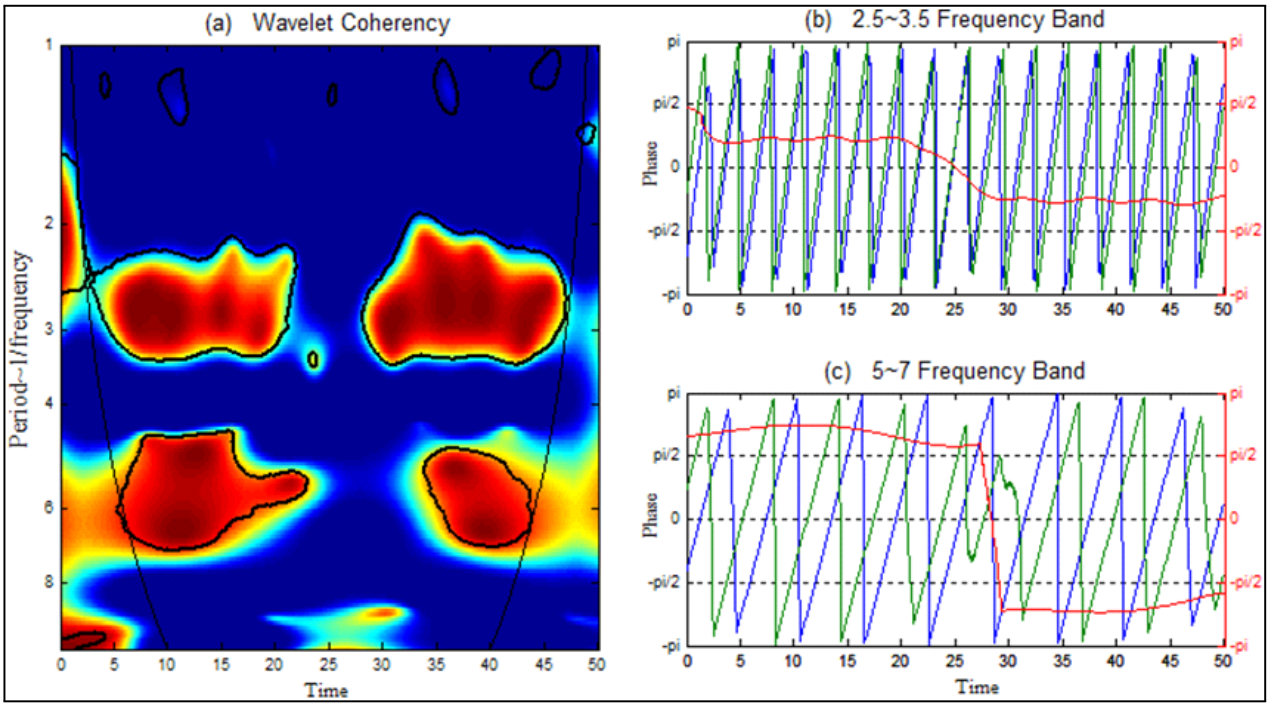


Figure 6: On the Left: Cross-Wavelet Coherency - The cone of influence is shown with a thin line. The thick contour designates the 5% significance level. Coherency ranges from blue (low coherency) to red (high coherency). On the right: Phase and phase-difference. The green line represents the y_t phase, the blue represents x_t's phase and the red represents the phase difference.

3 War Cycles

The tools of wavelet analysis we have presented in the previous section can be applied to time series data that have long been relevant to political scientists. We will start by looking at cycles of war. The question of whether wars occur cyclicaly has intrigued observers at least since the 16th century (Clark 1958), but the answer has remained extremely elusive for modern social scientists. On the one hand, variations between both data sources and dimensions of the phenomenon of war – outbreak, duration, magnitude, or severity, for example – seemed to yield rather different answers (Wright 1942; Sorokin 1947; Singer and Small 1972; Houweling and Siccama 1988; Mansfield 1988). On the other hand, theories that might explain why cyclical rhythms were likely to be found in wars were never in abundant supply.

However, the focus on Great Power wars contributed for some theoretical and empirical focus to be gained on the potential cyclicality of warfare, including Modelski and Thompson's "long cycles theory"(Modelski 1978; Modelski and Thompson 1988), Doran's "power cycle theory"(1991), Gilpin's theory of hegemonic war (Gilpin 1981), or Goldstein's theory of long cycles (1985, 1988 and 1991).[12] Goldstein's approach, in particular, offers both a cyclical theory of major wars and a cyclical explanation for their intensity. Building on Kondratiev's observation that the most extensive wars tend to occur in the rising phases of economic long-cycles (cycles that range approximately from forty to sixty years in length), Goldstein argued for a basic relationship between major wars and the economy in the last five centuries. His theory features a "lagged negative feedback loop"of economic growth and war: upswings in production, investment and innovation lead both to competition for scarce resources and to increases in the "war chests"of great powers, making war both desirable and possible. War severity, in turn, leads to rises in prices and declines in real wages, but as wars come to an end, economic recovery reinitiates, leading us back to where we started. Using intuitive methods, together with the autocovariance function analysis and VAR modelling, Goldstein identified a 50 to 60-year cycle in Levy's (1983) war severity series and a relationship between war fatalities and long-wave prices' upswings.

Several theoretical, empirical and methodological questions have been raised, however, about these general propositions. On the one hand, it seems implausible that the same cyclicality should prevail throughout the entire historical period under examination. Structural changes are likely to have fundamentally changed the cycles of war and their relationship with other phenomena, such as trends in economic growth or technological innovation. For example, the Industrial Revolution has allowed states to improve their

[12] For reviews of cyclical theories of war, see Rosecrance (1987), Levy (1991), Kohout (2004) or Jacoby (2007), particularly chapter 10.

power in the system without resorting to war, increased the overall destructiveness of warfare, and permitted the creation of permanent military establishments, thus changing the overall incentives for warfare and breaking the connection between rises in production and war capabilities (Rosecrance 1987; Levy 1991). On the other hand, the 50∼60 year cycle for which Goldstein found empirical support seems, in fact, non-existent on the basis of classical techniques. As Beck (1991) shows, using spectral analysis, although there is some evidence for coherence between inflation and war severity – especially in the post-1815 period – the war series lacks a strong periodic component itself (see also Levy 1983, Conybeare 1990, and Silverberg 2006 for similar conclusions). And if rigorous testing does not allow rejecting the null hypothesis of no predominance of a long period cycle, "we would normally conclude that a cycle does not exist"(Beck 1991, 471).

Goldstein's response, however, is that cycles in war (and other social and political phenomena) do not have to be characterized by the fixed periodicity typical of physical phenomena, and rather by"repeating sequences"and a "inner dynamic that gives rise to repetition"(Goldstein 1988: 176). In a single time series, only periodicity allows us to conclude that cycles exist. "But when ups and downs correlate throughout a worldwide political economic-system, it is safe to conclude that there is a deeper cyclical dynamic at work, not just a scatter of random ups and downs"(Goldstein 1988, 177). Thus, Goldstein proposes shedding techniques that presume fixed periodicity (such as spectral analysis – Goldstein 1991b, 478) and, instead, detects cycles that are spaced irregularly in time: while the 50∼60 year cycle does characterize the period between the beginning of the 17th and the 19th century, other periodicities characterize the periods immediately before and after (1988, 239-242). And, crucially to establish cyclicality in his own definition, Goldstein poses that war leads prices by one to five years (1988, 249-254).

Do we need to presume fixed periodicity in cycles in order to perform rigorous statistical testing concerning their existence? Or should we reject"orthodox statistical inferences"(Goldstein 1991b: 480) when testing a theory proposing a time-varying cyclicality? Both spectral analysis and ARMA models allow us to estimate coefficients that summarize the dynamics of the time series for the chose time period, but they have assumptions about the nature of the series that may render them incapable of capturing all the features of the data one may be interested in. Wavelet analysis, however, is flexible from the outset. First, it does not assume that a time series can or cannot be decomposed as a sum of periodic cycles, and while it does not assume fixed periodicity, it does not rule it out either. Second, it does not need stationarity, and thus does not need to transform the series to achieve stationarity. Third, because it assumes neither fixed periodicity nor stationarity, it is well equipped to detect the possibility of transient and evolving cycles and that particular movements are characteristic of particular time periods. Fourth, with the

wavelet coherency we can investigate the relationship between war and inflation without imposing many of the assumptions of other techniques. And finally, it allows all this with straightforward rigorous statistical testing.

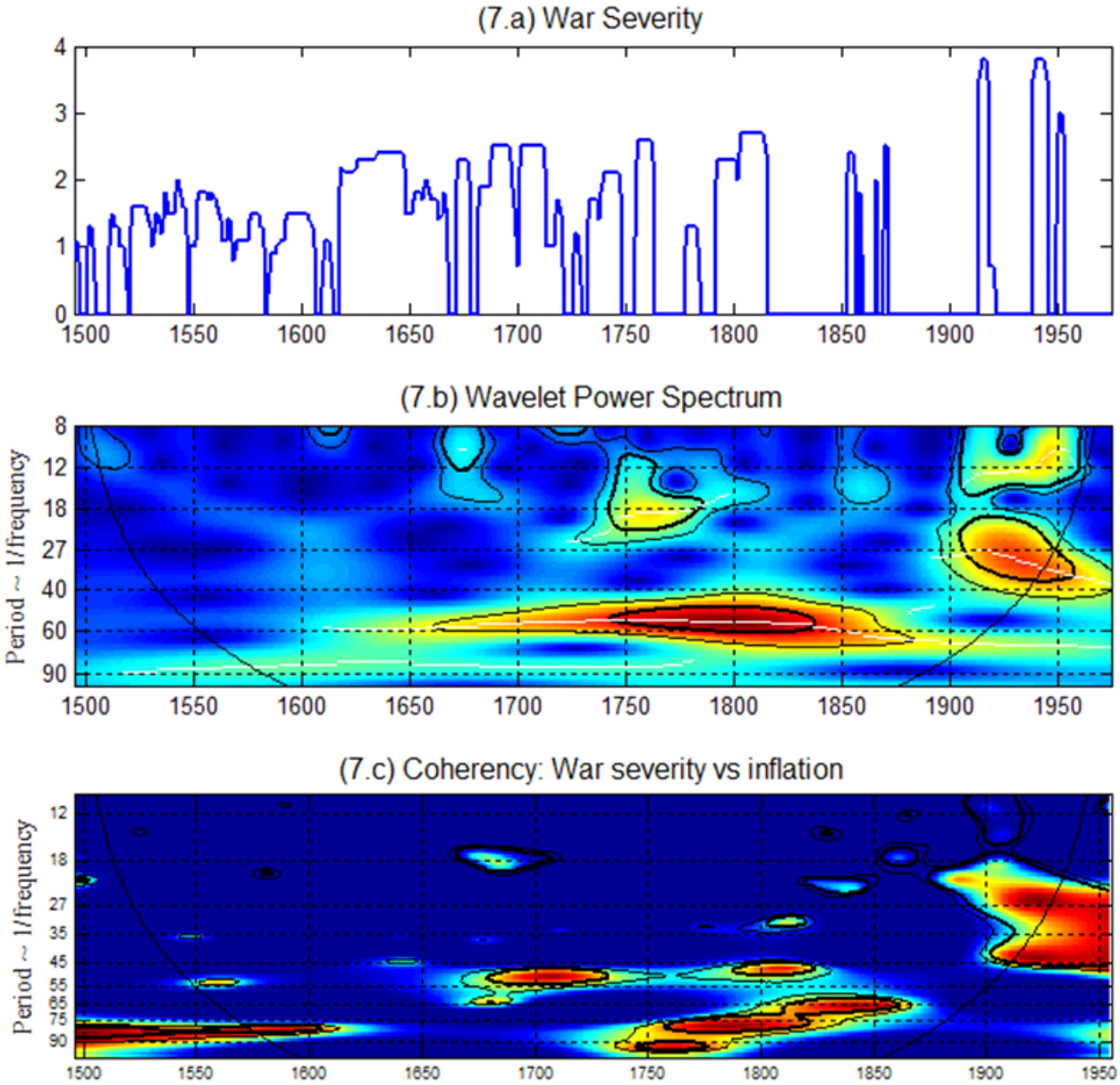


Figure 7: (a) War Severity. (b) War Severity Wavelet Power Spectrum. (c) Wavelet Coherency between war severity and inflation. The thick/thin black contour designates the 5%/10% significance level. The cone of influence, which indicates the region affected by edge effects, is shown with a thin black line. The color code for power ranges from blue (low power) to red (high power). The white lines show the maxima of the undulations of the wavelet power spectrum.

Our data on war severity is taken from Goldstein (1988). It measures fatalities in great power wars and runs from 1495 to 1975 (see Figure 7.a). For comparison with Goldstein, Beck (1991) analyzes the full sample, however he warns that caution must be used when interpreting his results. This is so because of the structural change that occurred around 1815. As he points out, the time series is not stationary because the mean and variance of the time-series before 1815 are, respectively, 1.45 and 0.72, while after 1815 they become 0.46 and 1.28. Because of this non-stationarity, rigorous Fourier spectral analysis would require the sample to be divided in two. As we have pointed out earlier, this is one major advantage of wavelet analysis: stationarity is not required. Therefore, in our setup, there is no reason to break the sample in two.

Figure 7.b shows the wavelet power spectrum of the war severity series. The first and most striking result is that, in accordance with Goldstein's proposal, wavelet analysis does uncover a statistically significant cycle of period around 60 years, as we can see by the white stripe and by the dark red area for that frequency. Furthermore, such cycle is shown to be localized in time, rather than prevalent throughout the entire series: the thin black contour identifies the 10% statistically significant region for the 60-year cycle, which runs from around 1660 to late 1800s, but is replaced in the 20^{th} century by a shorter and also significant cycle, with a period around 30 years. Another transient cycle, although not as strong, is also identified between 1750 and 1800, at the 20-year frequency band. In short, unlike what had been suggested so far on the basis of spectral analysis or ARMA techniques, we do have enough evidence to clearly reject the null hypothesis that there are no predominant cycles in the war severity data. Furthermore, wavelet analysis provides us with a new a previously unexplored set of explanandums concerning war: the absence of cyclicality until the mid 17th century, the prevalence of a 60-year cycle until the late 19th century, and the structural change that brings about a shorter periodicity afterwards.

Wavelet analysis also allows us to explore possible explanations of these cycles in war severity, particularly the proposed relationship between war severity and inflation, again using the Goldstein 1988 data. Figure 7.c gives us the wavelet coherency between war severity and British inflation, again using the Goldstein 1988 data also employed in Beck 1991. Here, the results are mostly consistent with theirs. Beck found the series to be significantly coherent after 1815, a pattern that we also observe in the $18 \sim 45$ frequency band (but only after 1875). Before that period, we observe high coherency at frequencies around 75 years between the mid-18th and the mid-19th centuries, as well as small regions of high coherence in the $45 \sim 55$ frequency band around 1700 and 1800. These, however, are not regions of high power in the war series, as we see in Figure 7.b. Therefore, on this regard, we have to convene that our analysis does not add much to the existing results nor does it lend particular credence to the notion that cycles in war severity consistently

precede cycles in inflation (and much less that they are preceded by upswings in economic growth). This is hardly surprising. There are several economic theories that attempt to explain inflation. Some relate prices to the monetary base, others consider interest rates, government expenditures, unemployment rate, output gap, agents' expectations, etc. One should not expect to produce substantial claims about the influence of war on inflation without controlling for several of these possible causes.

4 Cycles in American National Electoral Politics

Our second substantive application of wavelet analysis concerns American electoral politics, and particularly the question of whether election returns exhibit cyclical features. Arguments and evidence in favor of such cyclicality have been considerably stronger in this regard than in the case of war cycles. However, the ways in which those cycles are supposed to manifest themselves, their periodicity and how they should be uncovered have been the object of considerable debate. Since very early on, the most favored view seemed to be that these cycles had a relatively regular but short periodicity. As early as 1898, Lowell proposed the existence of a "regular political oscillation" in presidential election results, which he interpreted as a short-term reaction against the party in power, caused by the "coalition of minorities" problem and by the inevitable discrepancy between heightened public expectations and policy achievements (Lowell 1898, 94-96). Later, Bean would argue that the ebb and flow in election returns was related to the business cycle (Bean 1948), while Schlesinger had suggested that electoral cycles were related to changes in public mood, in reaction to conservative or liberal governmental policies (Schlesinger 1939; see also Schlesinger Jr. 1986), changes that would occur about every fifteen years.[13]

A number of recent works has tended to validate these views with the help of more sophisticated statistical techniques. Norpoth (1995), for example, identified a second-order autoregressive process in the presidential two-party vote, through which, since 1860, peaks in electoral support for a particular party tended to be reached in four to five-term intervals, a phenomenon attributed to a combination between the"costs of ruling"and the two-term limit for presidents. Lin and Guillén (1998), combining an AR(2) model and spectral analysis for election returns and an event history model for party control of the White House, lent further empirical support to this notion. And more recently, Merrill, Grofman, and Brunell (2008), using spectral analysis, not only confirmed Lin and Guillén's findings concerning the presidency (a 13 to 14-year half-cycle of dominance by a particular party since the mid-19th century) but also that similar prominent cycles characterized the series of Democratic seat shares in the House and the Senate, leading the authors to recover Schlesinger's – and Stimson's (1999) – focus on shifting ideological"public mood"Although these results do not imply the kind of deterministic view of cycles that tended to prevail in the earlier scholarship,[14] they all point to the notion that a "pendulum"in election returns

[13]See also Munro (1928). For a fascinating account of these early cyclical theories and the public and academic debate around them, see Rosenof (2003), particularly chapter three.

[14]Forecasting models of election results have accommodated both the notion of an inherent cyclicality in election returns and the fact that actual predictions also need to take short-term conditions into account (Norpoth 1995 and 2002; Lebo and Norpoth 2007). Similarly, in spectral analysis, the detection of predominant electoral cycles does not preclude "idiosyncratic perturbations that partially disrupt the overall pattern"(Merrill, Grofman, and Brunell 2008), while event history analysis explicitly models losses

does exist, that it has oscillated with apparent regularity through time, and that such oscillation has been relatively quick.

There is, however, another strand of scholarship, particularly the one linked to electoral realignment theories, which has proposed somewhat different views of the cyclicality in American elections. Burnham, for example, suggested that "critical realignments" – abrupt changes in the voter coalitions behind the American political parties – occurred "with a remarkable uniform periodicity" (Burnham 1970, 8), "approximately once every thirty years" (Burnham 1967, 288), i.e., in a 60-years rather than a 26 to 30-year full cycle. Burnham-style "critical realignment" cycles have come under attack, as evidence concerning the existence of such realignments – and, thus, of their periodicity – seems limited (Lichtman 1976; Mayhew 2000 and 2004). Others, however, using a more nuanced approach that allowed for realigning "periods" or "eras", have continued to observe long-cycles in American politics. Beck, for example, suggests the existence of a generational replacement pattern that leads to the "cyclical creation, stabilization, and decay of mass electoral coalitions over the last one hundred and fifty years of American politics", through which "realignments have appeared about every thirty years" (Beck 1979: 131). Later works done under the basic electoral realignment framework have continued to yield a picture of party system dynamics that allows for longer cycles. Campbell (2006), for example, using period dummies and least median squares regression for the 1868-2004 presidential results, finds four historical "party systems" separated by three realignments. While two of them were characterized by a roughly 30-year half-cycle of dominance of a particular party, the other periods (before 1896 and since 1968) were instead characterized by an evenly balanced and highly competitive system. Besides, analyses of House elections suggest an even more complex picture: unlike what occurred in presidential elections, the prevalence of the Democrats since the 1930s ended up being extended all the way to 1994, a result both of the increased advantage of incumbency in House elections and the late establishment of the Republican party in the South (Campbell 2006, 364). Other studies also detect periods without the extended dominance of any party (Robbins and Norpoth forthcoming, 9) and conclude that, although long periods of dominance by particular parties do exist, realigning periods seem "less regular in their periodicity than claimed by classic realignment theory" (Norpoth and Rusk 2007, 402).

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of party tenure in the presidency as a stochastic process (Lin and Guillén 1998).

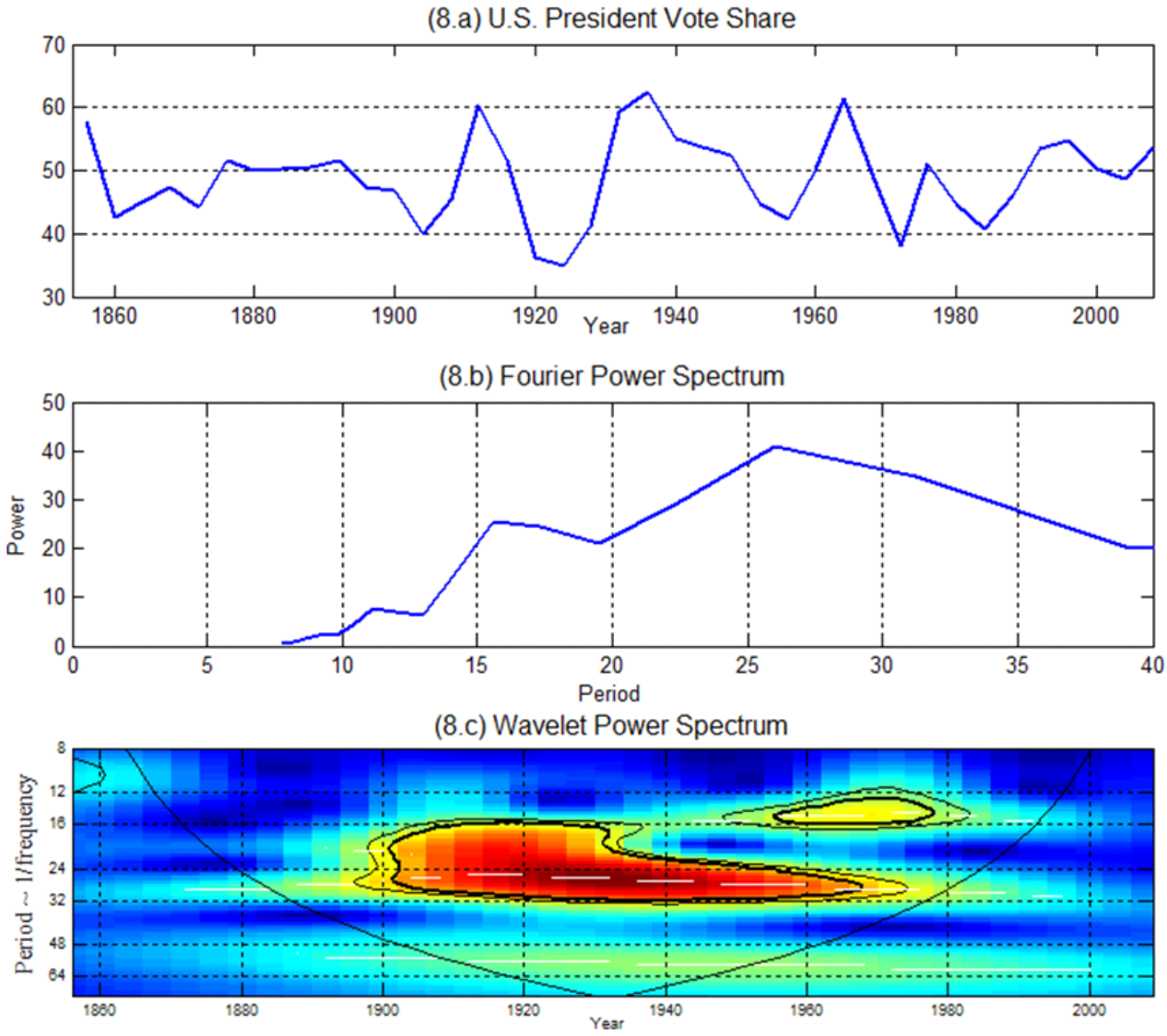


Figure 8: (a) U.S. president democratic vote share (b) Fourier Power Spectrum (c) Wavelet Power Spectrum. The thick/thin black contour designates the 5%/10% significance level. The cone of influence, which indicates the region affected by edge effects, is shown with a thin black line. The color code for power ranges from blue (low power) to red (high power). The white lines show the maxima of the undulations of the wavelet power spectrum.

The two previous generic views about the dynamics of electoral change in the US seem difficult to reconcile. While one proposes relatively short cycles with fixed periodicity, common to both presidential and congressional elections, another proposes not only the existence of longer cycles but also that, throughout American electoral history, certain periods characterized by electoral dominance of a particular party are preceded or followed by others where parties remain evenly balanced for prolonged periods of time, or where fundamental political and institutional changes affect the cyclical nature of American politics,

and in different ways for presidential and congressional elections.

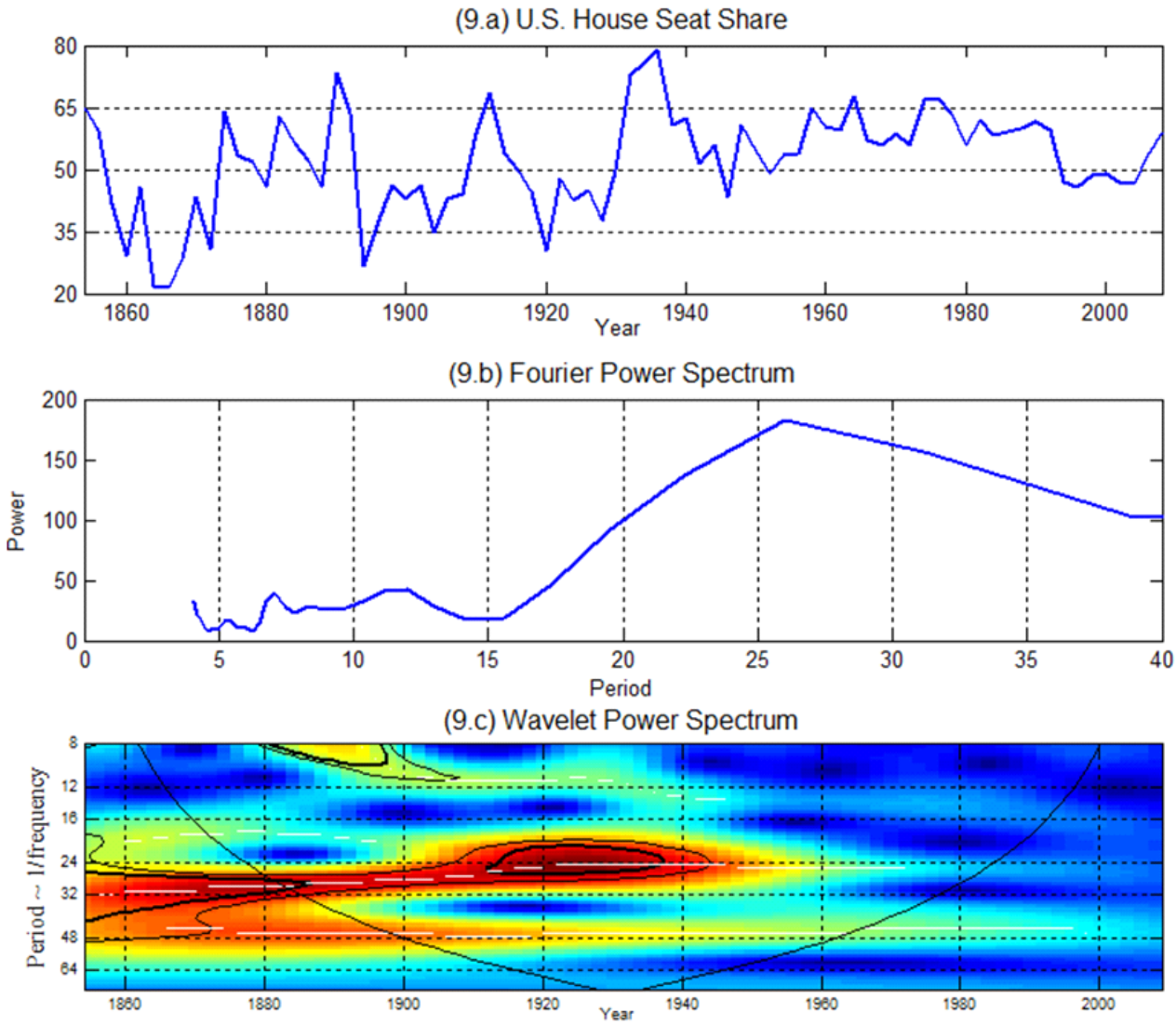


Figure 9: (a) U.S. House seat democratic share (b) Fourier Power Spectrum (c) Wavelet Power Spectrum. The thick/thin black contour designates the 5%/10% significance level. The cone of influence, which indicates the region affected by edge effects, is shown with a thin black line. The color code for power ranges from blue (low power) to red (high power). The white lines show the maxima of the undulations of the wavelet power spectrum.

We use wavelet analysis to explore these issues. Our data is essentially the same used by Merrill, Grofman and Brunell (2008): the share of the Democratic vote in presidential elections since 1856 and the Democratic share of the House seats since 1854, including, in our case, the 2008 results. First, we compute the wavelet power spectrums for the presidential and House results. Second, we explore the varying relationship between these

two series, not only through the already presented tools of cross-wavelets but also by looking at phase differences, which give us a measure of the delays of the oscillations between them.

Figure 8.a displays the data for the presidential elections. In Figure 8.b, the Fourier power spectrum of this series identifies a predominant cycle with 26-years periodicity, delivering essentially the same results found by Merrill, Grofman and Brunell (2008) In Figure 8.c, however, the estimation of the wavelet power spectrum allows several new pieces of information to emerge. First, the wavelet spectrum identifies not only the 26-year cycle, but also a transitional 15-year cycle between the late 1950s and 1980, where the electoral results of a particular party seem to have improved (or worsened) for slightly less than two consecutive terms. Second, there is some (albeit weak) evidence that they coexist with a long cycle of 60 years, basically equivalent to the one argued for in the realignment literature. Although statistically significant only at the 15% level, the very fact that with 150 years of data one is able to see some evidence of a 60-year cycle is quite striking. Third, the statistically significant evidence for the strongest cycle – the 26-year one – is, in our wavelet spectrum, temporally localized. It starts in the turn of the 20th century but dissipates by the end of the 1960s.

In Figure 9, we can observe the same graphs, but this time for the House of Representatives.[15] Again, in Figure 9.b, we observe Merrill, Grofman and Brunell (2008)'s cycle with a periodicity around 26 years. But again, in the wavelet power spectrum in Figure 9.c, we conclude that such cycle is not predominant throughout the entire series: for the House of Representatives the cycle is present since 1854, but evidence for the existence of such cycle disappears after the 1940s.

We also estimate the wavelet coherency and the phase-differences between both series.[16] As we can see in Figure 10.a, the main region of high coherency occurs between slightly before 1900 and 1940 at the $18 \sim 30$-year frequency band. This corresponds roughly to the intersection of the high power regions of Figures 8.c and 9.c. The phase information for those period and frequencies (Figure 10.b) tells us that the series were in phase, i.e. their cycles were highly coordinated. But apart from that particular region, we do not observe relevant areas of high coherency. It is, therefore, fair to conclude that different elections have different dynamics, and that the biggest divergence started around 1940s.

[15] We use biannual data from 1854 to 2008.

[16] Because presidential elections only occur each 4 years, to be able to match the biannual data for the House election, we linearly interpolated the data on presidential elections. If, instead, we had chosen to delete the in-between observations for the House elections, similar conclusions would have been reached.

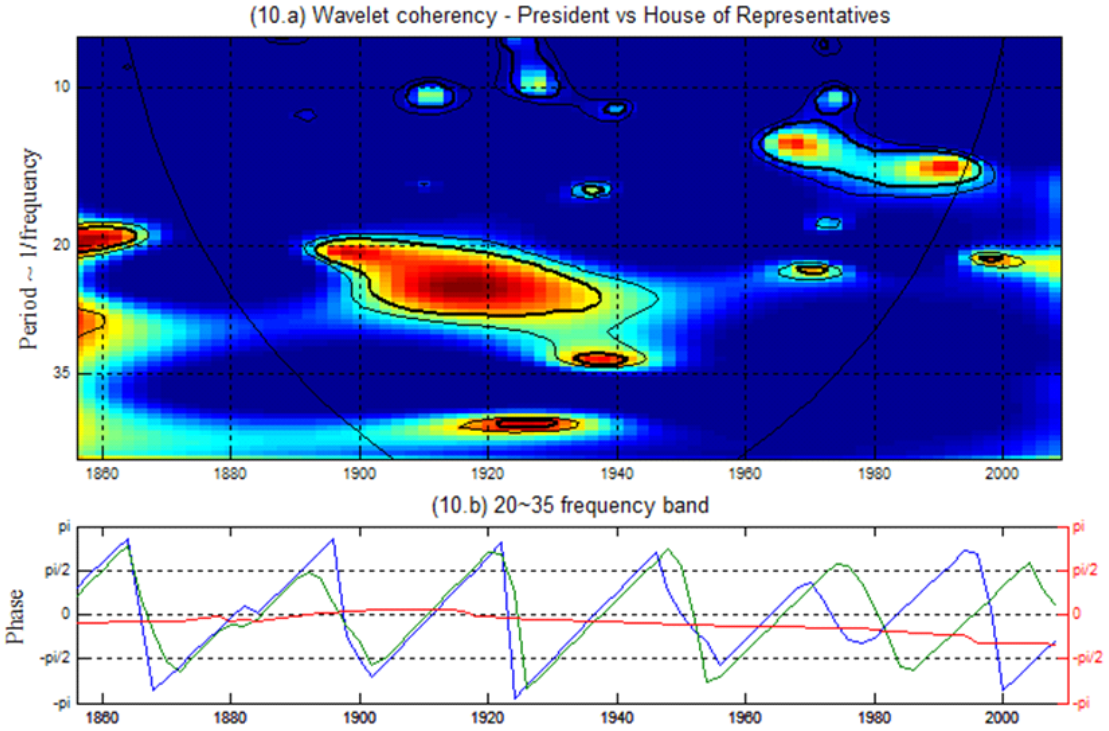


Figure 10: Presidential Election vs House of Representatives. (a) Wavelet Coherency — The thick/thin black contour designates the 5%/10% significance level. The color code for coherency ranges from blue (low coherency — close to zero) to red (high coherency — close to one). (b) Phase and Phase-difference – The green line represents the House of Representatives phase, and the blue line represents the Presidential election's phase. The red line gives us the phase difference between the two series.

This set of findings obtained using wavelet analysis allows us to address the controversies about the cyclicality of election returns in the United States under a new light. Besides uncovering the possibility that more than one cycle component (a 60-year"long"one as well as a 26-year "short"one) contributes to explain the variance in the series, wavelet analysis provides two additional contributions. First, it shows that the cyclicality of election returns is not a feature of the entire series but, instead, is temporally localized. In the case of presidential elections, the"system of 1896"that followed the 1893 economic panic and the ensuing regional redistribution of partisan support does not seem to be entirely a fiction engendered by realignment theorists. Instead, it marks the beginning of a cyclical pattern in election returns of which we do not have evidence for earlier periods. Similarly, by the end of the 1960s, that cyclicality dissipates, something that spectral analysis misses altogether. Although our aim in this paper is not to explain the causes of these structural breaks, phenomena such as the dramatic partisan transformation of the South in the 1960s or the disturbance in the trade-off between inflation and unemployment brought about

by the oil shocks of the early 1970s (a trade-off that, until then, had arguably played an important role in fostering regular alternation in office – Hibbs 1987) are potentially interesting candidates as explanatory factors.

Second, wavelet analysis also allowed us to uncover that the dynamics in presidential and House elections are less similar than conventional methods would lead us to believe. While the 26-year cycle for the presidential elections started in the 1900s and lasted until, at least, late 1960s, a cycle of the same periodicity characterizes the House election series from the very beginning but only until the 1940s, something that we would have missed altogether had we relied solely on classical spectral analysis. Similarly, the wavelet coherence and the phase difference tools clearly show that the synchronism in the cycles in the presidency and the House is also a localized phenomenon, that takes place only between shortly before 1900 until the 1940s, with the latter displaying an increasing lag since then. Again, although the purpose of this paper is not to test explanations for those patterns, there is an obvious candidate that may account for that discrepancy: the surge in incumbency advantage in House elections that took place beginning in the 1950s (Gelman and King 1990), putting an end to the regular ebb and flow that House elections returns had exhibited until then and disturbing the relationship between its short-term cyclical component and that of presidential elections. This also suggests that, if we want to study cycles in national elections, it is clearly not indifferent to know which specific election we are talking about, raising another objection to the analysis of Merrill, Grofman and Brunell (2008): it is unlikely that the same model can be used to explain both the cyclical behavior in presidential and congressional elections.

5 Conclusions

So far, ARMA models and spectral analysis have served political scientists quite well in summarizing the dynamics of time series for a particular period in countless studies and applications. However, although very useful, they cannot capture all the features of the data one may be interested in. Recent approaches, such as the development of dynamic conditional correlation (DCC) models under the ARCH/GARCH framework (Engle 2002), have allowed reseachers to explore the notion that correlations between series are time-varying, a development that has already found applications for the study of the relationship between evaluations of the econony and presidential approval or of international cooperation in the Middle East (Lebo and Box-Steffensmeier 2008). Wavelet analysis is, we submit, another important breakthrough in addressing time-varying features of time series data. Not only it is particularly adapted to the detection of cycles, but can also, unlike spectral analysis, be applied whenever one is interested in estimating the spectrum as a function of time, revealing how the different periodic components of the time-series change over time. It allows researchers to analyze transient dynamics, both for single time-series or for the association between two time-series. Furthermore, unlike DCC (and other time-varying approaches, like Kalman filtering), wavelet analysis explicitly addresses relationships across frequencies.

In this paper, we used three tools in wavelet analysis that, to our knowledge, have not yet been used by political scientists: the wavelet power spectrum, the cross-wavelet coherency and the phase-difference. While the wavelet power spectrum quantifies the main periodic component of a given time-series and its time evolution, the cross-wavelet transform and the cross-wavelet coherency are used to quantify the degree of linear relation between two non-stationary time-series in the time-frequency domain. Phase analysis is a nonlinear technique that makes possible to study synchronization and delays between two time-series across different frequencies. We applied these tools to the study of cycles in war severity and presidential and congressional election returns in the United States. In both cases, we uncover patterns that spectral analysis (or, for that matter, ARMA models) have kept hidden in the existing literature. First, we find evidence of statistically significant – albeit transient – predominant cycles in war severity, which spectral analysis has led other reasearchers to conclude as being altogether absent. Second, both in presidential and house elections, we find that the predominant cycles that have been identified in previous studies are in fact also transient, failing to characterize the entire period under examination. Finally, we show that, in spite of similar predominant short-term cycles in presidential and congressional elections, the periods in which they prevail are only partially overlapping between the two institutions. These results, while raising themselves important

explanatory questions for future analysis, are already suggestive in and on themselves, considering their congruence with existing findings and hypotheses in the literature about potential causes of structural breaks in both the war severity and the U.S. election results.

Our focus on war and elections provides just two of many possible illustrations of the potential of wavelet analysis. Political scientists have gained increasing access to historical politically relevant time series data going sometimes several centuries back (such as Vanhanen 2000; Acemoglou, Johnson, and Robinson 2005; Marshall and Jaggers 2009; or Gates et al. 2006, just to give a few examples), as well as to a wealth of high frequency time series data – coming either from official statistics or surveys – about contemporary politics in many different societies, and there are good reasons to believe that such wealth and the diversity of its sources is likely to increase quite dramatically in the near future (King 2009). The alluring search for cyclical patterns in those sorts of data, however, has been made particularly difficult by the assumptions of the kind of techniques employed so far, which have prevented attention to contextual variations over time, short-term dynamics, and structural breaks, thus leading either to negative findings or to positive findings of questionable face validity. As Kennedy notes, wavelet analysis addresses these problems "so successfully that it has become very widely used in a broad range of disciplines, and is becoming more common in econometrics"(2008, 317). In this paper, we tried to show that its potential also extends to questions of interest to political scientists and other social scientists more generally.

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