

Universidade do Minho
Escola de Engenharia

Amirhossein Mohammadi

**Data-driven analytical and ML-based
models for shear-strengthened RC beams
using FRP systems**



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Data-driven analytical and ML-based models for shear-strengthened RC beams using FRP systems

Doctoral Thesis

Doctoral Program in Civil Engineering

work developed under the supervision of

Professor Joaquim A. O. Barros

Professor José Sena-Cruz

April 25

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University of Minho, 01/04/2025

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Amirhossein Mohammadi

To my beloved mother,
and in loving memory of my father, Ali, and my brother, Farhad—forever in my heart.

With knowledge, the soul ascends,
With effort, the spirit transcends.
Through patience and pursuit,
We harvest the fruits of truth.

Rumi

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ABSTRACT

Reinforced Concrete (RC) structures continue to dominate the construction industry due to their extensive use, durability and economic efficiency. However, as safety formats and performance demands evolve, many of these structures fail to comply with current requirements. Demolition, though sometimes necessary, is often suboptimal due to substantial economic costs and severe environmental consequences, such as waste generation and increased carbon emissions. Consequently, structural strengthening has emerged as a sustainable and cost-effective alternative.

This thesis focuses on enhancing the shear resistance of RC beams using Carbon Fibre-Reinforced Polymer (CFRP) reinforcements, with a focus on two widely employed techniques: the Externally Bonded Reinforcement (EBR) and Near-Surface Mounted (NSM) techniques. Existing predictive models for these techniques exhibit insufficient accuracy, often resulting from oversimplified assumptions and failure to capture the complex, nonlinear interactions among various structural parameters. To address these limitations, this research introduces a suite of innovative models designed to significantly improve predictive accuracy and reliability.

A comprehensive experimental database was compiled, and advanced data preprocessing methods, including outlier detection and data transformation, were implemented to improve model performance. Analytical models were developed with integrated calibration factors to minimize error variability and ensure consistent predictions across diverse beam characteristics. Additionally, a Reliability Analysis framework was established to calibrate resistance reduction factors, ensuring the proposed models align with safety and cost-effectiveness standards for practical design applications. Furthermore, Machine Learning models were developed, demonstrating substantial advancements in predictive accuracy and reliability compared to traditional analytical approaches.

Overall, this research presents significant contributions to the field, offering improved design models that promote the sustainable and safe use of CFRP for shear strengthening, while laying the groundwork for future advancements.

Keywords: CFRP strengthened RC beam, Design model for shear resistance, Reliability analysis, Resistance reduction factor, Machine learning

RESUMO

As estruturas de Betão Armado (BA) são amplamente utilizadas na construção devido ao seu desempenho, durabilidade e competitividade económica. No entanto, com a evolução das exigências normativas, muitas deixam de estar em conformidade. A demolição, embora necessária em alguns casos, apresenta custos elevados e impactos ambientais significativos. Assim, o reforço estrutural surge como uma alternativa mais sustentável e económica.

Esta tese desenvolve modelos analíticos para o dimensionamento de soluções em Polímeros Reforçados com Fibras de Carbono (CFRP) no reforço ao corte de vigas de BA. O estudo foca-se nas técnicas de colagem externa (EBR) e inserção em entalhes no betão de recobrimento (NSM). A avaliação dos modelos existentes revela limitações devido a simplificações que não captam adequadamente as interações complexas do problema. Para superar essas limitações, esta investigação propõe modelos inovadores que melhoram a precisão e fiabilidade das previsões.

Uma base de dados existente de vigas reforçadas ao corte com as técnicas EBR e NSM foi analisada e ampliada com dados de publicações recentes. Métodos avançados de pré-processamento, como a deteção de outliers e a transformação de dados, foram aplicados para melhorar o desempenho dos modelos. Modelos analíticos com fatores de calibração integrados foram desenvolvidos para reduzir a variabilidade dos erros e garantir previsões consistentes. Também foi aplicada uma Análise de Fiabilidade para calibrar fatores de segurança, assegurando a conformidade com as normas de projeto e a eficiência económica das soluções de reforço. Além disso, modelos de Aprendizagem de Máquina foram desenvolvidos, proporcionando ganhos significativos em precisão e fiabilidade face às abordagens analíticas tradicionais. Em suma, esta investigação oferece contribuições significativas na área do dimensionamento de soluções de reforço ao corte de vigas de BA com CFRP, disponibilizando modelos com maior fiabilidade capacidade de preditiva. As ferramentas e metodologias utilizadas podem ser replicadas em outras áreas de utilização de CFRP no reforço de estruturas de BA.

PALAVRAS-CHAVE: Viga de betão armado reforçada com CFRP, Modelo de projeto para resistência ao corte, Análise de confiabilidade, Fator de redução da resistência, machine learning

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LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

ACI	American concrete institute
AdaBoost	Adaptive boosting
AFRP	Aramid fibre reinforced polymer
BFRP	Basalt fibre reinforced polymer
CDC	Critical diagonal crack
CDM	Core drilled mounted
CFRP	Carbon fibre reinforced polymer
CNR	Italian advisory committee on technical recommendations for construction
CoV	Coefficient of variation
DT	Decision tree
DLI	Double loop iteration
EBR	Externally bounded reinforcement
EC2	European code 2
EDA	Exploratory data analysis
ELBO	Evidence lower bound
ERT	Extremely randomized trees
ETS	Embedded through section
FCT	Fundação para a ciência e tecnologia
FE	Finite element
FIB	Fédération Internationale du béton/International federation for structural concrete
FRP	Fiber reinforced polymer
GANS	Generative adversarial networks
GB	Gradient boosting
GFRP	Glass fiber reinforced polymer
IF	Isolation forest
IS	Importance sampling
ISISE	Institute for sustainability and innovation in structural engineering
jpdf	Joint probability density function
KNN	k-nearest neighbours

LightGBM	Light gradient boosting machine
LOF	Local outlier factor
LRFD	Load and resistance factor design
MAPE	Mean absolute percentile error
MCS	Monte Carlo simulation
MD	Mahalanobis distance
ML	Machine learning
MLP-ANN	Multilayer perceptron artificial neural network
NN	Neural network
NSM	Near surface mounted
NZEBs	Nearly zero-energy buildings
PCE	Polynomial chaos expansion
RA	Reliability analysis
RC	Reinforced concrete
RFECV	Recursive feature elimination with cross validation
RF	Random Forest
RMSE	Root mean squared error
SMCFT	Simplified modified compression field theory
SS	Subset sampling
SLS	Serviceability limit state
UMinho	University of Minho
ULS	Ultimate limit state
XGBoost	Extreme gradient boosting

SYMBOLS

b_w	Width of the beam's web (mm)
C_f	Configuration type factor
d	Dimension of random variable
D_f	Failure domain
d_s	Internal arm of the beam (mm)
d_g	Depth of the groove (mm)
E_f	Elastic modulus of CFRP reinforcement (MPa)

$E_f \rho_f$	Axial stiffness of CFRP reinforcement
$f_X(\mathbf{x})$	Joint probability density function
f_{swy}	Yield strength of stirrups (MPa)
f_{cm}	Concrete mean compressive strength (MPa)
$g(\mathbf{X})$	Limit state function
h_f	height of FRP
k_v	Tensile stress factor
L_{Rf}	Average available resisting bond length
M	The total number of scenarios
N	Total number of samples/scenarios
n_f	Number of failed beams
P_f	Probability of failure
$P_{f,required}$	Required probability of failure
$R(\mathbf{X})$	Distribution of resistance
R^2	Coefficient of determination
r_P	Pearson coefficient of correlation
$S(\mathbf{X})$	Distribution of actions (demand)
ν_c	Contributions of concrete to shear strength (N/mm ²)
ν_s	Contributions of stirrups to shear strength (N/mm ²)
ν_f	Contributions of CFRP reinforcement to shear strength (N/mm ²)
ν_n	Nominal shear strength of RC beam (N/mm ²)
V_n	Nominal value of shear resistance (N)
V_d	Design value of shear resistance (N)
$\mathbf{X}$	Vector of random variable
φ	Resistance reduction factor
β_T	Target reliability index
Φ^{-1}	Inverse cumulative distribution function
ρ_{sw}	Ratio of steel stirrups
ρ_l	Ratio of longitudinal steel reinforcement
ρ_{slr}	Ratio of tensile steel reinforcement
α_f	Inclination of FRP fibres

α	Loading condition ratio
θ	CDC inclination angle
ρ_f	Ratio of CFRP reinforcement
ε_{fe}^{model}	Predicted value of effective strain
χ	Model error

LIST OF CUMULATIVE PAPERS OF THE THESIS

1. Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2023. "A new model for predicting the shear strength of RC beams strengthened with externally bonded FRP sheets." *Journal of Composite Structures*, 319, 117081, <https://doi.org/10.1016/j.compstruct.2023.117081>. [NC= 13] (Scopus at 04-11-2024)

Impact Factor: **6.3** (2023)

2. Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2024. "Multiobjective optimization of an unbiased model for externally bonded CFRP system contributions to shear resistance in RC beams." *Journal of Composites for Construction*, 28, no. 6, 04024066, <https://doi.org/10.1061/JCCOF2.CCENG-4814>. [NC= 1] (Scopus at 04-11-2024)

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3. Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2024. "An interpretable machine learning-based model for shear resistance prediction of CFRP-strengthened RC beams using experimental and synthetic dataset." *Journal of Composite Structures*, 35, 118632, <https://doi/10.1016/j.compstruct.2024.118632> . [NC= 1] (Scopus at 04-11-2024)

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4. Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2024. " A novel design model for predicting the shear resistance of reinforced concrete beams strengthened with EBR-CFRP systems." *Journal of Composite Structures*, 331, 117901, <https://doi.org/10.1016/j.compstruct.2024.117901>. [NC= 4] (Scopus at 04-11-2024)

Impact Factor: **6.3** (2023)

CHAPTER 1

1. INTRODUCTION

This chapter provides an overview of the subject addressed in this PhD thesis. It begins with a concise background on the shear strengthening of Reinforced Concrete (RC) beams using Carbon Fiber Reinforced Polymer (CFRP) materials, followed by an identification of key gaps in the existing literature that underscore the motivation for this study. Finally, the research objectives are outlined, and the overall structure of the thesis is presented.

1.1. BACKGROUND AND MOTIVATION

While concrete has been used for millennia, the advent of steel-reinforced concrete (RC) in the late 19th century revolutionized the construction industry, meeting the growing demands of urbanization and the need for durable infrastructure. Today, RC structures are integral to modern construction in a wide range of applications, from residential buildings to infrastructure projects such as bridges and roads (Kovacs et al. 2019), with an estimated worldwide usage exceeding 10 billion cubic meters annually (Li et al. 2020; Coffetti et al. 2022). As can be seen in Fig. 1, the global consumption of building materials has tripled just from 2000 to 2017, with concrete, being the most commonly used material, highlighting its widespread adoption and significance in infrastructure development (Huang et al. 2020).

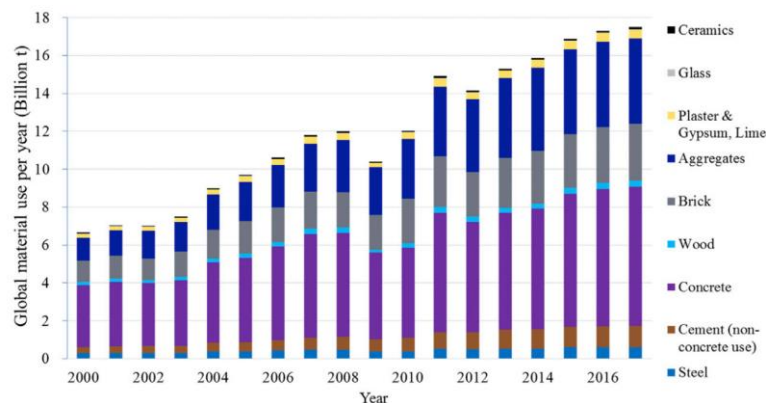


Fig. 1. Annual Global Building Material use during 2000–2017 (Huang et al. 2020)

Despite the advanced and well-established nature of RC technology today, it is crucial to remember that a significant portion of existing structures were designed and constructed before the advent of modern state-of-the-art design guidelines. These contemporary standards ensure structural reliability across various requirements, including higher and updated exigences on the seismic design, as well as expected

loads and long-term material behaviour. Consequently, many older structures may not meet current safety and performance criteria. Moreover, ageing of structures, deterioration due to exposure to aggressive environmental conditions, increased service loads leading to overloading, and damages caused by vandalism, further challenge the durability and performance of many existing RC structures. Demolishing these structures would result in substantial and avoidable losses of materials, energy, and financial resources, directly conflicting with actual sustainability principles. These principles emphasize resource conservation, waste reduction, and minimizing the environmental impact of construction activities. In line with the European Union's push for Nearly Zero-Energy Buildings (NZEBS) and zero-emission buildings (Zangheri et al. 2021), reducing the need for new construction by retrofitting and strengthening existing structures supports energy efficiency and the goal of decreasing greenhouse gases emissions. Extending the lifespan of existing structures aligns with these sustainability goals by conserving embodied energy and reducing the environmental impact of building material production and disposal.

Following the assessment of existing structures, appropriate strengthening interventions should be implemented to improve their overall performance in terms of resistance, serviceability, ductility, and durability. These interventions may target various aspects, such as enhancing flexural, shear, or axial resistance, or even increasing the ductility of the structure (Parvin and Wang 2001; Barros et al. 2011a; Dias et al. 2021). Shear deficiencies are often critical, as they can lead to sudden and brittle failure modes, posing significant risks to structural safety (Chaallal et al. 1998; Cervenka 2013; Firouzi et al. 2019). Therefore, this study specifically focuses on shear strengthening of RC beams.

Since the 1960s (L'hermite and Bresson 1967; Bresson 1971), steel plates have been employed to enhance the shear resistance of RC beams. These plates are typically attached to the sides of the beams using bolts or adhesives, providing additional reinforcement and improving the overall shear capacity of the structure (Castro et al. 2018). Moreover, in some cases shear strengthening has been achieved by installing U-shaped bars, which are welded to steel angles and then bolted to the slab.

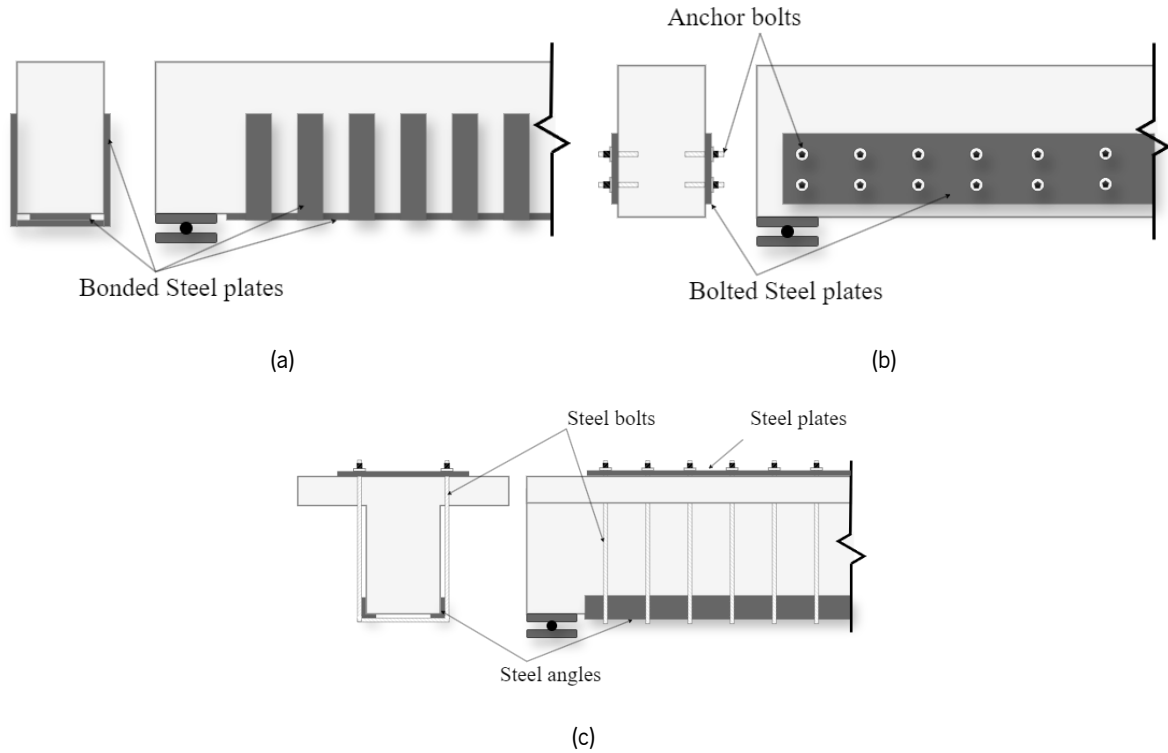


Fig. 2. Examples of strengthening of RC beams using: (a) bonded steel plates, (b) bolted side plates, and (c) U-shaped bars that from bottom are welded to steel angles and from top are bolted to the slab (Su et al. 2010; Sabahattin et al. 2013; MDRAP 2019)

The use of steel reinforcements for strengthening RC beams offers several advantages due to the relatively high tensile strength and modulus of elasticity of steel (when compared with plain concrete), which significantly enhances the shear capacity of beams. Steel is also a widely available, well-established material, making it a cost-effective solution in some applications. Additionally, steel provides good ductility, allowing structures to demonstrate greater ductility under loading conditions corresponding to the ultimate limit state (ULS) design load combinations. However, steel reinforcements also present notable disadvantages. Steel is prone to corrosion, particularly in aggressive environments, which can undermine its long-term durability. Due to high weight of steel, the installation process of strengthening system requires scaffolding and involves labour-intensive and time-consuming handling. This often leads to significant disruption in the usage of the structure during retrofitting. Architectural challenges can also arise due to the bulky nature of steel reinforcements, which can limit design flexibility or interfere with the aesthetic aspects of a building. Additionally, the relatively high dead weight of steel can increase the dead load on the structure.

Another method to increase the resistance of RC beams is adding RC jackets (Altun 2004). However, this technique has several significant drawbacks. RC jackets increase the beam's size, weight placing greater demands on the structure and often presenting architectural challenges. Moreover, the installation process is labour-intensive, requiring extensive work such as drilling, rebar preparation and installation, shuttering, and scaffolding. These demands complicate design considerations, increase execution time, and can negatively impact the overall aesthetics of the building, making this approach less practical in certain situations.

Strengthening techniques utilizing Fibre-Reinforced Polymer (FRP) reinforcements are increasingly replacing traditional methods such as steel and RC jackets due to their superior characteristics. FRP reinforcements offer several advantages over these conventional materials, including resistance to corrosion (Cruz et al. 2021; Dushimimana et al. 2024), lightweight properties, and ease of application. They are available in various forms and characteristics in the market, making them versatile for different applications. Additionally, FRP exhibits excellent fatigue behavior (Sena-Cruz et al. 2012), is inert to thermal and electrical effects, and has a high strength-to-weight ratio. Importantly, FRP reinforcements do not adversely affect the architectural appearance of the beams, allowing for a seamless integration into existing structures. Their rapid installation does not require heavy scaffolding, further minimizing disruption during the strengthening process.

The most used FRP materials in civil engineering include Glass Fiber Reinforced Polymers (GFRP), Carbon Fiber Reinforced Polymers (CFRP), Aramid Fiber Reinforced Polymers (AFRP), and Basalt Fiber Reinforced Polymers (BFRP). Among these, CFRP have gained prominence in strengthening applications for RC structures, despite being more expensive than the counterpart alternatives (Sen 2015). For these reasons, CFRP will be the primary focus of this study.

Several effective and widely used strengthening techniques have been introduced in the literature to enhance the shear resistance of RC beams using CFRP reinforcements. These techniques can generally be classified as: (i) Externally Bonded Reinforcement (EBR) technique (Czaderski and Meier 2018), which involves attaching CFRP sheets or prefabricated laminates to the surface of the structural member and was first proposed in the 1980s (Meier 1988); (ii) Near-Surface Mounted (NSM) technique (Alkhrdaji et al. 2000; Lorenzis et al. 2000; Barros et al. 2004), which involves embedding reinforcement into grooves cut on the concrete cover of the beam's lateral faces; and (iii) the Embedded Through Section (ETS)

technique or also known as Core Drilled Mounted (CDM) technique (Barros et al. 2008; Barros and Dalfré 2013; Breveglieri et al. 2014), which involves embedding FRP bars or ropes into holes drilled through the core of the member.

The EBR technique is indeed the most widely researched and applied technique for strengthening RC beams with FRP reinforcement. Despite its popularity, the method has known to be vulnerable to bond failure, which limits its effectiveness. To mitigate such issues, anchorage systems have been extensively studied and applied (Godat et al. 2020; Dias et al. 2021; Arslan et al. 2022). These techniques include FRP insertion, FRP extension, planting spikes, and metallic or FRP patches. FRP insertion involves embedding rods or laminates into grooves cut into the concrete, improving bonding and load distribution. FRP extension enlarges the bonded area, increasing load transfer and minimizing debonding. Spike anchors are mechanical devices driven into the concrete to physically secure the FRP layers, significantly enhancing bond performance. Metallic and FRP patches are placed over critical stress to provide additional reinforcement, improving shear capacity and preventing localized failures. However, the effectiveness of these techniques in improving the shear contribution of FRP reinforcements varies (Godat et al. 2020). In Fig. 3, are illustrated examples of RC beams strengthened solely using the EBR technique and those strengthened with both EBR and additional anchorage systems.

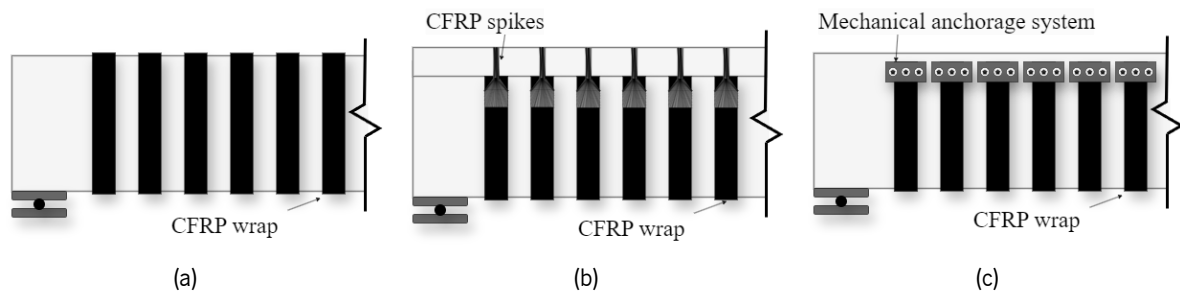


Fig. 3. Shear strengthening of RC beam using: (a) CFRP wet lay-up wraps (Barros and Dias 2003), (b) CFRP U-jackets anchored by CFRP spikes (Dias et al. 2021), and (c) CFRP wraps anchored by steel mechanical anchorage system (Belarbi et al. 2012)

The NSM technique was developed to overcome several limitations associated with the EBR technique (Al-Zu'bi et al. 2024). Unlike EBR, the NSM technique eliminates the need for extensive surface preparation, making installation faster and more efficient. With NSM, the bond between the FRP reinforcement and concrete substrate is significantly improved, without extra effort, due to the larger contact area and the distribution of stresses over a greater concrete volume (Bianco et al. 2009). This enhances the overall bond strength, addressing the common issue of debonding in EBR systems.

Furthermore, because the FRP reinforcements in the NSM method are embedded within grooves in the concrete, they are better protected against acts of vandalism, fire, mechanical damage, and environmental degradation. This protection ensures better durability and longevity compared to EBR systems, where the reinforcements are exposed. Another notable advantage of NSM is its minimal impact on the aesthetic appearance of the structure, as the reinforcements are hidden within the concrete, making this method particularly suitable for retrofitting applications where maintaining the original look is essential (Sena-Cruz et al. 2016; Kadhim et al. 2021). Fig. 4 shows two RC beams shear-strengthened with NSM CFRP reinforcement, illustrating the beam's condition before and after failure.

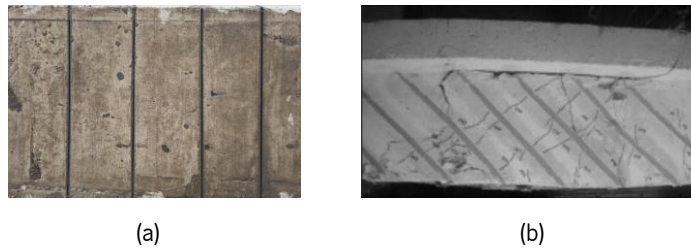


Fig. 4. Shear-strengthened RC beam with NSM CFRP reinforcement: (a) before insertion of CFRP reinforcement (Alwash et al. 2021); (b) finished appearance (after failure) (Dias and Barros 2013)

In the ETS technique, CFRP reinforcement is embedded deep within the concrete core of the beam (Breveglieri et al. 2016). This configuration allows the reinforcement to benefit from the confining stresses exerted by the surrounding concrete, significantly enhancing the bond behavior between the CFRP reinforcement and the concrete (Oehlers et al. 2008). The increased bond strength resulting from this confinement improves the overall performance and shear resistance of the strengthened RC beam. Although this technique holds great promise, due to the much larger experimental data available from the two other (EBR and NSM), no model will be developed for ETS, since the approach to be adopted in the present work requires as much as possible reliable experimental results.

Although numerical methods, such as supported in Finite Element (FE) analyses, can simulate the full range of behavior of RC beams strengthened with FRP reinforcements, these approaches are often impractical and not widely adopted for design purposes (Barros et al. 2013; Abbasi et al. 2022). Furthermore, the adoption of these methods requires the execution of Reliability Analysis (RA), which adds complexity to the design process. The draft version of the Eurocode 2 (EC2), in Annex F, provides supplementary guidance on conducting RA for this approach (prEN1992-1-1 2022). For this reason, the design guidelines usually propose straightforward analytical equations, supplemented by partial factors (adopted by European guidelines) or resistance reduction factors (adopted by American Concrete

Institute, ACI, guidelines), which eliminate the need for RA, to predict the shear resistance of RC beams (fib 2001, 2019; EN 1998-3 2005; CIDAR 2006; DAfStb 2012; CNR 2013; ACI 2023). Nevertheless, the assessment of these models revealed that their performance is inadequate (Barros et al. 2007; Sas et al. 2009; D'Antino and Triantafillou 2016; Kotynia et al. 2021; Ke et al. 2022; Mohammadi et al. 2023, 2024), resulting in either excessively conservative predictions or outcomes that are unsafe. Consequently, these predictions either prevents the achievement of strengthening goals or lead to solutions that lack feasibility. The assessment of these models not only highlights the need for improved prediction models but also reveals the inefficiency of the introduced partial factors or resistance factors. This underscores the potential benefits of applying more advanced RA approaches.

1.2. RESEARCH OBJECTIVES

Building on the issues discussed in the previous section, this PhD thesis aims to promote the application of shear strengthening techniques for RC beams using CFRP reinforcements (i.e., EBR, NSM) by developing more reliable predictive tools. These tools are designed to ensure practical, cost-effective, and reliable designs, addressing the limitations of existing models and contributing to more efficient and sustainable strengthening practices. Given the substantial experimental data available in the literature for these techniques, data-driven prediction models have been developed. These predictive tools incorporate not only traditional analytical methods but also Machine Learning (ML) algorithms, providing additional models that enhance the accuracy and reliability of the overall approach. Despite their accurate predictions, the developed models' potential to fail in providing safe predictions remains a critical issue to address. Given that these models are intended for being used by engineers in a deterministic format, this study introduces a framework for calibration of resistance reduction factors, ensuring that these models can be practically applied in design scenarios. This global objective of this study can be divided into several specific objectives, namely:

- **Data Collection:** The methodologies employed in this study are dependent on previously observed data. Therefore, the first milestone is to collect a comprehensive dataset from published literature. This collection involved identifying relevant studies and extracting pertinent information to ensure a robust foundation for subsequent analyses and model development;
- **Data Screening:** The mass collection of data from various sources, without a comprehensive analysis of the results and the underlying factors driving those results, may significantly impact the predictive performance of the models. Therefore, this study needs to employ various methods to identify and eliminate anomalous data from the collected dataset;

- **Evaluation of existing models:** A series of existing models gathered from the literature were reviewed and their predictive performance are evaluated using the collected and screened dataset. Their performance needs to be systematically assessed to identify strengths and weaknesses, providing insights into their accuracy and reliability for shear strengthening prediction in RC beams;
- **Proposing predictive models:** To develop predictive models for each technique estimating the shear resistance of strengthened RC beams;
- **Performing reliability analysis (RA):** To derive design values from the prediction models, RA needs to be conducted to calibrate the resistance reduction factors for each model.

1.3. STRUCURE OF Thesis

This PhD thesis is presented as a collection of papers published and submitted to peer-review in SCI international journals. These are presented alongside three introductory chapters followed by a chapter on preliminary results on the proposed model for NSM technique, a concluding chapter and an annex, as outlined below:

- **Chapter 1:** Provides an overview of the thesis, including the background, scope, motivation, and the general and specific objectives. It also outlines the structure of the thesis;
- **Chapter 2:** Presents the overall methodology employed to achieve the proposed objectives. This includes data collection, screening, model development, and evaluation, as well as the incorporation of machine learning techniques and reliability analysis;
- **Chapter 3:** Provides a summary and detailed justification for each paper included in this thesis, highlighting their relevance and contribution to the overall research objectives;
- **Chapter 4:** Provides preliminary results on the development of models for the NSM technique;
- **Chapter 5:** Summarizes the key findings, presents the concluding remarks, and outlines potential future research directions based on the work carried out in this doctoral thesis;
- **Annex:** Published papers

CHAPTER 2

2. RESEARCH METHODOLOGY

As outlined in the previous chapter, the primary aim of this investigation is to develop reliable design models for the shear-strengthened RC beams using CFRP reinforcements according to the EBR and NSM techniques. This chapter provides an introduction and detailed overview of the methodologies adopted in this research, facilitating a better understanding of the studies and analyses presented in the subsequent papers.

2.1. DATA COLLECTION, SCREENING, SYNTHETIC DATA GENERATION

From the outset of this PhD study, a primary focus has been on collecting credible data to strengthen the foundation for developing reliable predictive models. Initially, an existing dataset known as DABASUM (Barros et al. 2011b) was utilized. Subsequently, there has been an ongoing effort to add more recent data, with continuous gathering from the literature to further enrich DABASUM up until the completion of this document. This continued effort ensures that the models are also using the most current and comprehensive information available, thereby enhancing their accuracy and practical applicability.

Upon further evaluation, the collected dataset was suspected to contain some samples with abnormal data. Instead of relying on manual identification, it was decided to employ systematic approaches for detecting these anomalies. Throughout the various stages of the study, as evidenced in the published works, different anomaly detection techniques were examined, including Mahalanobis Distance (MD) (Todeschini et al. 2013), Local Outlier Factor (LOF) (Breunig et al. 2000), and Isolation Forest (IF) algorithm (Liu et al. 2012). These methods helped to ensure a more accurate and reliable dataset by effectively identifying and removing outliers that could negatively affect the model's predictive performance.

Moreover, based on the initial explanatory data analysis (EDA), a lack of sufficient experimental data was noted, prompting the need for additional experimental studies. However, considering the constraints of the project, conducting new experiments was not a feasible option. Instead, recent artificial intelligence techniques were explored, including Generative Adversarial Networks (GANs) and Variational Auto Encoders (VAEs) (Xu et al. 2019; Davila Delgado and Oyedele 2021). Upon evaluation, VAEs were chosen

for this study due to their superior performance on the dataset of beams strengthened using the EBR technique.

VAE represent a significant advancement in the realm of generative models¹, merging concepts from statistics and deep learning to create a powerful framework for unsupervised representation learning (Kingma and Welling 2014). As shown in Fig. 5, the foundational architecture of a VAE consists of two primary components: an encoder and a decoder. The encoder maps real data (beams available in the dataset) into a latent space, producing the parameters (μ and σ^2) of a Gaussian (AKA. normal) distribution. Latent variables (z) are then obtained by sampling from a standard normal distribution $\varepsilon \sim \mathcal{N}(0,1)$, and transforming this sample by multiplying ($\times$) it by the variance and adding (Σ) the mean. This process allows the latent variables to follow the Gaussian distribution defined by $\mathcal{N}(\mu, \sigma^2)$. The decoder then reconstructs the original data from these latent variables, aiming to minimize the reconstruction error while also regularizing the latent space to follow a prior distribution, typically a standard Gaussian. Once the model has been validated for accuracy, it can be used to generate synthetic, previously unseen data—representing beams that have not yet been physically constructed or tested but are anticipated to be realized. This capability allows to augment existing datasets with realistic synthetic samples, enhancing model performance.

The probabilistic foundation of VAEs allow them to incorporate uncertainty in the latent representations, distinguishing them from conventional autoencoders. This is accomplished through the Evidence Lower Bound (ELBO) as a loss function, which balances the reconstruction loss with a regularization term to encourage the latent distribution to approximate a standard normal distribution (Kullback-Leibler divergence loss) (Doersch 2016; Asperti et al. 2021). This approach not only enables the generation of new data samples but also facilitates meaningful interpolation within the latent space, enhancing the model's robustness and adaptability (Hadjeres et al. 2017).

¹ A machine learning model designed to capture the underlying patterns or distributions within data, enabling it to generate new data with similar characteristics.

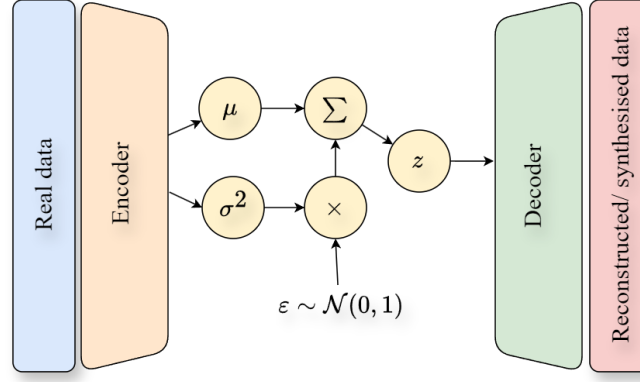


Fig. 5. Encoder and decoder in a VAE model

2.2. MODEL DEVELOPMENT STRATEGY

In this study, the Simplified Modified Compression Field Theory (SMCFT), originally developed for predicting the shear resistance of RC beams, is adapted to incorporate the effects of CFRP reinforcement (Bentz et al. 2007; Baghi and Barros 2018). In this approach, which is well-established in some design guidelines, the total shear strength is obtained as (i) the sum of the concrete's strength (v_c), which includes mechanisms such as aggregate interlock and dowel action, (ii) the transverse steel reinforcement (v_s), and (iii) the CFRP reinforcement (v_f). By following this principle, the developed models ensure compatibility with practical engineering design guidelines and align with current practices. Thus, the nominal shear strength (v_n) can be expressed as the sum of these three contributions:

$$v_n = v_c + v_s + v_f \quad (1)$$

Therefore, the nominal (V_n) and the design value of shear resistance (V_d) of the beam can be obtained from the following equations:

$$V_n = v_n b_w d_s \quad (2)$$

$$V_d = \phi V_n \quad (3)$$

where b_w and d_s are the width of the beam's cross section and the internal arm of the longitudinal steel reinforcement in tension, respectively. Additionally, ϕ is the resistance reduction factor, which is applied to convert the nominal shear strength into a design value.

The first two term of the Eq. (1) are well-established in the literature, representing the contributions from concrete and transverse steel reinforcement to the shear resistance of RC beams. However, the term

related to CFRP reinforcement requires derivation of a dedicated equation like the one employed for steel reinforcement. In the case of steel, a common and reasonable assumption is that the strain in the steel stirrups bridging the two sides of a Critical Diagonal Crack (CDC) reaches the yielding strain (given that the beam does not fail due to the concrete crushing of the struts). This assumption, however, does not hold for CFRP reinforcement, due to its linear elastic and brittle behavior and nonuniform strain profile/distribution along the CDC (Carolin and Täljsten 2005; Chen et al. 2013). As can be seen in Fig. 6, the strain and stress levels vary across different CFRP strips at the point of failure. Accurately predicting the contribution of CFRP reinforcement would require locating CDC, knowing the exact stress distribution along the CDC and finally determining the resultant shear force transferred by CFRP reinforcement. Since this process is impractical for design purposes, an alternative approach is typically employed, that is using an equivalent value known as effective strain to represent the nonuniform stress distribution. Using this simplification in this study allows for a more practical and efficient application in design while still accounting for the complex behavior of CFRP reinforcements in shear.

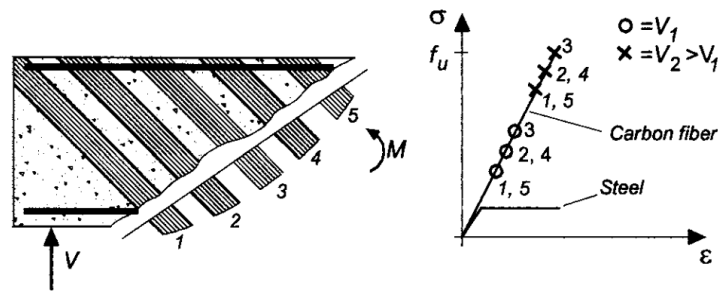


Fig. 6. Stresses and strains over height of cross section (Carolin and Täljsten 2005)

In this study, statistical models for determining the effective strain were developed using both analytical techniques and ML algorithms. The analytical models aim to capture and quantify the relationship between key parameters influencing effective strain in CFRP reinforcement, offering a direct mathematical representation. In parallel, ML algorithms were employed to enable a more advanced and nuanced approach, where patterns and correlations within larger datasets could be identified, thereby improving prediction accuracy and model robustness.

2.3. ML BASED MODELS

In this thesis, nine machine learning (ML) algorithms were trained to predict the effective strain in CFRP reinforcement for shear-strengthened RC beams using the EBR technique. These algorithms included Extreme Gradient Boosting, XGBoost(Chen and Guestrin 2016), Random Forest, RF(Breiman 2001),

Gradient Boosting, GB (Friedman 2001), k-Nearest Neighbours, KNN (Cover and Hart 1967), Adaptive Boosting, AdaBoost (Freund and Schapire 1997), Multilayer Perceptron Artificial Neural Network, MLP-ANN (Werbos 1974), Decision Tree, DT (Breiman et al. 1984), Light Gradient Boosting Machine, LightGBM (Ke et al. 2017), and Extremely Randomized Trees, ERT (Geurts et al. 2006). Among these, the XGBoost model demonstrated the best predictive performance, outperforming the other models. Building on these results, a new model was trained using XGBoost on an expanded synthetic dataset generated by a VAE, which led to further significant improvements in predictive accuracy.

2.4. RELIABILITY ANALYSIS

2.4.1. Probabilistic safety concepts

The deterministic models proposed in this thesis, while effective in enhancing predictive accuracy, inherently lack the capacity to account for the inherent uncertainties in material properties, geometrical entities, loading conditions, and model errors. The resistance of a structural element is not a deterministic value; rather, it is a random entity influenced by the stochastic nature of abovementioned uncertainties. Similarly, the acting loads on the structure are also random variables, typically governed by their own probabilistic distributions. To address these uncertainties, the limit state design principles are employed, which define the boundary between acceptable and unacceptable performance states of a structural element using a limit state probability function $g(\mathbf{X})$, given as:

$$g(\mathbf{X}) = \chi R(\mathbf{X}) - S(\mathbf{X}) \quad (4)$$

where χ is the model uncertainties distribution, $R(\mathbf{X})$ and $S(\mathbf{X})$ are the distribution of resistance and load demand on the beam, respectively. $\mathbf{X} \in \mathbb{R}^d$ represents the vector of random variables (including material properties, geometric entities, loading conditions and model error) with dimension of d .

If a realization of the limit state function, $\mathbf{x}$, results in $g(\mathbf{x}) > 0$, the structural element is deemed safe, indicating that its resistance exceeds the applied loads $S(\mathbf{x}) \leq R(\mathbf{x})$. Consequently, the sample space $D_{\mathbf{X}}$, representing all possible realizations of the basic random variables, can be divided into two distinct subsets: (i) Safe domain ($D_s \in D_{\mathbf{X}}$) and (ii) Failure domain ($D_f \in D_{\mathbf{X}}$), therefore:

$$D_s = \{\mathbf{x} \in D_{\mathbf{X}} \mid g(\mathbf{x}) > 0\} \quad (5)$$

$$D_f = \{\mathbf{x} \in D_X \mid g(\mathbf{x}) \leq 0\} \quad (6)$$

The boundary between these domains, defined by $g(\mathbf{x}) = 0$, represents the limit state surface. This division provides the foundation for reliability analysis, where the probability of failure (P_f) is calculated as the probability that a realization belongs to the failure domain:

$$P_f = P(\mathbf{x} \in D_f) = \int_{D_f} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (7)$$

where $f_{\mathbf{X}}$ is the joint probability density function (jpdf) of the random variables $\mathbf{X}$. Since the integration domain is not defined explicitly, a classifier is introduced to solve the integration (Marelli et al. 2024):

$$1_{D_f}(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in D_f ; g(\mathbf{x}) \leq 0 \\ 0 & \mathbf{x} \notin D_f ; g(\mathbf{x}) > 0 \end{cases} \quad (8)$$

Therefore:

$$P_f = \int_{D_X} 1_{D_f} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (9)$$

In Paper 4, crude Monte Carlo Simulation (MCS) is used to estimate P_f . MCS is a robust and versatile approach, capable of handling a wide range of problems by repeatedly sampling from jpdf. It utilizes sample-based estimates of the integral to accurately assess the probability of failure, as demonstrated by:

$$P_f^{MCS} = \frac{1}{N} \sum_{i=1}^N 1_{D_f}(\mathbf{x}^i) = \frac{n_f}{N} \quad (10)$$

where N is the number of simulations and n_f is the number of samples found from failure domain.

While MCS is a highly accurate and straightforward approach, its computational cost escalates rapidly when dealing with rare events characterized by very small failure probabilities (P_f). This makes MCS

computationally expensive for problems requiring an exceedingly high level of reliability, as it necessitates an extremely large number of simulations to obtain statistically significant results.

To address the computational challenges of estimating very small probabilities of failure, alternative methods such as Importance Sampling (IS), Subset Sampling (SS), and Metamodel-Based Adaptive Methods are promoted for their efficiency. IS improves Monte Carlo Simulation by biasing the sampling distribution toward the failure region, ensuring more relevant samples are collected and reducing variance. SS divides the rare event simulation into a sequence of less rare intermediate events, progressively sampling these subsets to reach the failure domain with fewer total simulations. Metamodel-based adaptive methods, such as Kriging, Polynomial Chaos Expansion (PCE), or Neural Networks (NN), approximate the limit state function using surrogates that are iteratively refined near the failure domain (Allahvirdizadeh et al. 2024).

2.4.2. Load resistance factors design concepts

To quantify uncertainties and ensure a safe design model, the load and resistance factor design (LRFD) methodology was adopted. This approach, or similar methods widely employed in design guidelines, addresses the uncertainty problem within a deterministic framework, thereby sparing engineers from direct involvement in reliability analysis. Instead, the methodology recommends load and resistance factors calibrated to account for uncertainties in material properties, geometries, loading conditions, and other variables. In this method factored loads (S_d) must be less than the factored resistance ($V_d = \phi V_n$) as given by:

$$S_d \leq V_d \rightarrow \sum_{i=1}^k \gamma_i S_{ni} \leq \phi V_n \quad (11)$$

where S_d is the design value of load effect, S_{ni} is the i^{th} of k load effect (Dead load, Live load, etc.), γ_i is the i^{th} of k load factor, V_n is the shear resistance obtained from Eq. (2) and ϕ , as previously introduced, is the resistance reduction factor.

Given that at limit state the design value of resistance and design value of load effects are equal ($S_d = V_d$) the specified load effects can be determined as:

$$\forall j \in \{1, \dots, k\}: S_{nj} = \frac{\varphi V_n}{\sum_{i=1}^k \gamma_i \frac{S_{ni}}{S_{nj}}} = \frac{\varphi V_n}{\sum_{i=1}^k \gamma_i \alpha_{ij}}; \text{ where } \alpha_{ij} = \frac{S_{ni}}{S_{nj}} \quad (12)$$

By adopting the recommended values of γ_i from previous studies, such as those provided in building codes (CEN 2002; ACI 2019), the nominal value of the acting loads can then be expressed as a function of φ , V_n , load factors, and the vector of loading conditions α_{ij} 's (α).

The distribution of resistance, $R(\mathbf{X})$ can be derived from the probabilistic distribution of the variables in $\mathbf{X}$. Moreover, using the nominal values of load effects obtained from Eq. (12), their distribution type, expected service life, and their variances, the overall load effect distribution $S(\mathbf{X})$ can be obtained:

$$S(\mathbf{X}) = \sum_{i=1}^k S_i(\mathbf{X}, \varphi, \alpha) \quad (13)$$

Therefore, having these distributions, the probability of failure in the element under given conditions can be obtained. Subsequently, the resistance reduction factor, φ , can be optimized to ensure that the probability of failure remains below the required threshold, $P_{f,required}$.

This process is a critical step in achieving a balance between economic feasibility and structural safety. Therefore, the structural element can withstand most conditions while accepting a very low probability of failure under rare extreme scenarios (e.g., maximum loads combined with minimum material strength). This probabilistic framework can consider the service life, the consequences (consequences classes) and types of failure, distinguishing between the ULS and the Serviceability Limit State (SLS) through differing acceptable failure probabilities.

According to EN1990 (CEN 2002), for a reference period of 50 years and within consequence class CC2 (reliability class RC2), the target failure probabilities are specified as 7.2×10^{-5} (for ULS) and 6.7×10^{-3} (for SLS).

The reliability index (β) is often used as an alternative to directly expressing failure probability because it provides a more intuitive measure of reliability and safety. The β index is related to P_f through the cumulative distribution function (CDF) of the standard normal distribution, determined by:

$$\beta = \Phi^{-1}(1 - P_f) = -\Phi^{-1}(P_f) \quad (14)$$

where Φ^{-1} is the inverse of cumulative distribution function (also known as the quantile function) of the standard normal distribution. A higher reliability index corresponds to a lower failure probability, indicating a safer structure. Therefore, to ensure compliance with design standards, the calculated β of the structural element must be greater than the Target reliability index (β_T), expressed as $\beta \geq \beta_T$.

Similar to $P_{f,required}$, β_T is also influenced by several factors, including the severity of potential failure, the costs of implementing safety measures, and the intended lifespan of the structure (CEN 2002; fib 2023).

2.4.3. Calibration of resistance reduction factor

While determining φ to ensure the safety for a single design scenario is relatively straightforward, the challenge arises when attempting to find a unique φ value that can be applied across a variety of scenarios. Each design scenario in this thesis means a RC beams with distinct geometries, loading conditions, and material properties. To address this complexity, φ must be calibrated in a way that minimizes the deviations of the reliability indices obtained for each beam (β_i) for that given φ from required β_T across all scenarios, known as the calibration points. The objective is to ensure that, for each scenario, the calculated reliability indices remain close to the required β_T , ensuring both safety and cost-efficiency. The calibration process for φ is performed using the least squares method, where the optimal values of φ are determined by minimizing the mean squared difference between the reliability indices predicted for each design scenario and the target reliability index. This process can be expressed as:

$$H = \frac{1}{M} \sum_{i=1}^M (\beta_i - \beta_T)^2 \quad (15)$$

where M is the total number of scenarios.

Fig. 7 presents the overall flowchart of the reliability analysis framework proposed in this research.

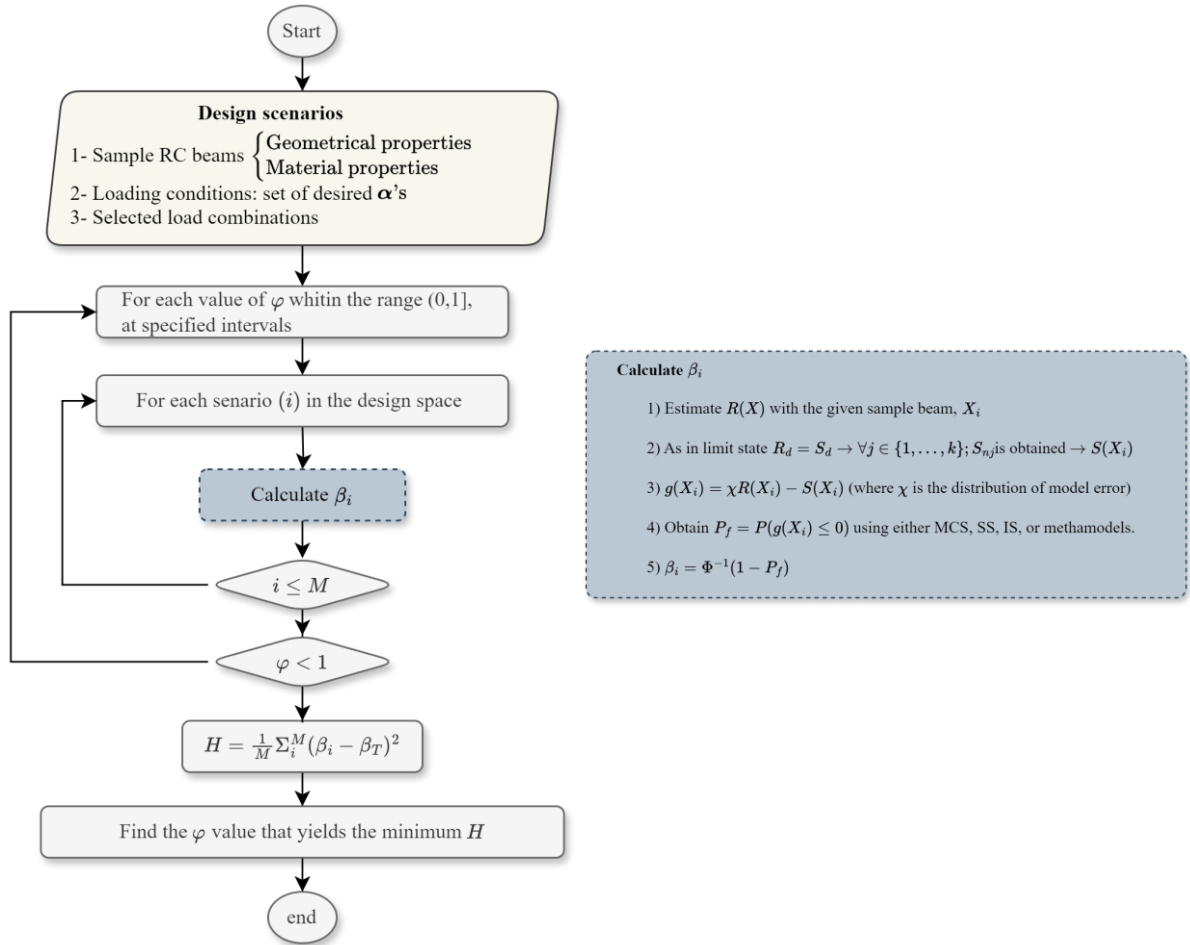


Fig. 7. The proposed RA framework

CHAPTER 3

3. SUMMARY OF PAPERS

This chapter presents a summary of the papers published in international journals, in consequence of the work performed in this PhD thesis. Therefore, a general overview and the interconnection between the papers is explained. Finally, the key points addressed in each paper is also given.

3.1. INTRODUCTION

Table 1 presents an overview of how each study objective was or to be addressed across the papers included in this thesis. Each paper aligns with specific objectives, outlining the methodologies, analyses, and findings that contribute to the overarching goal of developing reliable design models for RC beams strengthened in shear with CFRP reinforcement. The initial goal of this study was to develop design models for shear strengthening techniques including the EBR and NSM techniques. However, it was strategically decided to focus on the EBR technique first, given the larger volume of experimental data available for this type of strengthening. This approach allowed for a thorough exploration and completion of the tasks for the EBR technique, resulting in four papers (Paper 1, 2, 3, 4), which have been finalized and published. Once the objectives regarding EBR strengthening technique were fully achieved, the study advanced to the NSM technique. Partial progress on NSM technique, which was submitted as the Paper 5 is documented in the next chapter. Remaining objectives for the NSM technique are currently in development and include developing ML-based predictive models and conducting reliability analyses for these models. The outcomes of these efforts are anticipated to be documented in Papers 6 and 7. Three additional publications are planned to address these objectives, which are expected to be completed and submitted soon.

Table 1. Papers conducted in the scope of this PhD thesis

Type of strengthening technique	Analytical model	ML model	Partial factor
EBR	(Paper 1, Paper 2) ¹	Paper 3 ¹	Paper 4 ¹
NSM	Paper 5 ²	Paper 6 ³	Paper 7 ³

1: Published, 2: Submitted, 3: Expected in future

3.2. PAPER 1

Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2023. "A New Model for Predicting the Shear Strength of RC Beams Strengthened with Externally Bonded FRP Sheets." *Journal of Composite Structures*, 319: 117081. <https://doi.org/10.1016/j.compstruct.2023.117081>

In Paper 1, a comprehensive dataset of 344 RC beams strengthened with the EBR technique was assembled, accompanied by a descriptive statistical analysis to understand the distribution and variability within the data. The main goal of this study was to evaluate the predictive accuracy of several well-known analytical models from major guidelines including *fib* bulletin 90 (fib 2019), CNR-DT 200 R1 (CNR 2013), and ACI 440.2R-17 (ACI 2017). By assessing these models against an extensive set of experimental results from the literature, the study aimed to highlight limitations and areas for potential improvement in existing prediction models.

Using the mean values of material properties and setting the partial safety factors, resistance reduction factors, and other long-term modification factors to 1.0, the models were expected to produce outputs that closely matched the experimentally measured values of V_f . However as shown in Fig. 8 and detailed in Table 2, the models demonstrated significant inaccuracies, resulting in large errors. Among these, the ACI model exhibited the best performance, with evaluation metrics of root mean squared (RMSE), mean absolute percentile error (MAPE), coefficient of determination (R^2), and Pearson coefficient of correlation (r_p) recorded as 59.1, 74.6%, 0.70, and 0.71, respectively. Additionally, it was observed that these models fail to account for critical features, such as the interaction between internal steel stirrups and externally bonded FRP reinforcement, which was identified as a key factor contributing to their limited predictive accuracy.

The paper also introduced a preliminary predictive model intended to investigate the feasibility of developing an enhanced model that could outperform the other abovementioned models. This new model demonstrated better predictive accuracy (RMSE = 51.4, MAPE = 61.8, R^2 = 0.74 and r_p = 0.76), suggesting significant promise for future refinement.

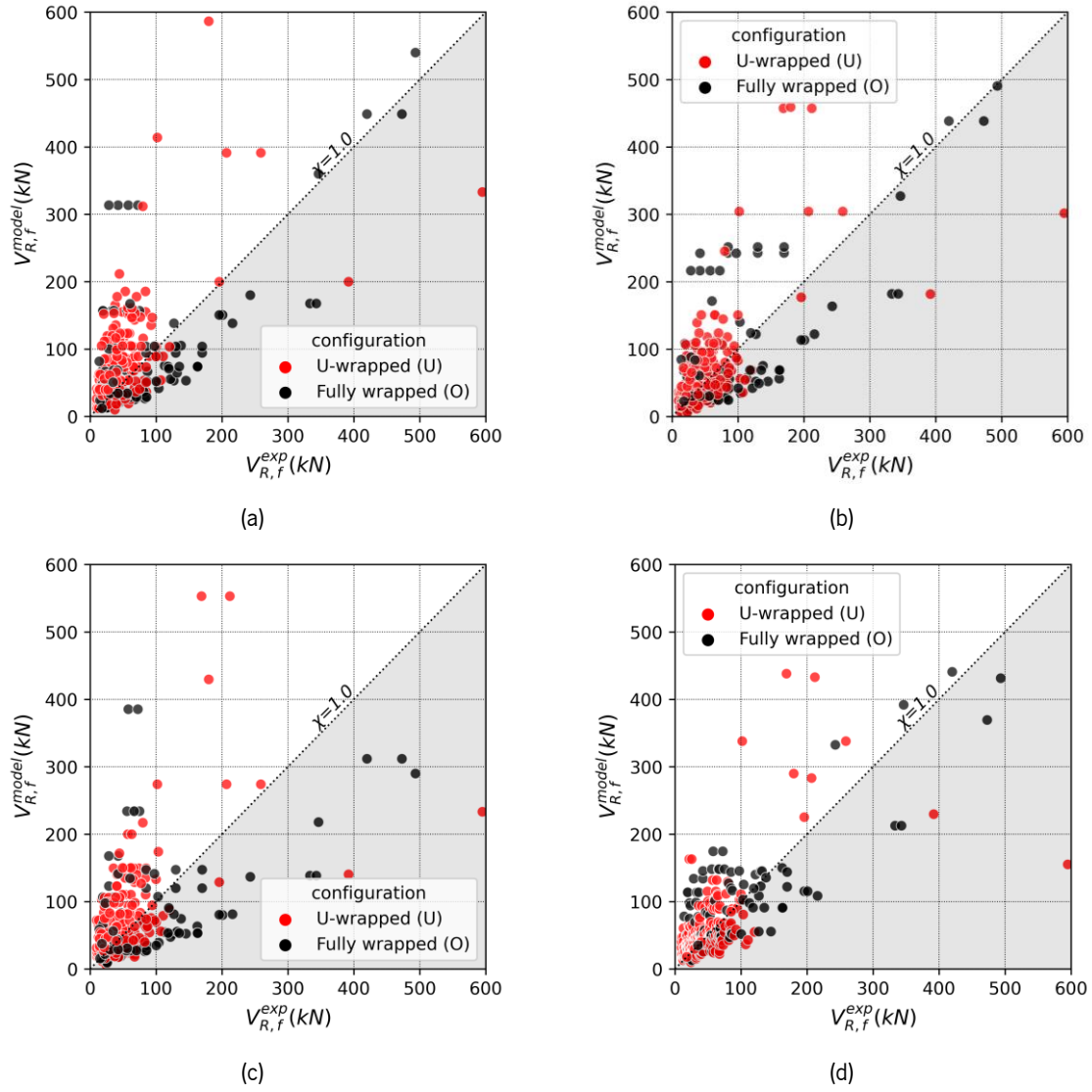


Fig. 8. Predicted versus experimentally measured value of V_f in RC beams strengthened with EBR-CFRP: (a) *fib* bulletin 90 (b) ACI 440.2R-17 (c) CNR-DT 200 R1 (d) preliminarily proposed model

Table 2. Predictive performance of the proposed model and considered models in guidelines

Model	RMSE	MAPE (%)	R^2	r_p
<i>fib</i> bulletin 90	78.1	99.7	0.63	0.63
CNR-DT 200	72.7	91.4	0.52	0.53
ACI 440.2R-17	59.1	74.3	0.7	0.72
Proposed	51.4	61.8	0.74	0.76

3.3. PAPER 2

Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2024. "Multiobjective Optimization of an Unbiased Model for Externally Bonded CFRP System Contributions to Shear Resistance in RC Beams." *Journal of Composites for Construction*, 28(6): 04024066.

<https://doi.org/10.1061/JCCOF2.CCENG-4814>

In Paper 2, a new predictive model was developed for RC beams strengthened with EBR-CFRP reinforcement (fully or U-wrapped), expanding on the preliminary findings of Paper 1. The procedure for developing this model was described in detail, outlining the rationale behind each step. Initially, a basis model was established using a simple regression fit that focused on the key feature influencing the effective strain of CFRP reinforcement. Subsequently, the model errors across various features were analysed. To reduce variations in model errors across different beam features, a calibration factor was introduced. This factor was designed not only to reduce these variations but also to enhance predictive accuracy and stabilize the mean model error around 1.0. A multi-objective optimization technique, was then employed to determine the parameters of the calibration factor, ensuring an optimal balance between accuracy and consistency in predictions.

To highlight the advancements introduced by the proposed model, its predictive performance for the shear-strengthening contribution of CFRP-EBR was rigorously compared with several well-established models, including those outlined in *fib* Bulletin 90 (fib 2019), CNR-DT 200 R1 (CNR 2013), ACI PRC-440.2-23 (ACI 2023), TR55 (Concrete Society 2012), and CIDAR (CIDAR 2006). In Fig. 9, a series of scatterplots exhibits the prediction of these models against the experimentally measured values.

As can be seen in Fig. 10, the proposed model outperformed these benchmarks, demonstrating superior predictive accuracy. Notably, it showed a strong linear correlation between predictions and experimental results, as reflected by an r_p value of 0.79. Moreover, the model delivered well-balanced predictions, achieving an R^2 of 0.62, and exhibited the lowest frequency of erratic predictions, as indicated by a MAPE of 54.4%. Furthermore, it recorded the smallest significant prediction error, with an RMSE of 35.8 kN. Additionally, the proposed model produces model errors with a mean value of 1.0 and a CoV of 0.48 positioning it as a superior choice compared to the other existing models.

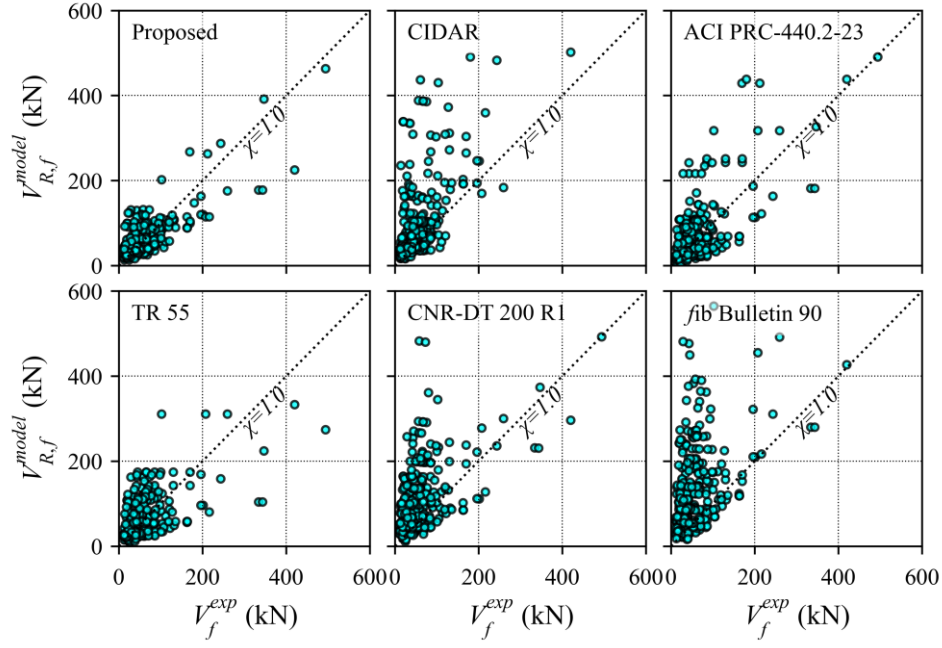


Fig. 9. Predicted *versus* experimentally measured value of V_f in RC beams strengthened with EBR-CFRP (Proposed, CIDAR, ACI PRC-440.2-23, TR 55, CNR-DT 200 R1, *fib* bulletin 90)

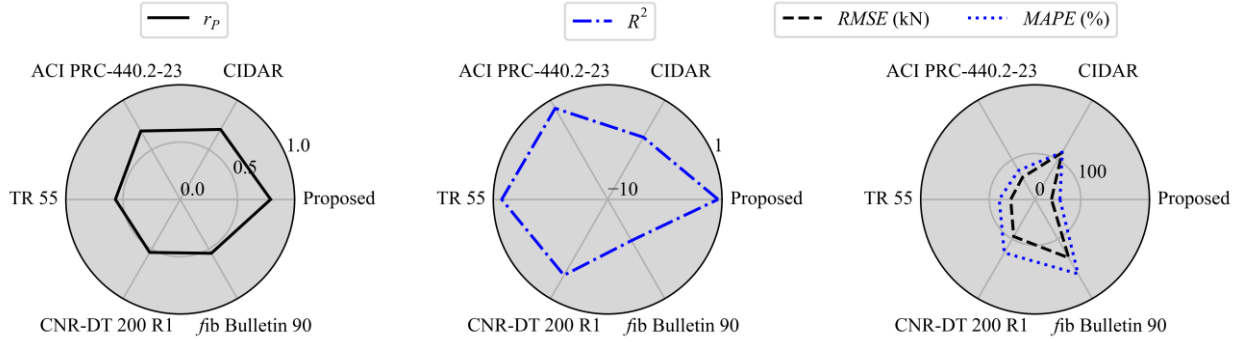


Fig. 10. Comparison of the models' prediction performances using performance metrics

3.4. PAPER 3

Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2024. "An interpretable machine learning-based model for shear resistance prediction of CFRP-strengthened RC beams using experimental and synthetic dataset." *Journal of Composite Structures*, 35: 118632.

<https://doi/10.1016/j.compstruct.2024.118632>

In Paper 3, alternative ML algorithms, including XGBoost, RF, GB, KNN, AdaBoost, MLP-ANN, DT, LightGBM, and ERT, were employed to develop models aimed at improving the prediction of CFRP reinforcement contributions to the shear resistance of RC beams strengthened using the EBR-CFRP technique, whether fully wrapped, U-wrapped, or side-bonded. The influential features were selected using recursive feature elimination with cross validation (RFECV) analysis. These features include axial stiffness of CFRP reinforcement ($E_f \rho_f$), concrete mean compressive strength (f_{cm}), height of FRP (h_f), ratio of steel stirrups (ρ_{sw}), ratio of longitudinal steel reinforcement (ρ_l), configuration type factor (C_f : 0, 1, and 2 for Fully wrapped, U-wrapped, and side-bonded beams, respectively), inclination of FRP fibres (α_f), and yield strength of stirrups (f_{swy}). Finally, as exhibited in Fig. 11, the XGBoost model, trained and tested using the experimental dataset, emerged as the best-performing model. It demonstrated superior predictive capabilities, achieving an R^2 of 0.87, a r_p of 0.96, a RMSE of 19.0 kN, and a MAPE of 27.7%.

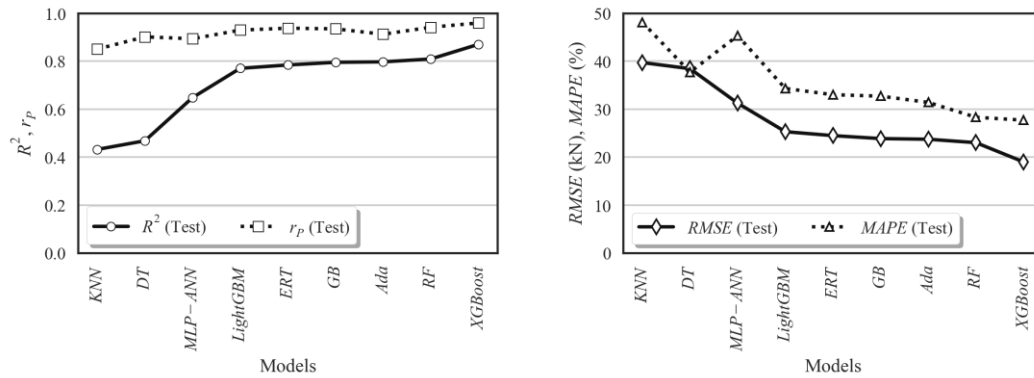


Fig. 11. Model performance statistical indicators (for test datasets)

To further enhance the model's predictive performance, the recognized gaps and limitations in the experimental dataset were addressed through the generation of a synthetic dataset using VAEs (as introduced previously). Multiple verification steps were conducted to ensure that the synthetic dataset closely resembled the characteristics of the original experimental dataset while effectively addressing

underrepresented regions. Leveraging this augmented dataset, a new XGBoost model was developed, resulting in significant improvements in predictive performance. The enhanced model (XGBoost_{syn}) achieved a R^2 of 0.94, a r_p value of 0.97, a RMSE of 12.37 kN, and a MAPE of 18.3%. As shown in Fig. 12, the XGBoost_{syn} surpasses in accuracy both existing analytical models and the ML model developed based on experimental dataset (XGBoost_{real}).

Furthermore, based on the SHAP value analysis conducted in this study, the proposed model successfully captures the underlying physics of the problem represented in the dataset. As a result, this model has proven to be a reliable alternative to traditional parametric models, offering both accuracy and robustness.

This research introduced an accessible and user-friendly online tool (accessible at <https://shear-frp-dabasum.streamlit.app/>), enabling practical use of the ML model for design purposes. The tool's interactive interface supports broader adoption in engineering applications.

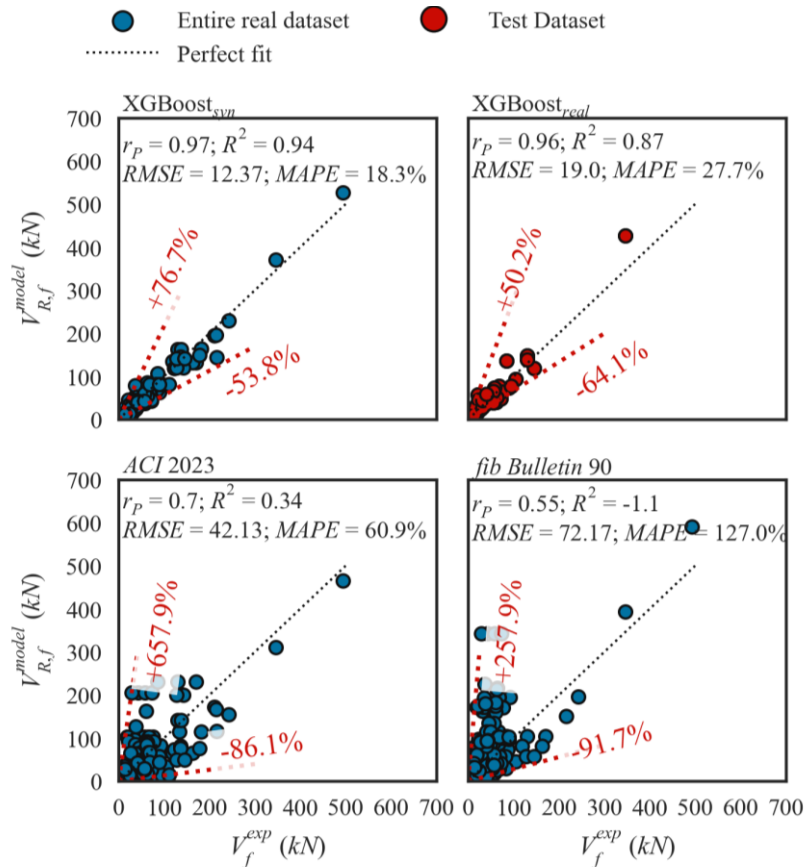


Fig. 12. Predicted *versus* experimentally measured value of V_f in RC beams strengthened with EBR-CFRP (XGBoost_{syn}, XGBoost_{real}, ACI PRC-440.2-23 2023, *fib* Bulletin 90)

3.5. PAPER 4

Mohammadi, A.; Barros, J.A.O.; Sena-Cruz, J. 2024. "A novel design model for predicting the shear resistance of reinforced concrete beams strengthened with EBR-CFRP systems." *Journal of Composite Structures*, 331: 117901. <https://doi.org/10.1016/j.compstruct.2024.117901>.

In Paper 4, the SMCFT analysis was extended by incorporating a term accounting for the contribution of EBR-CFRP reinforcement, resulting in a comprehensive model for predicting the total shear resistance of RC beams strengthened with EBR-CFRP. Key parameters, including the tensile stress factor (k_v) and the inclination angle of the diagonal compressive stress in the web of the section (θ), were computed for all RC beams in the experimental dataset. A sensitivity analysis was subsequently conducted to identify the relationships between these parameters and the features of the strengthened beams. Using the most influential features identified from the sensitivity analysis, a hyperplane was first fitted to predict the tensile stress factor, as shown in Fig. 13a. Building on the strong correlation between the tensile stress factor and the inclination angle of the diagonal compressive stress, seen in Fig. 13b, an additional model was developed to predict the latter based on the former. The given models for these parameters closely replicate the values obtained from SMCFT. Specifically, the approximate equation for tensile stress factor achieves an RMSE of 0.013, a MAPE of 4.60%, an R^2 of 0.96, and an r_p of 0.98. Similarly, the equation for the inclination angle of the diagonal compressive stress results in an RMSE of 1.71, a MAPE of 3.51%, an R^2 of 0.86, and an r_p of 0.93. These models eliminate the need for the iterative procedure typically required in the SMCFT, thereby enhancing the practicality of the proposed approach.

According to the reliability analysis procedure described in the previous chapter, the resistance reduction factors for the proposed model were calibrated using MCS to achieve various target reliability levels, ensuring safe design applications. Specifically, reliability indices (β) of 3.1, 3.4, 3.8, and 4.1 correspond to resistance reduction factors (φ) of 0.62, 0.57, 0.52, and 0.49, respectively. In line with EN 1990 (CEN 2002), a target reliability index (β_T) of 3.8 is recommended for designing new structural elements with a medium consequence of failure and a 50-year service life. Consequently, a resistance reduction factor of $\varphi = 0.52$ was adopted to enable a comparative evaluation of the proposed model's performance against existing design models. The proposed design model is the only one among the evaluated models to consistently provide safe predictions. Furthermore, it exhibits less dispersion compared to the existing models, as evidenced by its high correlation coefficient, r_p , of 0.92.

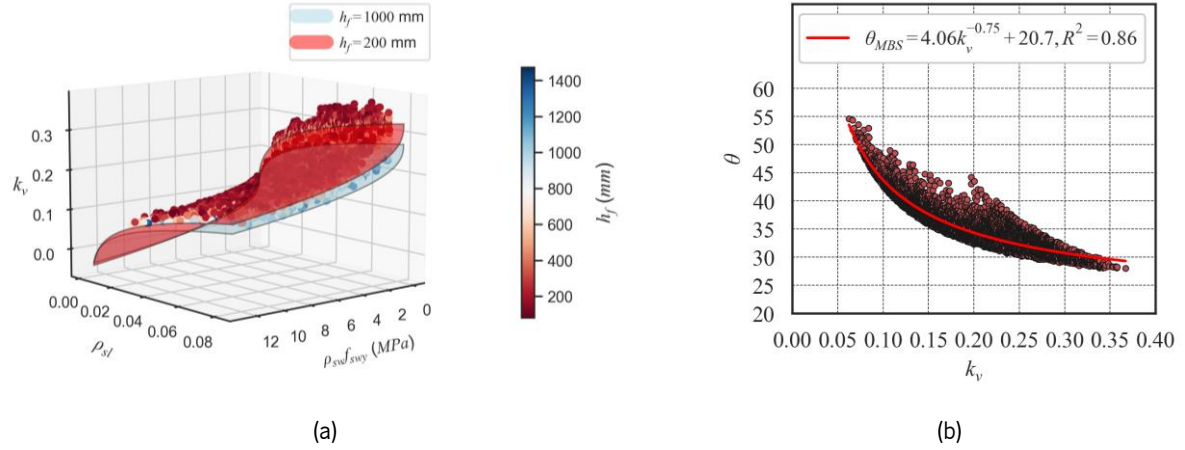


Fig. 13. (a) Hyperplane fitted to predict the tensile stress factor; (b) regression model to predict the inclination angle of the diagonal compressive stress

While the results were successfully obtained, the computational expense associated with MCS was substantial. Consequently, this study advocates for more efficient alternatives, such as IS, SS, or metamodel-based approaches, for future reliability analyses. The proposed framework retains its adaptability, allowing for the calibration of resistance reduction factors for other models, including those developed for NSM techniques in this thesis.

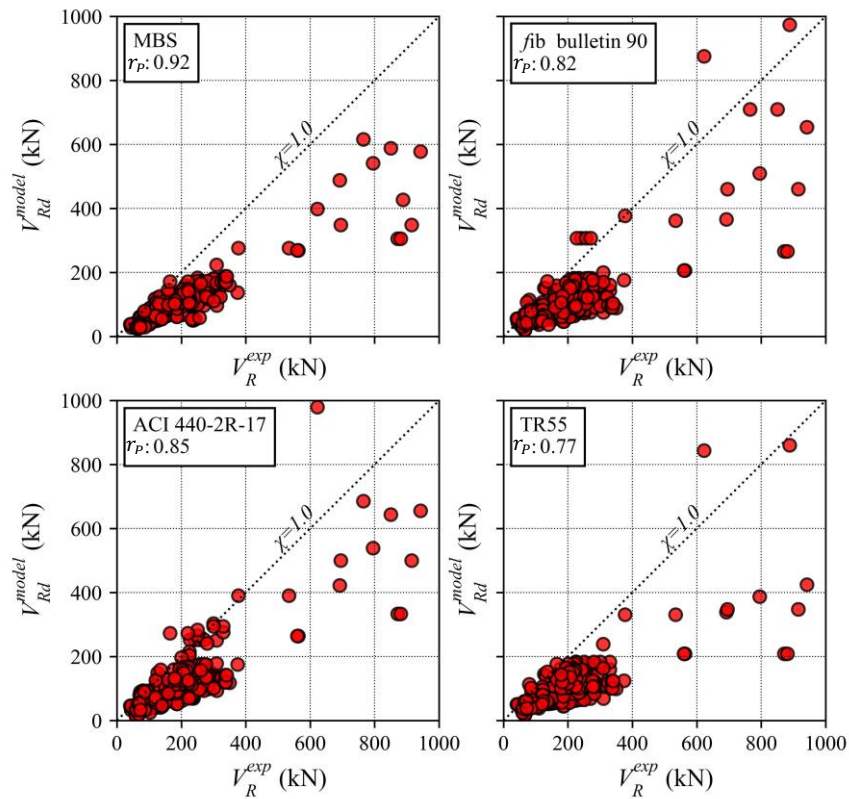


Fig. 14. Predicted against experimentally measured shear resistance of RC beams strengthened with EBR-CFRP

CHAPTER 4

4. PRELIMINARY EFFORTS ON DEVELOPING MODELS FOR NSM TECHNIQUE

This chapter presents the model developed for NSM strengthening technique and assesses its predictive performance by comparing to some existing ones, considering experimental results available in the dataset.

4.1. METHODOLOGY

The methodology from Papers 1, 2 and 4 was extended to study RC beams strengthened with NSM-CFRP reinforcement. Initially, an extensive dataset of 195 beams from experimental studies was compiled, which includes beams with different cross-sections, steel reinforcement ratios, different strengthening (CFRP) reinforcement type, ratio, etc. Outliers of this dataset were identified and removed using the IF technique, resulting in a refined dataset of 124 beams. Of these, 68.5% were strengthened with CFRP laminates, and 31.5% with CFRP bars. Fig. 15 presents the characteristics of the dataset.

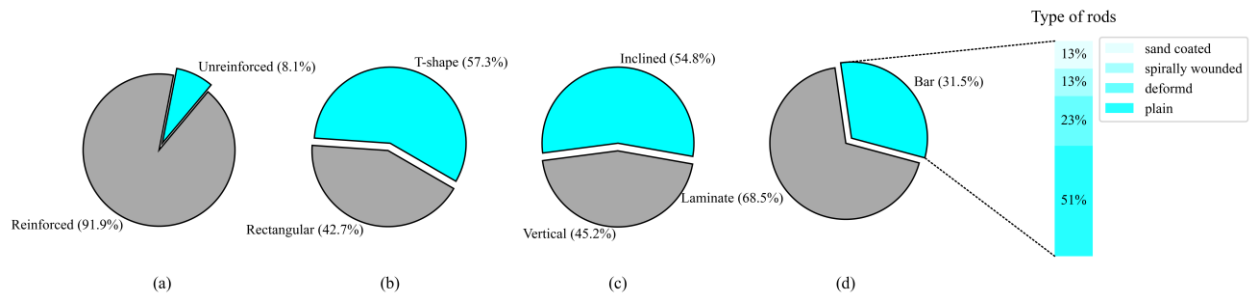


Fig. 15. The distribution of RC beam specimen characteristics in the dataset: (a) Transversal steel reinforcement state, (b) cross-section shape, (c) fibre orientation, (d) type of reinforcement

A truss analogy was also used for this strengthening technique which requires the effective strain estimation. Fig. 16, shows the relationship between the effective strain and various beam features using the Spearman correlation coefficient. The analysis indicates that the effective strain is potentially influenced by numerous features of the strengthened beam, including concrete mean compressive strength (f_{cm} , in MPa), the ratio of longitudinal steel bars in tension (ρ_{sIT}), the depth of the groove (d_g , in mm), the average available resisting bond length $L_{Rf} = h_f \sin \theta (\cot \theta + \cot \alpha_f) / 4 \sin(\theta + \alpha_f)$ (Bianco et al. 2014a)), $\rho_{sw} f_{swy}$, $\rho_f E_f$ and α_f . Here ρ_{sw} and f_{swy} (in MPa) denote the steel stirrup ratio and yield

strength, and ρ_f and E_f represent the ratio and elastic modulus of CFRP reinforcement. Therefore, it was proposed to use the following equation to predict the effective strain:

$$\varepsilon_{fe}^{model} = \kappa_m \varepsilon_{fe}^{basis} \quad (16)$$

where ε_{fe}^{basis} is a basis model accounting for the effect of the most influential features given by Eq. (17), and κ_m is a calibration factor to account the rest of influential features that is given by Eq. (18).

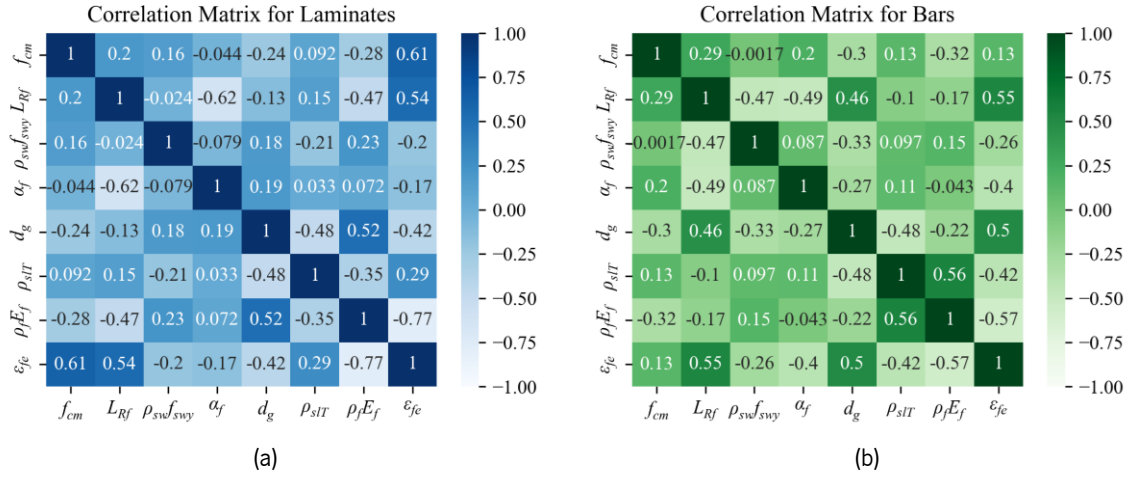


Fig. 16. The Spearman correlation heatmap between beam features and ε_{fe} for dataset of beams strengthened with CFRP type of: (a) laminates, (b) bars

$$\varepsilon_{fe}^{basis} = p'(\rho_f E_f)^q \quad (17)$$

where p' and q are the basis model parameters to be determined.

$$\kappa_m = \prod_{i=1}^{NP} a_i x_i + b_i = \prod_{i=1}^{NP} \left(\frac{a_i x_i + b_i}{b_i} \right) \prod_{i=1}^{NP} b_i = B \prod_{i=1}^{NP} (m_i x_i + 1); \text{ where } m_i = \frac{a_i}{b_i}; B = \prod_{i=1}^{NP} b_i \quad (18)$$

where $\mathbf{x}$ is the set of influential features, and $x_i \in \mathbf{x}$ is the i^{th} influential feature. a_i , b_i , m_i and B are the parameters of the calibration factor for the i^{th} feature, and NP is the number of features recognized as influential. However, unlike the models developed for the EBR technique, where influential features affecting effective strains were identified prior to optimizing the modification factor parameters, in this study, a novel Double Loop Iteration (DLI) procedure was introduced. This approach integrates the truss analogy, SMCFT (Bentz et al. 2007), and multi-objective optimization techniques to achieve the most accurate prediction of effective strain. Notably, it ensures that only those features are included whose exclusion would result in a decline in the model's accuracy.

As presented in Fig. 17, The DLI procedure begins with the assumption that the CDC inclination angle is 45° . Given these assumptions, the experimental values of effective strain, ε_{fe}^{exp} can be determined from the following equation:

$$\varepsilon_{fe}^{exp} = \frac{V_f^{exp}}{A_{fp} h_f E_f (\cot \theta + \cot \alpha_f) \sin \alpha_f} \quad (19)$$

At this stage, the decision regarding which features should be included in the influential feature set has not yet been made. Therefore, x is initially taken as an empty set and the Eq. (16) is fitted to the obtained ε_{fe}^{exp} with the assumption that κ_m is equal to 1. Consequently, the values of θ are estimated using the SMCFT, based on the fitted equation, ε_{fe}^{model} .

Next, the model error of the predictions, $\chi_{\varepsilon_{fe}}$ is evaluated and its correlation with strengthened beam features is calculated based on Pearson's coefficient of correlation. The feature exhibiting the greatest correlation with the model error is then added to the feature set x . The values of ε_{fe}^{exp} are also updated using Eq. (19).

Given the new values of ε_{fe}^{exp} and new set of influential features, the ε_{fe}^{model} given in Eq. (16) is refitted and parameters of Eqs. (17) and (18) are optimized using the non-dominated sorting algorithm (NSGA-II), and the values of θ are updated through SMCFT procedure.

This iterative process continues until the correlation between the model error and the beam features becomes negligible.

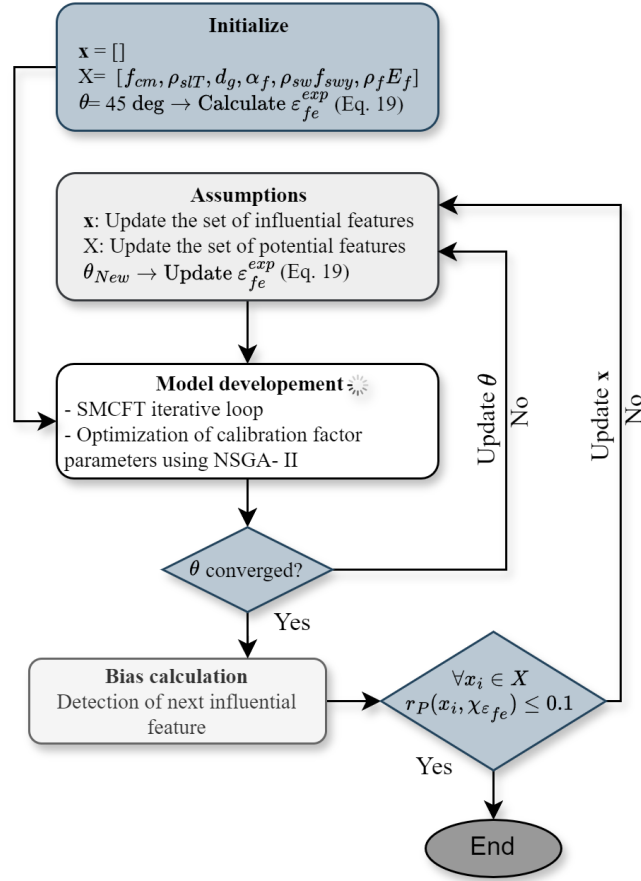


Fig. 17. An overview of DLI procedure

4.2. OUTCOMES AND DISCUSSION

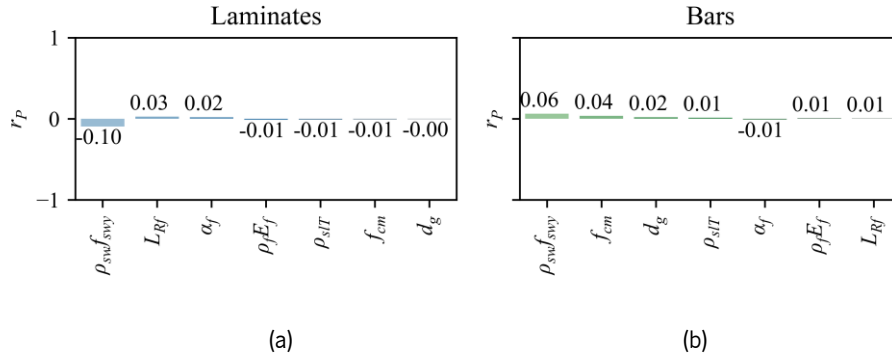
The parameters of this model obtained from this procedure and are given in Table 3 and Table 4. As presented in Fig. 18, the DLI procedure seems that successfully achieved its goals, as the Pearson correlations between the features and the model error $r_P(x_i, \chi_{\varepsilon_{fe}})$ were found to be negligible ($r_P \leq 0.1$).

Table 3. Parameters of basis model parameters for CFRP laminates and bars

CFRP Type	p'	q
Laminates	0.098	-0.643
Bars	0.028	-0.525

Table 4. Optimized values of parameters for calibration factor for laminates and bars

i	Feature (x_i)	Laminates			Bars			
		a_i	b_i	m_i	B	a_i	b_i	m_i
1	f_{cm}	0.06	0.71	0.08	0.12	-	-	-
2	L_{Rf}	5.4e-4	1.03	5.2e-4		2.8e-4	0.41	6.9e-4
3	α_f	-5.8e-3	1.95	-3.0e-3		-7.1e-3	1.73	-4.1e-3
4	d_g	0.025	0.66	0.038		0.15	0.22	0.68
5	ρ_{sIT}	7.17	0.49	14.5		7.4	1.38	5.4
6	$\rho_f E_f$	4.3e-5	0.25	1.7e-4		-1.2e-4	0.52	-2.3e-4

**Fig. 18.** Correlation between model error and features after applying calibration factor: (a) laminates, (b) bars

Additionally, regression equations for the CDC angle (θ) and the tensile factor (k_v) were derived, as presented in Eqs. (20) and (21). These equations, developed using the existing dataset, eliminate the necessity of estimating these parameters through the iterative SMCFT procedure.

$$\theta = \begin{cases} 0.62 \left(40\rho_{sIT}^{-0.06} + 1.1\rho_{sw}f_{swy}^{3.1} \right) (1+0.001f_{cm})(1+0.00034L_{Rf}) \left(1+0.00004\rho_f E_f \right) & \text{laminates} \\ 0.38 \left(58\rho_{sIT}^{-0.08} + 0.32\rho_{sw}f_{swy}^{1.7} \right) (1+0.0002f_{cm})(1+0.0007L_{Rf}) \left(1+0.00001\rho_f E_f \right) & \text{Bars} \end{cases} \quad (20)$$

$$k_v = \begin{cases} 3.5 \times 10^4 \theta^{-3.4} & \text{Laminates} \\ 11 \times 10^4 \theta^{-3.8} & \text{Bars} \end{cases} \quad (21)$$

The predictive performance of the proposed model for the shear-strengthening contribution of CFRP-NSM reinforcement was compared with several well-established models, including those proposed by Dias and Barros (Dias and Barros 2013), Bianco et al. (Bianco et al. 2014b), Mofidi et al. (Mofidi et al. 2023), Lorenzis and Nanni (De Lorenzis and Nanni 2001), and Rizzo and Lorenzis (Rizzo and De Lorenzis 2009).

Table 5 provides the comparison of the predictive performance metrics of each model, evaluated through four different indicators. The results are presented separately for RC beams strengthened with CFRP bars and those strengthened with CFRP laminates, highlighting the superior predictive accuracy of the

proposed model in both categories. For the model proposed by Dias and Barros, the values for beams strengthened with CFRP bars could not be obtained, as this model was not developed for this type of reinforcement.

Table 5. Prediction performance of the proposed models and the existing models

Model	Laminate				Bars			
	R^2	r_p	RMSE	MAPE	R^2	r_p	RMSE	MAPE
Proposed	0.73	0.86	15.3	21%	0.72	0.85	14.8	26%
Dias and Barros 2013	0.57	0.78	19.4	28%	-	-	-	-
Bianco et al. 2014	-0.19	0.74	32.2	43%	0.19	0.78	25.1	40%
Mofidi et al. 2023	-0.10	0.48	30.9	44%	-0.13	0.59	29.6	71%
Lorenzis and Nanni 2001	-0.94	0.31	41.2	57%	-0.51	0.61	34.2	73%
Rizzo and Lorenzis 2009	-0.99	0.54	41.7	56%	-1.27	0.65	42.0	80%

Ensuring consistency in model predictions across various failure modes is essential. To achieve this, failure modes were identified using information from the literature where the experimental data were sourced, and the specimens were labelled accordingly. The proposed model's performance was then assessed for each specific failure mode. As highlighted in the literature, multiple failure modes have been documented for NSM reinforcements, as illustrated in Fig. 19. These failure modes are FRP rupture (FR), interfacial debonding (ID), Concrete cover separation (CCS), and hybrid mode combining both interfacial debonding and concrete cover separation (ID+CCS). The beams for which the failure mode was not specified in the reference were labelled as (NA). The performance of the proposed model and existing models for each failure mode was evaluated using the coefficient of variation (CoV) of the model error for predicting the contribution of CFRP ($\chi_V = V_f^{exp} / V_f^{model}$). As shown in Fig. 20, which provides a histogram of model error for each failure mode along with CoV values for all models, the proposed model exhibited the lowest CoV across all failure modes compared to existing models. Specifically, the overall CoV for the proposed model was 0.27, significantly outperforming the models by Dias and Barros (0.37), Bianco et al. (0.65), Mofidi et al. (1.14), De Lorenzis and Nanni (0.83), and Rizzo and Lorenzis (0.53). This demonstrates the superior consistency and predictive reliability of the proposed model.

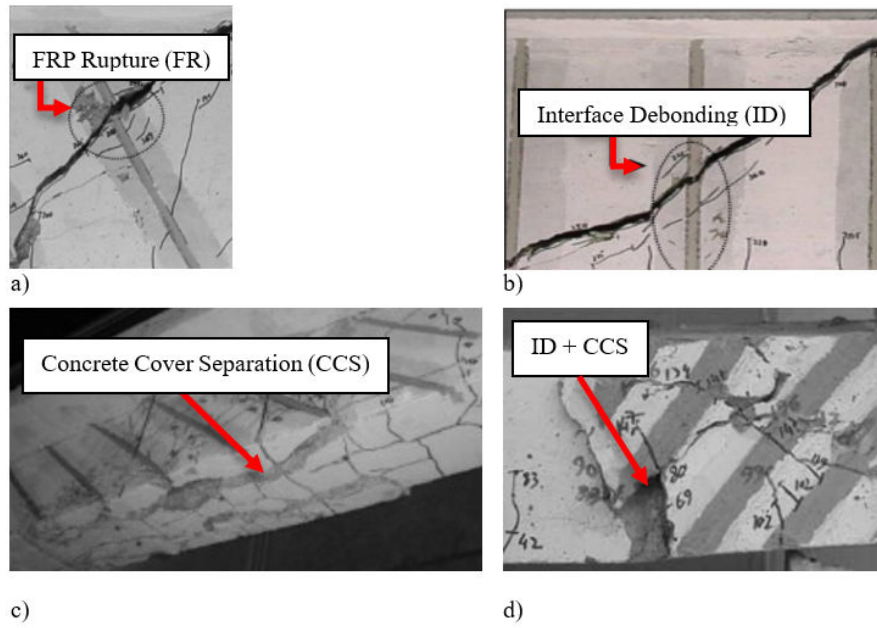


Fig. 19. Different failure modes observed in RC beams strengthened in shear with NSM CFRP reinforcements: (a) FRP Rupture (Dias and Barros 2013), (b) Interface debonding (Dias and Barros 2013), (c) Concrete cover separation (Dias and Barros 2013), and (d) Combined ID and CCS failure mode (Barros and Dias 2006)

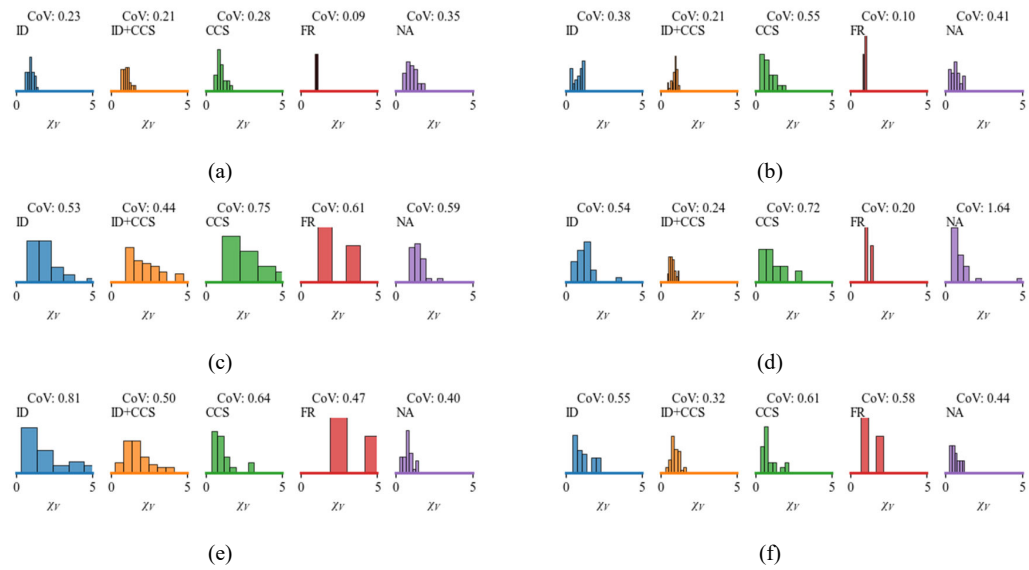


Fig. 20. Histogram of model error for: (a) the proposed model, (b) the model by Dias and Barros, (c) the model by Bianco et al., (d) the model by Mofidi et al., (e) the model by De Lorenzis and Nanni and (f) the model by Rizzo and Lorenzis

CHAPTER 5

CONCLUSION AND FUTURE WORK

The shear strengthening of RC beams using CFRP reinforcement significantly enhances structural sustainability. By preventing brittle failures, this strengthening technique reduces the need for extensive repairs and prolongs the life cycle of structures, thereby cutting costs and lowering the environmental impact associated with construction and maintenance. For these benefits to be effectively realized, robust design guidelines must be established to ensure reliability and economic efficiency in strengthening practices. This study aimed to develop a comprehensive and practical design framework for the two most widely used techniques: EBR and NSM. Across four papers published, and one paper submitted as a result of this thesis, significant progress was made in optimizing predictive models, calibrating reliability factors, and integrating advanced methodologies. The following sections summarize these findings and propose directions for future research and practical improvements.

5.1. CONCLUSIONS

Need for data preprocessing: In this study, particular emphasis was placed on the preprocessing of data to enhance model accuracy. The curation process involved multiple advanced methods to handle and refine the experimental data collected. It was identified that not all experimental results were suitable for inclusion, as certain data points, considered outliers, could adversely impact model performance. Given the high-dimensional nature of the data, simple visualization or basic statistical methods were inadequate for effective outlier detection. Instead, more sophisticated techniques (such as MD, LOF, and IF) were employed, leading to marked improvements in model reliability and predictive accuracy.

Moreover, a critical analysis revealed that experimental datasets often lack representation of full-scale structural elements due to cost constraints, resulting in datasets dominated by small-scale specimens. This imbalance typically causes a right-skewed distribution of the target variable, such as the effective strain in CFRP-EBR, as illustrated in Fig. 21a. To address this issue, applying data transformation techniques, like a logarithmic transformation, was shown to mitigate the effects of skewness, enhancing the overall predictive performance of the models. As illustrated in Fig. 21b, the log-transformed effective strain exhibits a more balanced data distribution. Furthermore, the Kolmogorov-Smirnov (K-S) test,

conducted at a significance level of 0.05, confirmed that the log-transformed data follows an Extreme Type I (Gumbel) distribution, as indicated by a p-value of 0.45.

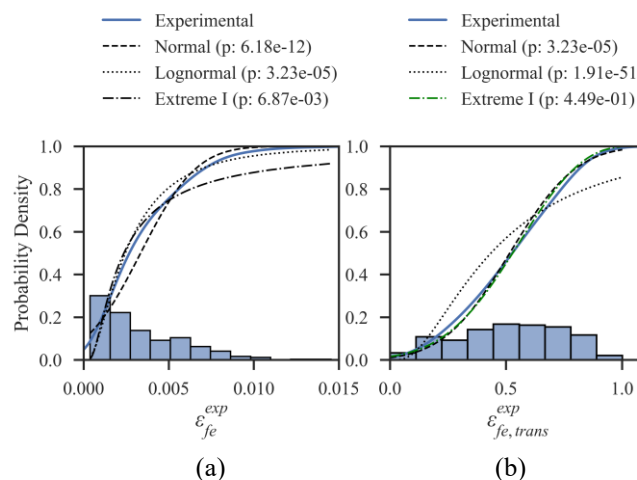


Fig. 21. Histogram, cumulative distribution and fitted PDFs and CDFs for (a) effective strain, (b) log-transformed effective strain

Finally, it was also highlighted that existing datasets from the literature often contain gaps in specific parameter ranges, which can hinder comprehensive model development. To address this limitation, the potential of ML techniques, specifically VAEs, was explored. The application of VAEs in this research demonstrated the ability to generate synthetic data, effectively filling these gaps and enhancing the model's coverage. This data augmentation enabled the development of more accurate predictive models, as evidenced in Paper 3. Incorporating this technique improved the XGBoost model's performance metrics, with the R^2 increasing from 0.87 to 0.94, the r_p value from 0.96 to 0.97, the RMSE decreasing from 19.0 kN to 12.37 kN, and the MAPE reducing from 27.7% to 18.3%.

Weak predictive performance of the available models: One of the primary conclusions drawn from this research, through a detailed comparison between experimental data and predictions from existing models used for designing RC beams strengthened with EBR and NSM CFRP techniques, is that these models demonstrate inadequate predictive accuracy. Specifically, for beams strengthened using the EBR technique, the existing models yielded a mean absolute percentage error (MAPE) ranging from 72.8% to 209.8%. Similarly, for the NSM technique, the MAPE varied between 28% and 110%. This substantial inaccuracy underscores the pressing need for the development of more precise and reliable models, ensuring enhanced safety and efficiency in structural strengthening practices.

Overlooking the impact of influential features in a model for the sake of simplification

comes with a cost: In this research, it was demonstrated that simplifying complex, multidimensional problems by focusing only on a few main features while disregarding other influential variables leads to inaccurate predictions and inconsistent model performance for beams with varying characteristics. Such oversimplification elevates uncertainty and complicates reliability analysis since model uncertainty becomes inconsistent, hindering the straightforward calibration of resistance reduction factors. A thorough examination highlighted those features often neglected in existing models, such as beam size, steel reinforcement ratios, concrete strength, cross-sectional type, and CFRP reinforcement configuration, may significantly influence the contribution of CFRP to shear resistance. This oversight may explain the suboptimal predictive performance of existing models. The research methodology involved first fitting a simplified regression model to the experimental data and then analysing model error variations across these features. Subsequently, a calibration factor was defined to improve predictive accuracy and stabilize model performance across diverse beam characteristics. The proposed models for both EBR and NSM systems exhibited marked improvements compared to existing models, achieving MAPE of 54.4% for EBR-strengthened beams, and 21% and 26% for NSM-strengthened beams with CFRP laminates and CFRP bars, respectively.

ML algorithms offer enhanced accuracy, interpretability, and reliability: Although the analytical models developed in this research have significantly improved predictive performance, the results suggest that a more complex, nonlinear relationship exists between beam features and the CFRP reinforcement's contribution to shear resistance. This complexity requires more sophisticated model structures for better accuracy. This study demonstrated that ML models could more effectively capture these relationships, enhancing predictive accuracy. Additionally, it was shown that ML techniques could help address data gaps in specific feature ranges by providing the mean to generate synthetic data, resulting into the creation of more comprehensive and reliable models. To be more specific, the ML model developed for beams strengthened using the EBR technique significantly reduced the CoV in model error from 0.47, achieved with the analytical model, to 0.28, underscoring the superior efficiency of ML algorithms. Furthermore, incorporating a synthetic dataset generated using VAE algorithm further reduced the CoV to 0.22, demonstrating the added value of comprehensive data augmentation in enhancing model reliability and precision.

Success of the proposed reliability analysis framework in calibration of resistance

reduction factor: Using the developed framework for reliability analysis in this study, a resistance reduction factor was specifically calibrated for the models proposed for beams strengthened with the EBR technique. The findings indicated that, when benchmarked against experimental data, the proposed model is the only one among existing design models that consistently delivers 100% safe predictions while simultaneously being the model that delivers the least extremely conservative predictions (only 3% of beams). This dual achievement highlights the strength of the proposed model, which merges reduced uncertainties with a robust RA foundation, yielding a design tool that is both safe and economically efficient. The advancements presented in this study promote the broader application of shear strengthening techniques for RC beams using CFRP reinforcements. By ensuring safer and less conservative designs, these enhancements support extending the lifespan of structures and align with sustainable development goals, contributing to more resource-efficient and environmentally friendly construction practices.

5.2. FUTURE WORK

- Despite the efforts made in this study to provide reliable predictive tools for RC beams strengthened using externally bonded CFRP reinforcements, there is still a need for further understanding of the failure mechanisms of these beams. Additionally, the model should be refined to reduce errors, ensuring more cost-effective and reliable predictions. Further research is also required to assess the safety verifications of the developed models.
- The model proposed for NSM techniques still requires further work, particularly a comprehensive reliability analysis, to ensure that it can be confidently adopted by designers. Addressing this gap is crucial for future research, given that existing design guidelines, such as *fib* Bulletin 90 (*fib* 2019), currently lack a reliable design model for NSM shear strengthening. Moreover, observations from the preliminary results indicate that, while the model error exhibits a mean close to 1.0 for beams across different failure modes, the coefficient of variation in model error varies slightly. This suggests the potential benefit of developing an advanced model capable of predicting failure modes, which would allow for the calibration of distinct resistance reduction factors tailored to the uncertainty associated with each failure mode. Such an approach could significantly reduce the conservatism, making design practices more efficient and less conservative.

- In a manner similar to the approach used for the EBR technique, it is anticipated that machine learning tools can also serve as highly accurate predictive models for the NSM technique. To this end, a Conditional Variational Autoencoder (CVAE) is planned for development, designed to function as both a predictive tool and an inverse design framework for RC beams shear-strengthened with the NSM technique. The CVAE approach will allow for the prediction of CFRP reinforcement contribution to shear resistance (V_f) while also enabling the design of the required CFRP reinforcement for a target increase in shear resistance (V_f). Unlike VAEs, CVAEs condition the latent space with additional features, allowing the model to generate targeted synthetic data that is both consistent with the underlying distribution and aligned with specific design requirements; thereby supporting optimal reinforcement configuration for the NSM technique. In addition to the reconstruction loss and Kullback-Leibler divergence loss, the objective function of the CVAE model includes two additional components: one term that enhances the accuracy of the predictive model, and another term that ensures decorrelation between the latent space variables z (representing the encoded features) and V_f (representing the conditional variable) (Balmer et al. 2024).

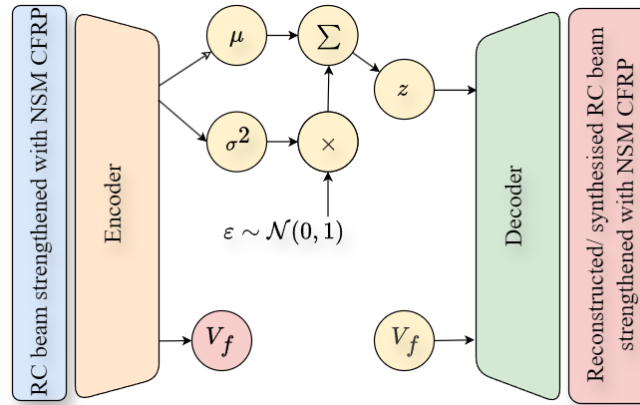


Fig. 22. Architecture of the CVAE network acting as predictive and generative model simultaneously

- The RA framework established in this research, based on MCS, has proven to be robust and effective for similar studies. However, its applicability becomes increasingly limited as model complexity rises, the dimensionality of the problem expands, or when analyses necessitate a larger number of samples from the design space or target very low probabilities of failure. Consequently, for more demanding scenarios, it is advisable to enhance the current RA framework by incorporating more efficient techniques for probability of failure estimation.

Advanced methods such as importance sampling, subset sampling, or the implementation of surrogate models are recommended to address these challenges and improve computational efficiency. These methods significantly reduce the computational burden by limiting the need for repeated evaluations of the true structural model while maintaining accuracy, making them suitable for high-reliability structural design. Therefore, for future models, it is recommended to explore these advanced methods rather than relying solely on MCS.

- In this thesis, two widely used shear strengthening techniques were investigated. It is recommended that future research extends this work by exploring other methods, such as the ETS technique and anchored EBR systems.

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ANNEX I – CUMULATIVE PAPERS OF THE THESIS

Paper 1

TITLE: A New Model for Predicting the Shear Strength of RC Beams Strengthened with Externally Bonded FRP Sheets

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A new model for predicting the shear strength of RC beams strengthened with externally bonded FRP sheets

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ABSTRACT

Externally bonded fibre reinforced polymer (FRP) reinforcements have been widely used for upgrading the shear resistance of reinforced concrete (RC) structures. This technique has been widely studied and numerous analytical models currently exist to predict the contribution of FRP for the shear resistance of RC beams. In this study, some of the most well-known models recommended in guidelines (including *fib* bulletin 90, CNR-DT 200 R1/2013, and ACI 440.2R-17) are selected, and their predictive performance are assessed using a large database of 344 beams compiled from published works. In addition, this study introduces a novel model that accounts for a missing influential predictive component in the existing models, namely the ratio of existing steel stirrups. The comparison of the results obtained with this model and those from the current models demonstrates the best predictive performance of the new one.

1. Introduction

Reinforced concrete (RC) structures have been widely built over the past several decades. However, these structures may fail to meet the nowadays requirements of design codes and need strengthening [1]. Degradation of mechanical properties [2,3] (due to environmental condition, accidents, etc.) and increase in the demanded load capacity caused by increase in seismic or service loads [4–6] are some of the common reasons of widespread use of strengthening techniques. RC beams are one of the prevalent elements in RC structures that require (flexural and/or shear) strengthening [7–10]. Shear failure in the RC beams is brittle and occurs suddenly, generally causing large damages. In the past three decades, fibre reinforced polymer (FRP) materials have been extensively used as Externally Bonded Reinforcement (EBR) to increase the shear resistance of RC beams. To date, many empirical, conceptual, and numerical models have been developed to predict the shear capacity of these beams; nonetheless, this task remains challenging. Due to its simplicity, most design guidelines use a conventional truss model to evaluate the contribution of FRP reinforcement, analogous to the one used to predict the contribution of steel stirrups [11–13].

Like in the steel shear reinforcement, some models assume the strain mobilized in FRP at the failure of a shear strengthened RC beam (currently designated by effective strain) is a constant value [14–16].

However, it has been demonstrated by many researchers that this effective strain is not constant and depends on some variables. In 1998, Triantafillou [17] was among the first researchers to propose a model with a varying effective strain for FRP reinforcement as a function of the axial stiffness of FRP reinforcement ($\rho_f E_f$). This model was later modified by Triantafillou and Antonopoulos [18] in 2000 and was adopted by *fib* bulletin 14 [19]. The continuous research has shown that the effective strain of FRP shear reinforcements also depends on the beam's cross-section corners where FRP is installed [20] and concrete substrate quality [21,22]. It was verified that the bond strength and ductility between FRP and concrete substrate decrease with the quality of the concrete substrate, generally correlated with the concrete compressive strength, resulting premature debonding with smaller FRP effective strain. The FRP bond length is another important variable discussed in the literature, since below what is designated as effective bond length (length above which the maximum bond force is not exceeded), the maximum strain capable to be mobilized decreases with the bond length [22–24]. The shear span to effective depth ratio was also shown affecting the FRP effective strain [21,25]. It was observed that at the failure of a beam shear strengthened with a relatively high FRP reinforcement ratio, existing steel stirrups might not reach the yielding strain, therefore their shear strengthening potential is not fully mobilized [26–29]. Furthermore, it was shown that the crack inclination angle is not a constant value [30–33], which has a significant influence on the FRP shear

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Nomenclature			
A_{fwc}	Effective FRP cross-section area (mm ²)	r	Pearson coefficient of correlation
a_t	Reduction factor to account for the long-term effects	$RMSE$	Root mean squared error (kN)
b_w	Width of the beam's web (mm)	s_f	Spacing of FRP strips (mm)
$C:p \times p$	symmetrical covariance matrix	s_{0k}	Characteristic ultimate deformation capacity (mm)
CoV	Coefficient of variation	SD	Standard deviation
d	Effective depth of the beam (mm)	t_{fe}	Effective thickness of FRP (mm)
d_i	Mahalanobis distance of i^{th} observation	t_f	Thickness of one layer of FRP (mm)
d_{fv}	Effective depth of FRP (mm)	$V_{R,f}^{exp}$	Shear contribution of FRP according to experimental results (kN)
E_f	Elastic modulus of FRP (MPa)	V_R^{ref}	Overall shear strength of the reference beam (kN)
$E_f t_f$	Axial stiffness of FRP (N/mm)	V_R^{str}	Overall shear strength of the strengthened beam (kN)
f_{cm}	Concrete mean compressive strength (MPa)	$V_{R,f}^{mod el}$	Nominal shear contribution of FRP predicted by each model (kN)
f_{fe}	Effective stress in the FRP reinforcement (MPa)	$V_{R,c}$	Shear contribution of concrete (kN)
$f_{fw,c}$	FRP strength attributed to a failure caused by FRP rupture (MPa)	$V_{R,s}$	Shear contribution of transversal steel (kN)
f_{fbw}	Bond strength of FRP reinforcement (MPa)	$V_{R,f}$	Shear contribution of FRP reinforcement (kN)
f_{fbk}	FRP debonding strength according to <i>fib</i> bulletin 90 (MPa)	$V_{R,max}$	Shear strength of concrete compression strut (kN)
f_{fee}	FRP debonding stress according to CNR-DT 200.R1-2013 (MPa)	w_f	Width of FRP strips (mm)
h	Height of the beam (mm)	X	Matrix of n observations of p variables
h_f	Height of FRP (mm)	x_i	Vector of variables for the i^{th} observation
h_{fe}	Effective height of FRP (mm)	α	Inclination of FRP fibres
h_w	Height of the beam's web (mm)	ϵ_{fe}	Effective strain in the FRP composite
k_b and k_G	Empirical coefficients	ϵ_{fu}	Strain at the failure of the FRP coupon
l_{ed}	Effective bond length (mm)	θ	Crack inclination angle
l_e	Effective bond length of FRP (mm)	κ_R	Modification factor accounting for the stress distribution in the FRP
$MAPE$	Mean absolute percentage error	κ_1	Modification factor that accounts for the quality of the concrete substrate
m_F	Modification factor	κ_2	Modification factor accounting for the bond mechanism
n_f	Number of FRP layers	μ	Vector of mean values for p variables
p	Number of variables	ρ_{sw}	Stirrup ratio (%)
Q_1	Lower quartile	ρ_{sl}	Longitudinal steel ratio (%)
Q_3	Upper quartile	τ_{b1k}	Characteristic ultimate bond strength
R	Radius at the corner of the beam's cross-section (mm)	χ	Model error
R^2	Absolute Fraction of Variance		

contribution. However, the prediction of this variable is still not straightforward.

There are already models that account for the most influential variables. For instance, in 2012 Chen et al. [28] proposed an analytical model that can predict the shear contribution of FRP with high accuracy. In addition, they proposed a modification factor to account for the negative effect of interaction between internal steel stirrups and external FRP shear reinforcement, hence increasing the predictive performance of the model. However, these types of models require knowledge of a broader set of parameters to solve the equations, making their use for design purposes difficult, if not impossible. In addition, the application of the Modified Compression Field Theory (MCFT) [34] may also improve the predictive performance since the formulation integrates the evaluation of the crack inclination angle. Due to the iterative procedure of MCFT equations, more complexity is introduced, making these models less practical from a design standpoint. These models are rarely adopted in the current design guidelines. The models adopted in the design guidelines are straightforward and practical, yet they produce less accurate predictions. This may be due to the fact that these models exclude some influential variables, such as the reinforcing ratio of steel stirrup.

In this work, a large dataset composed of 344 experimental results of beams shear strengthened with the EBR technique is collected from the published literature. Based on this data, the predictive performance of some well-known recent models is evaluated, and a new model is

proposed. The results show that the accuracy of the proposed model has outperformed current models, yet the procedure is practical for its use in design practice. Given the fact that some guidelines do not include equations for the side-bonded configuration due to its lower efficiency, this research focuses exclusively on beams strengthened by either fully wrapped or U-wrapped beams.

2. Construction of the dataset

A comprehensive database of over 900 tested RC beams that are transversely reinforced with FRP materials was compiled [35]. This database includes detailed information on test specimens such as geometry (e.g., the length, cross-section, test set-up), material properties (e.g., the concrete compressive strength and the mechanical properties of flexural and shear reinforcement), FRP and conventional steel reinforcements details, and test results (e.g., shear capacity before and after strengthening with FRP, and failure mode). Due to the diverse range of experiments included in this dataset, the updated version of the database that contains 383 specimens excluded those that displayed any of the following characteristics: (1) specimens with missing data that are required for the analysis; (2) specimens where the FRP were not in carbon fibres (CFRP); (3) specimens reinforced with the NSM technique; (4) specimens with a negligible shear contribution of FRP for the shear resistance $V_f^{exp} < 10$ kN (suspicion of some incorrection on the strengthening process); (5) specimens with two-sided FRP

reinforcement (as this technique is not considered in some guidelines, such as the case of *fib* bulletin 90); (6) specimens that are strengthened with FRP systems with multidirectional fibres, e.g. fabrics; (7) and, specimens with any kind of end anchorage (since the effectiveness is too dependent of the type of anchorage adopted).

2.1. Outlier detection

The eventual existence of outliers in the compiled dataset was checked, i.e., data points that deviate significantly from normal data, which may influence, detrimentally, the model's predictive performance. Outliers exist due to several factors, such as data entry or measurement errors, sampling problems and unusual conditions, and natural variation. Univariate methods are often used to detect the outliers. Although these methods are easy to implement, they may fail to detect an outlier in multivariable data. In this study selected predictive variables and the contribution of FRP to shear resistance of the beam form a multivariate space. This study employs Mahalanobis Distance (MD) Method, an effective approach for [36] detecting outliers in multivariate space. The MD is a useful multivariate distance metric to evaluate the distance between a single data point and a distribution, which can be found as follow:

$$d_i^2 = (\mathbf{x}_i - \boldsymbol{\mu})^T (\mathbf{C}^{-1}) (\mathbf{x}_i - \boldsymbol{\mu})$$

$$\mathbf{x}_i = \{\mathbf{x}_i^1, \dots, \mathbf{x}_i^p\}; \boldsymbol{\mu}_i = \{\boldsymbol{\mu}_i^1, \dots, \boldsymbol{\mu}_i^p\}; \mathbf{C} = \begin{bmatrix} \text{VAR}(\mathbf{x}_i^1) & \dots & \text{Cov}(\mathbf{x}_i^1, \mathbf{x}_i^p) \\ \vdots & \ddots & \vdots \\ \text{Cov}(\mathbf{x}_i^p, \mathbf{x}_i^1) & \dots & \text{VAR}(\mathbf{x}_i^p) \end{bmatrix}$$
(1)

In this equation, d_i is the Mahalanobis distance of observation i ; $\mathbf{x}_i$ is a vector of p variables representing the n observations ($i = 1, 2, \dots, n$); $\boldsymbol{\mu}$ is the vector of mean values for p variables, and $\mathbf{C}$ is $p \times p$ symmetrical covariance matrix. According to the proposed model presented later, $\mathbf{x}_i$ consists of the predictive and target variables as follow:

$$\mathbf{x}_i = \left\{ (E_f \rho_f / f_{cm}^{2/3}), \rho_{sw}, \rho_{sl}, \epsilon_{fe} \right\}$$
(2)

where $E_f \rho_f / f_{cm}^{2/3}$ is the normalized axial stiffness; $\rho_{sw} = A_{sw} / (b_w d_s)$, $\rho_{sl} = A_{sl} / (b_w d_s)$ and ρ_f are the reinforcing ratio of steel stirrups, the reinforcing ratio of tensile steel reinforcement, and the strengthening ratio of FRP reinforcement (later will be defined), respectively. In Eq. (1), ϵ_{fe} is the effective strain of FRP as defined later in Eq. (30) and E_f is the elastic modulus of FRP, while f_{cm} is the mean concrete compressive strength.

If the d^2 follows a chi-square distribution, a critical value for d^2 to recognize an specimen as an outlier is determined using a significance level of 0.05 (as there is four degrees of freedom, which is equal to the number of variables in the $\mathbf{x}_i$ vector, the critical value of 9.49 is determined). Specimens with d^2 higher than the chi-square critical value will be considered outliers. The d^2 for each specimen is depicted in Fig. 1, where outliers were shown in red colour (10% of the initial total updated dataset). The remaining 344 specimens [9,20,32,33,37–100,110,111] are used for developing a new model, as well as to validate both this model and existing models.

2.2. Statistical characteristics of some important variables

The model developed in this study should be applied with caution to specimens whose variables fall outside the range examined in this study, as generalization beyond this range may result in inaccurate results. The statistical characteristics of some important variables are given in Table 1.

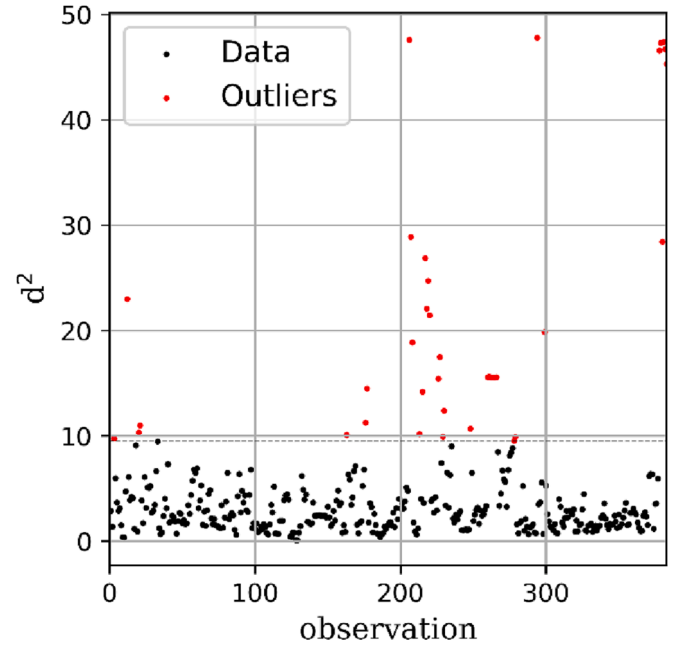


Fig. 1. Outlier detected using Mahalanobis Distance Method.

3. Review of the current design provisions for the contribution of externally bonded FRP shear reinforcement

Although it is known that there is a certain interdependency between the shear strength contributions provided by concrete, stirrups, and FRP [28,29,101], the superposition principle is generally used in the models proposed in the guidelines. The shear contribution of FRP is, therefore, calculated independently and added to the other contributions [102–105]. The overall shear resistance is then expressed as:

$$V_R = V_{R,c} + V_{R,s} + V_{R,f} \leq V_{R,max}$$
(3)

where $V_{R,c}$, $V_{R,s}$ and $V_{R,f}$ are the shear contribution of concrete, steel stirrups and FRP reinforcement, respectively, while $V_{R,max}$ is the shear strength provided by the concrete crushing of the compressive strut formed at beam's failure.

The guideline models utilize truss analogy and propose similar equations for finding the FRP contribution. Their principal difference is how to calculate the effective strain or stress in the FRP composite (ϵ_{fe} or f_{fe}) at beam's shear failure. The predictive performance of some of the most renowned models is evaluated in this study, including *fib*-Bulletin 90 [12], CNR-DT200 [13], and ACI 440-2R [11]. In the following section, the equations used to calculate the shear contribution of FRP in accordance with the aforementioned design guidelines are concisely presented.

3.1. *fib*-bulletin 90 (*fib*-TG5.1)

The *fib* TG5.1 [12] has proposed a model that can be used for fully (O configuration) or U-wrapped beams.

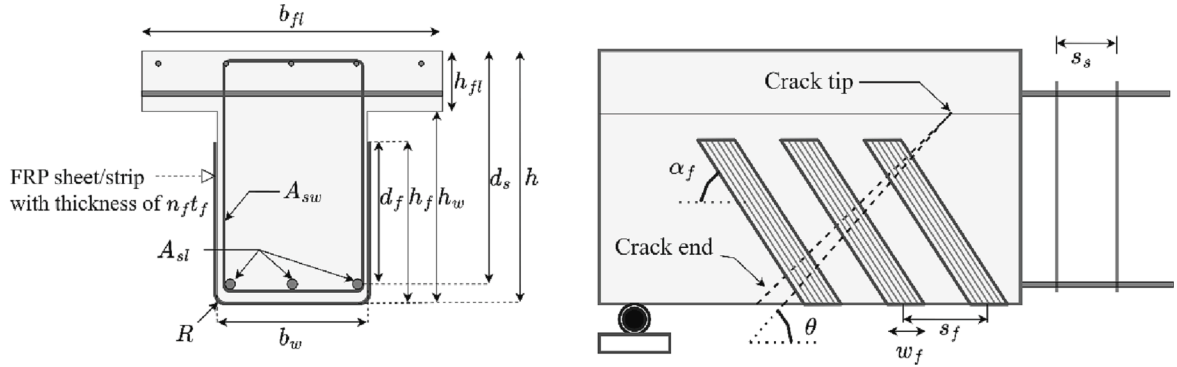
$$V_{R,f} = A_{fwc} h_{fe} f_{fe} (\cot \theta + \cot \alpha_f) \sin \alpha_f$$
(4)

where A_{fwc} is the effective FRP cross-section area calculated as $2w_f t_{fe} / s_f$ for strips and $2t_{fe} \sin \alpha_f$ for continuous FRP sheets, being w_f , t_{fe} and s_f the width, the effective thickness and the spacing of FRP strips, and α_f the angle between the longitudinal axis of the beam and the fibres of these strips, as depicted Fig. 2. In Eq. (4) h_{fe} is the effective height of FRP calculated as a minimum of h_f and $h - 0.1d_s$, where h_f , h , and d_s stand for the height of FRP, the height of the beam's cross section and the effective depth of the beam's longitudinal tensile steel reinforcement,

Table 1

Statistical summary of some crucial variables for strengthened beams in the experimental dataset.

Variable	Min	Max	Mean	Q ₁ ¹	Median	Q ₃ ²	Std ³	CoV ⁴
Concrete mean compressive strength f_{cm} (MPa)	10.6	61.3	33.7	27.4	32.8	38.0	10.1	0.3
Axial stiffness of FRP $E_f \rho_f$ (N/mm ²)	38.4	3339.7	494.3	190.1	340.4	593.9	457.37	0.92
Stirrup ratio ρ_{sw} (%)	0	0.84	0.164	0	0.106	0.26	0.194	1.18
Longitudinal steel ratio ρ_{sl} (%)	0.3	5.8	2.6	1.55	2.4	3.7	1.2	0.46
Width of the beam's web b_w (mm)	76	600	180.2	122.2	150	250	81.4	0.45
Height of the beam's web h_w (mm)	150	762	341.9	252.5	305	420	116.1	0.34

¹ Q₁ lower quartile, ² Q₃ upper quartile, ³Std standard deviation, and ⁴CoV coefficient of variation.**Fig. 2.** Representation of the physical meaning of the variables adopted in the formulations.

respectively. The effective thickness, t_{fe} , is equal to $n_f t_f$ for up to three layers of FRP and is equal to $(n_f t_f)^{0.85}$ for four and more layers (n_f is the number of layers), being t_f the thickness of one layer of FRP. In Eq. (4) θ is the crack inclination angle, and f_{fe} is the effective strength of FRP, which is governed by either FRP rupture or debonding failure modes. The model assumes that in fully wrapped beams FRP fails by its tensile rupture:

$$f_{fe} = f_{fw,c} = a_t k_R f_{fu} \quad (5)$$

$$k_R = \begin{cases} 0.5 \frac{R}{50} \left(2 - \frac{R}{50} \right) & R < 50 \text{ mm} \\ 0.5 & R \geq 50 \text{ mm} \end{cases} \quad (6)$$

where k_R is a modification factor accounting for the stress gradient due to the effect of rounding the corner of the beam's cross-section with a certain radius (R in mm). The a_t in Eq. (5) is a reduction factor to account for the long-term effects (taken equal to 1.0 in this study), and f_{fu} is the value of the tensile rupture of the FRP.

For beams strengthened with U-wrapped FRPs:

$$f_{fe} = \min(f_{fw,c}, f_{fbw}) \quad (7)$$

where f_{fbw} is the bond strength of shear strengthening system.

For continuous FRP sheets:

$$a) f_{fbw} = \left[1 - \frac{1}{3} \frac{l_e}{(h_{fe}/\sin \alpha_f)} \right] f_{fbk} \quad \text{for } x \geq l_e \quad (8)$$

$$b) f_{fbw} = \frac{2}{3} \frac{h_{fe}/\sin \alpha_f}{l_e} f_{fbk} \quad \text{for } x \leq l_e \quad (9)$$

For discrete FRP strips:

$$a) f_{fbw} = f_{fbk} \quad \text{for } x \geq l_e; \text{ and } l_e \leq y \leq x \quad (10)$$

$$b) f_{fbw} = \left[1 - \left(1 - \frac{2}{3} \frac{ms_f}{l_e} \right) \frac{m}{n} \right] f_{fbk} \quad \text{for } x \geq l_e; \text{ and } y \leq l_e \quad (11)$$

$$c) f_{fbw} = \frac{2}{3} \frac{(ns_f)/[(\cot \theta + \cot \alpha_f) \sin \alpha_f]}{l_e} f_{fbk} \quad \text{for } x \leq l_e; \text{ and } y \leq x \quad (12)$$

where x is calculated as $h_{fe}/\sin \alpha_f$, y is $s_f/[(\cot \alpha_f + \cot \theta) \sin \alpha_f]$, n and m are the integer part of $h_{fe}(\cot \alpha_f + \cot \theta)/s_f$, and $l_e(\cot \alpha_f + \cot \theta) \sin \alpha_f/s_f$, respectively. In previous equations f_{fbk} is the characteristic value of the FRP debonding strengthened l_e is the effective bond length of FRP determined from:

$$f_{fbk} = \sqrt{\frac{E_f s_{0k} \tau_{b1k}}{t_{fe}}} \quad (13)$$

$$l_e = \frac{\pi}{2} \sqrt{\frac{E_f t_{fe} s_{0k}}{\tau_{b1k}}} \quad (14)$$

where E_f is the elastic modulus of FRP, s_{0k} and τ_{b1k} are characteristic ultimate slip and bond strength, respectively.

3.2. Cnr-DT 200 R1/2013

The Italian guideline for FRP (CNR-DT 200 R1/2013) [13] has basically adopted and modified the model proposed by Monti and Liotta [20].

$$V_{Rf} = 0.9 d_s f_{fe} 2 n_f t_f (\cot \theta + \cot \alpha) \frac{w_f}{s_f \sin \alpha} \quad (15)$$

The effective strength of FRP is computed according to Eq. (16) for fully wrapped and anchored U-wrapped beams, and Eq. (17) for U-wrapped and side-bonded beams.

$$f_{fe} = f_{fec} \left[1 - \frac{1}{6} \frac{l_e \sin \alpha_f}{\min\{0.9 d_s, h_w\}} \right] + \max \left\{ 0, \left\langle \frac{1}{2} (\kappa_R f_{fu} - f_{fec}) \left[1 - \frac{l_e \sin \alpha_f}{\min\{0.9 d_s, h_w\}} \right] \right\rangle \right\} \leq 0.005 E_f \quad (16)$$

$$f_{fe} = f_{fec} \left[1 - \frac{1}{3} \frac{l_e \sin \alpha}{\min\{0.9 d_s, h_w\}} \right] \leq 0.005 E_f \quad (17)$$

In the above equations, κ_R is a reduction factor due to local stress in

corners, which is obtained by Eq. (18). l_e is the effective bond length computed according to Eq. (22).

$$\kappa_R = 0.2 + 1.6 \frac{R}{b_w}, \text{ with } 0 \leq \frac{R}{b_w} \leq 0.5 \quad (18)$$

f_{fe} is the FRP debonding stress, which is computed according to:

$$f_{fe} = \sqrt{\frac{2E_f \Gamma_{fe}}{n_f t_f}} \quad (19)$$

where

$$\Gamma_{fe} = k_b k_G \sqrt{f_{cm} f_{cm}} \quad (20)$$

being k_G an empirical coefficient, whose mean value is equal to 0.037 for wet-layup sheets and 0.023 for precured FRP composites, and

$$k_b = 1 \leq \sqrt{\frac{2 - w_f / (s_f \sin \alpha_f)}{1 + w_f / (s_f \sin \alpha_f)}} \leq 1.18 \quad (21)$$

In Eqs. (14) and (15)

$$l_e = \max \left\{ \frac{1}{f_{be}} \sqrt{\frac{\pi^2 E_f n_f t_f \Gamma_{fe}}{2}}, 200 \text{ mm} \right\} \quad (22)$$

with

$$f_{be} = \frac{2\Gamma_{fe}}{s_u} \quad (23)$$

In the case of using continuous sheets, w_f is substituted with $\min(0.9d, h_w) \sin(\alpha_f + \theta) / \sin \theta$, and s_f is substituted with $w_{fv} / \sin \alpha_f$. s_u is the ultimate FRP-support slip, taken as 0.25 mm.

3.3. Aci 440.2R-17

According to the American Concrete Institute [11] the contribution of a FRP system for the shear strengthening of RC beams is obtained from:

$$V_{R,f} = \frac{2n_f t_f w_f f_{fe} d_f (\sin \alpha_f + \cos \alpha_f)}{s_f} \quad (24)$$

$$f_{fe} = E_f \varepsilon_{fe} \quad (25)$$

where d_f and ε_{fe} are the effective depth and the effective strain of FRP, which is equal to the minimum of 0.004 and $0.75 \varepsilon_{fu}$ in case of fully wrapped beams, and is computed according to eq. (26) for U-wrapped beams.

$$\varepsilon_{fe} = \frac{\kappa_1 \kappa_2 L_e}{11900} \varepsilon_{fu} \leq \min(0.75 \varepsilon_{fu}, 0.004) \quad (26)$$

where ε_{fu} is the ultimate tensile strain of the FRP, while κ_1 and κ_2 parameters are determined from:

$$\kappa_1 = \left(\frac{f'_c}{27} \right)^{2/3} \quad (27)$$

$$\kappa_2 = \frac{d_{fv} - L_e}{d_{fv}} \quad (28)$$

for taking in consideration the quality of the concrete substrate and geometry shear strengthening configuration, respectively, while L_e is the FRP effective bond length:

$$L_e = \frac{23300}{(n_f t_f E_f)^{0.58}} \quad (29)$$

4. Proposed model

The proposed model considers the parameters that highly affect the

contribution of FRP for the shear strength of a RC element, including the interaction between existing steel stirrups and FRP shear reinforcement. In this study, a truss analogy-based model is adopted, by estimating the effective strain in the FRP reinforcement through a regression-based prediction equation that considers the experimentally FRP shear contribution, V_f^{exp} :

$$\varepsilon_{fe} = \frac{V_f^{\text{exp}}}{A_{fwc} h_{fe} E_f (1 + \cot \alpha_f) \sin \alpha_f}$$

where A_{fwc} and h_{fe} are as introduced in fib bulletin 90. It is assumed the critical diagonal crack has an inclination angle of 45 degrees. Firstly, the correlation between each predictive variable and the effective strain of FRP (ε_{fe}) is explored and reported to comprehend their relationship, Fig. 3. Then, a regression equation accompanied by several modification factors is proposed. As shown, a five-by-five matrix is established, with the figures in its diagonal representing the histogram for the distribution of each variable. The information below and above this diagonal represent the scatter plots and Pearson correlation coefficient, respectively. To emphasize the significance of relationship between the normalized axial stiffness of FRP and effective strain by assistance of linear correlation, the logarithm of $\rho_f E_f / f_{cm}^{2/3}$ is used to convert a nonlinear relation to a linear one. The linear correlation is evaluated using the Pearson correlation coefficient represented by the symbol “r” as depicted in the figure. According to Fig. 3 the effective strain of FRP generally decreases as the axial stiffness of FRP rises which can be justified by the no integral shear strengthening mobilization of the existing steel stirrups with the increase of $E_f \rho_f$. Furthermore, it is recognised that by decrease in the concrete compressive strength lower value of effective strain is obtained. These effects were considered together in the model taking $\rho_f E_f / f_{cm}^{2/3}$ variable. The obtained results show a decrease of in the effective strain of FRP with the increase of the reinforcement ratio of existing steel stirrups ($r = -0.20$). This can be justified by mechanisms contributing for the premature debonding of FRP when the stirrups are spaced in shorter distance, as demonstrated elsewhere [106]. Moreover, as expected from the physical understanding, data generally exhibit higher values of effective strain for the case of fully wrapped specimens ($r = 0.33$). The database however is unable to show the expected behaviour in the case of corner radius where it is physically proven that higher corner radiuses will avoid premature FRP rupture due to stress concentration. This could be attributed to the limited and uncertain information available in the literature regarding the corner radius of the beam's cross-section. Which, consequently, leads to observe no direct positive linear relationship ($r = -0.02$) with the estimated effective strain. Nonetheless, it has been conceptually shown that an increase in this variable would result in a rise in the effective strain [105]. Later, a modification factor will be proposed for this variable by considering this reality.

According to the truss analogy and assuming a crack inclination angle of 45°, Eq. (31) can be proposed for the contribution of FRP in the shear capacity of the beam:

$$V_{R,f} = A_{fwc} h_{fe} E_f \varepsilon_{fe} (1 + \cot \alpha_f) \sin \alpha_f \quad (31)$$

The proposed equation for the effective strain (ε_{fe}) is expressed as follows:

$$\varepsilon_{fe} = m_F \times 0.038 \times (E_f \rho_f / f_{cm}^{2/3})^{-0.765} \quad (32)$$

In this equation m_F is the recommended modification factor:

$$m_F = \kappa_{sw} \kappa_R \kappa_{O/U} \quad (33)$$

$$\kappa_{sw} = 1 - 24.1 \rho_{sw} \quad (34)$$

$$\kappa_R = 0.17 \left(\frac{R}{50} \right) + 0.93 \leq 1.1 \quad (35)$$

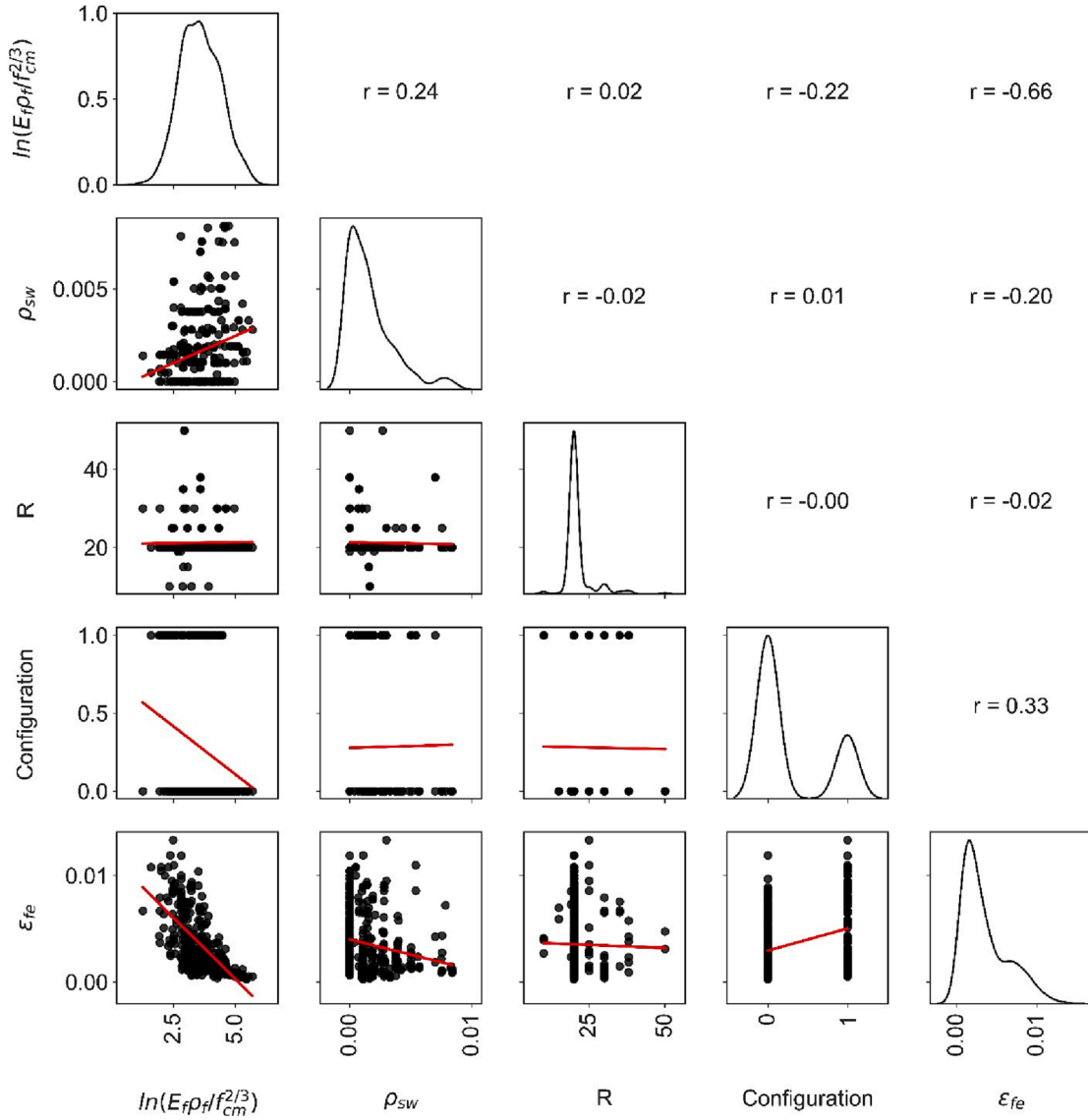


Fig. 3. Scatterplots of independent variables and effective strain.

$$\kappa_{O/U} = 0.92 + 0.28\kappa \quad (36)$$

In the above equations, κ_{sw} is a modification factor accounting for the interaction between internal steel and external FRP shear reinforcement; κ_R is a coefficient for considering the stress concentration in the corners (in which R is Radius at the corner of the beam's cross-section, measured in millimetres); $\kappa_{O/U}$ accounts for the type of FRP shear strengthening configuration; ρ_{sw} is the reinforcement ratio of steel stirrups; κ is the strengthening configuration factor, assuming 1.0 and 0 for fully wrapped and U-wrapped beams, respectively.

5. Validation and discussion

In this section, the performance of the above described existing design models is compared with the model proposed in this study. For each of the models, the nominal shear contribution of FRP will be computed through the given equations (i.e., $V_{R,f}^{\text{model}}$) and then will be compared with the experimental value V_f^{exp} . This last term is calculated by subtracting the shear strength of the reference beam, V_R^{ref} from the overall shear strength of the strengthened beam, V_R^{str} :

$$V_f^{\text{exp}} = V_R^{\text{str}} - V_R^{\text{ref}} \quad (37)$$

It must be noted that for obtaining the predictions of the models, average values of the material properties are used. The partial and global safety factors and/or long-term modification factors were taken to be equal to 1.0. The inclination of the shear crack is assumed to be 45 degrees (the assumed value in the ACI 440.2R-17 model) to ensure comparable values between all the equations, and also due to the fact this information is generally not provided in the bibliography supporting the database. Whenever the corner radius of the beam's cross-section was not provided, a value of 20 mm was assumed, since it was the average of the available values.

The model uncertainty was evaluated by the ratio between the value recorded experimentally and the predicted one:

$$\chi = \frac{V_{R,f}^{\text{exp}}}{V_{R,f}^{\text{model}}} \quad (38)$$

Fig. 4 shows the performance of the models in predicting the contribution of FRP for the shear strength of RC beams strengthened with fully wrapped and U-strip configurations. For each model, three columns of plots are provided. The first plot is a representation of $V_{R,f}^{\text{model}}$

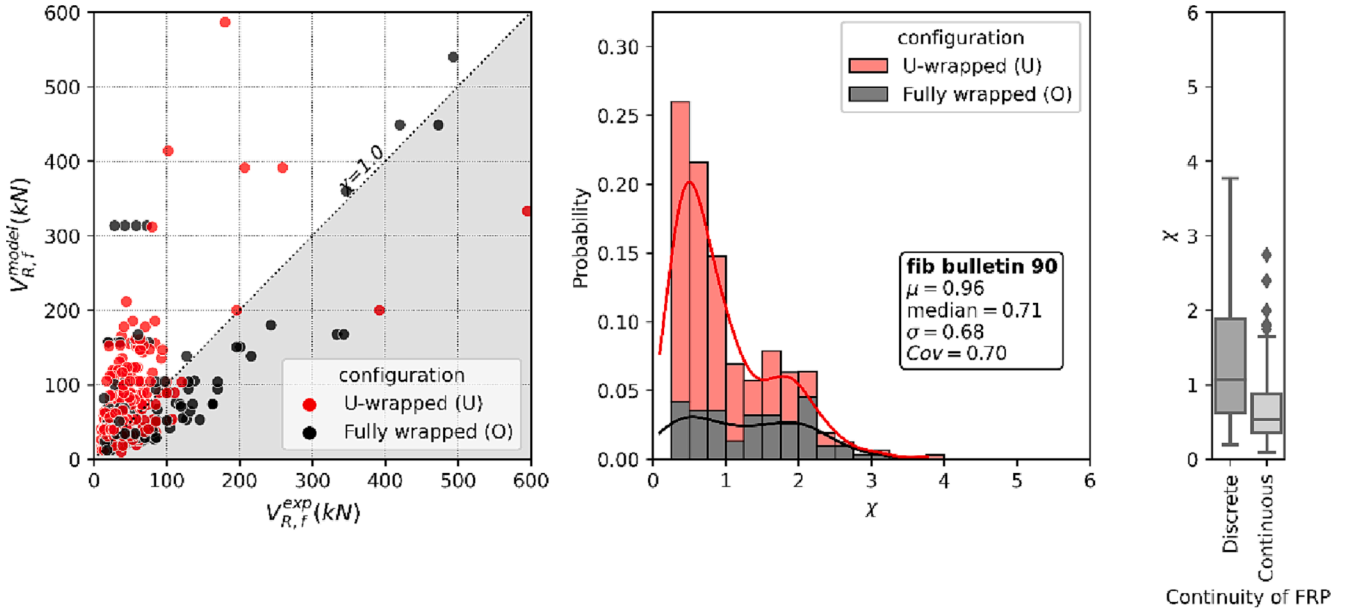


Fig. 4. V_{Rf}^{model} versus V_{Rf}^{exp} (left), distribution of model error probability in entire dataset χ (middle) and boxplot representation of χ for continuous and discrete configuration (right) for: (a) *fib* bulletin 90 [12], (b) CNR-DT 200 [13], (c) ACI 440.2R-17 [11], and (d) proposed model.

versus V_{Rf}^{exp} . Careful inspection of these figures reveals that the proposed model produces the most straight-line like predictions. The second plot shows the distribution of model error probability for both configurations of strengthened beams (i.e., fully wrapped and U-wrapped). The third plot shows the model error for beams strengthened with continuous FRP sheets and beams strengthened with discrete FRP strips. The mean value (μ), standard deviation (σ), median, and coefficient of variation (COV) of χ are reported in Fig. 5.

Fig. 4 reveals that the current models generally result in unconservative responses, which raises serious concerns, despite the safety factors of the models may significantly attenuate the unsafe level of the predictions. The scatter plot of the theoretical versus experimental values of shear strength provided by FRP, shows that the prediction models have dispersed results including the proposed model, which may be justified by the difficulty of assuring uniform application methodologies of the strengthening systems, size effects and inadequate loading conditions, since inadequate a/d can avoid the occurrence of a failure due to the formation of a critical diagonal crack. A different percentage of the flexural reinforcement can introduce a distinct profile for the shear failure crack and dowel effect resisting mechanisms, aspects not considered explicitly in major part of the formulations. Fig. 5 shows that of the considered existing models, *fib* bulletin 90 achieved the lowest standard deviation ($\sigma = 0.68$), however its median value of 0.71 indicates that it tends to give overestimated predictions. ACI 440.2R-17 has the second smallest standard deviation ($\sigma = 0.74$), but its median ($=0.85$) indicates a tendency toward overpredictions. CNR-DT200 has the highest standard deviation ($\sigma = 0.77$), but its median ($=0.81$) indicates that it also produces unconservative predictions. The proposed model on the other hand, outperforms the considered existing models and produces the lowest standard deviation ($\sigma = 0.54$). In addition, having a median of 0.92, this model makes less unconservative predictions in comparison with existing models.

The histograms for each model present the probability of model error χ falling within each interval of 0.25. According to these plots, predictions made by CNR-DT200 and *fib* bulletin 90 are more likely to be overestimated. However, the proposed model shows a better performance for both fully and U-wrapped beams.

The plot in the third column of Fig. 4 shows if the models are biased

for beams strengthened with continuous sheets and discrete FRP strips. According to these plots, all considered existing models have more tendency to overestimate beams strengthened with continuous FRP sheets. This may suggest difficulties of assuring proper bond conditions of the FRP to the concrete substrate when using continuous FRP sheets. Fig. 4d shows that unlike other models, the proposed model has unbiased performance for beams reinforced with either FRP strips or continuous FRP sheets.

The performance of models in predicting the shear contribution of FRP is further explored by determining other indicators of accuracy, namely: root mean squared error (RMSE), mean absolute percentage error (MAPE), Absolute Fraction of Variance (R^2), and Pearson coefficient of correlation (r):

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (V_{Rf,i}^{model} - V_{Rf,i}^{exp})^2} \quad (39)$$

$$MAPE = \frac{100\%}{n} \sum_{i=1}^n \left(\frac{|V_{Rf,i}^{model} - V_{Rf,i}^{exp}|}{V_{Rf,i}^{exp}} \right) \quad (40)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (V_{Rf,i}^{model} - V_{Rf,i}^{exp})^2}{\sum_{i=1}^n (V_{Rf,i}^{model})^2} \quad (41)$$

$$r = \frac{\sum_{i=1}^n (V_{Rf,i}^{model} - \bar{V}_{Rf}^{model})(V_{Rf,i}^{exp} - \bar{V}_{Rf}^{exp})}{\sqrt{\sum_{i=1}^n (V_{Rf,i}^{model} - \bar{V}_{Rf}^{model})^2} \sqrt{\sum_{i=1}^n (V_{Rf,i}^{exp} - \bar{V}_{Rf}^{exp})^2}} \quad (42)$$

A model with a perfect prediction has a null value for MAPE and RMSE and the unitary value for r and R^2 .

The values for these indicators obtained for all models are given in Table 2.

According to Table 2, ACI 440.2R-17 has the highest R^2 and r , and smaller MAPE and RMSE among the considered existing models. Concerning the proposed model, it has almost a r value of 0.8, which means a significant linear correlation between the model predictions and experimental results [107]. The proposed model has also the lowest

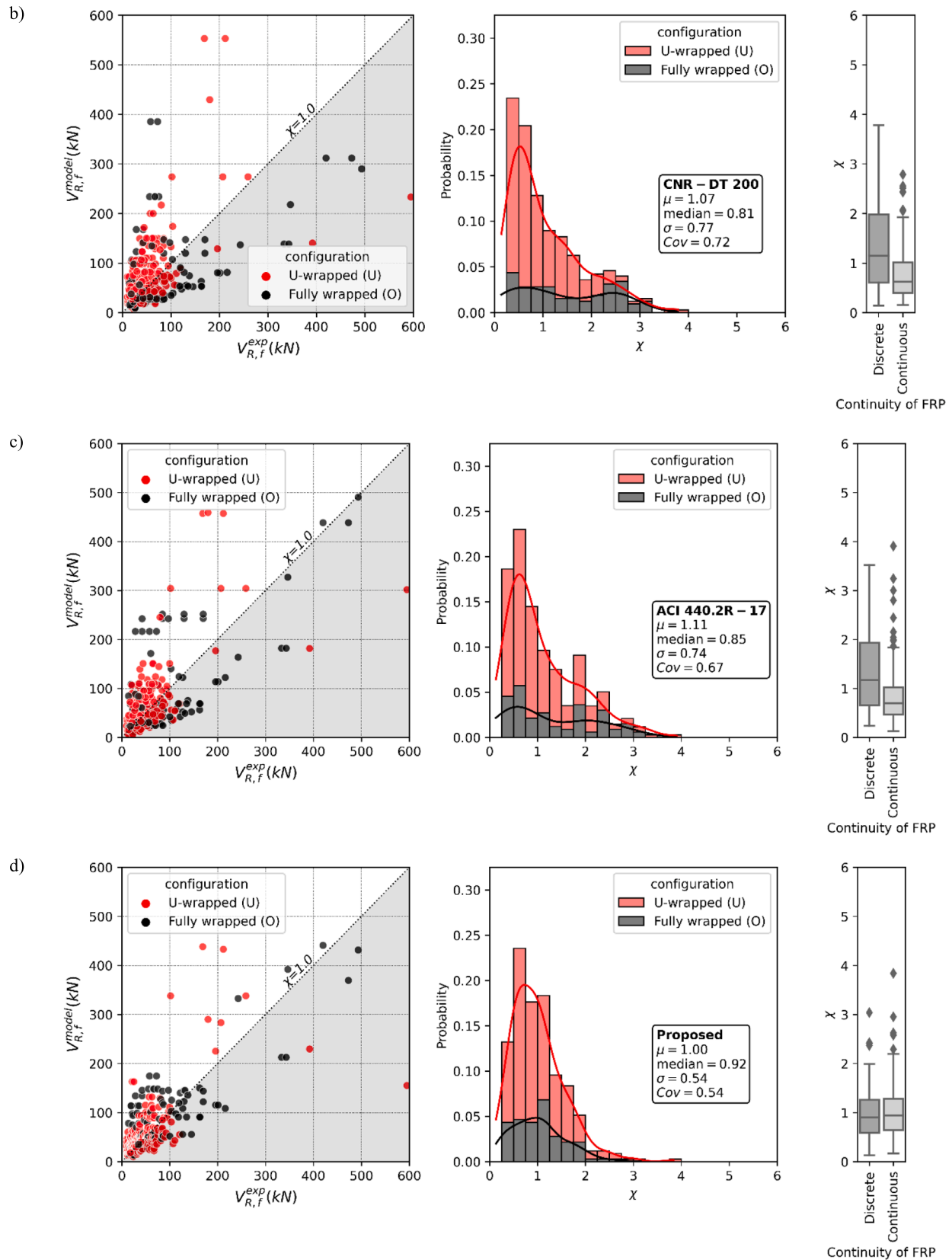


Fig. 4. (continued).

MAPE and RMSE values, while *fib* bulletin 90 conducted to the largest values.

The considered models, as well as major part of available ones, do

not take into consideration the detrimental influence of the percentage of existing steel stirrups on the FRP shear strengthening effectiveness, whose mechanisms are discussed elsewhere [28]. Fig. 6 presents the

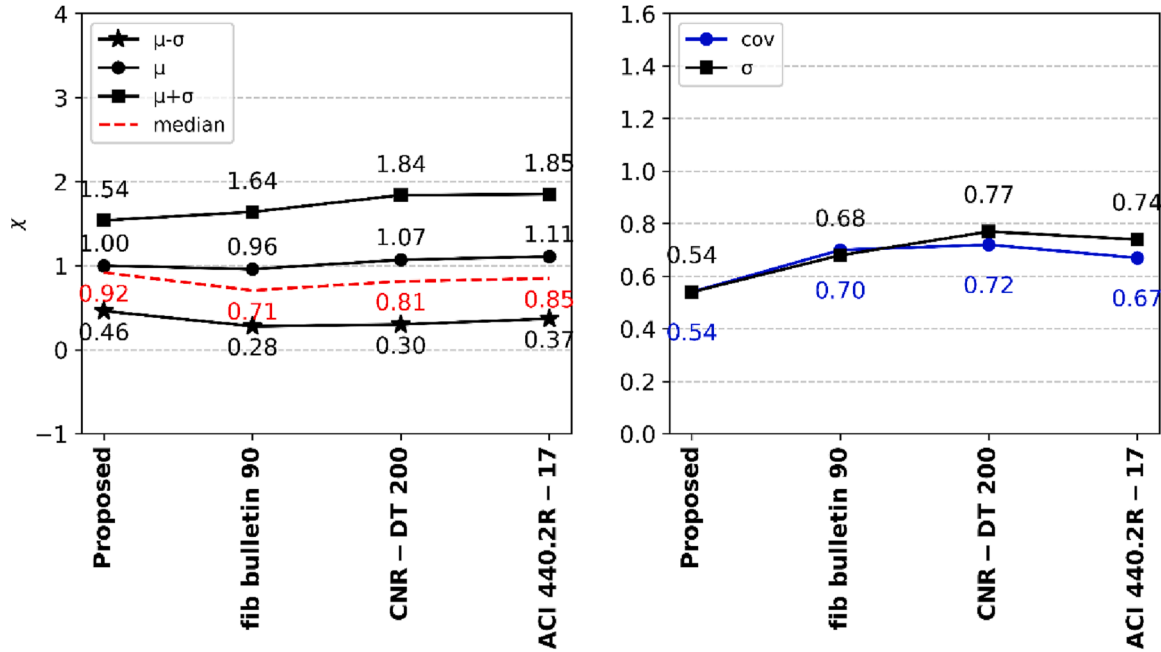


Fig. 5. Ratio between the value recorded experimentally and the predicted one (χ) for the different models analysed.

Table 2

Prediction performance of the proposed model and the design models.

Model	RMSE	MAPE (%)	R2	r
fib bulletin 90	78.1	100.4	0.63	0.63
CNR-DT 200	72.7	90.4	0.52	0.53
ACI 440.2R-17	59.1	74.6	0.70	0.71
Proposed	51.4	61.8	0.74	0.76

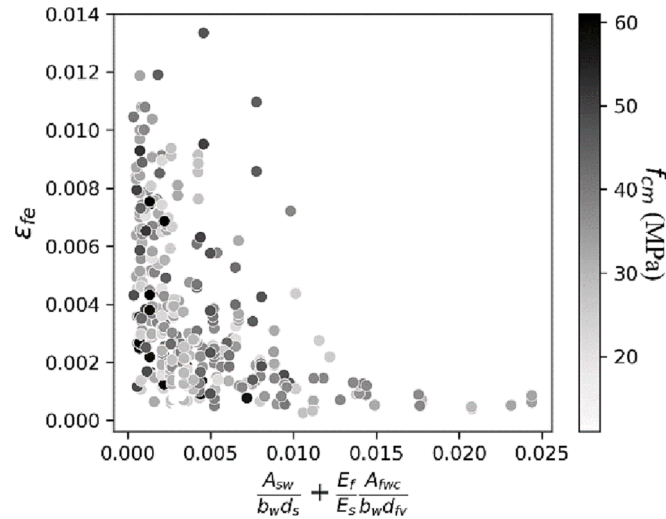


Fig. 6. Relation between steel-FRP interaction factor and effective strain in FRP reinforcement.

relation between the steel-FRP interaction factor ($A_{sw}/b_w d_s + (E_f/E_s)A_{fw}/b_w d_{fw}$) and the χ parameter. As can be seen, the effective strain generally decreases as the ratio of steel stirrups or FRP reinforcement increases. This can be justified by the fact that in beams strengthened with a large amount of FRP, the crack opening is restricted, meaning that steel stirrups may fail to reach the yield strain upon failure. Consequently, maximum capacity of steel stirrups is not mobilized and

using the yielding stress for their design is incorrect. As a result, it is expected that current models overestimate the results in beams strengthened with a high Steel-FRP interaction factor, since this detrimental effect is not accounted by them. Fig. 7 presents the scatterplot of model errors along the steel-FRP interaction factor and the red line shows the mean of the model error along this factor. p -values at a significance level of 0.05 are calculated and displayed to reject or accept the null hypothesis of unbiased estimates. Only in the proposed model with a p -value of 0.4, the mean value of the χ parameter shows no discernible pattern, indicating that the model is not biased with respect to the Steel-FRP interaction factor. In the existing models, the χ decreases with the increase of the Steel-FRP interaction factor and/or the decrease of the concrete strength. This means that the overpredictions of existing models increase with the Steel-FRP interaction factor and/or the decrease of concrete strength. Therefore, additional considerations should be taken into account when applying these models to beams with a relatively high Steel-FRP interaction factor. As Fig. 7d evidences, this detrimental effect is significantly attenuated in the proposed model, since the percentage of existing steel stirrups, ρ_{sw} , is considered via κ_{sw} parameter (Eq. (34)).

6. Reliability analysis

It should be noted that, from the standpoint of structural reliability, a classification system based solely on the main descriptive statistics measures regarding the behaviour of χ may not provide enough information to assess the reliability of a design proposal. In fact, a χ equal to 0.5 is worse than a χ equal to 2, which is not considered in the statistical analysis provided in the previous section [108]. For this reason, a weighted penalty categorization method, called Demerit Point Scale Methodology, is used [109], where a penalty point is assigned to each range of χ . The total demerit point score for each of the models is obtained by summing the products of the percentage of χ placing in each range times the demerit points attributed to that range divided by 100. The result of this score will be a number between 0 and 10. The smaller total demerit point score the better the model performs. Table 3 introduces the range of χ and their penalties, as well as the percentage of specimens that fall within each range. Finally, the overall performance of each model is assessed by a cumulative penalty score. Table 3 shows

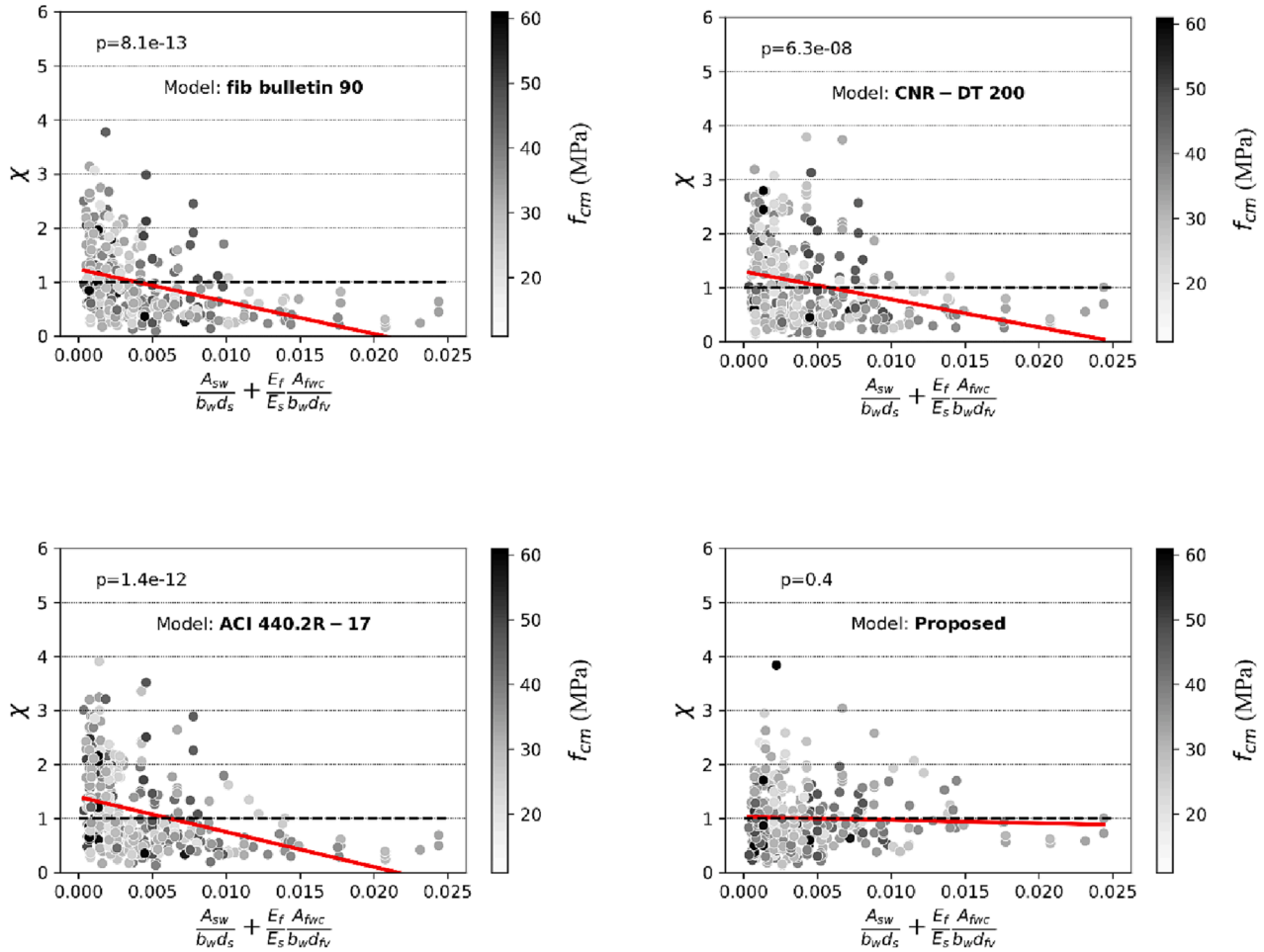


Fig. 7. Ratio of experimental to analytical model.

Table 3
Demerit point classification.

χ	Classification	Penalty	<i>fib</i> bulletin 90	CNR-DT 200	ACI 440.2R-17	Proposed
<0.5	Extremely dangerous	10	31	27	21	16 ^a
0.5–0.65	Dangerous	5	15	13	14	13
0.65–0.85	Low safety	2	10	11	16	15
0.85–1.3	Appropriate safety	0	17	18	16	33
1.3–2	Conservative	1	17	16	19	19
>2	Extremely conservative	2	10	15	14	4
Total demerit point score			4.42	4.03	3.59	2.82 ^b

a: Percentage of specimens with χ laying in the range.

b: $((16 \times 10) + (13 \times 5) + (15 \times 2) + (33 \times 0) + (19 \times 1) + (4 \times 2))/100 = 2.82$

that compared to the considered existing models, the largest percentage of the predictions fall within the appropriate safety range (33% *versus* 17%, 18%, and 16%), while the least percentage of predictions fall inside the dangerous and extremely dangerous ranges (29% *versus* 46%, 40%, and 35%). The total demerit point score of the proposed model is also lower than the considered existing models. Therefore, it is more reliable. The ACI 440.2R-17, with 21% of the predictions in the extremely conservative range, is the most conservative model. On the other hand, 46% of *fib* bulletin 90's predictions fall in dangerous and extremely dangerous ranges, which makes it the less reliable model.

7. Conclusions

This paper assessed the performance of some of the most used guideline models for predicting the shear strength of reinforced concrete

(RC) beams strengthened with externally bonded fibre reinforced polymer (FRP) reinforcement systems, namely the *fib*-bulletin 90, CNR-DT 200 and ACI 440.2R-17. For this purpose, a comprehensive database formed by results from scrutinized 344 RC beams was set from available literature. In the present phase, the shear strengthening FRP systems of this database are limited to those applied as full wrapping and three-sided (U-wrapped) configurations. In addition, a new model was proposed, and its prediction performance was compared with the above-mentioned existing guideline models. Based on the results and discussions presented in this paper, the following conclusions can be drawn:

- 1- Among the existing models, ACI 440.2R-17 has the least prediction errors and highest accuracy, supported by the values obtained in RMSE, MAPE, R^2 and r , namely, 59.1, 74.6%, 0.70, and 0.71,

respectively, and supported by the $\chi = V_{Rf}^{\text{exp}}/V_{Rf}^{\text{model}}$ parameter, where V_{Rf}^{exp} and V_{Rf}^{model} are the FRP shear contribution registered experimentally and predicted one by the model, respectively. Nevertheless, it has a high standard deviation ($\sigma = 0.74$), showing that this model has high dispersed prediction errors. The proposed model not only has the best predictive indicators (RMSE = 51.4, MAPE = 61.8, $R^2 = 0.74$ and $r = 0.76$), but also the standard deviation (0.54).

- 2- None of the current models considers the interaction between the internal steel stirrups and externally bonded FRP reinforcement. Ignoring this effect has contributed to overestimate the shear strengthening capacity of FRP strips, with an increase of the over-estimation prediction with the axial stiffness of the FRP system. The proposed model improved this shortcoming of the considered existing models by considering in the formulation a factor that depends on the steel stirrups reinforcement ratio.
- 3- According to the reliability analysis performed in this study, the proposed model produces the most reliable predictions in comparison with current models. Nevertheless, there is a 29% probability of dangerous and extremely dangerous predictions, which is still a concern that requires further research, mainly to have rational justification.
- 4- For close future research the authors plan to use the existing database for determining $\chi_d = V_R^{\text{exp}}/V_{Rd}^{\text{model}}$, where V_R^{exp} and $V_{Rd}^{\text{model}} = V_{Rd,c} + V_{Rd,s} + V_{Rd,f} \leq V_{Rd,max}$ are beam's shear capacity registered experimentally and predicted by the model, respectively. For the V_{Rd}^{model} , the design values of the shear contribution provided by concrete ($V_{Rd,c}$), steel stirrups ($V_{Rd,s}$) and FRP ($V_{Rd,f}$) will be evaluated according to the formulations of the corresponding design guidelines, in order to have a realistic scenario of the χ_d in real design practice.

CRediT authorship contribution statement

Amirhossein Mohammadi: Conceptualization, Methodology, Formal analysis, Investigation, Visualization, Writing – original draft. **Joaquim A.O. Barros:** Conceptualization, Validation, Supervision, Writing – review & editing. **Jose Sena-Cruz:** Validation, Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Data availability statement.

All data, models, and code generated or used during the study appear

in the submitted article.

Appendix A. Supplementary material

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.compstruct.2023.117081>.

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Paper 2

TITLE: Multiobjective Optimization of an Unbiased Model for Externally Bonded CFRP System Contributions to Shear Resistance in RC Beams

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Multiobjective Optimization of an Unbiased Model for Externally Bonded CFRP System Contributions to Shear Resistance in RC Beams

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Abstract: Predicting the shear resistance of reinforced concrete (RC) beams strengthened with externally bonded reinforcement using carbon fiber–reinforced polymer (CFRP) materials poses a significant challenge due to the complex nonlinear relationship with beam features and the inherent randomness in shear failure events, driven by their brittle nature. This paper proposes a model employing a multiobjective optimization technique that both, enhances the accuracy and ensures unbiased predictions. To highlight the achieved improvement, a comparison is made between the performance of the proposed model and a series of well-established models in the literature. The proposed model presents superior predictive performance and a smaller coefficient of variation in the model error distribution than the models in the literature. In addition, it exhibits unbiased predictions, highlighting the advantages of incorporating multiobjective optimization. Furthermore, a modified demerit point classification is introduced and applied to all models, highlighting the superiority of the proposed model in producing more reliable and cost-effective predictions. **DOI:** 10.1061/JCCOF2.CCENG-4814. © 2024 American Society of Civil Engineers.

Author keywords: Shear strengthening; Carbon fiber–reinforced polymer (CFRP) reinforcement; Multiobjective optimization; Model error; Guideline models.

Introduction

Over the past three decades, carbon fiber–reinforced polymer (CFRP) materials have been extensively employed to enhance structural behavior, such as improving the flexural and shear resistance of reinforced concrete (RC) beams (Barros et al. 2007; Abbasi et al. 2022). The goal of predicting the contribution of CFRP strengthening to shear resistance in RC beams is challenging due to the inherent complexity of shear behavior in RC beams. Additionally, different types of potential failure modes, nonuniform strain profiles along the CFRP composites, and significant uncertainties associated with the strengthening systems further contribute to this complexity (Chen et al. 2022; Mohammadi et al. 2023b). Design guidelines often resort to employing a conventional truss model to simplify the representation of the shear contribution from CFRP reinforcement. Unlike steel reinforcement, where the stress in the stirrups bridging the critical diagonal crack (CDC) can be assumed to reach the yield strength, CFRP reinforcements do not follow this straightforward behavior. As shown by advanced numerical simulations and complex analytical models, the strains in CFRP reinforcements at the point of shear failure in beams vary significantly among these reinforcements crossing the CDC

(Chen et al. 2013; Abbasi et al. 2022). Furthermore, the maximum strain level allowed to be mobilized by the CFRP shear reinforcements should allow the mobilization of other shear resisting mechanisms, like aggregate interlock. To attend to these aspects in a relatively simple approach, the concept of effective strain (ϵ_{fe}) for these reinforcements has been used (Barros et al. 2007; Mohammadi et al. 2024). Consequently, the contribution of CFRP to the shear resistance of the beam can be obtained by the following equation:

$$V_f = A_{fpl} h_f E_f \epsilon_{fe} (\cot \theta + \cot \alpha_f) \sin \alpha_f \quad (1)$$

where A_{fpl} = ratio of CFRP reinforcement cross-sectional area provided per unit length, determined as $2w_f t_f \sin \alpha_f / s_f$ for strips and $2t_f \sin \alpha_f$ for continuous CFRP sheets, with w_f (in mm), t_f (in mm), and s_f (in mm) being the width, the thickness, and the spacing of CFRP reinforcement, respectively. In Eq. (1), h_f (in mm), α_f , and E_f (in MPa) are the height, fiber orientation angle, and modulus of elasticity of the CFRP reinforcement, respectively. The variable θ is the inclination angle of CDC, for which a provisional value of 45° or more accurate estimates may be used (fib 2023).

In Fig. 1, the symbols of the considered geometric properties of the RC beams strengthened with externally bonded reinforcement (EBR)-CFRP systems are illustrated, as utilized in this study.

Different prediction models exist in the literature to obtain ϵ_{fe} , and consequently V_f . However, several studies have demonstrated that these models often fail to deliver reliable and accurate predictions. Additionally, these models tend to be highly sensitive to variations in input features such as the properties of the CFRP material and the geometric dimensions of the beams. This sensitivity results in inconsistent prediction performance, where the accuracy of the model can vary significantly depending on the specific values of the input parameters. This inconsistency highlights the models' inability to be applicable across different scenarios, making them less reliable for practical applications (Barros et al. 2007; D'Antino et al. 2020; Abbasi et al. 2022; Mohammadi et al. 2023a, 2024). The issue arises from the fact that these models ignore physically

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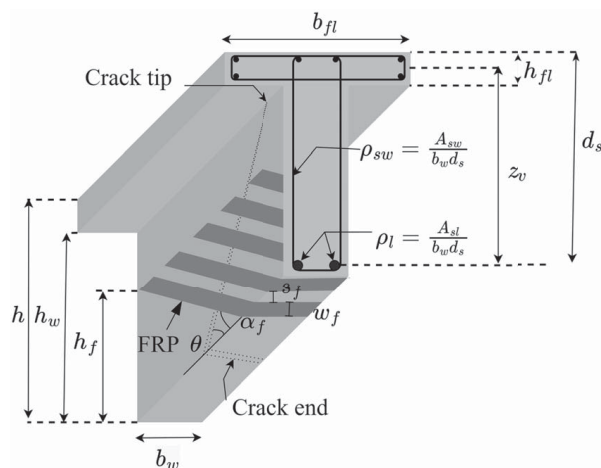


Fig. 1. Symbols adopted for the geometric properties of RC beam strengthened with the EBR-CFRP system.

impactful features, such as the interaction between CFRP and existing transverse steel reinforcement, resulting in inconsistent and biased predictions along these parameters. This study introduces the main influential features and discusses their impact on the predictions.

To address the limitations of the existing models in terms of their predictive performance, various design methods have been explored, including the utilization of machine learning algorithms such as XGBoost and artificial neural networks (ANNs), with promising results in various applications (Karimzadeh et al. 2024a, b; Mohammadi et al. 2023c). While these methods have the potential of providing more accurate predictions, they are

blackbox tools, not suitable for constituting design guidelines; therefore, in this study, a new empirical model utilizing multiobjective optimization is developed, considering influential features for providing unbiased predictions regarding these features.

To develop the model and compare it with other models, a sizable data set consisting of 323 experimentally tested RC beams shear strengthened with EBR-CFRP reinforcement was employed (Mohammadi et al. 2023b). Some statistics of the data set are given in Fig. 2. Most beams in the data set are U-wrapped (73.1%), including all beams with T-shaped cross sections, while the remaining beams are fully wrapped (26.9%). Nearly half of the beams (47.7%) utilize continuous CFRP sheets, and the rest of them (52.3%) are reinforced with CFRP strips. Among the beams, 69% exhibit a T-shaped cross section, while 31% have a rectangular cross section. In 88.2% of the beams, CFRP reinforcements are aligned vertically, and in the remaining 11.8%, CFRP reinforcements are applied with a certain inclination to the beam axis. Additionally, 65.9% of the beams are reinforced with steel stirrups, while the remaining beams lack this type of reinforcement.

Fig. 3 also presents boxplots for some primary features in the data set: the concrete compressive strength (f_{cm}), the longitudinal reinforcement ratio (ρ_l), the transverse reinforcement ratio (ρ_{sw}), the FRP reinforcement ratio (ρ_f), the height of the beam (h), E_f , and shear span to depth ratio (a/d_s). The values of f_{cm} ranges from approximately 10 to 53 MPa, with several cases beyond this range. The variable ρ_l shows a range mostly between 0.01 and 0.058, while ρ_{sw} predominantly falls between 0 and 0.002, but cases are up to 0.0085. The values of ρ_f ranges mostly between 0 and 0.005, with several cases reaching up to 0.025, while h varies widely from about 150 to 600 mm, indicating a diverse set of beam sizes in the data set. The variable E_f ranges from approximately 70,000 to 400,000 MPa, with a significant number of beams with

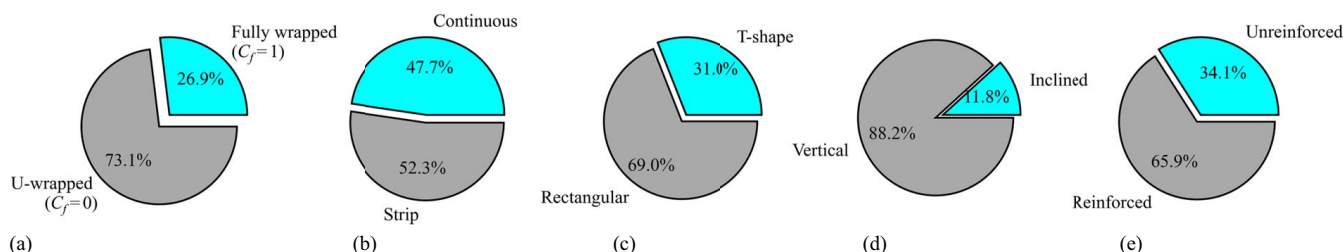


Fig. 2. Distribution of beam specimen characteristics in the data set: (a to e) CFRP strengthening configuration, CFRP sheet continuity, cross section shape, fiber orientation, and transversal steel reinforcement.

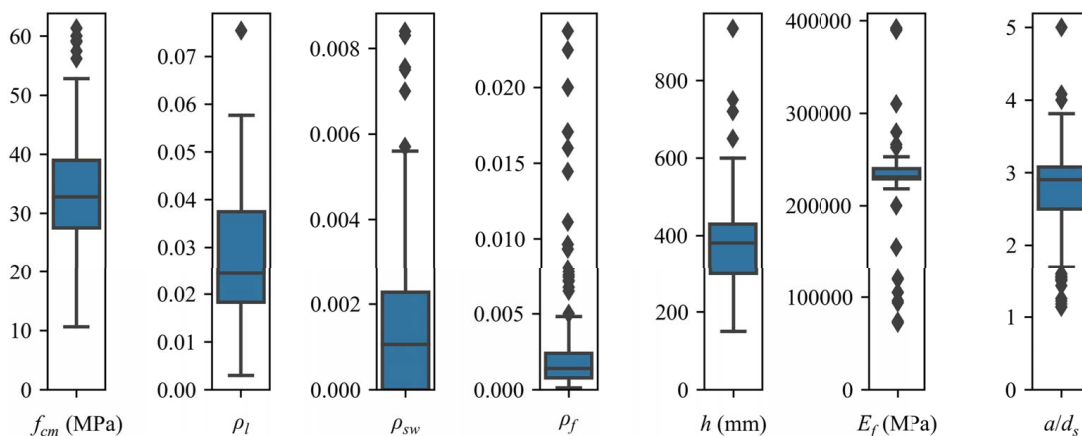


Fig. 3. Boxplots of the primary features in the data set.

E_f around 230,000 MPa, and a/d_s ranges from 1.0 to 5, with an approximate median of 3.0.

The predictive performance of the proposed model, along with several models commonly used by academics and designers, namely, *fib* Bulletin 90 (*fib* 2019), CNR-DT 200 R1 (CNR 2013), ACI PRC-440.2-23 (ACI 2023), TR55 (Concrete Society 2012), and CIDAR (2006), are evaluated using this data set. The formulation of these models is summarized in the Appendix. In addition to several statistical tests and comparisons to recognize the most accurate model, a modified demerit classification is introduced and utilized to facilitate the identification of the most reliable and cost-efficient model.

Determination of Effective Strain of CFRP Shear Reinforcement in EBR Shear-Strengthened RC Beams

This section outlines the development of the regression-based prediction model to estimate the effective strain of CFRP shear reinforcement in EBR shear-strengthened RC beams, denoted as ϵ_{fe} . The experimental values of ϵ_{fe} can be obtained from the collected data set. This involves calculating the experimental contribution of CFRP to shear resistance (V_f^{exp} in N) by subtracting the shear resistance of a reference beam from that of a strengthened beam, along with considering other features of the beam:

$$\epsilon_{fe} = \frac{V_f^{\text{exp}}}{A_{fp} h_f E_f (\cot \theta + \cot \alpha_f) \sin \alpha_f} \quad (2)$$

Assuming that FRP strengthening has a marginal effect on CDC angle (Mohammadi et al. 2024), θ can be determined as follows based on the longitudinal strain at the middepth of the section (ϵ_x) as recommended in *fib* Model Code 2020 (*fib* 2023):

$$\theta = 29 + 7,000\epsilon_x \geq \theta_{\min} = 20 + 4,000\epsilon_x \quad (3)$$

The variable ϵ_x is defined according to the following equation:

$$\epsilon_x = \frac{1}{2E_s A_{sl}} \left(\frac{M_E}{z_v} + V_E \right) \geq 0 \quad (4)$$

where E_s = elastic modulus of the longitudinal steel reinforcement, which is taken to be equal to 200×10^3 MPa for all beams; A_{sl} = its cross-sectional area; M_E and V_E = bending moment and shear forces caused by external loads, respectively; z_v = effective shear depth of the beam, which is defined as the maximum of $0.9d_s$ and $0.72h$.

To develop a regression-based model, the main step is to recognize the features that influence the target value. Data analysis could be employed to determine the important features that influence ϵ_{fe} . However, the literature review shows that an increase in ρ_f and E_f promotes premature debonding, which means a decrease in ϵ_{fe} (Khalifa et al. 1998; Aghabagloo et al. 2024). Conversely, an increase in f_{cm} , or using full wrapping configuration rather than U-wrapping configuration, retards debonding failure, which increases ϵ_{fe} (Triantafillou and Antonopoulos 2000; Chen and Teng 2003). It is shown that when ρ_{sw} increases, the CFRP shear reinforcement becomes less effective. This is because, at the CFRP failure, strain in the steel stirrups crossing the CDC may fall below the steel yielding strain (ϵ_{swy}), thereby the reinforcement potential of these stirrups is not fully exploited (Chen et al. 2017; Li et al. 2018; Mohammadi et al. 2023b, 2024). Another important feature is ρ_l , which can significantly alter the CDC angle and crack end opening width, thereby affecting ϵ_{fe} (Mohammadi et al. 2024). It is reported that the ϵ_{fe} relies on the internal arm of the steel reinforcement (d_s in mm); known as size effect in the shear behavior of RC beams strengthened in shear with externally bonded FRP (Abbasi et al. 2022). As far as the authors are aware, this can be mainly attributed to the fact that in a beam with larger d_s , the maximum crack width that leads to the failure of the CFRP occurs at a smaller crack tip opening angle. This results in a less uniform stress distribution profile, consequently resulting in a relatively smaller ϵ_{fe} . Moreover, it is demonstrated that rounding the corners with a larger corner radius (R_c in mm) helps reduce stress concentration and local rupture in the CFRP system (Li and Leung 2016; *fib* Bulletin No. 90, *fib* 2019). Fig. 4 presents a scatterplot that displays the relationship between ϵ_{fe} and the introduced features. Fitted lines along with the Pearson coefficient of correlation (r) values are displayed on the plot to assess the level of correlation between

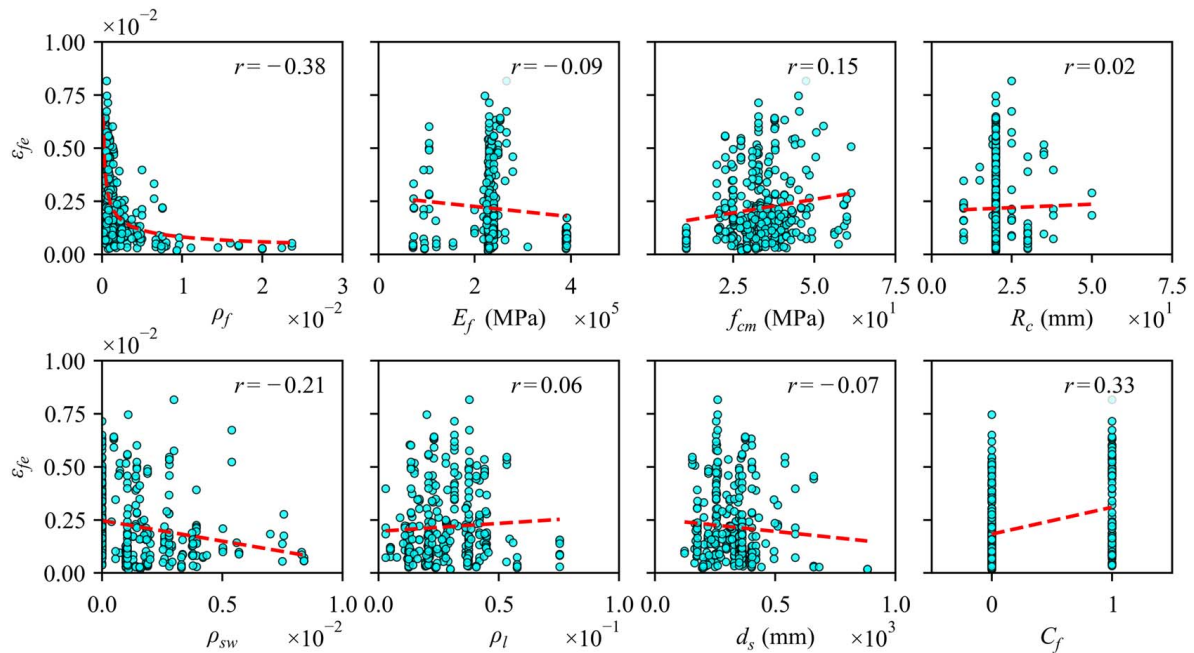


Fig. 4. Scatterplot displaying the relationship between ϵ_{fe} and considered features.

ε_{fe} and each parameter. The observed data support the aforementioned presumed dependence of ε_{fe} on indicated features.

Different approaches have been utilized by existing design guidelines for the estimation of ε_{fe} . This study introduces a methodology for developing a regression model. Initially, a basis model is fitted between the most influential input features and the target value. Subsequently, modification factors are implemented in the model to ensure consistent and unbiased performance across the remaining influential features. As shown in Fig. 4, a robust correlation between ρ_f and ε_{fe} is evident, establishing it as the most influential individual feature among others. However, previous studies (Triantafyllou and Antonopoulos 2000; Mohammadi et al. 2023b) show that combining ρ_f , E_f , and f_{cm} forms a composite feature (i.e., $K = \rho_f E_f / f_{cm}^{2/3}$) with more robust correlation with ε_{fe} . This observation has led to the decision to develop the basis model based on this feature. Trying to fit different functions to 323 experimental data points, the best-fitting function was determined to be a power function (i.e., $\varepsilon_{fe} = aK^b$). This choice was made not only based on its accuracy but also because it ensures prediction values remain above zero for even large values of K , which is more realistic. To ensure that the fitted function can produce equally accurate predictions for future unseen data, a 10-fold cross-validation was performed using the least squares method for fitting the data within each subset. This method involves dividing the data set into 10 subsets, training the model on nine subsets, and validating it on the remaining one, with the process repeated so each subset serves as the validation set once. The results of the 10-fold cross-validation showed that the fitted parameters (a and b) exhibited low coefficient of variation (CoV) (6.64% and 3.51%, respectively), indicating consistent performance across different folds. The mean value of r was 0.66 with a CoV of 10%, demonstrating a good linear relationship between predicted and actual values. Although the R^2 values displayed more variability with a CoV of 29.7%, the mean R^2 value of 0.4 indicates a reasonable fit. Therefore, it is concluded that the regression model, fitted using the average values from the cross-validation, is reliable and is expected to perform well on new unseen data as well. Fig. 5(a) shows the power function with the average values of fitted parameters along with the scatterplot of experimental data points. The power function with the average values of fitted parameters is as follows:

$$\varepsilon_{fe} = 0.0126 \left(\frac{E_f \rho_f}{f_{cm}^{2/3}} \right)^{-0.52} \quad (5)$$

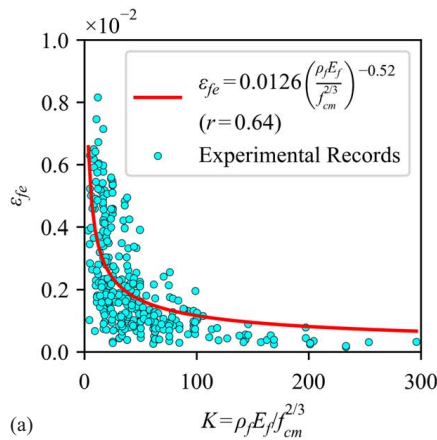


Fig. 5(b) shows the scatterplot comparing the predictions of the base model with the experimental values, resulting in an r of 0.64 and an R^2 of 0.40.

As discussed, the effective strain is influenced by various features beyond K , which renders the proposed model susceptible to bias concerning these additional features, namely, R_c , ρ_{sw} , ρ_l , d_s , and C_f (strengthening configuration factor being 1.0 and 0 for fully wrapped and U-wrapped beams, respectively). Furthermore, as shown in Fig. 5(b), the basic model leads to biased results, attributed to its inability to capture the entire nonlinear relationship between ε_{fe} and the associated features in K (i.e., ρ_f , E_f , and f_{cm}). Therefore, it appears necessary to adjust the basis model to ensure consistent results across these parameters. Fig. 6 shows the distribution of model error $\chi_e = \varepsilon_{fe}^{\text{exp}} / \varepsilon_{fe}^{\text{pred}}$ across different values of these features, allowing an evaluation of their impact on model performance. A line is fitted between model error and the corresponding feature, with the r value indicating the level of correlation, to facilitate the recognition of bias in model error for each feature. As is evident, the predictions generated by Eq. (5) exhibit bias, with model error showing a clear correlation to the different values of these features. To address this issue, it is common practice to adjust the basis model using modification factors (Mohammadi et al. 2023b; Shayanfar et al. 2023). These modifications aim to ensure consistent predictive performance across all feature ranges. Nevertheless, modifying the model for individual features one by one in a sequential manner may reintroduce significant bias within the features for which the model was initially modified. Therefore, this study presents a novel multiobjective optimization method aiming to maximize the overall predictive performance of the final model while simultaneously minimizing introduced bias across all the input features.

Bias Handling with Multiobjective Optimization

A predefined parametric model is introduced for predicting ε_{fe} , which includes Eq. (5) as the basis and a modification factor κ_m , as follows:

$$\varepsilon_{fe} = \kappa_m \left(0.0126 \left(\frac{E_f \rho_f}{f_{cm}^{2/3}} \right)^{-0.52} \right) \quad (6)$$

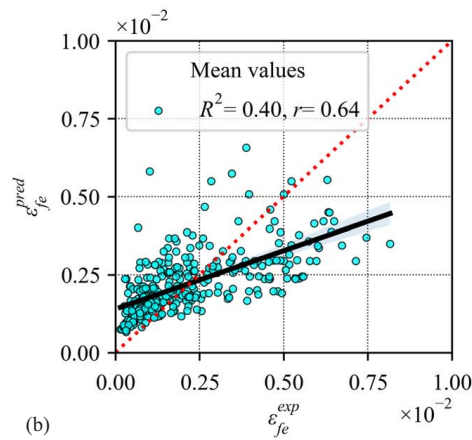


Fig. 5. (a) Scatterplot of experimental values of ε_{fe} against $\rho_f E_f / f_{cm}^{2/3}$ and fitted equation (basis model); and (b) scatterplot of predicted against experimental values of ε_{fe} .

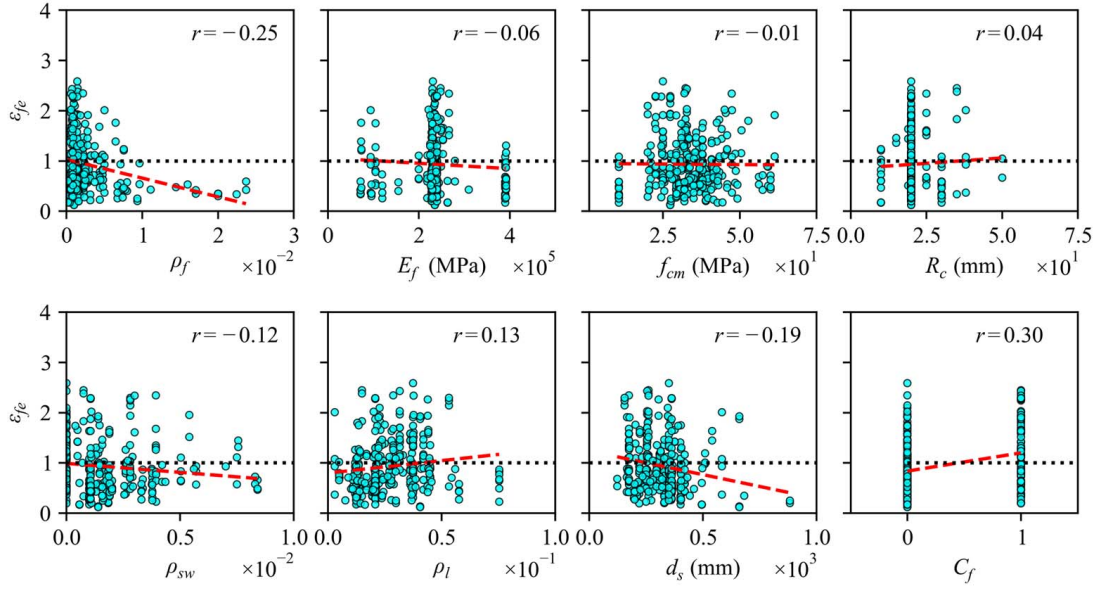


Fig. 6. Scatterplot of model error (χ_e) against considered features before modification [Eq. (5)].

The following is the global model modification factor (κ_m) calculated as the product of a series of linear equations

$$\kappa_m = \prod_{j=1}^p (a_j x_j + b_j), \text{ where} \\ x = [d_s, C_f, \rho_l, \rho_f, f_{cm}, \rho_{sw}, R_c, E_f] \text{ and } p = 8 \quad (7)$$

where $\mathbf{x}$ = vector of selected features; and p = number of selected features (in this case $p = 8$). The parameter vectors are defined as $\mathbf{a} = [a_1, \dots, a_p]$ and $\mathbf{b} = [b_1, \dots, b_p]$, where a_j and b_j are the parameters associated with each feature x_j , and j is the index of the selected features.

Optimization is used to obtain the unknown parameters of the previous equation. To enhance the optimization process and capture multiple facets of the problem, the following multiobjective function (Ω) is used in this study:

$$\Omega = \frac{\sum_{i=1}^N \sqrt{(y_i - \gamma_i)^2}}{N} + \sum_{j=1}^p |\rho_{x_j, \chi}| + \left| \frac{\sum_{i=1}^N y_i / \gamma_i}{N} - 1 \right| \quad (8)$$

where $\mathbf{y}$ and $\boldsymbol{\gamma}$ = vector of known and predicted values of ε_{fe} , respectively; i = index of beam sample in the data set; and N = number of samples. Traditionally, optimization is focused solely on minimizing the sum of squared errors between predicted and known values [first term in Eq. (8)]. While this approach promises the highest accuracy in predictions, it may overlook the bias in model errors regarding the features influencing the variable in analysis (in the present case, the ε_{fe}). A second objective is introduced to address this limitation by minimizing the correlation coefficients between the model error and the considered features [second term in Eq. (8)]. Finally, to ensure that the model error has a distribution with a mean value of unity, a third objective is adopted [third term in Eq. (8)]. By combining these three objectives, the optimization algorithm seeks to balance achieving unbiased predictions and improved predictive accuracy while assuring that the mean model error is close to 1.0.

A genetic algorithm developed in Python was employed to optimize the parameters of the model. A maximum number of iterations equal to 100, a population size of 100, a mutation probability

of 0.1, an elitism ratio of 0.01, a crossover probability of 0.5, and a parent's portion of 0.3 were used. These parameters were determined based on experimentation and empirical knowledge to ensure effective optimization performance, resulting in the parameters shown in Table 1.

After deriving the parameter vectors $\mathbf{a}$ and $\mathbf{b}$ through optimization analysis, Eq. (7) can be transformed to the following:

$$\kappa_m = \prod_{j=1}^p \left(\frac{a_j x_j + b_j}{b_j} \right) \prod_{j=1}^p b_j = B \prod_{j=1}^p (a'_j x_j + 1), \text{ where } B = \prod_{j=1}^p b_j \quad (9)$$

Therefore, using Eq. (9) and the value of B , Eq. (6) can be given as follows:

$$\varepsilon_{fe} = 0.016 \prod_{j=1}^p (a'_j x_j + 1) \left(\frac{E_f \rho_f}{f_{cm}^{2/3}} \right)^{-0.52}, \text{ where} \\ \mathbf{x} = [d_s, C_f, \rho_{sl}, \rho_f, f_{cm}, \rho_{sw}, R_c, E_f] \quad (10)$$

where $a'_j = a_j / b_j$. The sign of a'_j carries critical information. The positive sign is attributed to the inherent tendency of the basis model to underestimate the effective strain ($\varepsilon_{fe}^{\text{pred}} < \varepsilon_{fe}^{\text{exp}}$) with an increase in the corresponding feature. Consequently, the modification process entails multiplying the value obtained from the basis model by a factor greater than 1.0, which increases with the corresponding feature. Conversely, the negative sign indicates that the model tends to overestimate the effective strain ($\varepsilon_{fe}^{\text{pred}} > \varepsilon_{fe}^{\text{exp}}$) as the

Table 1. Optimized values of parameters

j	Feature	a_j	b_j	a'_j	B
1	d_s	-0.0006	0.88	-7.3×10^{-4}	1.24
2	C_f	0.59	1.25	0.47	
3	ρ_l	11.8	1.05	11.25	
4	ρ_f	-27	0.86	-31.4	
5	f_{cm}	-0.0056	1.28	-4.4×10^{-3}	
6	ρ_{sw}	-47.9	1.14	-42.1	
7	R_c	0.014	0.72	0.019	
8	E_f	-1.7×10^{-6}	1.18	-1.47×10^{-6}	

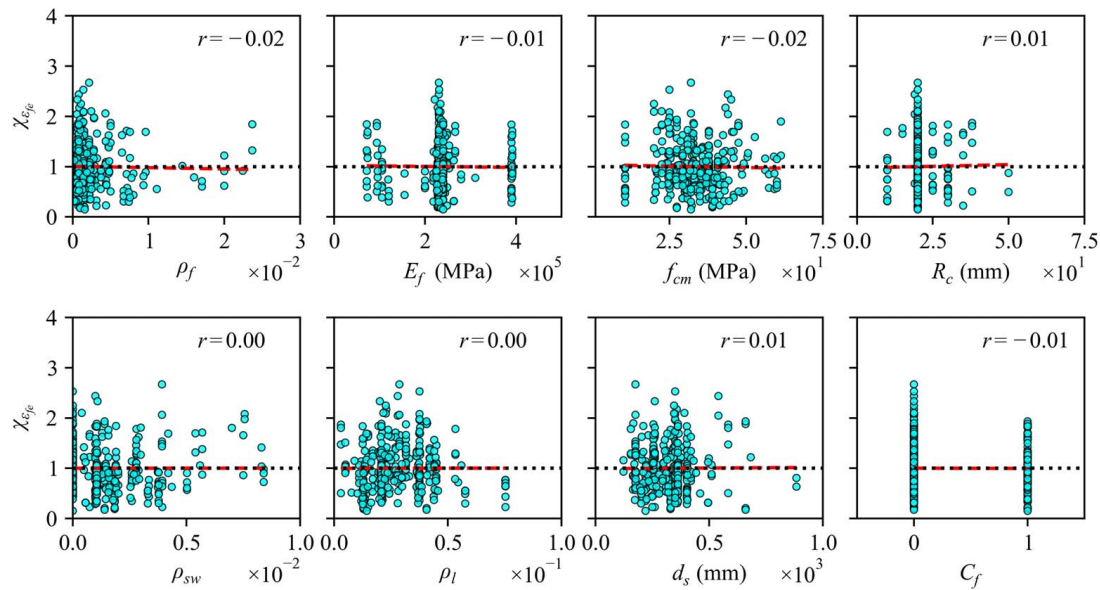


Fig. 7. Scatterplot showing the relation between model error (χ_ϵ) and considered features after modification [Eq. (10)].

corresponding feature increases. In such cases, a modification factor less than 1.0, which decreases with the increase of the corresponding feature, is utilized.

Fig. 7 shows the scatterplot of the model error obtained from Eq. (10) across different values of adopted features of the strengthened beams. The result demonstrates minimal bias concerning all the features, as evidenced by the r value between each feature and the model error. Fig. 8 shows the scatterplot of the model error against K and predicted against the actual value of ϵ_{fe} before and after taking the modification factor into account [Eqs. (5) and (10)]. As can be seen, the modification factor has decreased the CoV in the model from 0.57 to 0.48. Additionally, Eq. (10) provided consistent predictive performance along the K . The modified model is demonstrated to yield an improved coefficient of determination (R^2) and r , with values of 0.57 and 0.76, respectively, compared to 0.40 and 0.64 in the basis model.

Comparison of the Predictive Performance of Proposed and Existing Models

To showcase the predictive performance of the proposed model, a comparison is made with the ones provided by the following existing models: *fib* Bulletin 90, CNR-DT 200 R1, ACI PRC-440.2-23, TR55, and CIDAR. The performances of the proposed model and the considered existing models are evaluated by comparing their predictions in terms of the average CFRP shear contribution (V_{Rf}^{model}) to the experimental values (V_{Rf}^{exp}). A summary of these models' formulations for obtaining V_{Rf}^{model} is given in the Appendix. It must be noted that average values of material properties are used, and partial factors are set equal to 1.0. Throughout this study, the inclination of the shear crack is obtained from Eq. (3). Where not provided, R_c of 20 mm is assumed. In Fig. 9, a series of scatterplots demonstrates the predictive performance of these models. It is

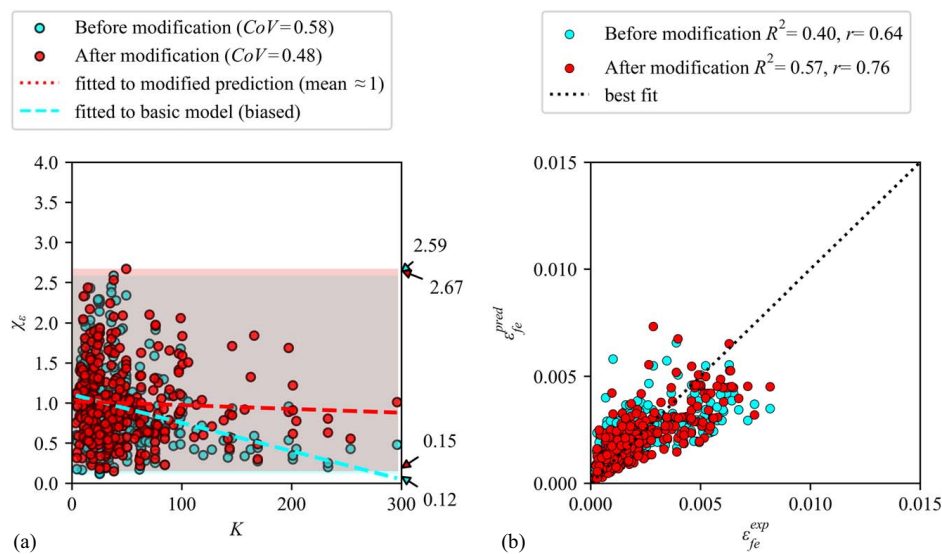


Fig. 8. (a) Scatterplots of model error (χ_ϵ) against K ; and (b) scatterplot of model predictions against experimental values of ϵ_{fe} without and with modification factor.

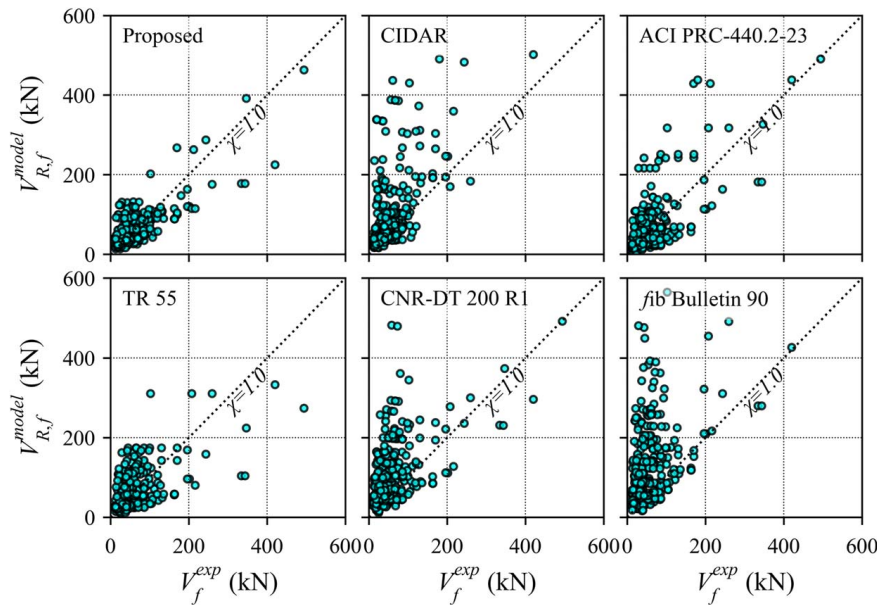


Fig. 9. Scatterplot of models' predictions against experimental values.

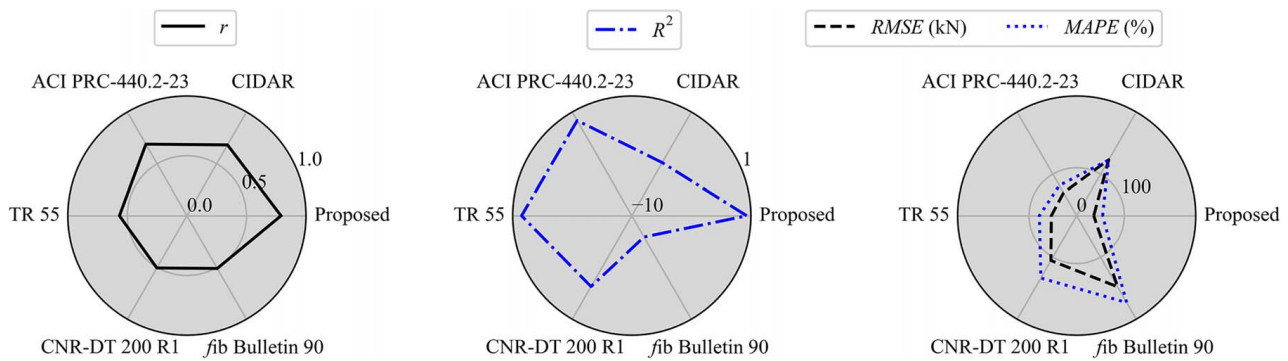


Fig. 10. Comparison of the models' prediction performances using performance metrics.

evident that the dispersion of predictions by the proposed model is comparatively lower than by other models.

A set of performance metrics is employed to quantify the predictive performance of each model, including r , R^2 , root mean squared error (RMSE), and mean absolute percentage error (MAPE), as defined in Eqs. (11)–(14), respectively. Each metric provides distinctive insights into various aspects of model performance, collectively contributing to a comprehensive assessment

$$r = \frac{\sum_{i=1}^n \left[(\gamma_i - \bar{\gamma})(y_i - \bar{y}) / \sqrt{\sum_{i=1}^n (\gamma_i - \bar{\gamma})^2 \sum_{i=1}^n (y_i - \bar{y})^2} \right]}{\quad} \quad (11)$$

$$R^2 = 1 - \left(\frac{\sum_{i=1}^n (y_i - \gamma_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \right) \quad (12)$$

$$\text{RMSE} = \sqrt{\sum_{i=1}^n (y_i - \gamma_i)^2 / n} \quad (13)$$

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^n (|(\gamma_i - y_i) / y_i|) \quad (14)$$

where y_i = known value for the i th record; $\bar{y}$ = mean of known values; γ_i = predicted value for the i th record; $\bar{\gamma}$ = mean of predicted

Table 2. Model performance metrics

Model	r	R^2	RMSE (kN)	MAPE (%)
Proposed	0.79	0.62	35.8	54.4
CIDAR	0.68	−4.4	134.7	134.1
ACI PRC-440.2-23	0.69	0.09	55.3	72.8
TR55	0.57	0.19	53.2	78.3
CNR-DT 200	0.50	−2.47	108.1	150.5
<i>fib</i> Bulletin 90	0.51	−7.74	171.7	209.8

Table 3. Statistical metrics for model errors

Models	Mean	Median	σ	CoV	Maximum	Minimum
Proposed	1.00	0.92	0.47	0.47	2.66	0.15
CIDAR	0.73	0.684	0.42	0.57	2.52	0.04
PRC-440.2-23	1.19	0.93	0.82	0.69	4.28	0.13
TR55	1.09	0.83	0.75	0.69	3.52	0.17
CNR-DT 200	0.66	0.51	0.46	0.69	2.36	0.11
<i>fib</i> Bulletin 90	0.58	0.44	0.43	0.75	2.36	0.06

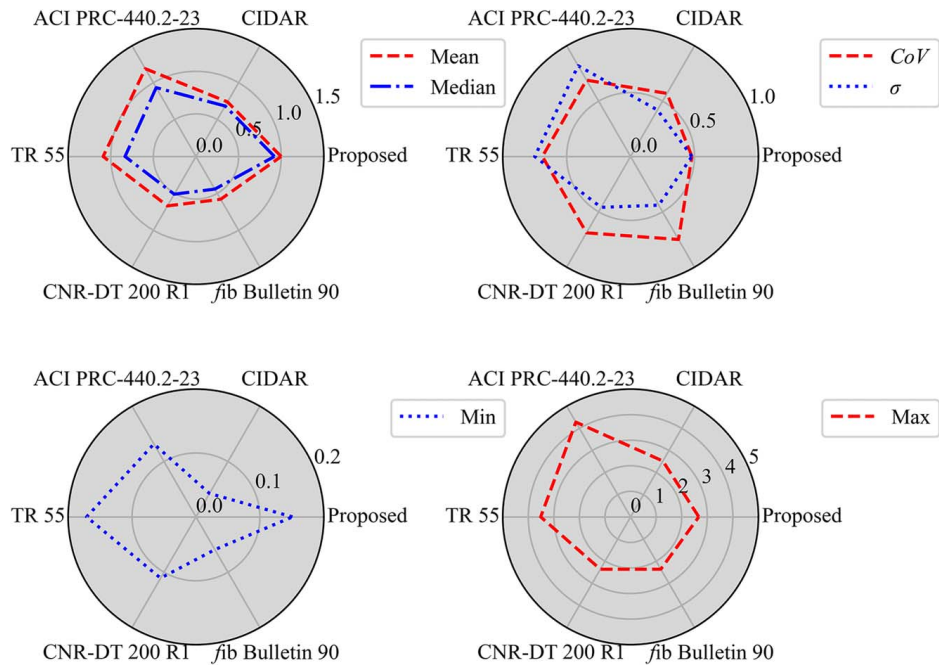


Fig. 11. Comparison of statistical metrics for model errors (χ_v) in considered models and proposed model.

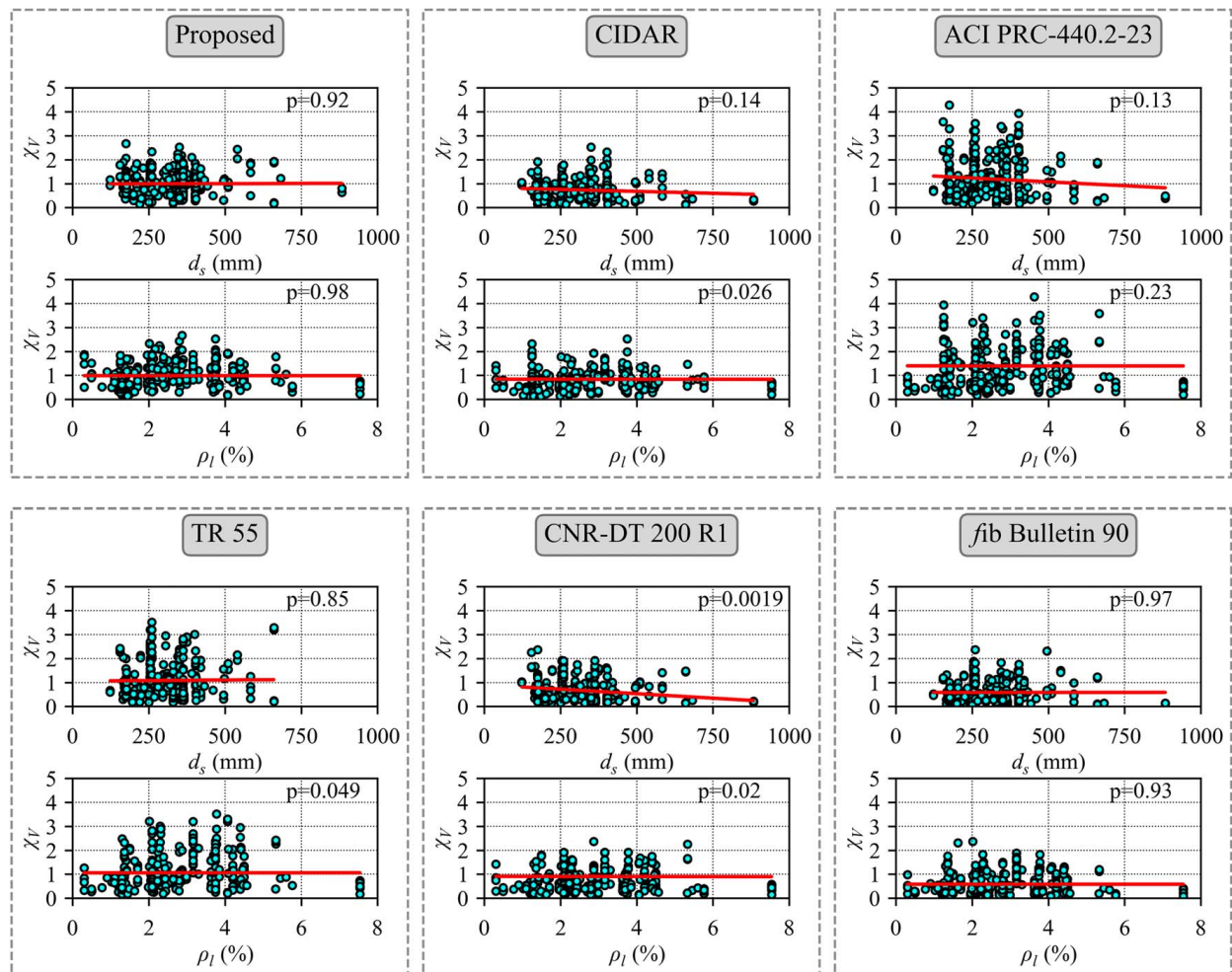


Fig. 12. Model error (χ_v) in considered models and proposed model against ρ_l and d_s .

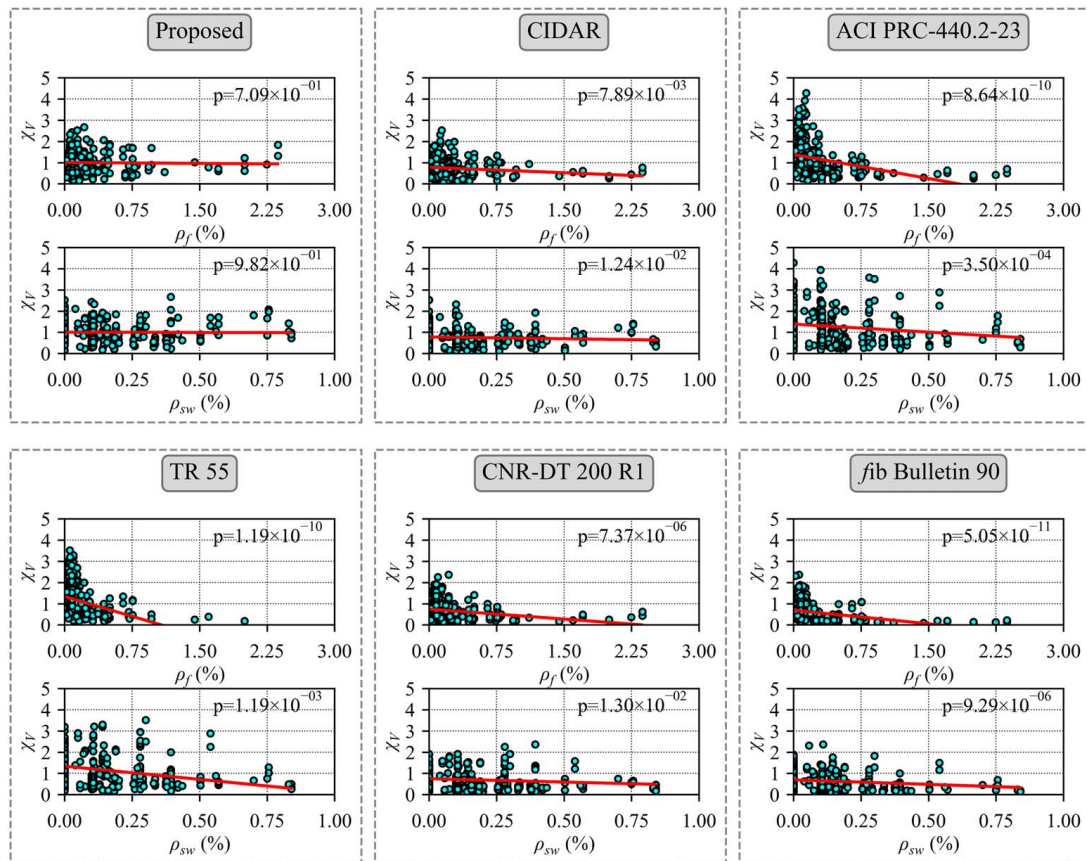


Fig. 13. Model error (χ_V) in considered models and proposed model against ρ_{sw} and ρ_f .

values; and n = total number of records. Fig. 10 shows that the proposed model has better predictive performance than the other models, exhibiting the highest values of r and R^2 along with the lowest RMSE and MAPE (perfect predictions correspond to $r = R = 1$, RMSE = MAPE = 0). For detailed metric values of all models, refer to Table 2.

Another crucial aspect of model evaluation involves determining whether the models make consistent predictions along beam features. To assess the degree of bias in each model, the model error is computed as the ratio of the experimental value to the average CFRP shear contribution, i.e., $\chi_V = V_f^{\text{exp}} / V_{R,f}^{\text{model}}$. Before assessing the consistency in the models' performance, the general distribution of model errors should be examined. The model with the best predictive performance has normal model error

distribution, with mean, median, minimum, and maximum the closest to the unitary value and the minimum standard deviation (σ) and CoV. According to Fig. 11 and Table 3, the proposed model has better predictive performance, having a mean, median, minimum, and maximum of 1.0, 0.92, 0.15, and 2.67, respectively, with σ and CoV of 0.48.

As demonstrated in the preceding section (Fig. 6), neglecting the impact of beam features may result in inconsistent predictive performance regarding these features. Similar to the analysis performed for effective strain, it is beneficial to evaluate whether the proposed model and the considered existing models are consistent along critical beam features on V_f prediction, namely, ρ_{sw} , ρ_f , ρ_b , and d_s . Fig. 12 shows the scatterplots of χ_V against ρ_f and d_s , while Fig. 13 presents the scatterplots of χ_V against ρ_{sw} and ρ_f .

Table 4. Modified demerit points classification

Classification	χ_V	p.p	Proposed	CIDAR	ACI PRC-440.2-23	TR55	CNR-DT 200	<i>fib</i> Bulletin 90
Extremely dangerous	<0.5	10	12 ^a	20	28	27	28	32
Dangerous	0.5–0.65	5	14	7	15	15	15	12
Relatively safe	0.65–0.85	2	16	18	13	14	13	12
Appropriately safe	0.85–1.3	0	34	30	15	17	17	17
Conservative	1.3–2	1	21	20	19	15	15	15
Extremely conservative	>2	2	3	5	10	11	13	13
Total demerit points			2.49 ^b	3.01	4.2	4.1	4.22	4.45

^aPercentage of specimens with χ_V laying in the range.

^b $[(12 \times 10) + (14 \times 5) + (16 \times 2) + (34 \times 0) + (21 \times 1) + (3 \times 2)] / 100 = 2.49$.

for the considered existing models and the proposed model. Each plot shows the relationship between the errors and different parameters on the x -axis for the different models. For clarity, a regression line is fitted to the data in each plot. The assessment of bias in the models involves examining whether there is a significant correlation between input features and χ_V . Therefore, a Pearson correlation test is conducted in which the null hypothesis is that no correlation between input feature and χ_V ($H_0: r=0$) exists and the alternative hypothesis ($H_1: r \neq 0$) is that the model error is dependent on the input features. The P -values are computed and displayed, using a significance level of 0.05, helping decide whether to reject or accept the null hypothesis. If the P -value is <0.05 , then the null hypothesis is rejected, indicating bias in predictions. Conversely, if the P -value is ≥ 0.05 , then the null hypothesis is accepted, indicating that there is no significant bias in the predictions of the model for that specific feature. The figures and the reported P -values for each model show that the proposed model produces no bias in the predictions corresponding to all the features. All considered existing models are deficient in making consistent predictions regarding ρ_{sw} and ρ_f . CNR-DT 200 R1 and CIDAR also exhibited a significant bias regarding ρ_l and d_s .

Modified Demerit Points Classification

The demerit points' classification analysis was performed to provide an additional fair and comprehensive evaluation of the model's predictive accuracy from a closer perspective of a designer. The traditional demerit points' classification is typically applied to model errors derived from design values rather than average values (Mohammadi et al. 2024). Given that the design values for the proposed model have not yet been determined due to the necessity for a thorough reliability analysis, a modified classification approach is introduced in this study. The modification ensures that the analysis remains valid and meaningful even in the absence of finalized design values. To ensure fairness in comparing model errors from average values, it is crucial that all distributions have a mean of 1.0. This is because in the reliability analysis, the bias in model error is taken into account; hence normalizing the distributions by their means ensures an unbiased comparison. Therefore, each distribution is divided by its mean to achieve this normalization. Similar to the traditional demerit points classification, the modified demerit points classification (MDPC) categorizes predictions for beam specimens into distinct levels, namely, extremely conservative, conservative, appropriately safe, relatively safe, dangerous, or extremely dangerous. The penalty points assigned to each category reflect the varying consequences of the model errors. For instance, extremely conservative predictions (which can lead to unnecessary costs) are penalized less severely than dangerous predictions (which can lead to failures). The total demerit points for each model are computed by summing the products of the percentage of specimens with χ_V falling within each defined range, multiplied by the corresponding penalty points for that range, and then divided by 100. The resulting score falls within the range of 0–10. A lower total demerit point indicates that the model produces more accurate and reliable predictions, highlighting the model's superiority. This classification provides a nuanced evaluation of model performance, highlighting not only the average error but also the distribution and severity of errors. Table 4 provides the breakdown of specimens with χ_V values falling into each classification, along with the total demerit points assigned to each model. The proposed model exhibits a notably lower

total demerit score. Among the considered existing models, CIDAR and *fib* Bulletin 90 record the lowest and highest total demerit points, respectively.

Conclusions

In this study, a data set comprising 323 RC beams shear strengthened with CFRP-EBR (fully or U-wrapped) was employed for the development of a regression-based model. A multiobjective optimization method was introduced to account for the diverse effects of various beam features and to avoid producing biased predictions regarding these features. The application of multiobjective optimization proved essential for successfully resolving biases in the regression model and markedly enhancing predictive performance. To underscore the improvement achieved by the proposed model, a rigorous comparison in terms of the predictive performance of the shear-strengthening contribution of CFRP-BR was conducted with well-established models in *fib* Bulletin 90, CNR-DT 200 R1, ACI PRC-440.2-23, TR55, and CIDAR. The proposed model has shown the best predictive performance. In fact, it exhibited a notable linear correlation between predictions and experimental measurements, as evidenced by r of 0.79. Additionally, the model achieved the most well-balanced predictions, with an R^2 of 0.62, the lowest occurrence of erratic predictions, reflected in an MAPE of 54.4%, and the minimum significant prediction error with an RMSE of 35.8 kN.

The evaluation of models was extended by examining a dimensionless parameter known as model error. Statistical metrics related to model errors revealed that the proposed model generates model errors with a mean value of 1.0 and a CoV of 0.48. These characteristics position the proposed model as a superior choice compared to the other considered models. Significant variability still remains, which is attributed to inherent experimental uncertainties and the simplicity of the current model. Efforts are underway to develop more advanced models, such as ANN and XGBoost, to further enhance predictive accuracy. Furthermore, the trend of model errors across various predictive features of the beam was analyzed using a Pearson correlation test, and the findings indicate that only the proposed model can generate consistent predictions.

A modified demerit point classification was introduced as a further step in the evaluation of the model's predictive performance. This classification facilitated a robust comparison of model reliability and cost-effectiveness. Notably, the proposed model presented the lowest total demerit points and the smallest percentage of predictions falling into the extremely dangerous (12%) and extremely conservative (3%) categories.

In summary, this study showcases the effectiveness of the proposed multiobjective optimization method in mitigating biases and enhancing predictive accuracy. The performance of the model, as evidenced by various metrics, positions it as a better tool for predicting the contribution of CFRP-EBR to the shear strengthening of RC beams. Despite its merits, the proposed model still demonstrates a notable margin of error in predictions, prompting consideration for the application of machine learning algorithms to enhance its performance. Additionally, a calibration of partial safety factors is deemed necessary to facilitate the utilization of the proposed model in design applications.

Appendix. Formulations of the Considered Models

Models	Formulation
<i>fib</i> Bulletin 90	$V_{R,f} = A_{fpl} h_{fe} f_{fwm} (\cot \theta + \cot \alpha_f) \sin \alpha_f$ $A_{fpl} = \begin{cases} \frac{2t'_f w_f}{s_f} & \text{strips} \\ 2t'_f \sin \alpha_f & \text{Continuous sheet} \end{cases}; t'_f = \begin{cases} t_0 n_f, & \text{if } n_f < 4 \\ t_0 (n_f)^{0.85}, & \text{if } n_f \geq 4 \end{cases}$ $h_{fe} = \min(h_f, h - 0.1d_s)$ <p>Fully wrapped: $f_{fwm} = f_{fwm,c} = k_R f_{fm}$ U-wrapped: $f_{fwm} = \min(f_{fbwm}, f_{fwm,c})$</p> <p>For continuous CFRP sheets:</p> $f_{fbwm} = \begin{cases} \left[1 - \frac{1}{3} \frac{L_e(fib)}{h_{fe}/\sin \alpha_f} \right] f_{fbm}, & \text{for } x' \geq L_e(fib) \\ \frac{2}{3} \frac{h_{fe}/\sin \alpha_f}{L_e(fib)} f_{fbm}, & \text{for } x' \leq L_e(fib) \end{cases}$ <p>For discrete CFRP strips:</p> $f_{fbwd} = \begin{cases} f_{fbm}, & \text{for } x' \geq L_e(fib); L_e(fib) \leq y' \leq x' \\ \left[1 - \left(1 - \frac{2}{3} \frac{m' s_f}{L_e(fib)} \right) \frac{m'}{n'} \right] f_{fbm}, & \text{for } x' \geq L_e(fib); y' \leq L_e(fib) \\ \frac{2(n' s_f)/[(\cot \theta + \cot \alpha_f) \sin \alpha_f]}{3 L_e(fib)} f_{fbm}, & \text{for } x' \leq L_e(fib); y' \leq x' \end{cases}$ $x' = h_{fe}/\sin \alpha_f$ $y' = s_f/(\cot \theta + \cot \alpha_f) \sin \alpha_f$ $n' = \text{int}[h_{fe}(\cot \theta + \cot \alpha_f)/s_f]$ $m' = \text{int}[L_e(fib)(\cot \theta + \cot \alpha_f) \sin \alpha_f/s_f]$ $L_e(fib) = \frac{\pi}{2} \sqrt{\frac{E_f t'_f s_{0m}}{\tau_{b1m}}}$ $\tau_{b1m} = 0.72 \sqrt{f_{cm} f_{ctm}}$ $s_{0m} = 0.24 \text{ mm}$ $f_{fbm} = \sqrt{\frac{E_f s_{0m} \tau_{b1m}}{t'_f}}$ $V_{R,f} = 0.9 d_s f_{fe} 2t_f (\cot \theta + \cot \alpha_f) \frac{w_f}{s_f \sin \alpha_f}$ <p>U-wrapped:</p> $f_{fe} = f_{fmm} \left[1 - \frac{1}{3} \frac{l_{em} \sin \alpha_f}{\min\{0.9d_s, h_w\}} \right]$ <p>Fully wrapped:</p> $f_{fe} = f_{fmm} \left[1 - \frac{1}{6} \frac{l_{em} \sin \alpha_f}{\min\{0.9d_s, h_w\}} \right] + \max \left(0, \frac{1}{2} (\kappa_R f_{fu} - f_{fmm}) \left[1 - \frac{l_{em} \sin \alpha_f}{\min\{0.9d_s, h_w\}} \right] \right)$ $\kappa_R = 0.2 + 1.6 \frac{R_c}{b_w}, \text{ with } 0 \leq \frac{R_c}{b_w} \leq 0.5; f_{fmm} = \sqrt{\frac{2E_f \Gamma_{fm}}{t_f}}, \Gamma_{fm} = k_b k_G \sqrt{f_{cm} f_{ctm}}; t_f = n_f t_0$ $k_b = \sqrt{\frac{2 - w_f/s_f \sin \alpha_f}{1 + w_f/s_f \sin \alpha_f}} \geq 1 \quad (\text{if } w_f/b_w < 0.25, k_b \text{ is equal to } 1.18)$ $k_G = \begin{cases} 0.037 & \text{Wet lay-up} \\ 0.023 & \text{pre-cured} \end{cases}$ <p>For continuous sheets:</p> $w_f = \min\{0.9d_s, h_w\} \frac{\sin(\theta + \alpha_f)}{\sin \theta}, s_f = w_f/\sin \alpha_f$ $l_{em} = \max \left(\frac{1}{f_{bm}} \sqrt{\frac{\pi^2 E_f t_f \Gamma_{fm}}{2}}, 200 \text{ mm} \right), f_{bm} = \frac{2\Gamma_{fm}}{s_u}, s_u = 0.25 \text{ mm}$

(Continued.)

Models	Formulation
ACI PRC-440.2-23	$V_{R,f} = \frac{A_{fw} f_{fe} d_{fv} (\sin \alpha_f + \cos \alpha_f)}{s_f} \quad A_{fw} = 2 t_f w_f; f_{fe} = E_f \varepsilon_{fe}$ <p>Fully wrapped: $\varepsilon_{fe} = 0.004 \leq 0.75 \varepsilon_{fu}$</p> <p>U-wrapped: $\varepsilon_{fe} = \kappa_v \varepsilon_{fu} \leq 0.004$</p> $\kappa_v = (\kappa_1 \kappa_2 L_e(\text{ACI})) / (11,900 \cdot \varepsilon_{fu}) \leq 0.75; L_e(\text{ACI}) = 23,300 / (t_f E_f)^{0.58}$ $\kappa_1 = (f_{ck}/27)^{2/3}; \kappa_2 = (d_{fv} - L_e(\text{ACI})) / d_{fv}$
TR55	$V_{R,d,f} = \frac{A_{fw}}{s_f} \left(d_{fv} - \frac{n_{ss}}{3} l_{t,\max} \cos \alpha_f \right) E_f \varepsilon_{fse} (\sin \alpha_f + \cos \alpha_f)$ $n_{ss} = \begin{cases} 0, & \text{fully - wrapped} \\ 1, & \text{U - wrapped} \end{cases}, l_{t,\max} = 0.7 \sqrt{\frac{E_f t_f}{f_{ctk}}}$ $\varepsilon_{fse} = \min \left\{ \varepsilon_{fu} / 2, 0.5 \sqrt{\frac{f_{ctk}}{E_f t_f}}, 0.004 \right\}$ <p>For continuous sheets: $w_f = \sin \alpha_f$, and $s_f = 1$</p> <p>Note: ε_{fse} should be > 0.002. if not, this formulation is not applicable.</p>
CIDAR 2006	$V_f = 2 f_{fem} t_f \frac{w_f}{s_f} h_{fe} (\cot \theta + \cot \alpha_f) \cdot \sin \alpha_f$ $f_{fem} = D_f f_{fm,\max}$ <p>Fully wrapped:</p> $f_{fm,\max} = \begin{cases} f_{fu}; & \varepsilon_f \leq 1.5\% \\ E_f \cdot \varepsilon_f; & \varepsilon_f > 1.5\% \end{cases}; D_f = 0.5 \left(1 + \frac{z_b}{z_b} \right), L_e(\text{CIDAR}) = \sqrt{\frac{E_f t_f}{f_{ck}}}$ <p>For U-wrapped:</p> $D_f = \begin{cases} \frac{2}{\pi \lambda} \cdot \frac{1 - \cos(\frac{\pi}{2} \lambda)}{\sin(\frac{\pi}{2} \lambda)}; & \lambda = \frac{L_{\max}}{L_e(\text{CIDAR})} \leq 1 \\ 1 - \frac{\pi - 2}{\pi \lambda}; & \lambda = \frac{L_{\max}}{L_e(\text{CIDAR})} > 1 \end{cases} \left(L_{\max} = \frac{h_{fe}}{\sin \alpha_f} \right)$ $f_{fm,\max} = \min \left\{ f_{fu}, 0.35 \times \beta_L \times \beta_w \cdot \sqrt{\frac{E_f \cdot \sqrt{f_{ck}}}{t_f}} \right\}, \beta_L = \begin{cases} \lambda, \lambda \leq 1 \\ 1, \lambda > 1 \end{cases}$ $\beta_w = \sqrt{\frac{2 - w_f / (s_f \cdot \sin \alpha_f)}{1 + w_f / (s_f \cdot \sin \alpha_f)}}$ $h_{fe} = z_b - z_t, z_b = 0.9 d_s, z_t = d_{ft}$

Data Availability Statement

Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

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Notation

The following symbols are used in this paper:

- A_{fpl} = ratio of CFRP reinforcement cross-sectional area provided per unit length (mm);
- A_{fw} = area of FRP for shear strengthening measured perpendicular to the direction of the fibers (mm²);
- A_{sl} = cross-sectional area of longitudinal steel reinforcement (mm²);
- a = shear span (mm);
- $\mathbf{a}_f$ = regression coefficient;
- $\mathbf{a}'_f$ = regression coefficient;
- $\mathbf{B}$ = regression coefficient;
- $\mathbf{b}_f$ = regression coefficient;
- b_w = breadth of beam web (mm);
- C_f = strengthening configuration factor;
- CoV = coefficient of variation;
- D_f = stress distribution factor;
- d_{fv} = effective depth of FRP shear reinforcement (mm);
- d_s = internal arm of steel reinforcement (mm);
- E_f = modulus of elasticity of the CFRP reinforcement (MPa);
- E_s = modulus of elasticity of longitudinal steel (MPa);
- f_{ck} = characteristic compressive strength of concrete (MPa);

f_{cm} = compressive strength of concrete (MPa);
 f_{ctk} = characteristic value of tensile strength of concrete (MPa);
 f_{ctm} = mean value of tensile strength of concrete (MPa);
 f_{fbwm} = mean value bond strength of FRP reinforcement (MPa);
 f_{fe} = effective stress in the FRP reinforcement as introduced in CNR-DT 200 R1 (MPa);
 f_{fem} = mean value for the FRP effective stress as introduced in CIDAR (MPa);
 f_{fm} = mean value of ultimate strength of FRP (MPa);
 $f_{fm,max}$ = mean value of maximum stress in FRP as introduced in CIDAR (MPa);
 f_{fimm} = FRP debonding stress according to CNR-DT 200 R1 (MPa);
 f_{fu} = ultimate strength of FRP (MPa);
 f_{fwm} = mean value of the average stress in the FRP intersected by the shear crack in the ultimate limit state (MPa);
 $f_{fwm,c}$ = mean value of FRP strength attributed to a failure caused by FRP rupture (MPa);
 H_0 = null hypothesis;
 H_1 = alternative hypothesis;
 h = height of beam (mm);
 h_f = height of CFRP reinforcement (mm);
 h_{fe} = effective height of FRP reinforcement as introduced in *fib* Bulletin 90 (mm);
 h_w = height of beam web (mm);
 K = composite feature of the CFRP reinforcement that is obtained from $\rho_f E_f / f_{cm}^{2/3}$;
 k_1, k_2 = parameters of the ACI PRC-440.2-23 model representing concrete strength, type of strengthening scheme, and bond reduction coefficient, respectively;
 k_b, k_g = geometrical corrective factor and additional corrective factor, respectively, as introduced in CNR-DT 200 R1;
 k_R = reduction factor to consider stress concentration at corners;
 $L_e(ACI)$ = active bond length as introduced in ACI PRC-440.2-23 (mm);
 $L_e(CIDAR)$ = effective bond length of FRP as introduced in CIDAR (mm);
 $L_e(fib)$ = effective bond length of FRP as introduced in *fib* Bulletin 90 (mm);
 l_{em} = effective bond length as introduced in CNR-DT 200 R1 (mm);
 $l_{t,max}$ = anchorage length required to develop full anchorage capacity (mm);
eMAPE = mean absolute percentile error;
 M_E = bending moment caused by external loads (N·m);
 N = number of specimens;
 n_f = number of FRP layers;
 n_{ss} = parameters of the TR55 model representing type of strengthening scheme;
 R^2 = coefficient of determination;
 R_c = corner radius of beam cross section (mm);
RMSE = root mean squared error (kN);
 r = Pearson coefficient of correlation;
 s_f = spacing of CFRP reinforcement (mm);
 s_u = interface slip corresponding to full debonding (mm);
 s_{om} = mean value of ultimate slip (mm);
 t_f = thickness of CFRP reinforcement (mm);
 t'_f = effective total thickness of FRP reinforcement as introduced in *fib* Bulletin 90 (mm);
 t_0 = thickness of each FRP layer (mm);
 V_E = shear load caused by external loads (N·m);
 V_f = contribution of CFRP reinforcement to shear resistance of RC beams (kN);

V_f^{exp} = value of V_f obtained from experiments (kN);
 $V_{R,f}^{model}$ = predicted value of V_f obtained from each specific model (kN);
 w_f = width of CFRP reinforcement (mm);
 x', y', m', n' = parameters of the *fib* Bulletin 90 model;
 x_j = input variable;
 y_i = known value for specimen i ;
 $\bar{y}$ = mean of known values;
 $\bar{\bar{y}}$ = mean of predicted values;
 α_f = fiber orientation angle;
 β_L = coefficient to compensate for insufficient FRP anchorage length;
 β_w = FRP width-to-spacing ratio;
 Γ_{fm} = bonded joint specific fracture energy;
 γ_i = predicted value for specimen i ;
 ϵ_f = strain in the FRP reinforcement;
 ϵ_{fe} = effective strain in CFRP reinforcement;
 ϵ_{fe}^{exp} = value of ϵ_{fe} obtained from experiments;
 ϵ_{fe}^{pred} = predicted value of ϵ_{fe} ;
 ϵ_{fse} = effective strain in the FRP for shear strengthening according to TR55;
 ϵ_{fu} = ultimate strain of FRP at rupture (MPa);
 ϵ_{swy} = steel yielding strain;
 ϵ_x = longitudinal strain at the middepth of the section;
 θ = inclination angle of CDC;
 θ_{min} = minimum inclination angle of CDC;
 κ_m = global model modification factor;
 λ = ratio of the maximum available bond length to the effective bond length according to CIDAR;
 ρ_f = ratio of CFRP reinforcement;
 ρ_l = longitudinal tensile steel reinforcement ratio;
 ρ_{sw} = ratio of steel stirrups;
 $\rho_{xj,\chi}$ = coefficient of correlation between j th parameter and model error;
 σ = standard deviation;
 τ_{b1m} = mean value of ultimate bond strength (MPa);
 χ_v = model error obtained for V_f ;
 χ_e = model error obtained for effective strain; and
 Ω = objective function.

Supplemental Materials

Data set S1 is available online in the ASCE Library (www.ascelibrary.org).

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Paper 3

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An interpretable machine learning-based model for shear resistance prediction of CFRP-strengthened RC beams using experimental and synthetic dataset

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ABSTRACT

Existing analytical models for predicting the shear resistance of RC beams strengthened with externally bonded CFRP reinforcements exhibit deficient performance due to their inability to accurately capture the complex resisting mechanisms. Combined with significant statistical uncertainties in shear failure, driven by its brittle nature, this further undermines the reliability of these models. To address these limitations, this study leverages Machine Learning (ML) to develop more robust and reliable predictive tool. A rigorous feature-selection process identified eight predictors as the most influential. Subsequently, nine ML-algorithms were trained on a refined experimental dataset comprising 239 beams, with XGBoost emerging as the top performer. This model also outperformed established models like fib Bulletin-90 and ACI 2023 models. However, the limited scope of the experimental dataset constrained the model's predictive performance especially when separately evaluated on beams strengthened with U-wraps, full wraps or side-bonded FRP configurations. Therefore, to achieve a more reliable model a synthetic dataset was generated using Tabular Variational Auto-Encoder. The XGBoost model trained with the synthetic dataset significantly improved the performance of the former model and exhibited better predictions for all strengthening configurations. Finally, to ensure the physical consistency of predictions, values obtained from the SHapley Additive exPlanations method were analysed.

Nomenclature and abbreviation

ACI	American concrete institute
AdaBoost	Adaptive boosting
ANFIS	Adaptive neuro-fuzzy inference system
ANN	Artificial neural network
BFRP	Basalt fibre-reinforced polymer
CDC	Critical diagonal crack
CFRP	Carbon fibre-reinforced polymer
CV	Cross-validation
DT	Decision tree
EBR	Externally bonded reinforcement
ERT	Extremely randomized trees
FRP	Fibre-reinforced polymer
GB	Gradient boosting
GPU	Graphics processing unit
GFRP	Glass fibre-reinforced polymer
KNN	K-nearest neighbour
LightGBM	Light gradient boosting machine
LOF	Local outlier factor

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ML	Machine learning
MLP-ANN	Multi-layer perceptron-ANN
PET-FRP	polyethylene terephthalate-FRP
RBPNN	Resilient back-propagating neural network
RC	Reinforced concrete
RF	Random forest
SHAP	Shapley additive explanation
XGBoost	Extreme Gradient Boosting model
A_f	CFRP cross-section area (mm ²)
A_{fpl}	CFRP cross-section area per unit length (mm)
A_{sw}	Steel stirrups cross-section area (mm ²)
b	Bias
b_w	Width of the beam's web (mm)
b_f	Width of the beam's flange (mm)
C_f	The strengthening configuration factor takes values of 0, 1, and 2 for Fully wrapped, U-wrapped, and side-bonded beams, respectively.
CoV	Coefficient of variation
Cs	Type of cross-section (Rectangular/ T-section)

(continued on next page)

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d_s	The internal arm of the beam (mm)
d_f	The effective height of FRP (mm)
E_f	Elastic modulus of CFRP (GPa)
f_{ib}	Fédération Internationale du Béton
f_{cm}	Concrete mean compressive strength (MPa)
f_{ctm}	Concrete mean tensile strength (MPa)
f_e	Weak learner prediction
f'_c	Concrete characteristic strength (MPa)
f_{fu}	The ultimate strength of FRP (MPa)
f_{swy}	Yield strength of stirrups (MPa)
f_{fm}	Mean value of FRP strength (MPa)
f_{fwm}	Mean value of FRP wrap strength (MPa)
$f_{fwm,c}$	Mean of FRP wrap strength due to rupture (MPa)
$f_{fwm,d}$	Mean of FRP wrap strength due to debonding (MPa)
FM	Failure mode
h	Height of the beam (mm)
h_f	Height of CFRP (mm)
h_{fe}	The effective height of FRP reinforcement (mm)
h_w	Height of the beam's web (mm)
h_{fl}	Height of the beam's flange (mm)
L_e	Effective length
$MAPE$	Mean absolute percentage error
m'	A factor for fib bulletin 90's model
n'	A factor for fib bulletin 90's model
n_f	Number of FRP layers
$RMSE$	Root mean squared error
$p.p.$	Penalty point
R^2	Coefficient of determination
R_c	Corner radius of beam (mm)
r	Pearson coefficient of correlation
s_{ok}	The characteristic value of ultimate slip (mm)
s_f	Spacing of CFRP strips (mm)
s_s	Spacing of stirrups (mm)
T	Number of weak learners
t_0	Thickness of each layer of FRP (mm)
t_f	Thickness of CFRP reinforcement (mm)
$V_{R,f}$	Shear contribution of FRP (kN)
V_f^{exp}	Shear contribution of FRP (kN) (experimental value)
w_f	Width of FRP strips (mm)
x	A factor for fib bulletin 90's model
y	Experimental value of target
y'	A factor for fib bulletin 90's model
α_f	Inclination of FRP fibres
β	Subsample of data
γ	Model prediction
γ_0	Initial prediction
ε_{fe}^{exp}	The effective strain of FRP in the experimental dataset
ε_{fu}	Strain at the failure of the FRP coupon
ε_{fe}	The effective strain in the FRP composite
$\varepsilon_{exp}^{fe, trans}$	The log-transferred ε_{fe}^{exp}
θ	Critical diagonal shear crack inclination angle
μ	Mean value
ρ_f	FRP reinforcement ratio
ρ_{sl}	Longitudinal steel ratio
ρ_{sw}	Transversal steel ratio
ρ_t	Ratio of longitudinal steel in tension
σ	Standard deviation
τ_{b1k}	The characteristic value of ultimate bond stress (MPa)
χ	Model error

1. Introduction

Shear failure, being much lesser ductile compared to flexural failure, is addressed in design guidelines by incorporating provisions to ensure that it is preceded by more ductile failures [1,2]. The application of Externally Bonded Reinforcement (EBR) using Carbon Fibre-Reinforced Polymer (CFRP) composites is a widely used technique to increase the shear resistance of Reinforced Concrete (RC) beams [3,4]. The shear resistance resulting from this intervention is typically estimated using empirical or semi-empirical models [4–6]. However, these models have been found to be inadequate in terms of performance [5,7,8]. This is primarily due to the complex nonlinear relationship between shear resistance and beam features, as well as the significant uncertainties

associated with shear failure events, which is a consequence of the brittle nature of such failures. Developing analytical models that can accurately capture this complex relationship and account for these uncertainties is a difficult task. Nevertheless, the utilization of Machine Learning (ML) algorithms holds the promise of constructing models with superior predictive performance by capturing the complex nonlinear relationship inherent in the system [9–14].

Prior to this study, several attempts were made to utilize different ML algorithms for predicting the contribution of CFRP reinforcement to shear resistance of RC beams ($V_{R,f}$), highlighting their potential advantages over existing empirical and semi-empirical models [10,15–18]. These models include Artificial Neural Network (ANN), Adaptive Neuro-Fuzzy Inference System (ANFIS), Resilient Back-Propagating Neural Network (RBPNN), eXtreme Gradient Boosting (XGBoost), Random Forest (RF), CatBoost, Light Gradient Boosting Machine (LightGBM), Adaptive Boosting (AdaBoost), k-Nearest Neighbour (KNN), and ElasticNet. More detail about these studies is provided in Table 1, which provides reference, considered types of FRP, configurations (Fully wrapped: F; U-wrapped: U; and Side-bonded: S), and beam cross-sections (rectangular: R; T-section: T) in the dataset of previous and current study. Additionally, it presents the algorithms tested, the input and target variables considered, the size of datasets in each study, the accessibility of the model, and the Coefficient of Variation (CoV) of the model errors, as determined by the available data in each particular study.

In Fig. 1, the symbols representing various geometric features of RC beams strengthened with EBR-CFRP systems are presented, as also utilized in this study (see also Nomenclature and abbreviation section).

While these models have shown promise, some challenges remain advising further exploration in this domain. One of the main challenges within the previous models is their limited accessibility for public use, as they exist in the form of black boxes and require accompanying software for utilization, unlike existing analytical models. Another issue with these models, which aim to directly predict the contribution of CFRP to their shear resistance (V_f), is their reliance on the data from small laboratory-scale beams. These models often struggle with extrapolation, as they are typically trained on limited datasets that do not encompass the variability and complexity of full-scale structures. Wang's model [17] additionally necessitates the inclusion of the Failure Mode (FM) as input parameters, which poses a challenge due to its unknown nature. Ezami's [18] model is developed solely based on RC beams with rectangular cross-sections, rendering it less applicable to real-world scenarios where T-section beams are more common.

The goal of this study is to propose a model with superior predictive performance that addresses the limitations of previous models as well. For this reason, a large and diverse dataset of beams with both rectangular and T-shaped cross-sections was collected to ensure the robust training and generalization of the model [19]. Furthermore, a hybrid approach combining physics-based and ML method was introduced which first predicts the effective strain (ε_{fe}) and then uses this value to predict V_f , thus enhancing the model's ability to generalize and reliably predict the shear resistance of larger, real-world beams.

A dataset comprising over 700 RC beams was compiled from the literature, accessible through the online repository DABASUM [20]. Yet, for enhanced performance, the dataset was resumed to 239 RC beams [21] by eliminating unnecessary data, such as beams strengthened with materials other than CFRP (this is the most used and effective FRP in real strengthening practice of RC structures), reinforcements with anchorage (too many different anchorages, explored in general at research domain), outliers, and unreliable sources (with evidence of abnormal occurrence in the test program). A feature selection analysis was performed and based on this analysis a set of beam features were identified for inclusion in the model. These features are as follows: the parameter $\rho_f E_f$ which combines $\rho_f = A_f / (b_w s_f)$, representing the ratio of CFRP reinforcement area, with E_f the modulus of elasticity of the CFRP reinforcement; height of FRP reinforcement (h_f); compressive strength of

Table 1
Available ML-based models to predict V_{Rf}

Model	Types of FRP	Strengthening configuration	Cross-section	Algorithms	Data size	Input features	Target	Access to model	CoV
Tanarslan et al. (2012) [15]	A/G/CFRP	U, F	R, T	ANN	84	$C_s, a/d_s, h_f, E_f, \alpha_f, b_{wp}, t_f, d_s, \epsilon_{fu}$	V_{Rf}	N/A	0.32
Naderpour and Alavi (2017) [10]	A/G/CFRP	U, F	R, T	ANFIS	89	$\epsilon_{fu}, h_f, f_c, w_f, S_f, \alpha_f, a/d_s, E_f, t_f$	V_{Rf}	N/A	0.23
Abuodeh et al. (2020) [16]	CFRP, GFRP	S, U	R, T	RBPNN	120	$b_{wp}, d_s, a/d_s, f_c, f_{swp}, A_{sp}, t_f, w_f, h_f$	V_{Rf}	N/A	0.29
Wang et al. (2023) [17]	CFRP	S, U, F	R, T	ANN, XGBoost, RF, CatBoost, LightGBM, AdaBoost	315	$w_f/S_f, f_{fu}, E_f, a/d_s, \rho_{sw}, h_f, t_f, C_f, FM, C_s, \alpha_f$	V_{Rf}	N/A	0.47
Ezami et al. (2023) [18]	C/G/B/PET-FRP	S, U, F	R	XGBoost, GB, RF, AdaBoost, KNN, ElasticNet	196	$b_{wp}, d_s, f_c, f_{swp}, A_{sp}, s_s, a/d_s, E_f, \epsilon_{fu}, f_{fu}, t_f, w_f, S_f, h_f, \alpha_f$	V_{Rf}	N/A	0.36
Proposed	CFRP	S, U, F	R, T	XGBoost, RF, GB, KNN, AdaBoost, MLP-ANN, DT, LightGBM, ERT	experimental 239 Synthetic 20,000	$E_f, t_f, f_{com}, \rho_{sw}, \rho_b, C_s, \alpha_f, f_{swp}, A_{sp}, h_f$	$\epsilon_{fe} \rightarrow V_{Rf}$	Web tool	0.28
							$\epsilon_{fe} \rightarrow V_{Rf}$	Web tool	0.22

concrete (f_{cm}); the ratio of steel stirrups ($\rho_{sw} = A_{sw}/b_w s_s$); the ratio of longitudinal reinforcement ($\rho_l = A_{sl}/b_w d_s$); the strengthening configuration factor (C_f being 0, 1, and 2 for Fully wrapped, U-wrapped, and side-bonded beams, respectively); the fibre orientation angle respect to the longitudinal axis of the beam (α_f); and the yielding strength of steel stirrups (f_{swy}). Notably, the failure mode was neither considered during the feature selection process nor used as a predictor in the model.

The characteristics of this dataset was thoroughly explained, and the gaps within this dataset were discussed. It was observed that scarcity of data and the presence of gaps in beams properties constraints the predictive performance of ML algorithms. In fact, most ML algorithms, especially tree-based ones, operate by returning predictions based on the closest known data points, and consequently, they may not yield accurate results, particularly when there are substantial gaps in the dataset. Achieving an enhanced predictive performance requires increasing and diversifying the dataset. However, collecting more data through experimental program is challenging and costly. The generative Artificial Intelligence (AI) known as Tabular Variational Autoencoder (TVAE) is a cutting-edge technique to tackle this challenge, which is utilized to increase the size of train dataset by generating synthetic data [22,23]. TVAe represents a specialized application of the Variational Auto Encoder (VAE), which finds extensive application across various data types, including time series [24], sequential data [25], images [26], and more.

To demonstrate the improvement achieved in predictive performance by incorporating the synthetic dataset, initially nine widely used ML-based algorithms were trained with experimental dataset, aiming to identify the best-performing model. A sufficiently large and representative portfolio of ML algorithms were selected, considering also their prevalence in similar studies [27], in order to have high probability of obtaining a model with higher predictive performance than existing ones. For the models trained with the experimental dataset, 80 % of the data was utilized for training, with the remaining 20 % reserved for testing. Subsequently, the analysis led to the conclusion that XGBoost emerged as the best-performing algorithm for experimental dataset demonstrating a Root Mean Square Error (RMSE) of 19.0 kN, Mean Absolute Percentile Error (MAPE) of 27.7 %, coefficient of determination (R^2) of 0.87, Pearson coefficient of correlation (r) of 0.96, and a model uncertainty distribution with CoV of 0.28 for the test dataset.

To determine whether employing the synthetic dataset generated by TVAe would enhance model performance, an XGBoost model was trained exclusively on the synthetic data. The model's effectiveness was then assessed using the experimental dataset, following the "Train on Synthesized – Test on Real" approach [28]. The overall predictive performance of the trained model has demonstrated significant improvement evidenced by RMSE of 12.37 kN, MAPE of 18.3 %, R^2 of 0.94, r of 0.97, and a model uncertainty distribution with CoV of 0.22 over the entire experimental dataset. This signifies a substantial enhancement compared to the XGBoost model trained with experimental dataset. Regarding the other models listed in Table 1, a fair comparison is challenging because assessing these models using the current study's dataset would require access to those models, which is not available. However, Table 1 presents the CoV of the model errors for each model based on the datasets used in their respective studies. This comparison indicates that the proposed model delivers superior performance and provides more reliable predictions. This model allows the design of safer and more cost-effective shear strengthening solutions than existing models.

It is noteworthy to mention that the hyperparameters of the models in this study were fine-tuned using Bayesian optimization technique [29–31] and 10-fold cross validation (CV). Additionally, to enhance predictive performance, the predictors were normalized, and the target variable was log-transformed.

To highlight the capabilities of the ML-based models in contrast to conventional models, the results of these models are compared against two renowned models, namely fib Bulletin 90 [1], and ACI 2023 [2], the

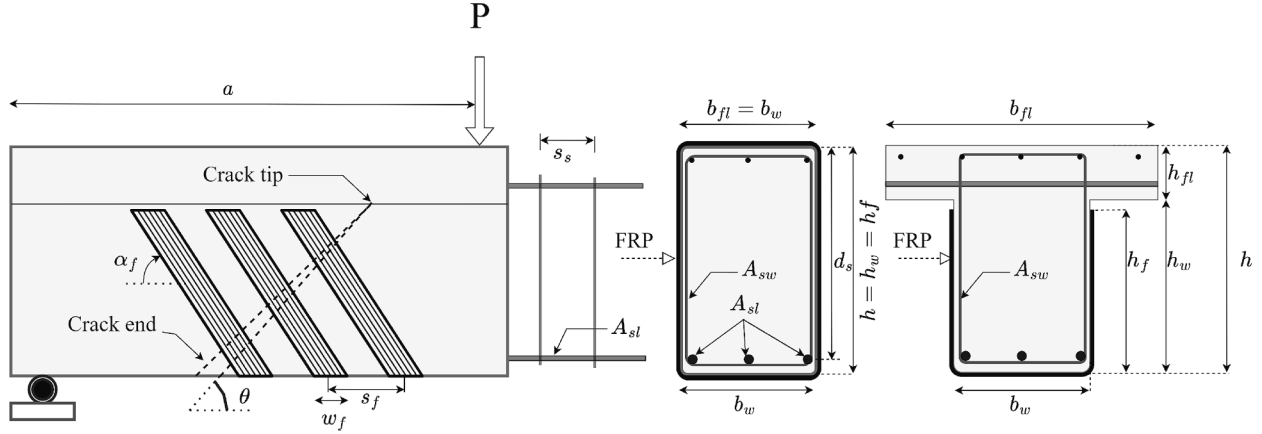


Fig. 1. Symbols for geometric properties of RC beam strengthened with the EBR-CFRP system. The side view (left), the cross-section of a rectangular beam (middle), the cross-section of a T-section beam (right).

formulations of which are provided in Appendix 1. Furthermore, Shapely Additive exPlanation (SHAP) [32] values are employed to assure that the proposed model is producing physical compatible predictions. Finally, to facilitate ease of access for predictions, an open-source web tool with graphical user interface (GUI) was developed that can be accessed through <https://shear-frp-dabasum.streamlit.app/>.

2. Dataset collection, screening, and synthetic dataset generation

The dataset used in this study was partially obtained from the established online data repository called DABASUM [20] and subsequently updated with recent data. The dataset archives comprehensive information about the tested beams, including their geometric properties (e.g., length, cross-section, shear span, and test set-up), mechanical properties (e.g., concrete compressive strength and yielding strength of stirrups), CFRP reinforcement details, and test results. To enhance the model's prediction performance, specimens in the dataset that exhibited the following characteristics were removed: (i) shear resistance improvement less than five percent (which could be attributed to improper implementation of CFRP reinforcement or other unforeseen issues); (ii) application of anchorage systems; (iii) any reinforcement

material other than CFRP; (iv); effective strain in the CFRP larger than ultimate strain; (v) beams with an exceptionally high flexural reinforcement ratio ($\rho_l > 10\%$); due to the rarity in real-world applications); (vi) shear span ratio (a/d_s) lower than 2.2 (in light of the study's focus on slender beams); and (vii) lacking required information.

2.1. Outlier Detection

There is yet the possibility that the screened dataset contains anomalous specimens known as outliers. These outliers could arise from various sources, including human error, inaccuracies in recording, or miscalculations. The presence of outliers in the dataset has the potential to adversely impact the accuracy of the prediction model [17,33]. Given that the beams in the dataset have multiple features, it is not advisable to classify a specimen as an outlier based solely on one of its features. To detect the outliers within such intricate data, it is prudent to use advanced mathematical procedures, such as the Mahalanobis [7,34] or Local Outlier Factor (LOF) algorithms [33,35]. These advanced methods excel in identifying anomalies that may prove challenging to distinguish through visual inspection, as opposed to more rudimentary techniques like the Inter Quartile Range (IQR) method. In this study, the LOF is utilized since it simplifies the multidimensional problem into a one-

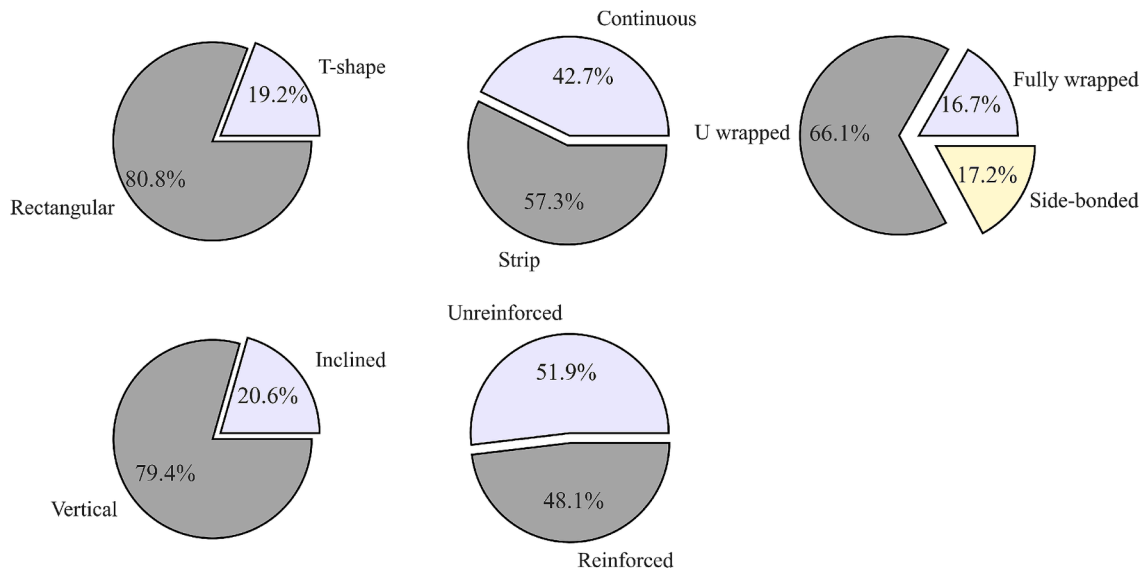


Fig. 2. The distribution of beam specimen characteristics in the dataset: cross-section shape (top left), discrete/continuum arrangement of the CFRP sheet (top middle), CFRP strengthening configuration (top right), fibre orientation (bottom left), and transversal steel reinforcement state (bottom right).

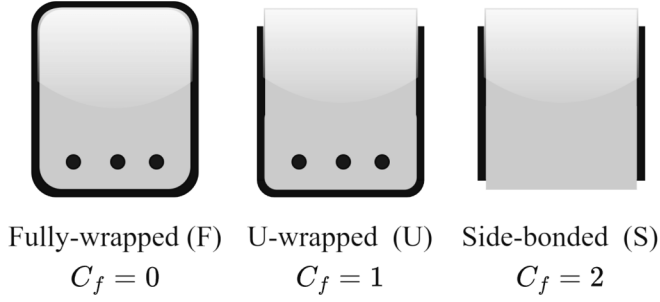


Fig. 3. Strengthening configurations: Fully wrapped (left), U-wrapped (Middle), Side-bonded (right).

dimensional problem. By evaluating the LOF value and using the IQR method, outliers are identified and subsequently removed from the database.

2.2. Data diversity and gaps

After applying this data screening process and removing outliers, 239 specimens remained [21]. As shown in Fig. 2, this dataset is composed of 19.2 % of RC beams with T-sections and 80.8 % of rectangular section beams. Among these, 57.3 % of the beams are strengthened with discrete strips, while 42.7 % are reinforced with continuous sheets. Additionally, 66.1 % of the beams are U-wrapped, 16.7 % are fully wrapped, and 17.2 % are side-bonded (a schematic representation of different strengthening configurations is given in Fig. 3). Moreover, within the dataset, 79.4 % of the beams were reinforced with CFRP materials oriented perpendicular to the beam axis, while the remaining 20.6 % were strengthened with CFRP fibres inclined at a 45-degree angle to the beam axis. Furthermore, 51.9 % of the beams lacked any transversal steel reinforcement, while the remaining 48.1 % featured varying ratios of transversal steel reinforcement. According to Fig. 4-left, there is a

comparable distribution of data concerning the presence or absence of existing transversal steel reinforcement in both rectangular and T-section RC beams. In contrast, the dataset predominantly comprises T-section beams that are U-wrapped, with no samples available for other strengthening configurations, except one that is side-bonded (Fig. 4-right). Even for beams with a rectangular cross-section, the majority are strengthened with U-wraps.

Fig. 5 also provides very important information about steel stirrup ratio, axial rigidity of CFRP reinforcement and dimension of RC beams (using the internal arm of the flexural reinforcement, d_s) with different strengthening configurations. The beams are organized into five categories based on the value of ρ_{sw} . This involves dividing the data into four intervals representing 0–25 % (Q1), 25–50 % (Q2), 50–75 % (Q3), and 75–100 % (Q4), with an additional group for beams with no steel stirrups. According to this plot (Fig. 5-left) higher values of $\rho_f E_f$ are used for side-bonded beams and U-wrapped beams compared to fully wrapped beams in this dataset. Except for two beams, those with side-bonded reinforcement in this dataset lack steel stirrups. In addition to the fact that this strengthening configuration is more prone to debonding, the absence of sufficient data on side-bonded reinforcement appears to have played a role in the decision not to integrate provisions for it in fib Bulletin 90 [1], CNR 2013 [36], and DAFStb guidelines. Additionally, a small variation in beam dimensions (represented by its internal arm, d_s in mm) is observed for beams with different values of $\rho_f E_f$, ρ_{sw} and C_f . Based on this observation, it becomes evident that there is a shortage of specimens representing various ranges of parameters. Expanding the dataset by incorporating more samples in the underrepresented zone, either through new experimental campaigns or synthetic data generation, is expected to significantly enhance the training of ML algorithms.

2.3. Generation of synthetic dataset with TVAE model

Tabular datasets, like the one collected in this paper, with N_c columns of continuous variables (C_i such as f_{cm}) and N_d columns of

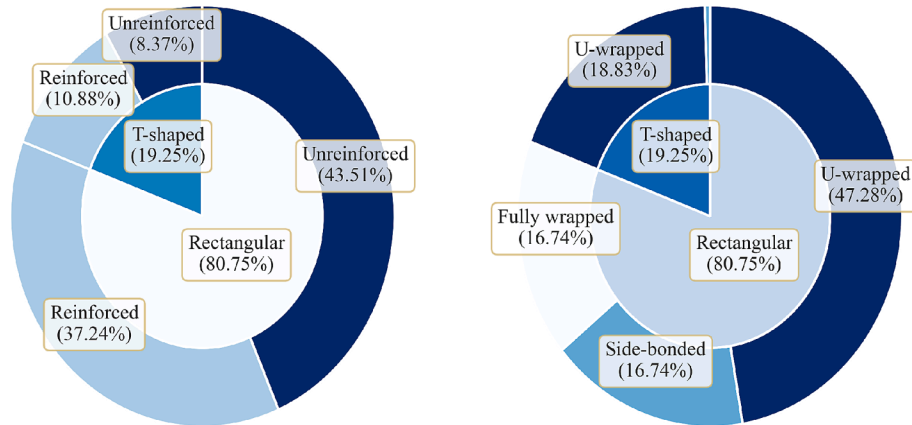


Fig. 4. Insights into the reinforcement status (left) and strengthening configurations (right) employed in rectangular and T-shaped RC beams.

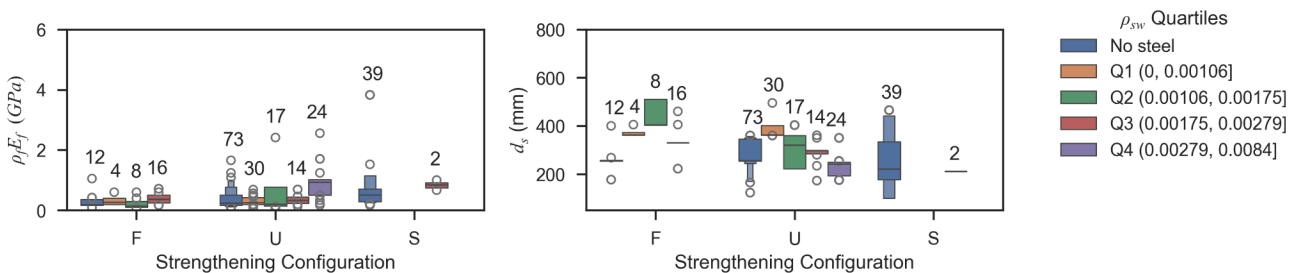


Fig. 5. Distribution of $\rho_f E_f$ and d_s across different strengthening configurations for varying ρ_{sw} , categorized into quartiles (experimental dataset).

categorical variables (D_i such as C_f), follow an unknown joint probability distribution function (jpdf), $P(C_{1:N_c}, D_{1:N_d})$. Being $r_j = \{c_{1,j}, \dots, c_{N_c,j}, d_{1,j}, \dots, d_{N_d,j}\}$ an observed row from this joint distribution. Sampling from this jpdf is interesting as it can generate additional data known as synthetic data. Employing synthetic data not only can enhance the size of training dataset for ML models but provides more samples from less represented areas as well, consequently improving the model's performance. Therefore, developing data synthesizer models can serve as an interesting alternative approach to the costly and time-consuming process of expanding the dataset size and addressing underrepresented samples through experimental studies. However, since these jpdfs are typically not defined in closed-form functions, sampling from them poses a significant challenge [37]. Among various techniques exist in the literature [38] to address this challenge, this study employs a specialized neural network generative model for tabular datasets known as TVAE [22,39]. It has demonstrated the ability to effectively learn the jpdf of tabular datasets with complex structures comprising columns of continuous and categorical (i.e., discrete) variables. Moreover, this model is engineered to generate suitable synthetic datasets, particularly addressing tabular datasets featuring non-Gaussian continuous columns and categorical columns with imbalanced distributions. These abilities are certainly advantageous for this study, where RC beams with fully wrapped or side-bonded strengthening are less presented compared to U-wrapped beams (making C_f an imbalanced categorical column). Additionally, as shown later in Fig. 12, the continuous columns of the dataset exhibit non-Gaussian distributions.

TVAE comprises two neural networks referred to as the probabilistic encoder and decoder. In this study, as represented in Fig. 6, the encoder is composed of three hidden layers, each containing 300 nodes ($h_{l,k}$; l being the layer number and k the node number) and employing a Rectified Linear Unit (ReLU) activation function, which learns a function h that maps entries of the model, r to a joint probability distribution q_θ of a latent Gaussian variable z ($q_\theta(z_j|r_j) \sim \mathcal{N}(\mu, \sigma^2 \mathbf{I})$; where $\mu = h_\mu(r)$ and $\sigma = h_\sigma(r)$). A similar architecture was also used for the decoder, consisting of three layers with 300 nodes at each, and utilizes ReLU activation function. In addition, the outputs of the final layer undergo transformations using tanh function for continuous variable and SoftMax function for categorical variables (see Fig. 6). The decoder learns a function g that re-maps the distribution of latent values, q_θ to a distribution over observations $p_\theta(r_j|z_j)$ given as follows:

$$p_\theta(r_j|z_j) = \prod_{i=1}^{N_c} P(\hat{\alpha}_{ij} = \alpha_{ij}) \prod_{i=1}^{N_c} P(\hat{\beta}_{ij} = \beta_{ij}) \prod_{i=1}^{N_d} P(\hat{d}_{ij} = d_{ij}) \quad (1)$$

where $\hat{\alpha}_{ij}$ is random variable of normal distribution ($\hat{\alpha}_{ij} \sim \mathcal{N}(\bar{\alpha}_{ij}, \delta_i)$) with a mean of $\bar{\alpha}_{ij}$ and standard deviation of δ_i . $\hat{\beta}_{ij}$ and $\hat{d}_{ij}$ are categorical random variables both of which follow categorical Probability Mass Function (PMF).

After training TVAE model using experimental dataset and utilizing the evidence lower-bound (ELBO) loss [22], a synthetic dataset with any size can be sampled from $p_\theta(r_j|z_j)$. The categorical variables are basically the obtained $\hat{d}_{ij}$ and the continuous variables ($c_{i,j}$) can be obtained from an inverse procedure used to normalize $c_{i,j}$ ($\alpha_{ij} = (c_{i,j} - \eta_k)/4\phi_k$) for the sampled mode with the mode of η_k and standard deviation of ϕ_k . The selected mode is recognized by $\hat{\beta}_{ij}$. The estimation of the number of modes for each continuous columns (m_i) and the corresponding η_k and ϕ_k of each mode is conducted using a variational Gaussian mixture model.

Performing this approach a synthetic dataset with 20,000 specimens (as more generated date had not significant impact on the predictive performance of the ML models) were generated. As shown in Fig. 7, the TVAE provided dataset with very similar statistical characteristics to experimental dataset. However, as can be seen in Fig. 8, the gaps present in the real dataset (see Fig. 5) have been effectively filled, significantly increasing the number of specimens available for training.

The heatmap of Pearson correlation coefficient for both datasets and the difference between the coefficients of these two datasets is illustrated in Fig. 9. As can be seen, the Pearson correlation coefficients in the synthetic dataset closely matches that of the experimental dataset, indicating a high degree of similarity between them. The colour bars on the right side of each plot indicate the Pearson correlation coefficient, with red representing 1.0 (perfect positive linear relationship) and blue representing -1 (perfect negative linear relationship). This visualization also confirms that the synthetic dataset closely mimics the correlation patterns observed in the experimental dataset. To further validate the quality of the synthetic dataset generated by the TVAE model, the Kolmogorov-Smirnov (K-S) test was applied. The K-S test measures the maximum difference between the empirical cumulative distribution functions (CDFs) of the real and synthetic datasets. The K-S statistic can range from 0 to 1, where a value of 0 indicates perfect alignment between the two distributions. As shown in Table 2, the small observed K-S statistics suggest a close match between the distributions of two datasets, indicating that the synthetic dataset adequately well replicates the statistical properties of the original dataset.

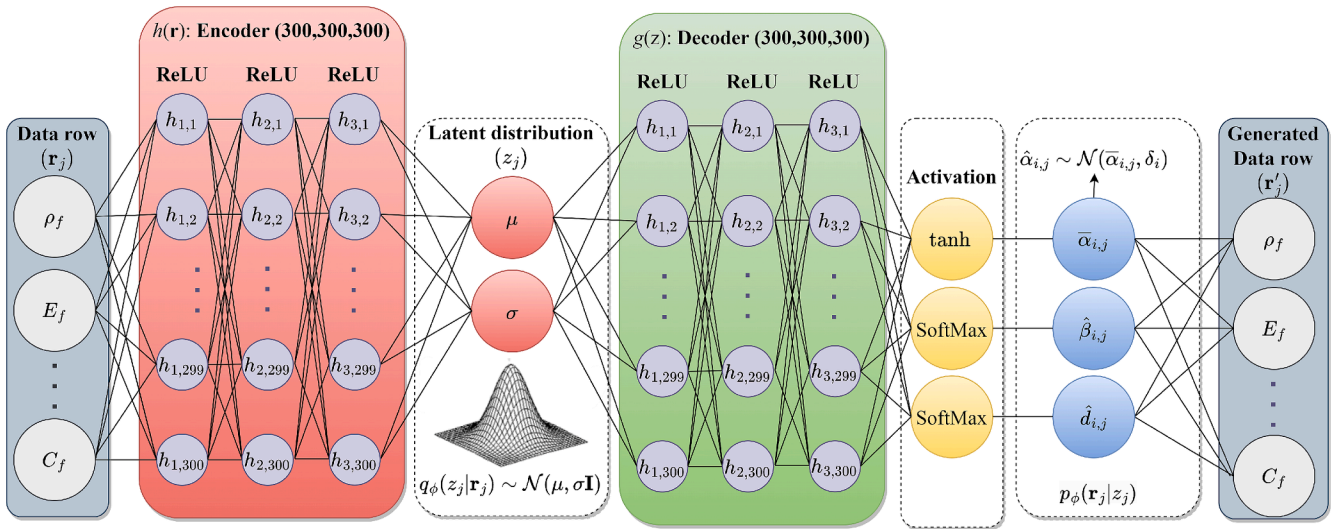


Fig. 6. TVAE model architecture used to generate synthetic data.

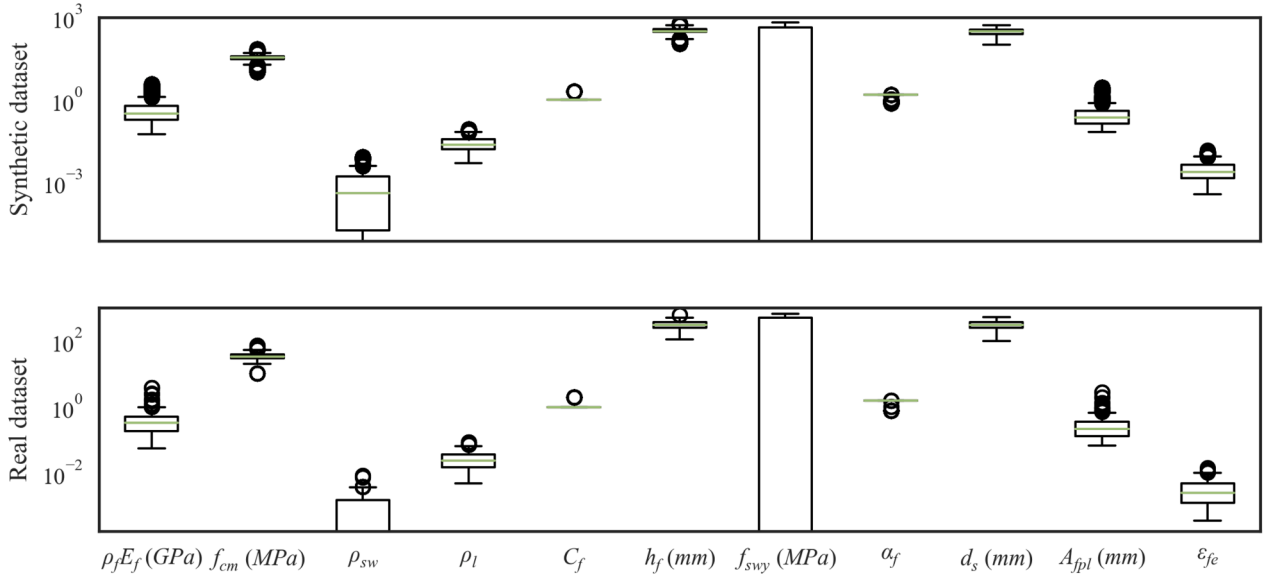


Fig. 7. Comparing the statistical characteristics of the synthetic and experimental datasets.

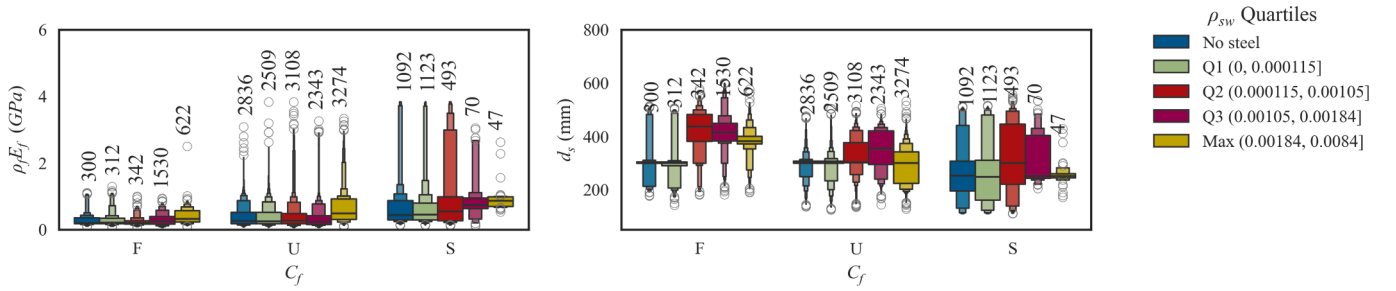


Fig. 8. Distribution of $\rho_f E_f$ and d_s across different strengthening configurations for varying ρ_{sw} , categorized into quartiles (synthetic dataset).

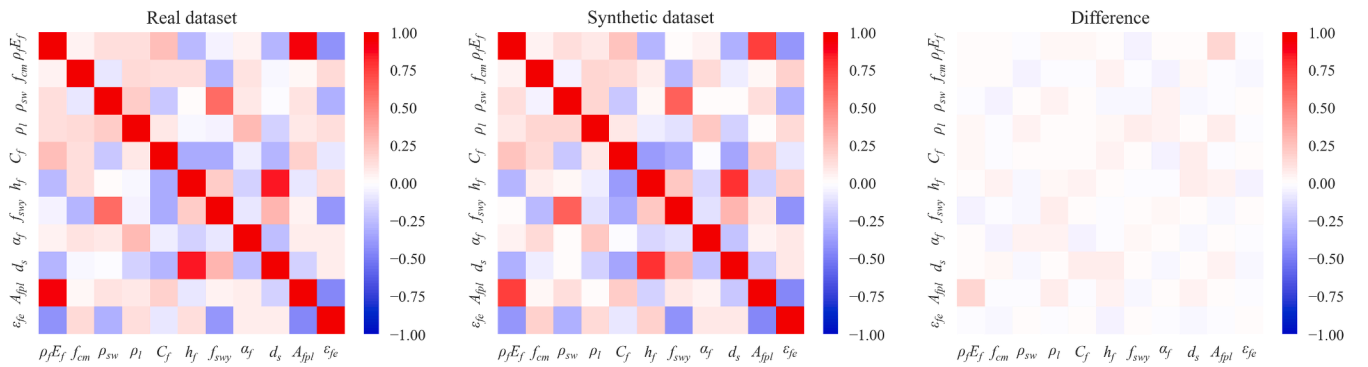


Fig. 9. Correlation between features of the datasets. Experimental dataset (left); Synthetic dataset (middle); Difference between correlations (correlation coefficients in experimental dataset minus correlation coefficients in synthetic dataset) (right).

Table 2

K-S test results for assessing the similarity between the CDF of synthetic and experimental datasets.

Parameters	$\rho_f E_f$	f_{cm}	ρ_l	ρ_{sw}	C_f	h_f	α_f	f_{swy}	A_{fpl}	d_s	ϵ_{fe}
K-S statics	0.06	0.14	0.08	0.32	0.05	0.16	0.08	0.10	0.09	0.08	0.07

3. Research methodology

3.1. Hybrid prediction model

Unlike previous studies, as shown in Table 1, the prediction of $V_{R,f}$ is obtained based on a hybrid model that combines physics-based principles and data-driven approaches. This approach results into better predictive performance, particularly for large-scale beams that are infrequently represented in the available dataset. This hybrid model entails a ML-based model to predict the effective strain (ϵ_{fe}), which will be further elaborated in the next section, followed by a truss-model to predict $V_{R,f}$, determined as follows:

$$V_{R,f}^{\text{model}} = A_{fpl} \epsilon_{fe} h_f E_f (\cot \theta + \cot \alpha_f) \sin \alpha_f \quad (2)$$

where A_{fpl} is the cross-sectional area of CFRP per unit length calculated as $2w_f t_f \sin \alpha_f / s_f$ for discrete strips and $2t_f \sin \alpha_f$ for continuous CFRP sheets, being w_f , t_f and s_f the width, thickness and spacing of CFRP reinforcement (see Fig. 1). For the inclination angle of the Critical Diagonal Crack (CDC), θ , since its determination is complex and generally is not available on the dataset, a provisional value of 45° is assumed for it through the course of this study. Although this approximation is not entirely accurate, it was explored that the result of the hybrid model is relatively insensitive to this variable. Therefore, to maintain simplicity in the model, the use of an accurate value for θ was avoided. While using the correct value of θ could lead to more precise predictions of ϵ_{fe} , its impact on the prediction of $V_{R,f}$ remains minimal.

3.2. Log transformation of effective strain

The value of effective strain obtained from experimental study $\epsilon_{fe}^{\text{exp}}$ can be derived from Eq. (2), which is given as follows:

$$\epsilon_{fe}^{\text{exp}} = \frac{V_f^{\text{exp}}}{A_{fpl} \times h_f E_f (\cot \theta + \cot \alpha_f) \sin \alpha_f} \quad (3)$$

where V_f^{exp} is the contribution of CFRP to the shear resistance of the beams recorded in the dataset, and it is the difference between the shear resistance of the strengthened and the reference beam.

The probability and cumulative distributions of $\epsilon_{fe}^{\text{exp}}$ is demonstrated in Fig. 10 (left). As it can be seen, $\epsilon_{fe}^{\text{exp}}$ has an imbalanced (left-skewed) distribution. Such imbalanced distribution can pose challenges for ML models, as they are typically optimized for symmetrically distributed data. In this study, the $\epsilon_{fe}^{\text{exp}}$ undergoes a logarithmic transformation, followed by min-max scaling as given in Eq. (4). This transformation

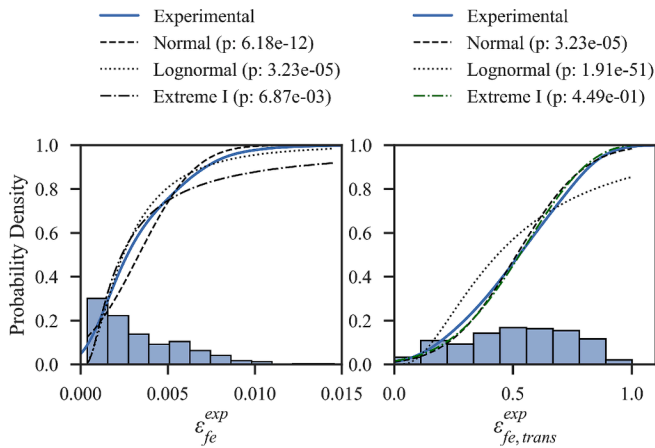


Fig. 10. Distribution of effective strain in the main dataset (left), and log transformed effective strain (right).

results into a more symmetric target variable.

$$\epsilon_{fe,trans}^{\text{exp}} = \frac{\ln(\epsilon_{fe}^{\text{exp}}) - \min(\ln(\epsilon_{fe}^{\text{exp}}))}{\max(\ln(\epsilon_{fe}^{\text{exp}})) - \min(\ln(\epsilon_{fe}^{\text{exp}}))} + 0.001 \quad (4)$$

In this equation a value of 1×10^{-3} is added to assure to have none zero results as some certain models instability problems may arise when the target value is exactly equal to zero [10,40].

The probability and cumulative distributions of the transformed effective strain ($\epsilon_{fe,trans}^{\text{exp}}$) are shown in Fig. 10 (right). The distribution of $\epsilon_{fe,trans}^{\text{exp}}$ appears well-balanced. To confirm that $\epsilon_{fe,trans}^{\text{exp}}$ has a more appropriate and balanced distribution compared to the original strain $\epsilon_{fe}^{\text{exp}}$, a hypothesis testing method using the K-S test was conducted. This test evaluated the fit of three commonly used distributions in practical engineering: Normal, Lognormal, and Extreme Type I distributions. In this paper, the K-S test was adopted with a significance level of 0.05. According to the K-S test, if the p-value is higher than 0.05, it can be inferred that the data likely follows the tested distribution. Conversely, if the p-value is lower than 0.05, the data does not follow that distribution. Additionally, a higher p-value indicates a closer fit of the hypothesis distribution to the actual data. The results of the K-S test are presented in Fig. 10. The determined p-value of 0.45 for the Extreme I distribution suggests robust evidence that $\epsilon_{fe,trans}^{\text{exp}}$ can be introduced by a common probability density function. This characteristic significantly enhances the predictive performance of ML-based models and, hence, is considered for training the models.

4. Developing Machine learning models

4.1. Overall framework

Fig. 11 presents the overall framework adopted in this study, comprising eight key components (the numbers correspond to those indicated in the blocks of Fig. 11):

- 1- Updating and enhancing the existing dataset of DABASUM.
- 2- Conducting various preprocessing steps, including data screening, outlier elimination, and transformation of the target variable (i.e., ϵ_{fe}).
- 3- Implementing a feature selection procedure to improve model performance by selecting statistically and physically meaningful features.
- 4- Training nine machine learning algorithms (XGBoost, RF, GB, KNN, AdaBoost, MLP-ANN, Decision Tree, LightGBM, and Extremely Randomized Trees) using 80 % of the dataset (Train dataset). Hyperparameters are tuned using Bayesian optimization [29,31,41] to maximize the R^2 with 10-fold cross-validation (CV) method.
- 5- Determining the best performing model based on various statistical evaluation metrics using the remaining 20 % of the dataset (Test dataset).
- 6- Training the selected best performing algorithm with the synthetic dataset.
- 7- Comparing the performance of the model trained with the synthetic dataset, with the one from the model trained with experimental dataset, as well as with the one from the most used models by the design community in shear strengthening of RC elements with EBR-CFRP systems, namely, ACI 2023 [2] and fib Bulletin 90 [1].
- 8- Utilizing a game theory-based interpretation tool called SHAP to provide deeper insights into the model.

The first two procedures were discussed in previous sections. In following sections firstly, a comprehensive study on the relation between the features and ϵ_{fe} is executed, then the feature selection process is introduced. Then, a succinct overview of machine learning algorithms

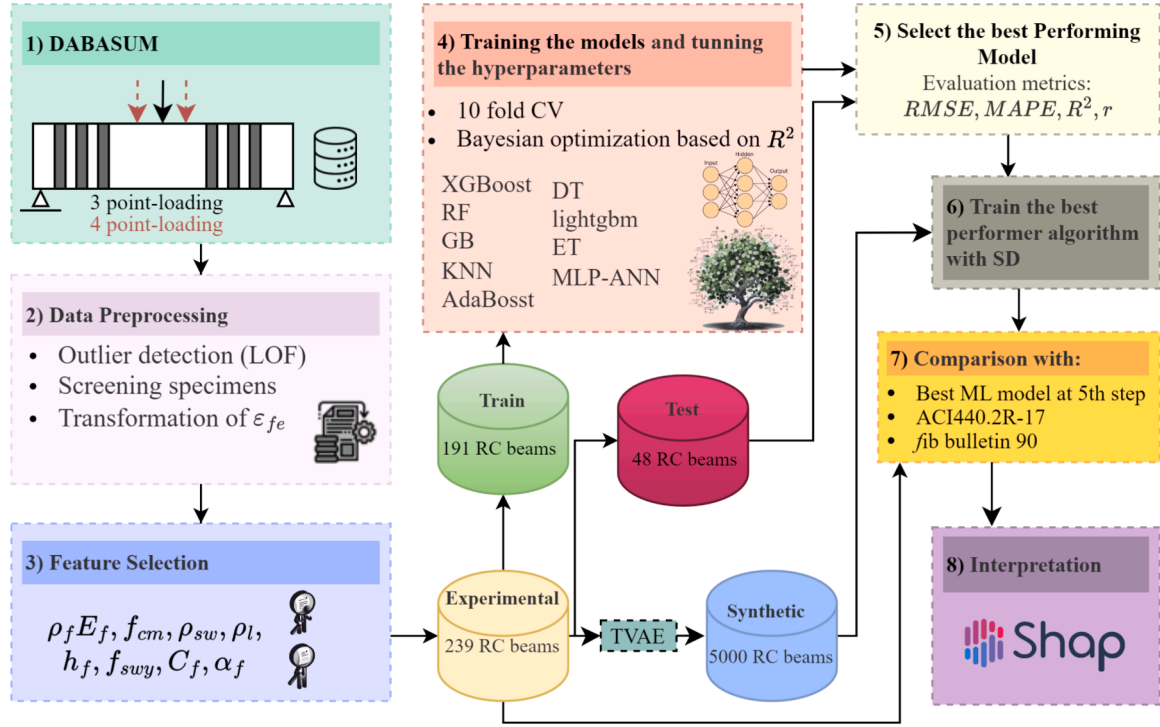


Fig. 11. Overall framework of this study.

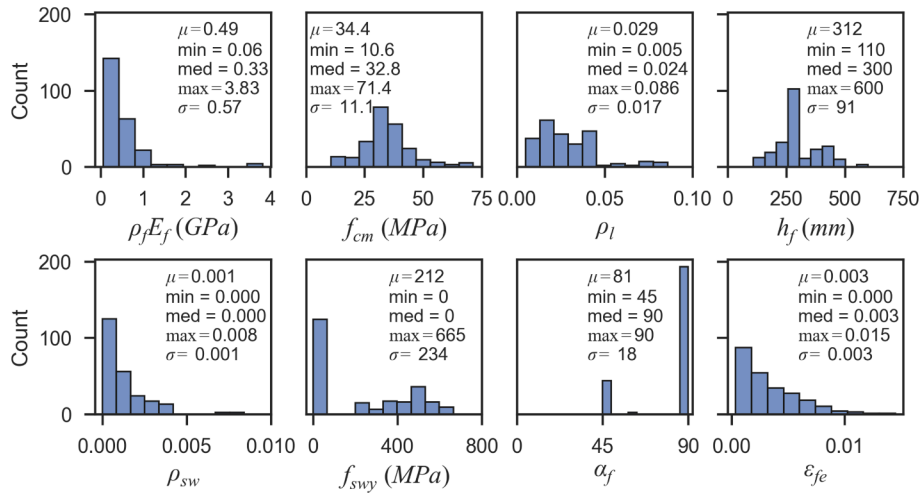


Fig. 12. Probability distribution of input features.

used in this study is provided. The Bayesian optimization, which is conducted for fine-tuning ML algorithms hyperparameters, is introduced in section 4.4. Furthermore, a short description of the formulation of two selected existing models (in ACI 2023 and fib Bulletin 90) is provided in Appendix 1. The best algorithm among the nine trained ML models is identified (XGBoost_{real}), and a new model was trained using generated synthetic dataset (XGBoost_{syn}). Subsequently, the predictive performance of this model is evaluated by comparing it to ACI 2023 and fib Bulletin 90 models, utilizing the experimental dataset, as well as with the best performer model from procedure (5) using the test dataset. Finally, SHAP is introduced as a tool for interpreting the model developed in this study [32].

4.2. Feature selection

Feature selection is crucial in data analysis and machine learning to streamline the modelling process, improve model performance, and enhance interpretability [42]. It helps combat the difficulties of dimensionality, reduces overfitting, and allows models to focus on the most influential predictors. As a result, a feature selection procedure is employed by choosing a subset of features that are already known to exhibit significant correlation with ε_{fe} . These selected features include: $E_f \rho_f$, f_{cm} , h_f , ρ_{sw} , ρ_l , C_f , α_f and f_{swy} . Fig. 12 displays the probability distribution and statistical details of each feature, including their mean (μ), minimum (min), median (med), maximum (max), and standard deviation (σ). Fig. 12 reveals a non-uniform distribution of input features, wherein certain values of some variables exhibit higher frequencies,

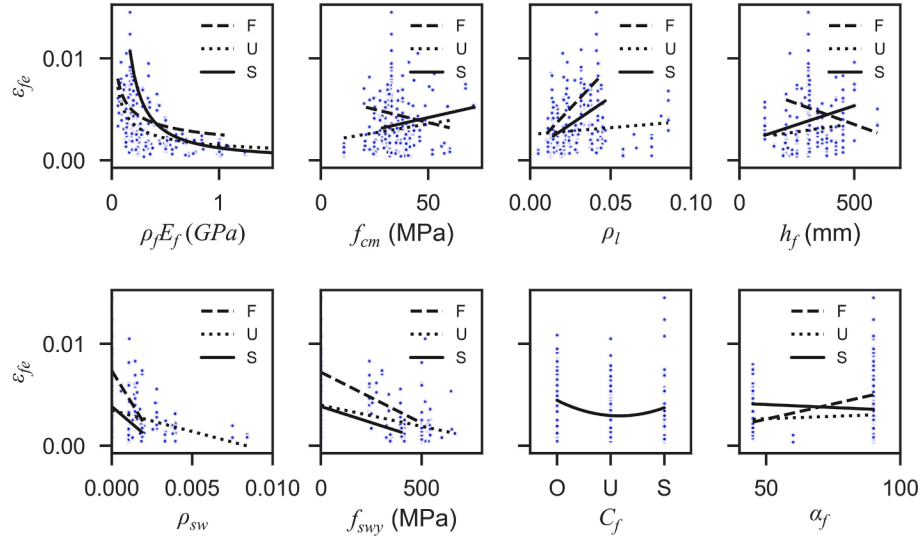


Fig. 13. Relation between effective strain and selected input variables.

such as in the case of α_f and f_{swy} . Conversely, variables like ρ_{sw} , $\rho_f E_f$, and ρ_l display values with rare occurrences. This observation underscores the advantage of using the synthetic dataset, which enhances predictive performance, particularly for these underrepresented observations. Meanwhile, in training stage the input variables were standardized and normalized to fall within the [0, 1] range [40], for improved accuracy and stability of models, especially relevant for the Multi-Layer Perceptron Artificial Neural Network (MLP-ANN) algorithm.

A comprehensive understanding of the relationship between the target variable (ϵ_{fe}) and individual input variables can be challenging, attributed to the variations in several input variables among the specimens in the collected dataset. This complexity further increases when the problem contains high levels of aleatory uncertainties and noisy observations. For example, in the collected dataset, the samples C2-EB2 and C2-EB2R [43] are identical RC beams reinforced with the same internal reinforcement and external FRP strengthening. Despite their similarities, V_f significantly varies, from 14.4 kN in C2-EB2 to 23.2 kN in C2-EB2R. However, to comprehend the effect of each of these input variables on ϵ_{fe} , the scatterplots of the specimens in the refined dataset (blue dots) are presented in Fig. 13. Three trend lines are plotted to offer a better insight into the variation of ϵ_{fe} with changes in the input variables. The dashed black line corresponds to fully wrapped beams, the dotted black line represents U-wrapped beams, and the solid line identifies side-bonded beams.

The following observations were made for each variable:

An increase in $\rho_f E_f$ significantly decreases ϵ_{fe} , which is physically justifiable since higher stiffness of the FRP systems introduces higher bond stresses to the concrete substrate, promoting premature debonding failure [44]

ϵ_{fe} increases with f_{cm} for U-wrapped and side-bonded RC beams, which is also physically justified, since in this type of configurations the rupture of the FRP is by cohesive debonding through concrete substrate (systematically observed in the post-inspection of the failed FRP [45,46]), which resistance increases with the quality of the concrete. However, slight declining trend is observed for fully wrapped beams, which may be caused by an increase of the stress gradient in the CFRP in the corner regions of the beam's cross section with the stiffness of the concrete substrate. Such conditions could potentially elevate the risk of premature CFRP failure [47];

Higher values of ρ_l result in an increase in ϵ_{fe} , attributed to the dowel effect. By increasing ρ_l the sliding of the two faces of the CDC decreases, which in its turn decreases a shear strain component in the CFRP crossing the CDC, which are subjected to tensile strain due to the crack separation component of the kinematic behaviour of shear cracks in RC elements failing in shear. By minimizing this shear strain component in these CFRP postpone their debond, which increase the ϵ_{fe} . This effect is more pronounced in fully confinement shear arrangements, since besides the previous favourable effect to increase ϵ_{fe} , by increasing ρ_l , the gradient of stresses installed in the CFRP where CDC crosses the flexural reinforcement (critical zones in terms of CFRP rupture) decreases

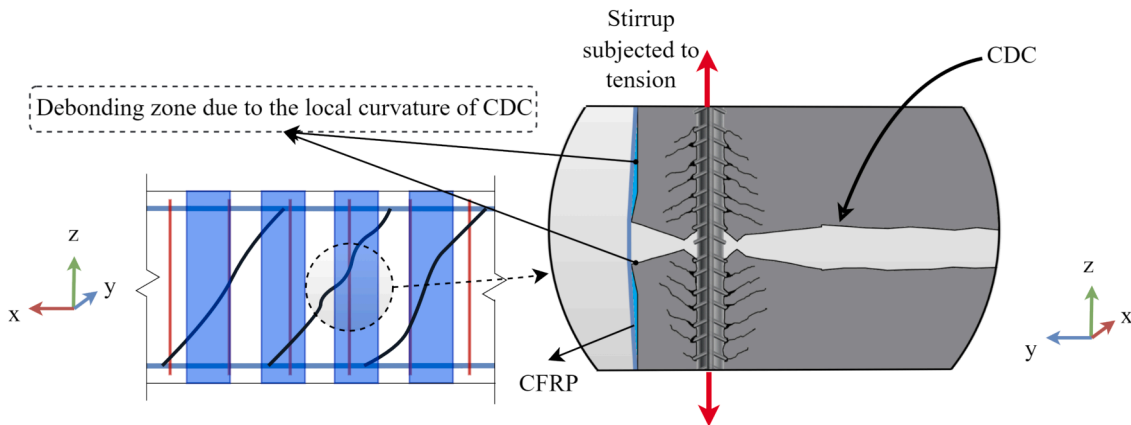


Fig. 14. Schematic representation of local curvature in the crack profile surrounding the leg of the stirrup.

In case of h_f , distinct behaviours are observed among beams that are fully wrapped and those with U-wrapped or side-bonded configurations. As h_f increases in the latter group, there is a corresponding increase in ε_{fe} [48]. The main failure observed for these beams is debonding failure, therefore this tendency can be attributed to the larger bond length available on both sides of the CDC in beams with larger h_f . In the case of fully wrapped beams, where rupture failure is more prevalent, the opposite trend is seen. In fully wrapped shear strengthening configurations, by increasing the height of the beam (which is highly correlated with h_f in the current dataset) means larger crack width profile for the same crack tip opening angle of the CDC. In these circumstances, the shear resisting mechanism provided by the aggregate interlock of the shear cracks decreases, being this resisting component to be supported by the CFRP crossing the CDC, which leads to an increase of biaxial strain field in the CFRP, in this region, promoting the premature rupture of the CFRP, and consequently, smaller ε_{fe} with the increase of h_f .

The ε_{fe} decreases with the increase of ρ_{sw} and f_{swy} , which is in good agreement with the previous observations [6,7,40]. By increasing the percentage of existing steel stirrups (ρ_{sw}), more shear cracks are formed, which decrease the bond length of the CFRP crossing these shear cracks. Furthermore, by increasing ρ_{sw} more stirrups are being pullout in the zones crossed by the shear cracks. This pullout process introduces damage to the surrounding concrete [49] and to a local curvature in the crack profile surrounding the leg of the stirrup [50], which introduces an extra increment of stress in the CFRP and local debonding (Fig. 14), leading to its premature rupture. As larger is f_{swy} , as pronounced is this effect, since yield initiation may not occur before the rupture of the CFRP. This tendency is more pronounced in the fully confined shear arrangements, since these damages and local debond of the CFRP crossing the shear cracks conduct to the migration of the stresses to the corner zones of the beam's cross section, with abrupt gradient of stresses and local premature failure. This adverse shear interaction between stirrups and CFRP reinforcement reduces the effectiveness of shear strengthening.

As expected, in general larger values of ε_{fe} are obtained for fully wrapped beams rather than when the other two strengthening configurations are adopted [51]. The plot might incorrectly suggest that side-bonding is as effective as U-wrapping. However, it is essential to consider that in the present dataset, most of the side-bonded beams lack steel stirrups. If a dataset were available with side bonded beams reinforced with stirrups, then a consistent observation with the prevailing belief that U-wrapping is more effective than side-bonding could be confirmed.

Finally, it is observed that in fully wrapped beams, ε_{fe} increases by increase in α_f . In these beams, CFRP tend to fail by tensile rupture, therefore as larger is α_f as smaller is the shear component introduced in

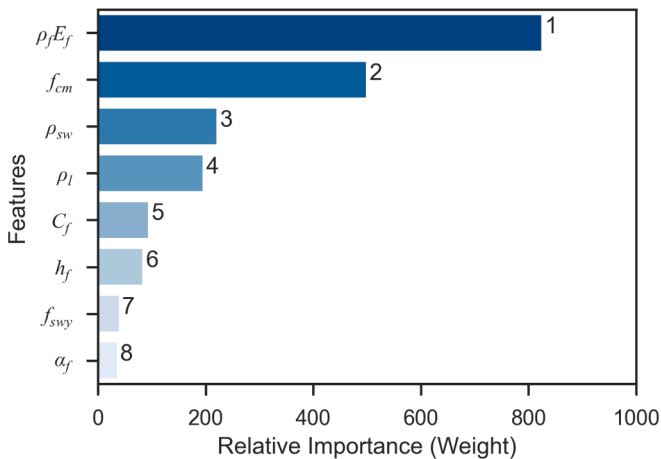


Fig. 15. Sorted relative importance of input variables.

the CFRP in the zone crossing the CDC, which postpone the rupture of the CFRP. In the case of the other shear strengthening arrangements, ε_{fe} seems to be not affected significantly by α_f , because failure is mainly governed by debonding. The orientation of the CDC (θ) is affected by the orientation of the CFRP, resulting similar maximum tensile strain in the critical region of the CDC.

The relative importance of features in ML-based models is determined through various techniques such as permutation importance, SHAP values, or feature importance scores provided by the model itself. These methods help in understanding which features have the most significant impact on the model's predictions. In this study, the relative importance of features is obtained from the weight score provided by XGBoost [52]. Fig. 15 shows these scores and ranks the features based on their importance in the model.

By including unnecessary features, the model becomes too complex and captures random noises in the training data rather than the actual patterns, which do not generalize well to unseen data. Therefore, it's crucial to identify and retain only those features that genuinely contribute to model prediction while eliminating the rest. This process, known as feature selection, aims to streamline the model by focusing on the most influential variables. Various feature selection approaches are available, such as Principal Component Analysis (PCA) [53], Sequential Feature Selection (SFE) [54], Correlation-based feature selection [55], and recursive feature elimination with cross-validation (RFECV). Initially, RFECV method was implemented using XGBoost algorithm and 10 folds CV. This method iteratively eliminates the least influential features at each step and trains the model on the remaining features to determine the optimal number of features. The result of this analysis is presented in Fig. 16 (left). As shown, taking all features in the model yields the highest R^2 and lowest dispersion in the CV scores. In a parallel study, a XGBoost model is initially trained using only the most influential variable, $\rho_f E_f$. Then, at each subsequent step, the next variable with the highest importance (according to Fig. 15) is added to the model inputs. At each step of this process, the mean and standard deviation of R^2 values, obtained from 10-fold CV, are evaluated. The result of this analysis is given in Fig. 16 (right) and agrees with the result obtained from RFECV method. As shown, while the improvement in predictive performance after incorporating five features is marginal, including all features yields the best predictive performance.

4.3. An overview on Machine learning algorithms

The array of machine learning algorithms available in the literature for constructing prediction models using tabular data, where a set of random variables serve as input to predict one or more target values, is extensive [27]. In this study, several models using XGBoost [40,52,56], RF [57], Gradient Boosting (GB) [58], k-Nearest Neighbors (KNN) [59], AdaBoost [60,61], MLP-ANN [17,40,62,63], Decision Tree (DT) [64], LightGBM [65], and Extremely Randomized Trees (ERT) [66] are developed. The rationale behind developing prediction models using various machine learning algorithms lies in the pursuit of adopting the best predictive model.

4.4. Bayesian optimization

Each ML algorithm is characterized by a set of parameters, commonly referred to as hyperparameters, which are set before the learning process begins. Optimization of these hyperparameter is a pivotal process for obtaining the highest possible predictive accuracy. Hyperparameter optimization methods can be manual or automatic. Manual search, guided by human expertise, relies on intuition and experience to fine-tune hyperparameters but lacks reproducibility and may be impractical for non-experts [31]. In contrast, automatic methods like grid search can systematically explore every combination of hyperparameter values, aiming to find the global optimum but often suffering from computational inefficiency and resource constraints [67].

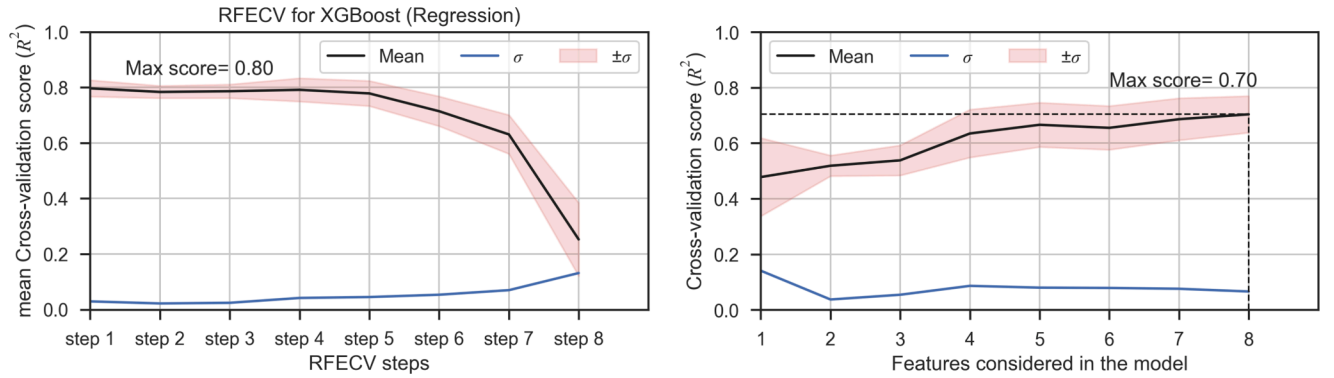


Fig. 16. CV score (R^2) against RFECV steps (left); and the number of features considered in the model (right).

Table 3

Optimized hyperparameters of ML-based prediction models.

Optimal Hyperparameters	ML-based Regression algorithms									
	XGBoost _{real}	XGBoost _{syn}	GBR	ERT	Ada	RF	LightGBM	KNN	DT	MLP-ANN
Number of trees	81	232	100	100	50	100	100	—	1	—
Maximum depth	3	8	3	—	—	No limit	No limit	—	13	—
Learning rate	0.194	0.027	0.1	—	1	—	0.1	—	—	0.001 (constant)
Fraction of features at each split (colsample_bytree)	0.63	0.5	—	1.0	—	1.0	1.0	—	1.0	—
Minimum sum of weights in a leaf (min_child_weight)	2	2	0.0	—	—	0.0	0.001	—	0.0	—
τ (reg_alpha)	3.2×10^{-10}	1×10^{-9}	—	—	—	—	0.0	—	—	—
λ (reg_lambda)	4.3×10^{-5}	10	—	—	—	—	0.0	—	—	—
Minimum samples leaf	—	—	1	1	—	1	20	—	4	—
Minimum samples split	—	—	2	2	—	2	—	—	2	—
Maximum tree leaves	—	—	—	—	—	—	31	—	No limit	—
Number of neighbours (K)	—	—	—	—	—	—	—	4	—	—
Activation function	—	—	—	—	—	—	—	—	—	ReLU
Neural network architecture	—	—	—	—	—	—	—	—	—	(100,100)
Maximum iteration	—	—	—	—	—	—	—	—	—	500

Random search is a more efficient approach, samples values within specified ranges, and focuses on more sensitive hyperparameters. However, challenges persist in achieving both precision and efficiency in automatic hyperparameter tuning, particularly for complex models [31].

Bayesian optimization is a powerful and efficient method for fine-tuning the hyperparameters of ML-based models with no closed-form expression for the objective function. It uses Bayesian inference and Gaussian processes to model the relationship between hyperparameters and the model's performance, allowing for more informed decisions about the next set of hyperparameters to iterate [29]. The hyperparameters of the ML algorithms analysed in this study were fine-tuned with Bayesian optimisation, which involved employing a 10-fold CV approach with R^2 as the optimization objective [31,41]. The optimal hyperparameters are reported in Table 3. For hyperparameters not listed in this table, default values were employed.

5. Validation of models and discussion

5.1. Selection of best performer ML models trained on experimental dataset

The shear contribution of CFRP reinforcements is derived for each model using the trained ML-based models and Eq. (1). In Fig. 17, a scatterplot is provided to compare the predictions of all models with the experimentally obtained values. The black dotted line in the plot illustrates the perfect fit between experimental and predicted values; hence, the model with more accurate predictions is characterized by a scatterplot that shows less deviation from this line. The red dotted lines represent the margins of minimum underestimation and maximum overestimation.

To evaluate the performance of models, a set of statistical indicators is employed, including $RMSE$, $MAPE$, R^2 , and r , defined:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \gamma_i)^2}{n}} \quad (5)$$

$$MAPE = \frac{100\%}{n} \sum_{i=1}^n (|(y_i - \gamma_i)/y_i|) \quad (6)$$

$$R^2 = 1 - \left(\frac{\sum_{i=1}^n (y_i - \gamma_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \right) \quad (7)$$

$$r = \sum_{i=1}^n \left[\frac{(y_i - \bar{y})(\gamma_i - \bar{\gamma})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 \sum_{i=1}^n (\gamma_i - \bar{\gamma})^2}} \right] \quad (8)$$

where y_i is the experimental value of $V_{R,f}$ for the i^{th} beam; $\bar{y}$ is the mean of experimental values of $V_{R,f}$; γ_i is the predicted value of $V_{R,f}$ for the i^{th} beam, and $\bar{\gamma}$ is the mean of predicted values of $V_{R,f}$. The rationale behind the selection of this ensemble of indicators is the need for a comprehensive evaluation of model performance. Each indicator offers unique insights into different aspects of model performance, collectively providing a well-rounded assessment [68]. A perfect prediction corresponds to $r = R^2 = 1$ and $RMSE = MAPE = 0$. The statistical indicators of each model are obtained based on test dataset and displayed on Fig. 17.

Fig. 18 shows the statistical indicators of ML-based models, ranked by the adopted indicated statistical indicators. XGBoost stands out as the leading performer among all models, excelling in all four key statistical indicators. On the test dataset, it achieves an R^2 of 0.87, an r of 0.96, an $RMSE$ of 19.0 kN, and an $MAPE$ of 27.7 %. KNN achieves the lowest prediction accuracy among all models. Therefore, XGBoost model is

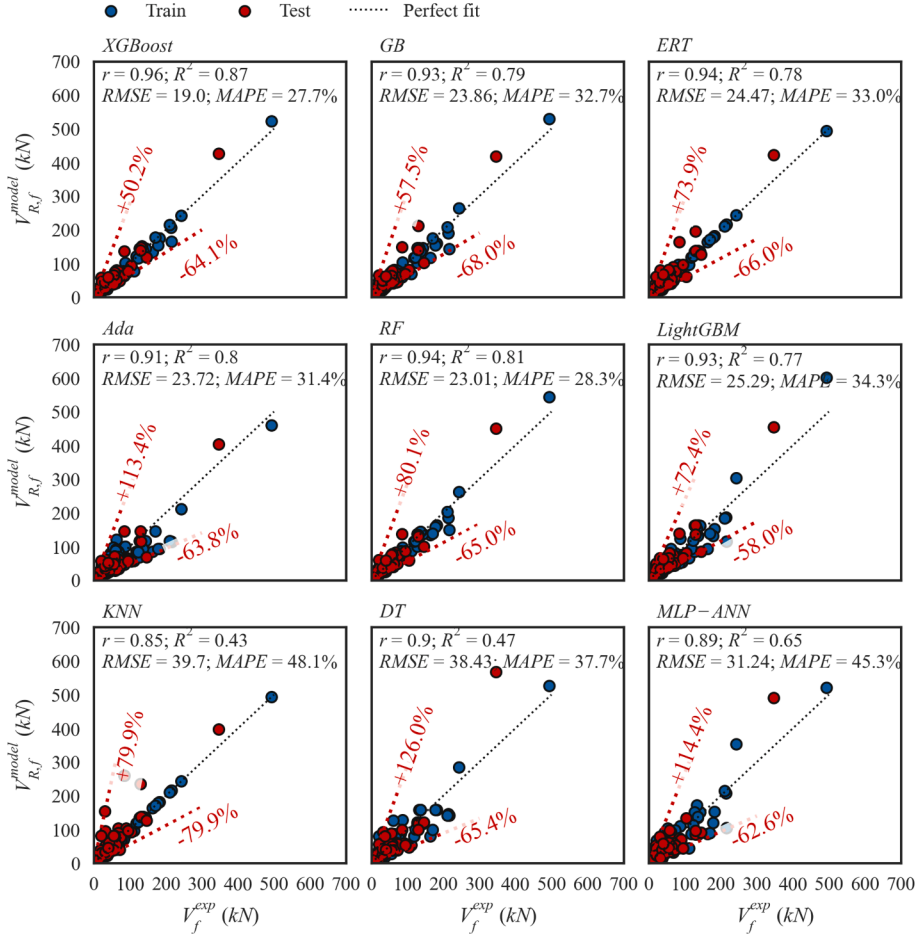


Fig. 17. The comparison of the experimental and predicted values of $V_{R,f}$ for the ML models trained on the experimental dataset.

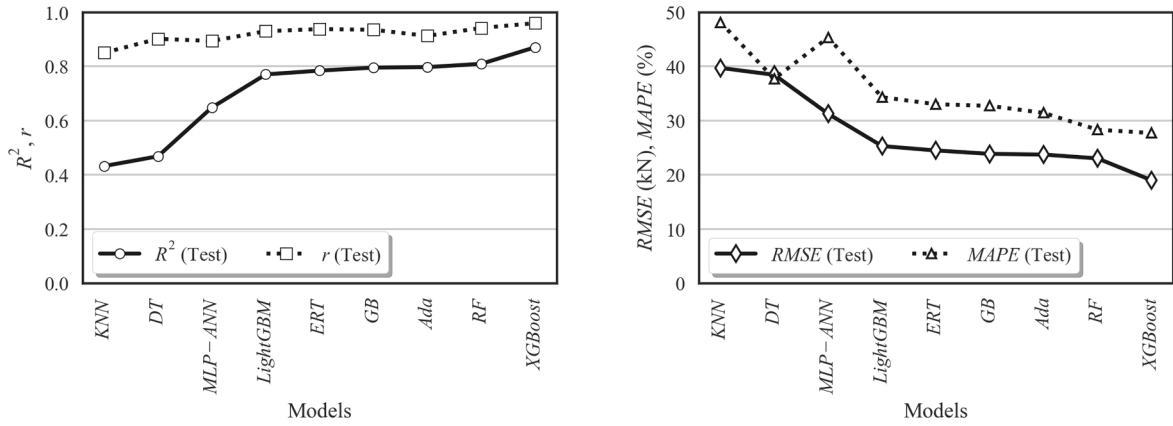


Fig. 18. Model performance statistical indicators (for training and test datasets).

nominated to be trained with the synthetic data generated by TVAE method.

5.2. Comparison of predictive performance of the models

This section compares the predictive performance of the XGBoost model trained on synthetic dataset (XGBoost_{syn}) with the one from XGBoost model trained with experimental dataset (XGBoost_{real}). The predictive performance of these approaches is also compared with the one from the ACI 2023 and fib Bulletin 90 formulations. The values of $V_{R,f}$ are derived for the experimental dataset using the XGBoost_{syn}, ACI

2023 and fib Bulletin 90, while XGBoost_{real} only considered the test dataset.

As evident from the results shown in Fig. 19, XGBoost_{syn} outperforms XGBoost_{real} with a r , R^2 , RMSE, and MAPE of 0.97, 0.94, 12.37 kN, and 18.3 %, respectively. Both XGBoost_{syn} and XGBoost_{real} exhibit significantly higher predictive performance compared to ACI 2023 and fib Bulletin 90.

Table 4 provides a detailed comparison of predictive performance metrics for the ML-based models across different strengthening configurations, namely full wrapped, U-wrapped, and side-bonded beams. It is clear from the table that XGBoost_{syn} consistently exhibits lower RMSE

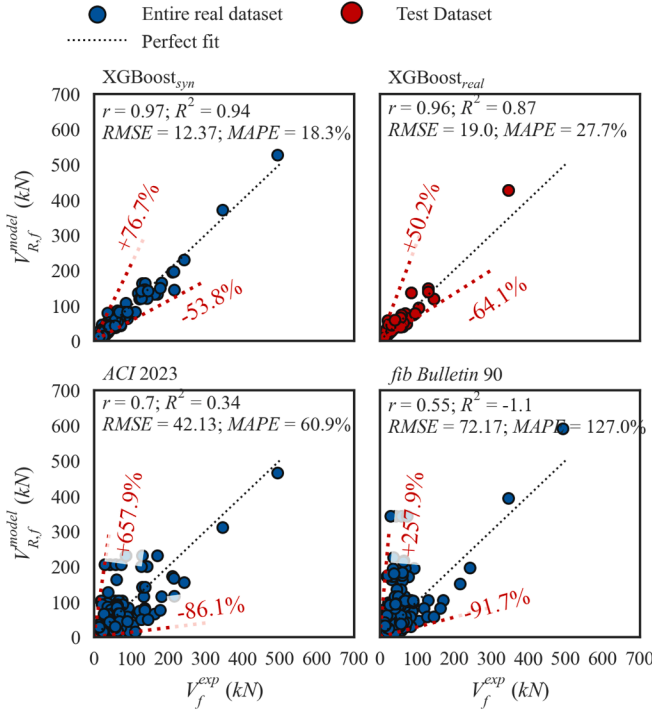


Fig. 19. The comparison of the experimental and predicted values of V_{Rf} using XGBoost_{syn}, XGBoost_{real}, ACI 2023, fib Bulletin 90.

and MAPE values while maintaining higher R^2 and r values across all configurations, indicating superior accuracy and reliability. For instance, in the full wrap configuration, XGBoost_{syn} achieves an RMSE of 18.6 kN and a MAPE of 17.2 %, compared to XGBoost_{real} with RMSE of 31.4 kN and MAPE of 36.5 %. Similarly, the correlation coefficient r for XGBoost_{syn} in the U-wrap configuration is 0.85, higher than the 0.79 obtained by XGBoost_{real}, demonstrating better predictive consistency for synthetic data across different beam types.

A complementary approach for assessing model performance is to analyse the model uncertainty. Model uncertainty is determined by taking the ratio of the experimentally recorded value to the predicted value, given by:

$$\chi = \frac{V_f^{exp}}{V_{Rf}^{model}} \quad (9)$$

The model uncertainty of an ideal model has a mean of (μ) 1.0 and a CoV approaching 0. Therefore, as revealed by the analysis conducted for each model and shown in Fig. 20 via scatterplots of model uncertainties against V_f^{exp} , it becomes evident that the XGBoost_{syn} and XGBoost_{real} models exhibit significantly better performance in comparison with the conventional models, as indicated by a mean of 1.03 and 0.97 and a CoV of 0.22 and 0.28, respectively. Both XGBoost_{syn} and XGBoost_{real} were capable to produce consistent prediction performance along different magnitude of V_f^{exp} . Among the existing models, in which unity partial factors and resistance factors are used to obtain the average values of V_f

for comparison reason, ACI 2023 has a μ of 1.53, which means the model tends to generally underestimate the V_f^{exp} and produce conservative results. Conversely, fib Bulletin 90 has a mean of model uncertainty of 0.89, which means the model generally has a slight tendency to overestimate the V_f^{exp} . Having a CoV in model error of 0.75, ACI 2023 exhibits slightly better prediction performance than fib Bulletin 90 with a CoV in model error of 0.77. The values shown in this plot underscores the capability of ML-based models to capture the intricate nonlinear relationships more effectively between input variables and target variable. It should be highlighted that the lower value of CoV translates to a more cost-effective reduction factor, rendering the developed XGBoost models as an interesting option for design applications [69].

Although a formal reliability analysis was not conducted in this study, the reliability of the model can be assessed using the modified demerit points classification method introduced in [70]. This classification framework helps provide insights into how ML algorithms, coupled with the utilization of synthetic datasets, can yield more reliable and cost-effective solutions. This classification method classifies the predictions based on the measured model uncertainties, to either extremely conservative, conservative, appropriately safe, relatively safe, dangerous, or extremely dangerous. Table 5 introduces the various classes in this classification, showing the corresponding model uncertainty ranges and the assigned penalty point (p.p) of specimens within each class. The total demerit points for each model are computed by summing the products of the percentage of specimens with χ within each defined range, multiplied by the associated penalty points for that range, and then divided by 100. The resultant score falls within the range of 0 to 10. As lower is the total demerit point score as more appropriate is the predictive performance of the model.

Table 5 provides the breakdown of specimens with χ ratios falling into each classification, along with the total demerit points assigned to each model. XGBoost_{syn} and XGBoost_{real} demonstrate significantly lower demerit points compared to ACI 2023 and fib Bulletin 90. Notably, XGBoost_{syn} shows the best performance with a total modified demerit point of 0.77.

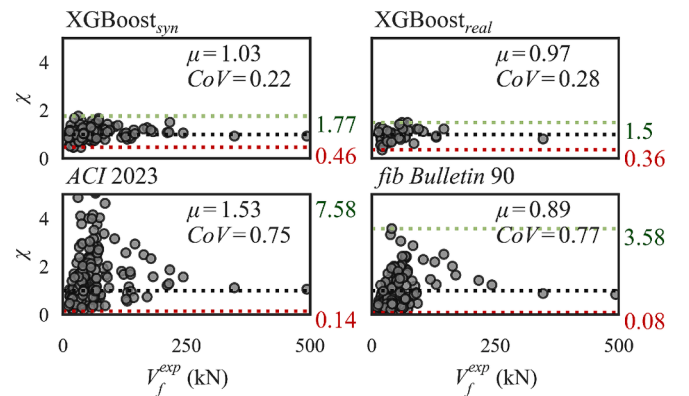


Fig. 20. Model uncertainties versus V_f^{exp} .

Table 4

Predictive performance metrics for XGBoost models trained on synthetic and real datasets across different strengthening configurations.

Models		XGBoost _{syn}			XGBoost _{real}		
Strengthening configuration		Full wrap	U wrap	Side-bonded	Full wrap	U wrap	Side-bonded
Metrics	RMSE (kN)	18.6	10.1	12.6	31.4	12.3	12.6
	MAPE (%)	17.2	19.6	14.3	36.5	25.8	19.4
	R^2	0.96	0.71	0.96	0.87	0.61	0.91
	r	0.98	0.85	0.98	0.97	0.79	0.97

Table 5
Modified demerit point classification.

Classification	χ	p-p	XGBoost _{syn}	XGBoost _{real}	ACI 2023 (%)	fib Bulletin 90 (%)
Extremely dangerous	<0.5	10	2	4	29	33
Dangerous	0.5–0.65	5	3	8	11	12
Relatively safe	0.65–0.85	2	17	19	14	10
Appropriately safe	0.85–1.3	0	70	56	20	16
Conservative	1.3–2	1	8	12	17	17
Extremely conservative	>2	2	0	0	8	12
Total modified demerit point			0.77	1.3	4.06	4.51

6. Physical interpretation of XGBoost_{syn} model using SHAP values

Based on the findings in the previous sections, XGBoost_{syn} consistently outperformed other models in terms of statistical indicators and model uncertainty distribution. Also, its modified total demerit point is 0.77, the lowest among the other models. Despite its high efficiency, XGBoost models are known for their challenges in result interpretation when compared to other predictive models like MLP-ANN or DT. Various techniques have been developed to address this issue, one of which is employed in this study and referred to as SHAP [32]. SHAP is rooted in game theory and was designed to interpret outputs generated by ML-based models, including black-boxed models such as XGBoost. This technique involves making predictions with and without each input variable, allowing for the measurement of the importance of each input variable by comparing these predictions. Several studies have employed this technique for interpreting their models, either to validate and reinforce existing knowledge or to uncover previously unknown underlying physical phenomena, which demonstrates its effectiveness in enhancing model transparency and interpretability. For instance, Mohammadi et al. [40], by utilizing SHAP values, demonstrated how a ground motion intensity measure (IM) varies with different predictors and that the most influential predictor can change across different IMs. In another study, Mohammadi et al. [62] demonstrated how the characteristics of both the structure and ground motion affect the maximum inter-storey and global drift ratios. Mangalathu et al. [71] demonstrated how each variable contributes to the punching shear strength, providing a comprehensive explanation of a specific tested slab. Zhang et al. [72] took a step further by obtaining additional insights from the relationship between variables and SHAP values, discovering that enhanced bond strength between FRP bars and seawater sea sand concrete occurs when high performance concrete is used.

Fig. 21 presents the SHAP values for the input features of the model using the synthetic and experimental datasets. On the x-axis, the SHAP value for each beam is shown. The y-axis in these graphs sorts the input variables from the most significant (at the top) to the least significant (at

the bottom), based on their respective SHAP values. SHAP values quantify the impact of each feature on the model's prediction, with higher absolute SHAP values indicating greater significance in influencing the output. It can be observed that the model yields relatively similar results for both datasets, with only a slight difference in the ranking of input features. However, given their very close values, this discrepancy is negligible. The values of input variables (feature values) are depicted on a scale ranging from the minimum to the maximum value of each input variable, with blue representing the lowest and red representing the highest value of the input variable. If the SHAP value is positive, it indicates that the target variable increases compared to the base value (the base value represents the mean of the model predictions for the training data). If it is negative, it suggests that the target variable is decreased compared to the reference value. Fig. 21 shows that the value of ϵ_{fe} typically decreases with an increase in the values of $\rho_f E_f$, f_{swy} , and ρ_{sw} . Conversely, it appears that an increase in h_f , ρ_l and f_{cm} generally results in an increase in ϵ_{fe} . Considering that larger C_f values are associated with side-bonded and U-wrapped beams, it is evident that the model effectively incorporates the previous knowledge that these configurations generally exhibit smaller ϵ_{fe} values compared to fully wrapped beams.

To obtain a more comprehensive understanding of the relationship between the SHAP values and other input variables, a scatter plot is presented in Fig. 22, showcasing the variation of SHAP values with changes in the input variables for both experimental and synthetic datasets. The correlation between variables and SHAP values is shown by fitting a curve to SHAP values obtained for both synthetic and experimental datasets. This highlights that the trend of variation of SHAP values remains consistent, further substantiating the high similarity between the synthetic dataset and the experimental dataset. Additionally, the trend of SHAP values illustrates how and to what extent the variation of input variables impacts the value of ϵ_{fe} . In general, the results of this analysis align with the observations made in Fig. 13 and discussed in section 4.2. As can be seen, increasing $\rho_f E_f$ up to 1.7 GPa leads to a significant decrease in the SHAP value and therefore ϵ_{fe} . It is observed that ϵ_{fe} also tend to increase with f_{cm} up to 27 MPa,

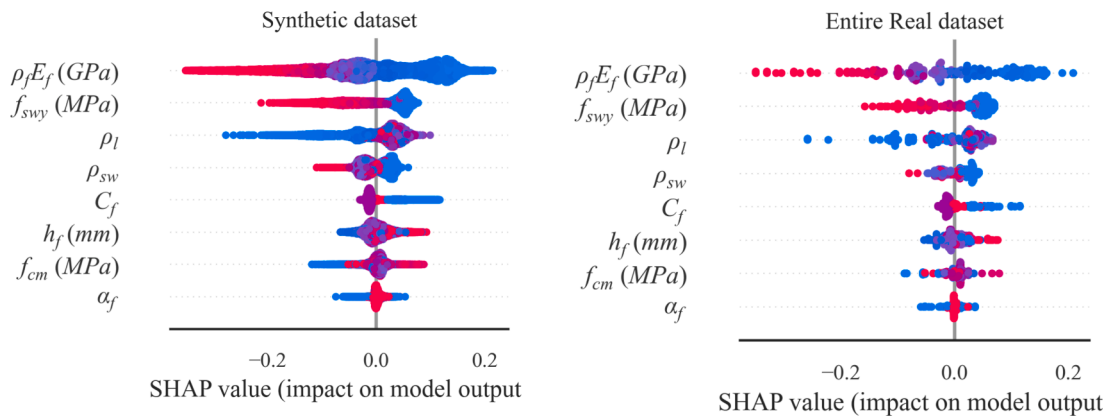


Fig. 21. SHAP plots of the developed XGBoostsyn model for synthetic and experimental datasets.

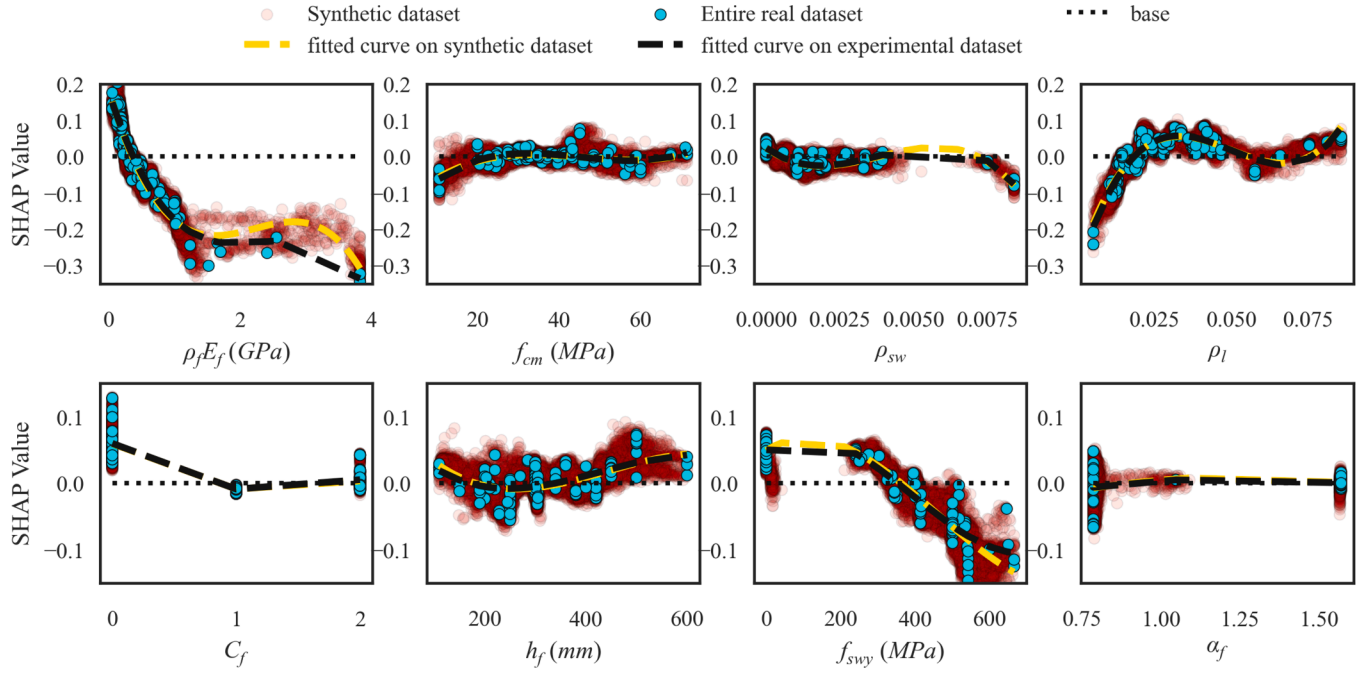


Fig. 22. SHAP values by variation in the input variables.

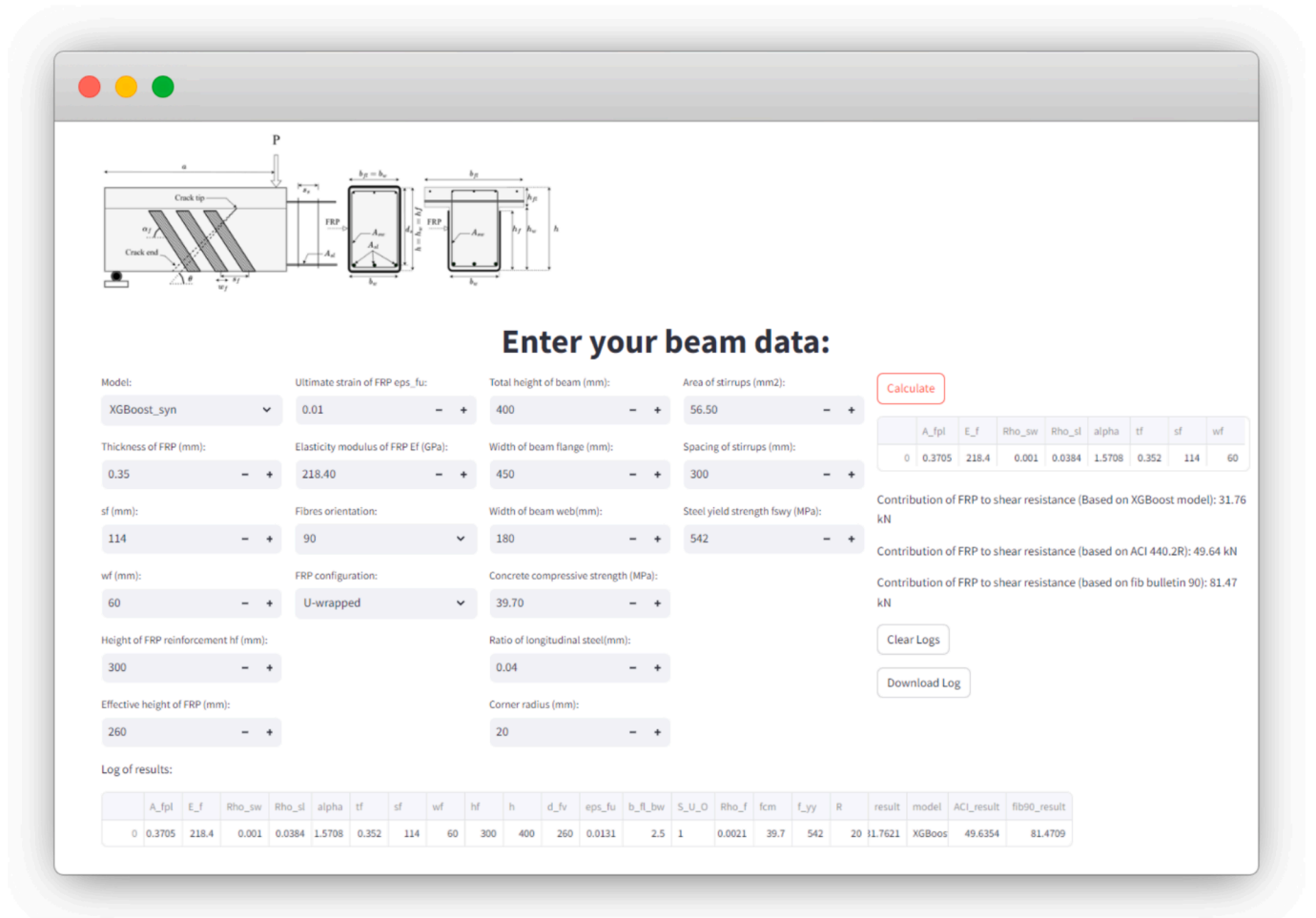


Fig. 23. A GUI web application for XGBoost_{syn} model to predict V_{Rf} .

which is expected since the debonding failure is postponed. An opposite trend occurs with the increase of ρ_{sw} , since a decrease of ε_{fe} is observed. This trend can be attributed to the additional stress in the CFRP caused by previously introduced local curvature in the crack profile, which accelerates local debonding and leads to its premature rupture. With f_{swy} exceeding 400 MPa, the SHAP value decreases, suggesting that ε_{fe} declines as f_{swy} increases. This behaviour can be attributed to the tendency of CFRP reinforcement to fail at relatively low strain levels. While the yield stress/strain of the internal steel reinforcement increases, it becomes less likely for the steel stirrups to reach their yielding point before the CFRP reinforcement fails. Consequently, the load-bearing contribution of the steel is reduced, thereby decreasing the overall effectiveness of the CFRP reinforcement. Additionally, it is observed that ε_{fe} tends to increase with ρ_l up to 0.025; however, ε_{fe} is not significantly affected beyond this ρ_l value. If the beams with very small h_f are neglected, ε_{fe} tends to increase with h_f , which is expected as explained before. In the case of $C_f = 0$, which corresponds to fully wrapped beams, the results indicate that this strengthening configuration is most effective one and when used the expected ε_{fe} is more than the other two configurations. The SHAP values obtained for α_f do not allow to draw any specific conclusions, therefore further analysis are needed to better understand its effect.”.

7. Web application

A Graphical User Interface (GUI) web application was developed with Streamlit package [73] to facilitate the use of the proposed model. Users can input the variables into the application and initiate the calculation process by clicking on the “calculate” button. The application is available to the public through a link, which means anyone can access and use it. The application requires 14 inputs, and once provided, it returns the contribution of FRP to the shear resistance of the beam. Illustrating the functionality of the application, an example is presented using inputs derived from specimen 2S-7 M(2) [74]. For this specific specimen, the experimental value yielded 39.8 kN, whereas the XGBoost_{syn} model predicted a value of 31.76 kN. The codes are available on GitHub at https://github.com/amirxdbx/DABASUM_MLM, and the user interface of the tool is depicted in Fig. 23. Access to the tool can be obtained through <https://shear-frp-dabasum.streamlit.app/>.

8. Conclusions

This study explores the potential of ML-based models in predicting the contribution of EBR-CFRP on the shear resistance of RC beams. Subsequently, it investigates the improvement in predictive performance of these ML models through the utilization of synthetic datasets.

For this purpose, a screened dataset comprising 239 beams [21] sourced from the literature was assembled. According to a feature selection analysis performed using XGBoost-RFECV analysis, $E_{ff}\rho_f$, f_{cm} , h_f , ρ_{sw} , ρ_b , C_f , α_f , and f_{swy} are identified to be influential. Using this experimental dataset, alternative ML algorithms, including XGBoost, RF, GB, KNN, AdaBoost, MLP-ANN, DT, LightGBM, and ERT, were trained to predict an introduced variable known as effective strain, which is recognized as the governing one for predicting the contribution of the CFRP for the shear strengthening of RC beams. This variable is then integrated into a truss model to predict the contribution of EBR-CFRP on the shear resistance of RC beams. XGBoost model trained and tested with experimental dataset emerged as the best performer model demonstrating a R^2 of 0.87, a r of 0.96, a $RMSE$ of 19.0 kN, and a $MAPE$ of 27.7 %. As a result, XGBoost was selected to further explore the potential of using the synthetic dataset.

Addressing the gaps and limitations identified in the experimental dataset, a state-of-the-art TVAE is utilized to generate a synthetic dataset. This synthetic dataset closely resembles the characteristics of the experimental dataset and effectively fills in the gaps, thereby

augmenting the available data, particularly in underrepresented regions. This augmentation ultimately leads to an enhancement in predictive performance of XGBoost model resulting in a r , R^2 , $RMSE$, and $MAPE$ of 0.97, 0.94, 12.37 kN, and 18.3 % respectively.

Furthermore, a comparison was made by two well-established existing models to assess their predictive performance against the proposed models. The considered existing models exhibited significantly lower predictive performance compared to ML models, indicating the high potential of ML models for more accurate predictions.

Model uncertainties were evaluated for all models, revealing that the XGBoost model trained with the synthetic dataset outperformed the others. It exhibited lower dispersion in the distribution of model uncertainty, measured by a CoV of 0.22, and achieved the lowest total demerit point score of 0.77 (for an interval of 0 to 10).

Based on the SHAP value analysis conducted in this study, the proposed model can replicate the underlying physics of the problem present in the dataset. Consequently, this model has demonstrated itself as a reliable alternative to parametric models.

It must be noted that, although this study aimed to address existing data gaps through the application of the TVAE model, conducting experimental program to further investigate significant areas of the design space is recommended. Current statistical models may not capture all relevant information that has yet to be observed experimentally.

A web application has been developed to facilitate the utilization of the created model, and the source codes for the model are accessible on GitHub for public use. However, users should note that the model has been trained exclusively on CFRP materials. Moreover, the model has not been validated for anchored reinforcement scenarios. While it may provide more conservative results, its use should be approached with caution and supplemented with further analysis.

Declaration of generative AI and AI-assisted technologies in the writing process

While preparing this work, the authors used ChatGPT and Grammarly to improve readability. After using these tools, the authors reviewed and edited the content as needed and takes full responsibility for the publication's content.

CRediT authorship contribution statement

Amirhossein Mohammadi: Writing – review & editing, Writing – original draft, Visualization, Validation, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Joaquim A.O. Barros:** Writing – review & editing, Writing – original draft, Validation, Supervision, Resources, Project administration, Funding acquisition, Data curation, Conceptualization. **José Sena-Cruz:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix 1. . An Introduction to existing models

To emphasize the superiority of ML-based models over existing conventional models in predicting the contribution of CFRP systems to the shear resistance of RC beams, a selection of well-known models has been made, and this selection includes ACI 2023 and fib Bulletin 90, as shown in Table A 1. The results of these models, which are obtained considering the average values of material properties and unit partial or resistant factors, will be assessed and compared alongside the ML-based models.

Table A1
Existing prediction equations.

Model	Parameters
ACI 2023	$V_{Rf} = \frac{A_f f_{fe} d_{fv} (\sin \alpha_f + \cos \alpha_f)}{s_f}$ $A_f = 2t_f w_f f_{fe} = E_f \varepsilon_{fe}$ <p>Fully wrapped: $\varepsilon_{fe} = 0.004 \leq 0.75 \varepsilon_{fu}$ U-wrapped: $\varepsilon_{fe} = \kappa_v \varepsilon_{fu} \leq 0.004$</p> $\kappa_v = (\kappa_1 \kappa_2 L_e) / (11900 \varepsilon_f) \leq 0.75; L_e = 23300 / (t_f E_f)^{0.58}$ $\kappa_1 = (f_c' / 27)^{2/3}; \kappa_2 = (d_{fv} - L_e) / d_{fv}$
fib Bulletin 90	$V_{Rf} = A_{fpl} h_{fe} f_{fwm} (\cot \theta + \cot \alpha_f) \sin \alpha_f$ $A_{fpl} = \begin{cases} \frac{2t_f w_f}{s_f} & \text{strips} \\ 2t_f \sin \alpha_f & \text{Continuous sheet} \end{cases}; t_f = \begin{cases} t_0 n_f & \text{if } n_f < 4 \\ t_0 (n_f)^{0.85} & \text{if } n_f \geq 4 \end{cases}$ $h_{fe} = \min(h_f, h - 0.1d_s) \quad \text{Fully wrapped: } f_{fwm} = f_{fwm,c} = k_R f_{fm}$ $\text{U-wrapped: } f_{fwm} = \min(f_{fbwm}, f_{fwm,c})$ <p>For continuous FRP sheets:</p> $f_{fbwm} = \begin{cases} \left[1 - \frac{1}{3} \frac{L_e}{(h_{fe}/\sin \alpha_f)}\right] f_{fbm} & \text{for } x' \geq L_e \\ \frac{2}{3} \frac{h_{fe}/\sin \alpha_f}{L_e} f_{fbm} & \text{for } x' \leq L_e \end{cases}$ <p>For discrete FRP strips:</p> $f_{fbwd} = \begin{cases} f_{fbm} & \text{for } x' \geq L_e; L_e \leq y' \leq x' \\ \left[1 - \left(1 - \frac{2}{3} \frac{m' s_f}{L_e}\right) \frac{m'}{n'}\right] f_{fbm} & \text{for } x' \geq L_e; y' \leq L_e \\ \frac{2}{3} \frac{(n' s_f) / [(\cot \theta + \cot \alpha_f) \sin \alpha_f]}{L_e} f_{fbm} & \text{for } x' \leq L_e; y' \leq x' \end{cases}$ $x' = h_{fe} / \sin \alpha_f \quad L_e = \frac{\pi}{2} \sqrt{\frac{E_f t_f s_{0m}}{\tau_{b1m}}}$ $y' = s_f / (\cot \theta + \cot \alpha_f) \sin \alpha_f \quad \tau_{b1m} = 0.72 \sqrt{f_{cm} f_{ctm}}$ $n' = \text{int}[h_{fe} (\cot \theta + \cot \alpha_f) / s_f] \quad s_{0m} = 0.24$ $m' = \text{int}[L_e (\cot \theta + \cot \alpha_f) \sin \alpha_f / s_f] \quad f_{fbm} = \sqrt{\frac{E_f s_{0m} \tau_{b1m}}{t_f}}$

Note: The utilization of average values for features and unit values for partial/resistant factors was chosen to ensure comparability of prediction with experimental values.

Data availability

Supplementary data to this article can be found online at <https://doi.org/10.17632/7fjd6jbff9.1> and additional data will be made available on reasonable request.

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Paper 4

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A novel design model for predicting the shear resistance of reinforced concrete beams strengthened with EBR-CFRP systems

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ABSTRACT

Shear resistance prediction of reinforced concrete (RC) beams strengthened with externally bonded reinforcements (EBR) is a challenging task due to the complex nature of shear resisting mechanisms and their intricate interaction. Existing design models often overlook these dependencies, leading to significant variations in prediction accuracy. In this paper, a design model is proposed by integrating the most relevant shear resisting mechanisms based on the evidence demonstrated by a large database of experimental results from RC beams shear strengthened with wet layup carbon fibre reinforced (CFRP) sheets applied according to the EBR technique. The developed approach integrates the simplified modified compression field theory (SMCFT) with a regression-based model. A global sensitivity analysis was conducted according to which closed form equations were derived to obtain the tensile stress factor in cracked concrete and the inclination angle of the critical diagonal crack. A reliability analysis is carried out, and resistance reduction factors are achieved for different levels of reliability index. The database, composed of 284 RC beams shear strengthened with EBR-CFRP technique, is utilized to validate the proposed model, and compare its performance against some well-established existing design models. The results show that the proposed model, with an adequate framework for its use on design practice, outperforms the other considered models.

1. Introduction

Carbon fibre reinforced polymer (CFRP) reinforcements applied according to the externally bonded reinforcement (EBR) have been demonstrated to be very efficient for increasing the shear capacity of reinforced concrete (RC) [1,2], being the unique technique with a design-based formulation in the *fib* bulletin 90 [3]. The literature offers various models for predicting the increase in shear resistance of RC beams strengthened with EBR-FRP systems. These models can be categorized into two main approaches. A semi-empirical type approach, where the governing variables, typically the effective strain or stress of the FRP system, are determined through curve fitting of experimental data [4,5] or by applying optimization algorithms using available data [6]. The second type of approach is mechanic-based, aiming to simulate the shear resisting mechanics [7,8]. The former approach is relatively simple to develop and easy to apply in design practice, but its predictive performance highly depends on the quality and relevance of the database. The mechanics-based type-approaches require more complex calculations, but have the potential of providing more reliable results.

Some ambiguity on how to consider the contribution of concrete, existing steel reinforcements and FRP for the shear resistance of a RC beam, and the use of partially safety factors for the materials or the global safety factor for the model introduces extra difficulties for the designer [6–9]. Design codes and guidelines have recommended some equations for predicting the design value of shear resistance in RC beams shear strengthened with EBR-FRP systems; however, recent studies have shown their debilities in terms of predicting the shear resistance with required accuracy and reliability [10,11]. Therefore, there is a need to develop more accurate and reliable design models.

Mohammadi *et al.* [10] proposed a model for predicting the contribution of CFRP to the shear resistance of RC beams strengthened with EBR-FRP reinforcements. Their model demonstrated some improvement in predicting the contribution of CFRP reinforcement over other selected existing models. This was assured by considering in the formulation the influence of various factors, such as the interaction between existing steel stirrups and FRP reinforcement, the shear strengthening configuration, and the radius of the corners in beam cross-section. In the present paper, the model of Mohammadi *et al.* [10] is improved by considering

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the influence of the ratio of longitudinal steel bars. The model then is integrated with the simplified compression field theory (SMCFT) to determine more rigorously the inclination of the critical diagonal crack (CDC), θ , and the tensile stress factor (ζ) that accounts the contribution of the cracked concrete for the shear resistance of RC beams shear strengthened with EBR-CFRP systems [12]. The application of SMCFT equations involves, however, an iterative process, posing additional challenges from a design standpoint [13]. To avoid this iterative process, a global sensitivity analysis is conducted to identify the key parameters influencing ζ and θ . The relation between these key parameters and ζ is established using a hyperplane fitting technique. Additionally, a strong correlation is observed between ζ and θ , enabling the estimation of θ through a curve fitting process that captures the relationship between these variables.

It has been verified that the web of T cross section type beams contributes significantly to their shear capacity [14–16]. In RC beams shear strengthened with EBR-CFRP systems this contribution can reach up to 45 % of the total beam's shear strength [17]. Therefore, the influence of the flange in beams with T-shaped cross-section on shear capacity of RC beams failing in shear is considered in the proposed model.

To ensure the practical applicability of the proposed closed-form model in design scenarios, a reliability analysis is conducted [18–20]. This analysis involves determining various resistance reduction factors corresponding to different levels of reliability indices. To validate the model, a reliable dataset composed of 284 experimental results of beams shear strengthened with the EBR technique is taken from a larger database called DABASUM [21]. This dataset served as the basis for evaluating the predictive performance of the proposed model, as well as comparing it with other well-known design models, namely: fib bulletin 90 (fib TG5.1) [3], ACI440-2R-17 [22] and TR55 [23]. The selected models are widely used by professionals and researchers in the field. A comprehensive introduction to the formulations of these models is given in Appendix A.

2. Proposed model (MBS)

In this section, a novel model for predicting the shear resistance of RC beams shear strengthened with EBR-CFRP sheets is described. This model, abbreviated by MBS, combines the SMCFT with the improved formulation of model proposed by Mohammadi et al. (2023) [10]. The shear resistance is evaluated using the following equation (the physical meaning of the adopted symbols is illustrated in Fig. 1):

$$V_{Rd}^{MBS} = \phi \nu b_w d_s \quad (1a)$$

$$\nu = \nu_c + \nu_s + \nu_f \quad (1b)$$

where ϕ is a resistance reduction factor, whose value will be determined by reliability analysis in section 4, b_w (in mm) is the width of the beam's web, d_s (in mm) is the depth of the beam's tensile reinforcement, ν is shear strength; ν_c , ν_s , and ν_f represent the respective contribution of concrete, steel, and FRP that can be determined from the following

equations:

$$\nu_c = k_f \zeta \sqrt{f_{cm}} \quad (2)$$

$$\nu_s = \rho_{sw} f_{swy} \cot \theta \quad (3)$$

$$\nu_f = \frac{V_f}{b_w d} = \frac{A_{fwc} h_f E_f \epsilon_{fe}}{b_w d_s} (\cot \theta + \cot \alpha_f) \sin \alpha_f \quad (4)$$

In Eqs. (2) and (3), f_{cm} (in MPa) is the concrete average compressive strength, k_f is a shape factor accounting for the extra shear fracture surface at the web-flange connection occurred in T-shaped section, proposed by RILEM TC162-TDF [24], and determined from Eq. (6), ζ is a factor to take into account that concrete is cracked in the shear failure region (also known as tensile stress factor), obtained from Eq. (7) [12], ρ_{sw} and f_{swy} (in MPa) are the ratio and the yield stress of existing transverse steel reinforcement. In Eq. (4) A_{fwc} is the effective CFRP cross-section area calculated as $2w_f t_f / s_f$ for strips and $2t_f \sin \alpha_f$ for continuous CFRP sheets, being w_f , t_f and s_f the width, the thickness and the spacing of CFRP strips (in mm); α_f is the angle between the longitudinal axis of the beam and the fibres of these strips, as depicted in Fig. 1. ϵ_{fe} is the effective strain in the CFRP, obtained from Mohammadi et al.'s model [10]. To enhance the prediction performance of this model, adjustments were made to its coefficients, and another important variable, the ratio of longitudinal bars in tension (ρ_l) was incorporated into the model. The ratio of longitudinal bars in tension are found to change the strain distribution at CFRP reinforcement along the CDC, consequently influencing ϵ_{fe} . These modifications led to a 10 % decrease in coefficient of variation (CoV) of the model uncertainty.

The modified prediction model for obtaining ϵ_{fe} is as follow:

$$\epsilon_{fe} = m_F \times (1 - 0.0011 E_f \rho_f / f_{cm}^{2/3}) \times (E_f \rho_f / f_{cm}^{2/3})^{-0.62}$$

with,

$$m_F = 0.013 \times \kappa_{sw} \kappa_R \kappa_{O/U} \kappa_{sl};$$

$$\kappa_{sw} = 1 - 25.4 \rho_{sw}; \kappa_R = 0.75 \left(\frac{R_c}{50} \right) + 0.7 \leq 1.45; \kappa_{O/U} = 1 + 0.52 \kappa; \kappa_{sl} = 1 + 12.5 \rho_l \quad (5)$$

where E_f and ρ_f are the elastic modulus and ratio of CFRP reinforcement; R_c is the corner radius of beam's cross section; κ is the strengthening configuration factor, being 1.0 for fully wrapped and 0 for U-wrapped beams, respectively.

$$k_f = 1 + n \frac{h_{fl}^2}{b_w d_s} \leq 1.5n = \frac{b_{fl} - b_w}{h_{fl}} \leq 3 \quad (6)$$

where b_{fl} and h_{fl} are the width and thickness of the beam flange.

$$\zeta = \frac{0.4}{1 + 1500 \epsilon_x} \frac{1300}{1000 + s_{xe}} \quad (7)$$

where s_{xe} is the crack spacing:

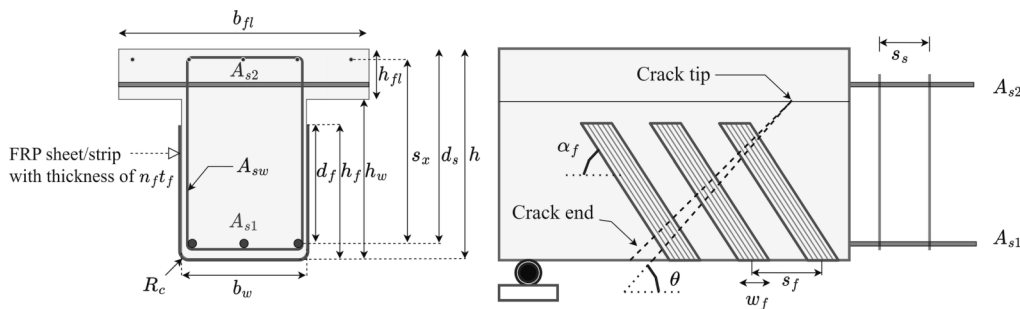


Fig. 1. The Symbols of geometric properties entities of a RC beam shear strengthened with of externally bonded CFRP on the strengthened RC beam.

$$s_{xe} = \frac{35s_x}{a_g + 16} \geq 0.85s_x \quad (8)$$

being s_x (in mm) the vertical distance between longitudinal bars; a_g (in mm) is the maximum aggregate size, and ε_x is the longitudinal strain at mid depth of the beam's cross section ($h/2$):

$$\varepsilon_x = \min\left(\frac{\nu \cot \theta - \nu_c / \cot \theta}{E_{sl} \rho_{sl}}, \varepsilon_{sy}\right) \quad (9)$$

where E_{sl} and ρ_{sl} are the modulus of elasticity and ratio of longitudinal reinforcements (tensile and compressive steel rebars, i.e., $\rho_{sl} = (A_{s1} + A_{s2})/b_w d_s$) and ε_{sy} is its yield strain.

θ in Eqs. (3), (4), and (9) is inclination angle, in degrees, of the CDC in the web of the section, determined from [12]:

$$\theta = (29 + 7000\varepsilon_x) \left(0.88 + \frac{s_{xe}}{2500}\right) \leq 75^\circ \quad (10)$$

Fig. 2 presents the steps of the iterative procedure to calculate the shear strength of a RC beam according to MBS model.

3. Sensitivity analysis

The initial formulation of MBS model requires an iterative procedure to determine the shear capacity of the beam. To avoid the inconveniences of an iterative procedure in design practice, a strategy is

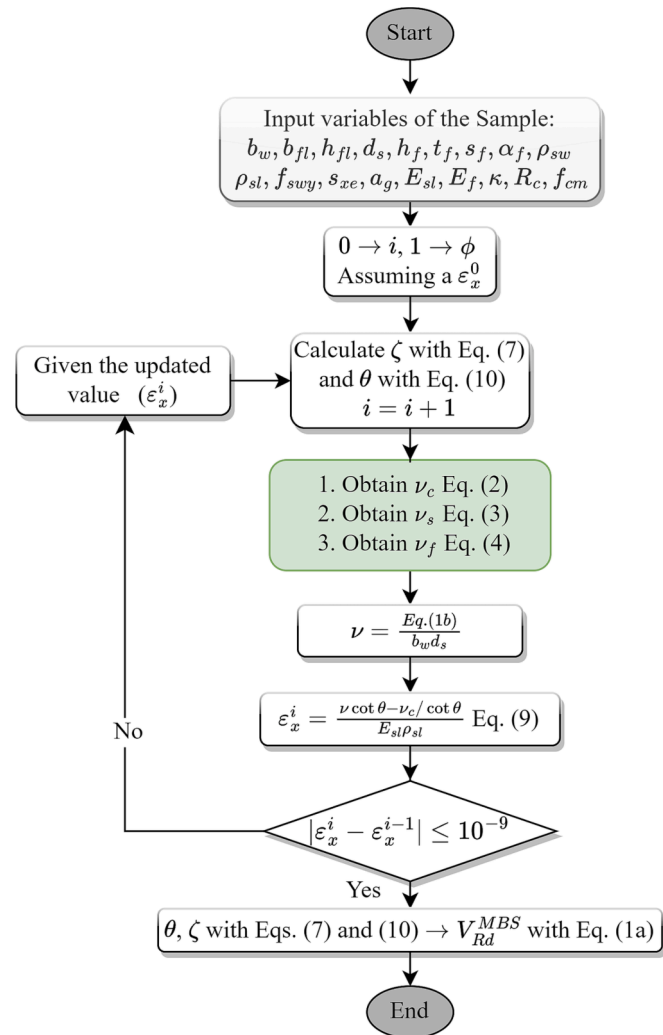


Fig. 2. Calculation procedure of MBS model.

adopted for obtaining ζ and θ , with minimum loss of accuracy, by employing a global sensitivity analysis [13]. Unlike local sensitivity analysis, global sensitivity analysis evaluates the output by simultaneously varying all input parameters [25]. This sensitivity analysis allows the assessment of the dependence of the model's output on the selected values of the input parameters. To perform the analysis, a random selection of input variables needs to be made, followed by the evaluation of the corresponding outputs (i.e., θ , ζ and ν) using the procedure outlined in Fig. 2. A sample set of 5000 beams was generated using a Latin Hypercube Sampling (LHS) method considering a uniform distribution of input variables (Δ), ensuring an equal probability of occurrence for each value within the adopted range for the considered variables, as shown in Table 1. This process results in the generation of a comprehensive dataset consisting of input–output pairs, which can be further analysed and investigated.

To assess the significance of each parameter on the outputs, a dimensionless parameter, denoted as σ , was employed. This parameter is obtained from the following equation [13]:

$$\sigma_{ij} = \frac{n \sum_{k=1}^n I_{ik} O_{jk} - \left(\sum_{k=1}^n I_{ik} \right) \left(\sum_{k=1}^n O_{jk} \right)}{\sqrt{n \left(\sum_{k=1}^n I_{ik}^2 \right) - \left(\sum_{k=1}^n I_{ik} \right)^2} \sqrt{n \left(\sum_{k=1}^n O_{jk}^2 \right) - \left(\sum_{k=1}^n O_{jk} \right)^2}} \quad (11)$$

where n is the number of samples ($n = 5000$); I_i is the i^{th} input parameter (e.g., f_{cm}); and O_j is the j^{th} output parameters (e.g., θ). The σ parameter can range from -1 to $+1$, with positive and negative signs indicating positive and negative linear correlations, respectively. A positive value of σ suggests that as the independent variable I_i increases, the dependent variable O_j also increases, and vice versa. Conversely, a negative value of σ indicates that O_j decreases with the increase of I_i , and O_j increases with the decrease of I_i . When the σ value is close to 0, signifies either a weak linear correlation or no linear correlation between the variables. The values of the σ for each input parameter and output entity is given in Fig. 3.

As can be seen, the ρ_{sw} , f_{swy} , ρ_{sl} and h_f exhibit the highest influence on the outputs. The b_w and b_{fl} also have significance, but they were intentionally omitted from consideration to avoid the potential multicollinearity issue and complexity of equations without improvement in the prediction accuracy. These selected parameters play a crucial role in a model for predicting ζ and θ . It is noteworthy that a strong correlation between ζ and θ is expected [13]. Consequently, by initially predicting one of these parameters and subsequently calculating the other one based on this predicted parameter, the iterative procedure of the model can be eliminated. In the following, an equation will be introduced to approximate the value of ζ found from Eq. (7), thereby facilitating the subsequent calculation of θ . Fig. 4 shows that there is a strong relation between ζ and the following composite parameters $x = \rho_{sw} f_{swy}$, $y = \rho_{sl} b$

Table 1

Design space of input variables (meaning of the symbols in Fig. 1).

Random variable	Range
Web width, b_w	[75–600] mm with $\Delta b_w = 5$ mm
Flange width, b_{fl}	$[b_w - 3b_w]$ mm with $\Delta b_{fl} = 0.5b_w$
Flange thickness, h_{fl}	{0, 50, 100} mm
Height of beam's cross section, h	$[b_w - 2b_w]$ mm with $\Delta h = 5$ mm
Corner radius, R_c	{10, 20, 30, 40} mm
Fibre's angle, α_f	{45, 60, 75, 90} degrees
Young modulus of elasticity of FRP, E_f	[60–400] GPa with $\Delta E_f = 20$ GPa
FRP strengthening ratio, ρ_{sf}	[0.8–20]%
Flexural reinforcement ratio of steel bars, ρ_{sl}	[1–7.5]%
Transverse reinforcement ratio of steel stirrups, ρ_{sw}	[0–1.8]%
Yield stress of steel stirrups, f_{swy}	[240–670] MPa with $\Delta f_{swyk} = 10$ MPa
Mean concrete compressive strength, f_{cm}	[20–100] MPa with $\Delta f_{cm} = 1$ MPa
Maximum dimension of aggregates, a_g	[10–40] mm with $\Delta a_g = 1$ mm

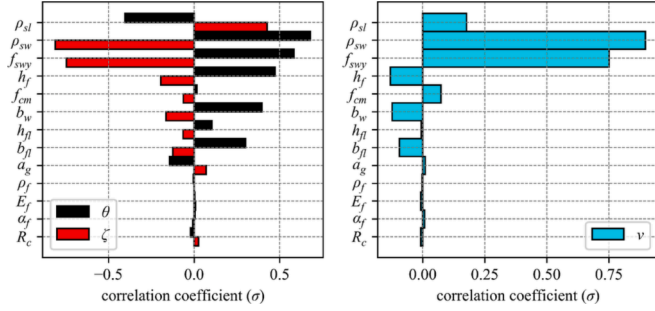


Fig. 3. Tornado plots; calculated via sensitivity analysis for MBS model.

and $w = h_f$. To find the relationship between ζ and these parameters a hyperplane should be fitted to the data. A hyperplane, described by the following form, is determined to be the best fit for the available data:

$$\zeta_{MBS} = (ax_i^b + cy_i^d) \times (w_i^e + f) \quad (12)$$

where ζ_{MBS} represents the fitted hyperplane to ζ (found by Eq. (7)) and a , b , c , d , e and f are the unknown constants of this hyperplane. To derive this hyperplane and to obtain the corresponding constants, an objective function is defined, and genetic algorithm was employed:

$$L = \sum_{i=1}^n \left| \frac{\zeta_i - \zeta_{MBS,i}}{\zeta_i} \right| \quad (13)$$

By minimizing the objective function, the constants are determined and the best fit of the hyperplane to the data is represented as follow:

$$\zeta_{MBS} = (0.49y^{0.19} - 0.066x^{0.37})(1.11 - 0.00021w) \quad (14)$$

where $0.05 < \zeta_{MBS} \leq 0.35$

Fig. 4a clearly illustrates the interrelation between θ and ζ . Considering this relationship, a power function is fitted to the data, resulting in a prediction model for θ based on the ζ value (Eq. (15)).

$$\theta_{MBS} = 4.06\zeta_{MBS}^{-0.75} + 20.7 \quad 29 < \theta_{MBS} \leq 60 \quad (15)$$

The goodness of the fit is shown by the following statistical metrics:

$$RMSE = \sqrt{\sum_{i=1}^n (y_i - \gamma_i)^2 / n} \quad (16)$$

$$MAPE = \frac{100\%}{n} \sum_{i=1}^n (|(y_i - \gamma_i)/y_i|) \quad (17)$$

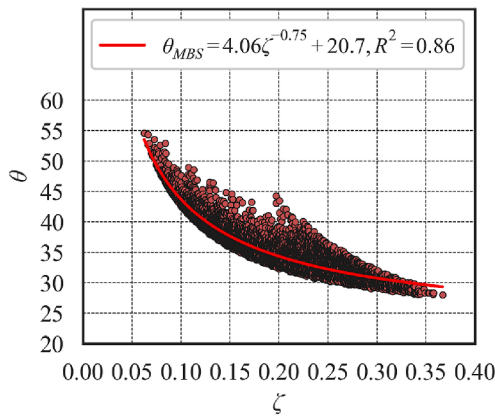
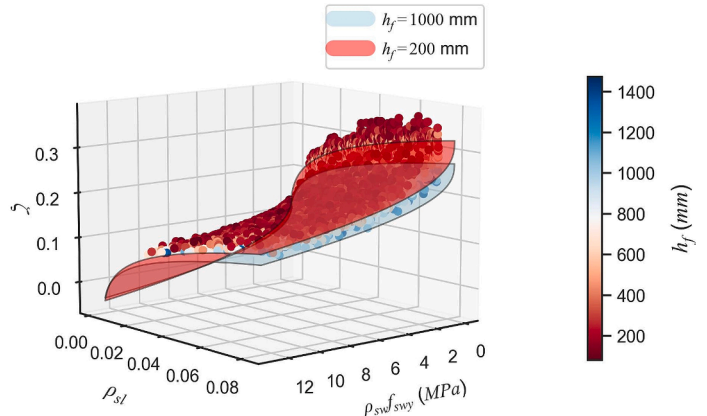


Fig. 4. A) interrelation between θ and ζ ; b) ζ values obtained from MBS model and the planes fitting the data with two different h_f .

$$R^2 = 1 - \left(\sum_{i=1}^n (y_i - \gamma_i)^2 / \sum_{i=1}^n (y_i - \bar{y})^2 \right) \quad (18)$$

$$r = \sum_{i=1}^n \left[\frac{(y_i - \bar{y})(\gamma_i - \bar{\gamma})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 \sum_{i=1}^n (\gamma_i - \bar{\gamma})^2}} \right] \quad (19)$$

where $RMSE$ is the root mean square error; $MAPE$ is the mean absolute percentile error; R^2 is the coefficient of determination; r is the Pearson correlation coefficient; y_i (i.e., ζ and θ obtained from MBS-SMCFT) is the actual value for the i^{th} sample; $\bar{y}$ is the mean of actual values; γ_i (i.e., ζ_{MBS} and θ_{MBS} obtained from approximate solution) is the predicted value for the i^{th} sample, and $\bar{\gamma}$ is the mean value of predicted values. A model with a perfect prediction has a null value for $MAPE$ and $RMSE$ and the unitary value for r and R^2 . The $RMSE$, $MAPE$, R^2 and r for the fitted hyperplane (ζ_{MBS}) for ζ are 0.013, 4.60 %, 0.96, and 0.98, respectively. As the fitted hyperplane cannot be easily visualized in a three-dimensional plot, two fixed values of w were chosen, and two different planes are plotted in Fig. 4b, corresponding to these values. The accuracy of the fitted curve (θ_{MBS}) for θ was assessed using introduced statistical metrics. The $RMSE$, $MAPE$, R^2 and r were 1.71, 3.51 %, 0.86, and 0.93, respectively.

4. Resistance reduction factor

In design applications, it is common to use a reduced value of resistance, commonly considered the design value of the resistance (R_d). This reduced value can be obtained by applying a resistance reduction factor (ϕ). The objective in this section is to calibrate strength reduction factors for the MBS model, taking into account uncertainties and variations, to ensure structural safety and to achieve desired target reliability indices (β_T). The choice of the target reliability index (β_T) in the context of the ultimate limit state is generally influenced by various factors, including the consequences of failure, relative costs of safety measures, and the reference period, as discussed in [26]. According to [26], the selection of the target reliability index for existing structures depends on the costs associated with implementing safety measures. It is recommended to consider a target reliability index within the range of 3.1 to 4.1, with different values corresponding to different cost considerations and specified reference periods. In this study, four levels of target reliability are considered to capture a range of safety and cost scenarios. The selected target reliability indices are $\beta_T = 3.1$, $\beta_T = 3.4$, $\beta_T = 3.8$ and $\beta_T = 4.1$. In the case of new elements a β_T of 3.8 is recommended [26], which is adopted in this study to compare the proposed model with other considered models. Monte Carlo Simulation (MCS) is a comprehensive and rigorous approach to determine the probability of failure and consequently ϕ values [18,27], which is also employed in

this study. The MCS is known for its accuracy in capturing the uncertainties and variability associated with the structural behaviour, material properties, etc. Although this approach can be computationally intensive and time-consuming.

Fig. 5 presents the flowchart of the reliability analysis performed in this paper using MCS method. In this analysis four different ϕ values are to determine considering the selected target reliability indices. The analysis initiates by generating 100 beam samples using LHS within specified parameter ranges as outlined in Fig. 5. The properties of each beam sample required for MBS model are subsequently stored in a vector denoted as $\vec{x}_m$, comprising p parameters. Then, for each value of ϕ (ranging from 0.1 to 1, with an increments of 0.01), a reliability index is obtained for all the samples adopting five different values of live load to dead load ratio ($\alpha = Q_{LL}/Q_{DL}$), as recommended in [18,20] (i.e., 0.5, 1.0, 1.5, 2.0, and 2.5). It should be noted that the selected number of samples are sufficient, as further increasing the sample size does not yield significant difference in the outcome.

The solution procedure to calculate β is presented in a nested flowchart in Fig. 5. Initially, the load distribution (Q) is obtained as the sum of the dead and live load distributions (Q_{DL} and Q_{LL}) without considering any load factors. The distribution of dead and live load can be obtained by taking the respective nominal values of dead load and live loads (Q_{DL} and Q_{LL}) as mean, and the statistical characteristics of dead and live load given in Table 2. To obtain Q_{DL} and Q_{LL} , an indirect method is used as described in [19,20]. The ultimate limit state load combination (Q_{ULS}), as defined by ACI 318 [28], is considered. This load combination considers the dead load (Q_{DL}) and live load (Q_{LL}) majored according to the following load factors:

$$Q_{ULS} = 1.2Q_{DL} + 1.6Q_{LL} \quad (20)$$

Based on the limit state design function (i.e., $R_d = Q_{ULS}$) and Eq. (20), Respective Q_{DL} and Q_{LL} can be expressed as:

$$Q_{DL} = \frac{R_d}{1.2 + 1.6\alpha} \quad (21)$$

$$Q_{LL} = \frac{\alpha R_d}{1.2 + 1.6\alpha} \quad (22)$$

The design resistance (R_d) is calculated using Eq. (1a) with the given $\vec{x}_m$ and the corresponding ϕ .

Simultaneously, the distribution of resistance (R) is obtained (using Eq. (1a) and ϕ of equal to 1.0) considering the distribution of random variables (X) with their respective means in $\vec{x}_m$ and the statistical characteristics in Table 2. The size of samples for MCS is denoted as N and is set to 2,000,000.

To consider the model uncertainty the distribution of model error should be identified. According to the Kolmogorov-Smirnov (K-S) testing model uncertainty $\chi = V_R^{\text{exp}}/V_R^{\text{MBS}}$ (with V_R^{exp} and V_R^{MBS} being the experimental values in the database introduced in the section 5.1 and the model prediction of the shear resistance, respectively) is identified to have an Extreme type I distribution with a mean of 1.0 and Cov of 0.3, as can be seen in Fig. 6.

Hence, a random variable of size N is sampled from this distribution to represent the model error distribution (χ). Having the R , Q , and χ distribution, the ultimate limit state function (g) of the beam sample is adopted to differentiate the acceptable and unacceptable structural state of the beam, which is given as

$$g(X) = \chi R(X) - Q(X) \quad (23)$$

An unacceptable state or often known as failure state can be represented as $g(X) \leq 0$. If the random vector of variables is represented by a joint probability density function $X \sim f(X)$, the failure probability p_f can be computed by

$$p_f = P(g(X) \leq 0) = \int_{\{X: g(X) \leq 0\}} f(X) dX \quad (24)$$

However, when using MCS the functional form of $f(X)$ is not available, therefore the expected value of p_f is calculated as the fraction of samples that belong to the failure domain n_f ($g(x) \leq 0$) over N .

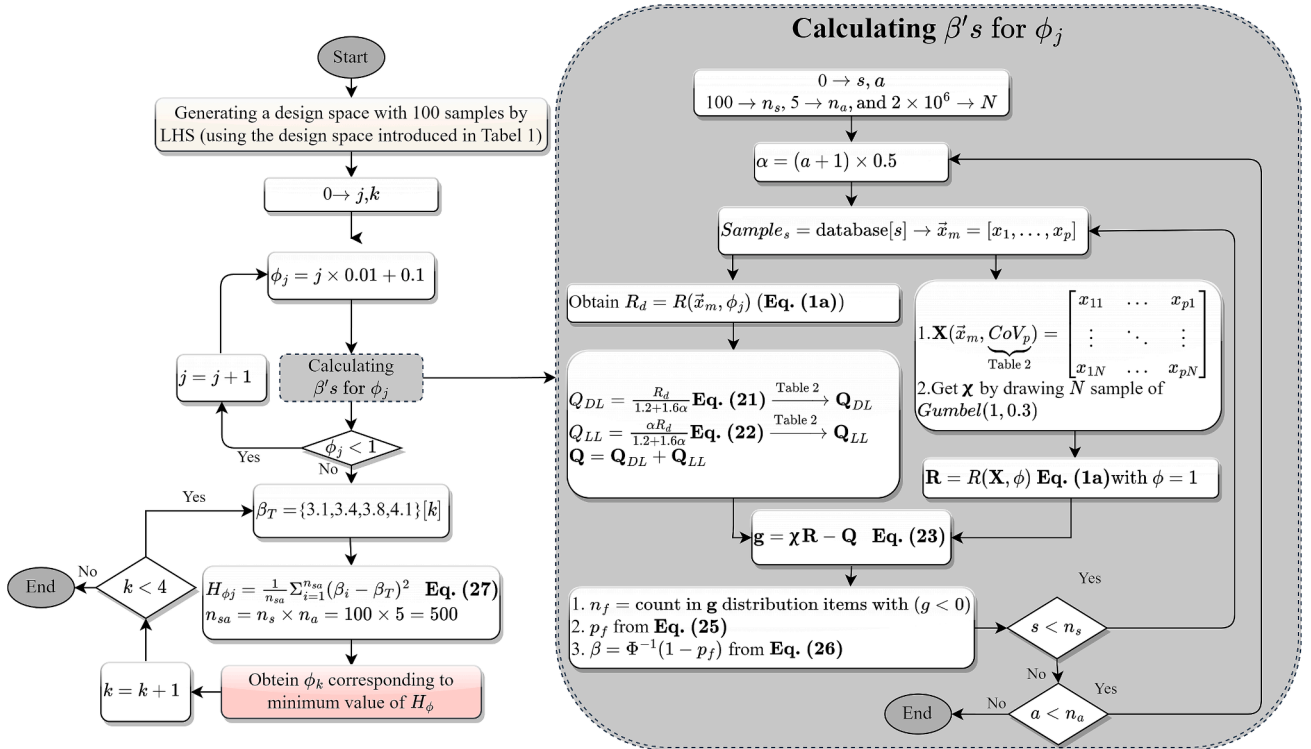


Fig. 5. Reliability index prediction and resistance reduction factor calibration.

Table 2
Statistical characteristics of random variables.

Random variable	Bias [29]	CoV	Distribution	Reference
Web width (b_w)	1	0.02	Normal	[30]
Total height (h)	1	0.01	Normal	[30]
Effective depth (d)	1	0.02	Normal	[30]
Corner radius (R_c)	1	0.05	Normal	assumed
Fibre's angle (α_f)	1	0.05	Normal	[31]
Young modulus of CFRP (E_f)	1	0.02	Normal	[20]
Young modulus of steel (E_s)	1	0.024	Normal	[32]
CFRP shear strengthening ratio (ρ_f)	1	0.03	Normal	[19]
Reinforcement ratio of longitudinal steel bars (ρ_{sl})	1	0.05	Normal	assumed
Reinforcement ratio of transversal steel bars (ρ_{sw})	1	0.05	Normal	assumed
Yield stress of steel stirrups (f_{swy})	1.1	0.075	Normal	[33]
Mean concrete compressive strength (f_{cm})	1.15	0.1	Lognormal	[34]
Model Error (χ)	1	0.3	Extreme I	k-s test
Dead load (Q_{DL})	1	0.1	Normal	[34]
Live load (Q_{LL})	1 + 0.14ln(t/ 50)*	0.18/(1 + 0.14ln(t/ 50))	Extreme I	[32]

* according to service life of 50 years a bias of 1.0 and CoV of 0.18 is taken for Live load.

$$p_f = \frac{n_f}{N} \quad (25)$$

The reliability index (β) of the beam sample is obtained from the following equations:

$$\beta = \Phi^{-1}(1 - p_f) \quad (26)$$

where Φ^{-1} is the inverse of standard normal cumulative distribution [18].

The probability distributions of R , Q_{DL} , Q_{LL} , Q , and scatterplot of R versus Q corresponding to a sample beam B10_M [35] are presented in Fig. 7. This illustration considers two distinct values of ϕ (0.9 and 0.5) and two different values of α (0.5 and 2.5). This example showcases the computation of the reliability index for the beam, considering various values of α and ϕ ratios. It underscores the dependency of the beam's reliability index on these parameters. The reliability indices of the beam

while α varies from 0.5 to 2.5 and ϕ is equal to 0.9 corresponds to reliability indices of 1.21 and 1.58, respectively. The reliability indices corresponding to ϕ of 0.5 are 3.92 ($\alpha = 0.5$) and 3.75 ($\alpha = 2.5$). As expected, increase in ϕ value leads to an increase in the reliability index. While the effect of α on reliability index depends on the value of ϕ . As ϕ value decreases, the R and Q distribution diverge, and increase in α leads to a larger p_f . On the contrary when R and Q distribution approach each other, increase in α results in a smaller p_f . This is primarily attributed to the fact that when larger values of α is used, the distribution of Q exhibits an elongated upper tail. This elongation is attributed to a greater influence from the Q_{LL} distribution, characterized by an extreme type I distribution with a large CoV. Conversely, when smaller α (0.5) is used, the Q distribution tends to resemble a normal distribution, primarily influenced by the Q_{DL} distribution.

Fig. 8 presents the average reliability indices of all 100 generated sample beams versus ϕ values, corresponding to different values of α . As discussed, an increase in ϕ consistently leads to a decrease in the reliability index. Nevertheless, the influence of α is contingent upon the ϕ value.

To calibrate the resistance reduction factor, the least square method is employed, which allows the determination of the optimal values of the resistance reduction factor that minimizes the difference between the predicted reliability index and the target reliability index:

$$H = \frac{1}{n_{sa}} \sum_{i=1}^{n_{sa}} (\beta_i - \beta_T)^2 \quad (27)$$

where β_i is the reliability index obtained from the analysis and n_{sa} is the total number of scenarios (i.e., 100 beams with 5 different α ratio, resulting 500 scenarios).

As presented in Fig. 9, the values of ϕ corresponding to specific β_T levels were found as follows: for $\beta_T = 3.1$, $\phi = 0.62$; for $\beta_T = 3.4$, $\phi = 0.57$; for $\beta_T = 3.8$, $\phi = 0.52$; and for $\beta_T = 4.1$, $\phi = 0.49$. These calibrated factors can be used in the design process to ensure the desired target reliability levels for shear failure.

5. Validation and discussion

5.1. Construction of dataset

A database comprising 284 RC beams strengthened with EBR-CFRP was constructed from literature [17,34–87], following a filtering process. The filtering criteria involved removing data with: (i) incomplete descriptions of material properties or sectional dimensions; (ii) CFRP shear strength contribution less than or equal to 10 kN; (iii) special configurations other than U-jacketing and fully wrapped; (iv) anchorage systems; (v) special configurations other than CFRP; and (vi) shear span-

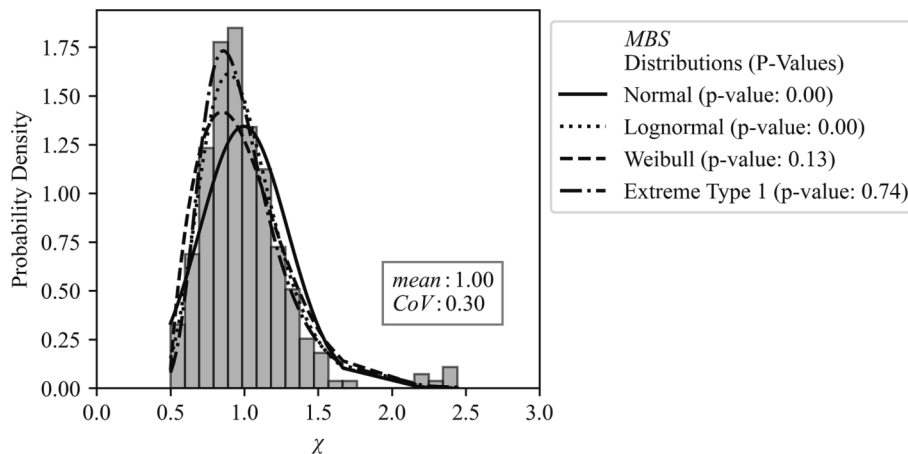


Fig. 6. K-S test result of the model error for MBS model.

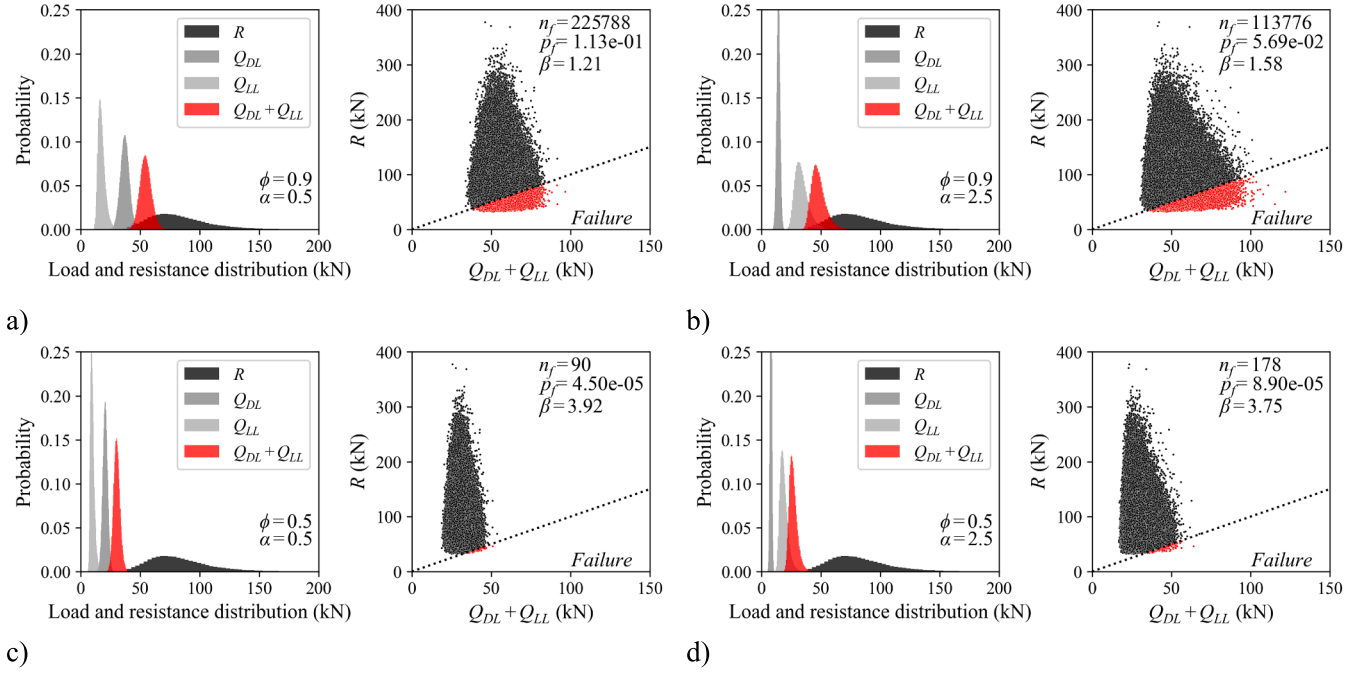


Fig. 7. Reliability analysis of beam B10_M [35] using: a) $\phi = 0.9$ and $\alpha = 0.5$, b) $\phi = 0.9$ and $\alpha = 2.5$, c) $\phi = 0.5$ and $\alpha = 0.5$, d) $\phi = 0.5$ and $\alpha = 2.5$.

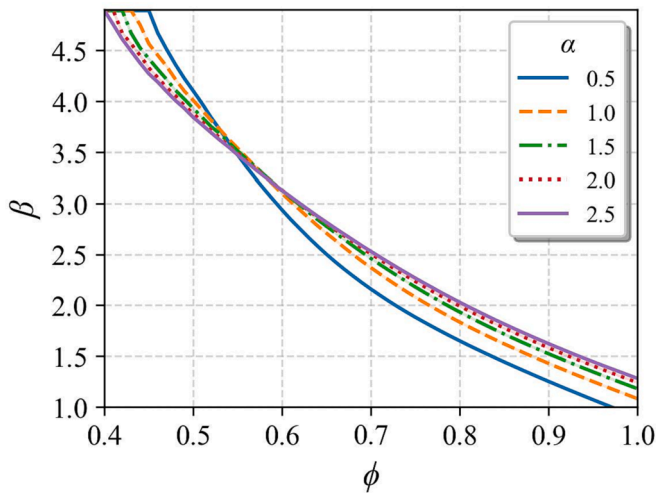


Fig. 8. Average reliability index vs ϕ , corresponding to different α values.

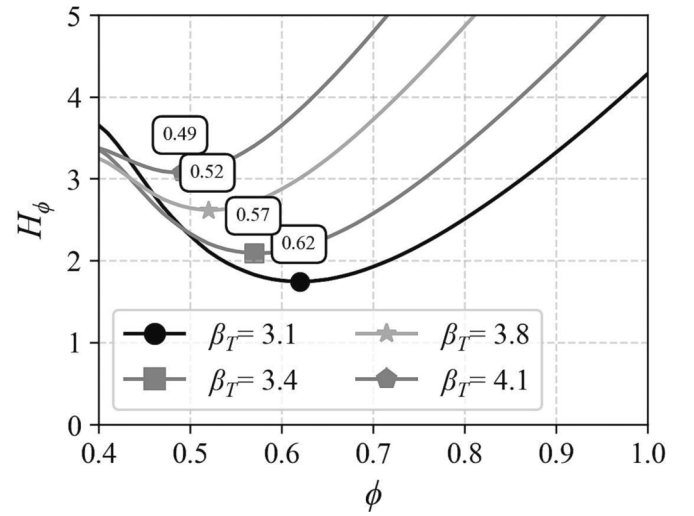


Fig. 9. Average deviation from target reliability index.

to-depth ratios smaller than 2.2. Out of the 284 beams, 211 were strengthened with U-wraps, while the remaining 73 beams were fully wrapped. Furthermore, the database included 202 beams with a rectangular cross-section and 82 beams with a T-section. This database serves as a valuable resource for validating both the proposed and existing models for RC beams strengthened with EBR-CFRP.

5.2. Assessment of models

In this section, the performance of the proposed model is assessed considering Eqs. (14) and (15) for ζ_{MBS} and θ_{MBS} , respectively. In accordance with MC2010 [26], the target reliability index of 3.8 is recommended for the design of new elements with a medium consequence of failure and a service life of 50 years. Hence, in this section, the resistance reduction factor $\phi = 0.52$ was utilized to facilitate a comparison of the proposed model's performance with existing design models. However, when considering strengthening purposes, resistance

reduction factors are available for various reliability levels, which can be chosen based on the information provided in Section 3.3.3 of MC2010 [26]. It should be emphasized that in the case of ACI 440-2R-17 [22], the shear strength contribution of concrete and steel is determined based on ACI 318-19 [28], and the design value incorporates the corresponding resistance reduction factor. In contrast, as recommended in fib bulletin 90 and TR55, Eurocode 2 (EC2) [90] is utilized to evaluate the shear contribution of the steel and concrete components in these models, considering the appropriate partial reduction factors of each models (refer to Appendix A, for more details).

Fig. 10 presents the comparison between the predictions of the proposed model and other existing design models with the experimentally measured values, using the data from the collected database, previously described. The primary objective of design models is to ensure safe predictions. The comparison of the predictions made by the models in Fig. 10 shows that each model has relatively addressed this goal, as there are only a few instances of unsafe predictions observed for each

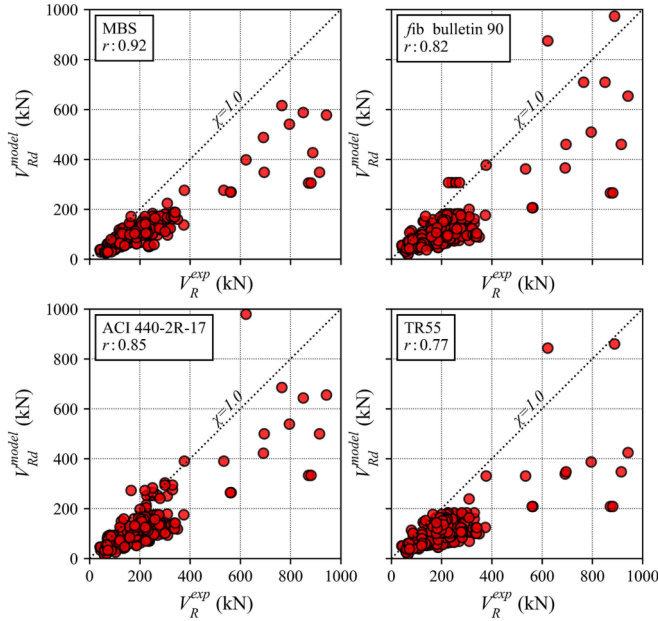


Fig. 10. Predicted versus actual shear resistance of RC beams strengthened with EBR-CFRP.

model. However, in addition to safety, a reliable model should exhibit low dispersion in its predictions, which can be assessed using previously introduced error metrics. The results demonstrate that the proposed model exhibits the least dispersion, as evidenced by its high Pearson correlation coefficient (r) of 0.92. Comparatively, the ACI 440-2R-17 and fib bulletin 90 design models also show relatively low dispersion, with r of 0.85 and 0.82, respectively, while the TR55 model presented the highest dispersion, with r of 0.77. To further analyse, the bias and dispersion of each model, the mean, standard deviation (σ) and CoV of model uncertainty ($\chi = V_R^{exp}/V_{Rd}^{model}$) were calculated and are presented in Fig. 11. The results reveal that the proposed design model exhibit less underestimated predictions on average (1.91), its predictions have less dispersion (with $\sigma = 0.57$, and $CoV = 0.3$) compared to the existing design models. This indicates a higher level of safety and reliability. These findings highlight the superior performance of the proposed model in terms of both accuracy and precision compared to the existing design models.

5.3. Demerit point score

It should be emphasized that relying solely on descriptive statistics, such as the mean of χ value, may not provide a comprehensive evaluation of the model reliability. Merely considering the mean χ value being closer to 1.0 does not necessarily indicate a better performance of the

model, as this model may still produce a significant number of extremely dangerous or overly conservative predictions. Additionally, it is well-established that extremely dangerous or dangerous predictions have more severe consequences compared to conservative predictions [91]. To overcome this limitation, the Demerit Point Scale Methodology, a weighted penalty categorization approach, is employed [10]. This method assigns penalty points to different ranges of χ and calculates the total demerit point score for each model by considering the percentage of χ falling within each range, the assigned demerit points, and normalizing the result. The resulting score ranges from 0 to 10, being the model performance as better as smaller is the score. It is important to note that, in this study, a modified demerit point classification is used as instead of mean values of shear resistance the design values are compared with the experimental values of shear resistance. This classification not only penalizes highly unsafe predictions, but also penalizes overly conservative predictions to in an attempt of considering both safety and cost arguments for the CFRP-based strengthening solutions. Table 3 gives the demerit points assigned to each model, revealing that the proposed model achieves superior performance with a score of 1.57. It is noteworthy that this model demonstrates a favorable outcome with 97 % of predictions falling within a low, appropriate safety and conservative level, indicating its enhanced reliability and cost-efficiency compared to other models.

5.4. Models feature sensitivity analysis

As previously mentioned, the shear mechanism in RC beams is a complex phenomenon influenced by multiple factors. However, existing models have generally overlooked these dependencies. Consequently, the prediction performance of these models can vary significantly when applied to beams with different conditions. Fig. 12 to Fig. 14 present the

Table 3
Demerit point classification.

χ	Classification	Penalty	MBS	fib bulletin 90	ACI 440-2R-17	TR55
<0.75	Extremely dangerous	10	0 % ^a	2 %	1 %	1 %
0.75–1.0	Dangerous	5	0 %	4 %	7 %	3 %
1.0–1.25	Low safety	0	7 %	10 %	10 %	8 %
1.25–1.75	Appropriate safety	1	35 %	35 %	30 %	29 %
1.75–3	Conservative	2	55 %	38 %	48 %	49 %
>3	Extremely conservative	4	3 %	11 %	5 %	10 %
Total demerits point score			1.57 ^b	1.95	1.91	1.92

^a : Percentage of specimens with χ laying in the range.

^b : $((0 \times 10) + (0 \times 5) + (7 \times 0) + (35 \times 1) + (55 \times 2) + (3 \times 4))/100 = 1.57$.

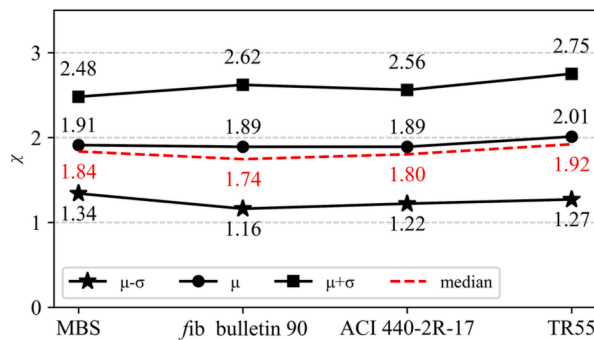
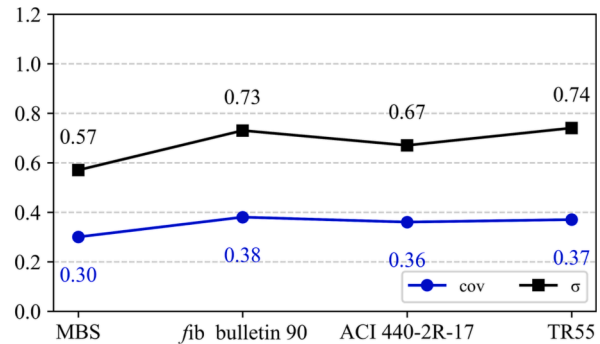


Fig. 11. Statistical analysis of model uncertainties.

boxplots of model uncertainties (for the proposed model and the considered existing design models) with a black line in the boxes that shows the median of χ corresponding to beams with different cross-section shapes (T-section or Rectangular), equivalent transversal reinforcement ($\rho_{sw} + \rho_f E_f / E_{sw}$; where E_{sw} is elastic modulus of steel stirrups), and the ratio of longitudinal reinforcement to the spacing of stirrups (ρ_{sl} / s_s). To quantify the performance of each model a baseline value (a value of 1.84 is used which is the median of χ for all models' predictions) is introduced. Then, the cumulative deviation of medians for each model from this baseline is measured across all input parameter ranges which provides an indication of the model's performance in the proposed analysis (Table 4).

Fig. 12 reveals a considerable underestimation in shear resistance of RC beams with a T-section using the existing models, which may be attributed to their disregard for the contribution of the beam flange and its impact on θ . The cumulative deviations of median values of χ from the baseline were assessed for Rectangular and T-shaped beams. The MBS model exhibited superior performance (0.097) in comparison to fib Bulletin 90 (0.457), ACI 440-R2-17 (0.523), and TR55 (0.431). Fig. 13 demonstrates the large variability in predictions among the current models concerning the impact of $\rho_{sw} + \rho_f E_f / E_{sw}$ on the shear resistance, which can be attributed to the omission of ρ_{sw} and ρ_f effect on θ (see Fig. 3). Assessing the cumulative deviations of median χ values from the baseline across various ranges of $\rho_{sw} + \rho_f E_f / E_{sw}$, the MBS model (0.316) outperforms other models such as fib Bulletin 90 (1.140), ACI 440-R2-17 (1.218), and TR55 (1.277). Additionally, Fig. 14 illustrates the existing models' tendency to underestimate the shear resistance of beams with the increase of ρ_{sl} / s_s . This is due to the increase of the dowel action with increase of ρ_{sl} and the decrease of the stirrup spacing s_s , which is not accounted by existing models in the parcel corresponding to the CFRP shear contribution. Comparing the cumulative deviations of χ medians from the baseline along different ranges of ρ_{sl} / s_s , the MBS model (0.541) demonstrates superior performance compared to other models, including fib Bulletin 90 (1.406), ACI 440-R2-17 (0.870), and TR55 (1.497). Fig. 12 to Fig. 14 demonstrate that the proposed model, which incorporates the influence of cross-section type, equivalent transversal reinforcement, and longitudinal reinforcement, shows improved performance in generating less variation in predictions compared to the baseline value.

6. Conclusion

The study presented in this paper aimed to develop a comprehensive design model for predicting the shear resistance of RC beams strengthened with EBR-CFRP reinforcement, applicable to U-jacketing and fully wrapped configurations. This was achieved by integrating the SMCFT approach and Mohammadi et al.'s [10] model, followed by conducting a reliability analysis. A key aspect of the model development was the

utilization of a global sensitivity analysis to derive non-iterative prediction equations for two important model parameters: the tensile stress factor in cracked concrete (ζ) and the inclination of the diagonal compressive stress in the web of the section (θ). Furthermore, a Monte Carlo reliability analysis was employed to determine the resistance reduction factor for the model.

To validate the proposed model, a thorough literature review was conducted, resulting in the compilation of a comprehensive database comprising 284 beams strengthened with EBR-CFRP reinforcement. This database served as the basis for comparing the performance of the proposed model against existing design models. Based on the results and discussions presented in this paper, the following conclusions can be drawn:

- 1- The proposed non-iterative equations for the parameters ζ and θ demonstrate a close resemblance to the values obtained from the SMCFT. The approximate equation for ζ exhibits an *RMSE* of 0.013, a *MAPE* of 4.60 %, an R^2 of 0.96, and an r of 0.98. Similarly, the approximate equation for θ yields an *RMSE* of 1.71, a *MAPE* of 3.51 %, an R^2 of 0.86, and an r of 0.93. These statistical measures indicate the strong agreement between the proposed equations and the SMCFT-based values, validating the accuracy and reliability of the non-iterative approach;
- 2- The proposed model exhibited superior predictive performance in estimating the shear resistance of RC beams strengthened with EBR-CFRP reinforcement, with r of 0.92;
- 3- Among the design models considered in this study, the proposed model exhibited the model uncertainties closest to unity, indicating more balanced and less biased predictions. Furthermore, the coefficient of variation (*CoV*) of the proposed model was 0.3, being the lowest one, indicating the proposed model provides more consistent and reliable estimates of shear resistance, making it a favourable choice for practical applications;
- 4- The reliability analysis conducted in this study reveals that different target reliability indices (β_T) can be achieved by employing specific resistance reduction factors (ϕ). Specifically, a target reliability index of 3.1 corresponds to a resistance reduction factor of 0.62, while indices of 3.4, 3.8 and 4.1 are associated with reduction factors of 0.57, 0.52, and 0.49, respectively;
- 5- According to the modified demerit point scale methodology performed in this study, the proposed model produces the most reliable and cost-effective predictions in comparison with the considered design models;
- 6- The considered existing design models overlook dependencies of shear resistance to a few crucial parameters, resulting in varied prediction performance for different beam conditions. The proposed model incorporates effects of several parameters including cross-section type, equivalent transversal reinforcement, and

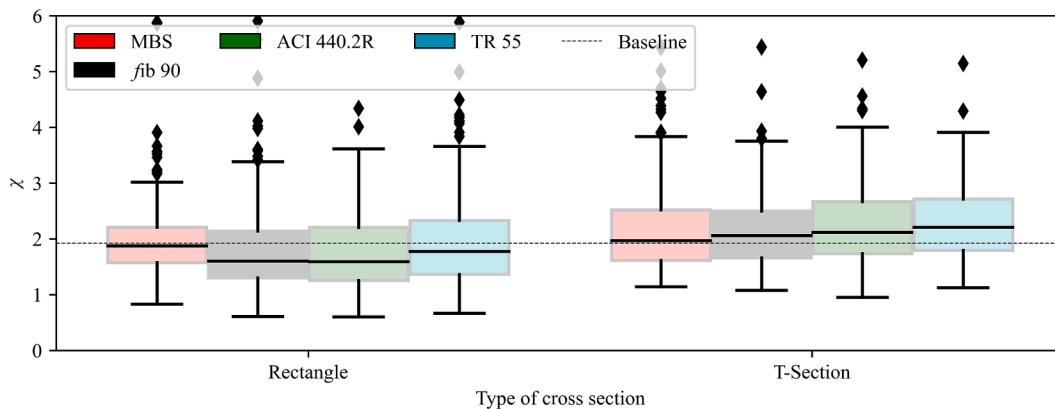


Fig. 12. Prediction performance of models for beams with different cross-section.

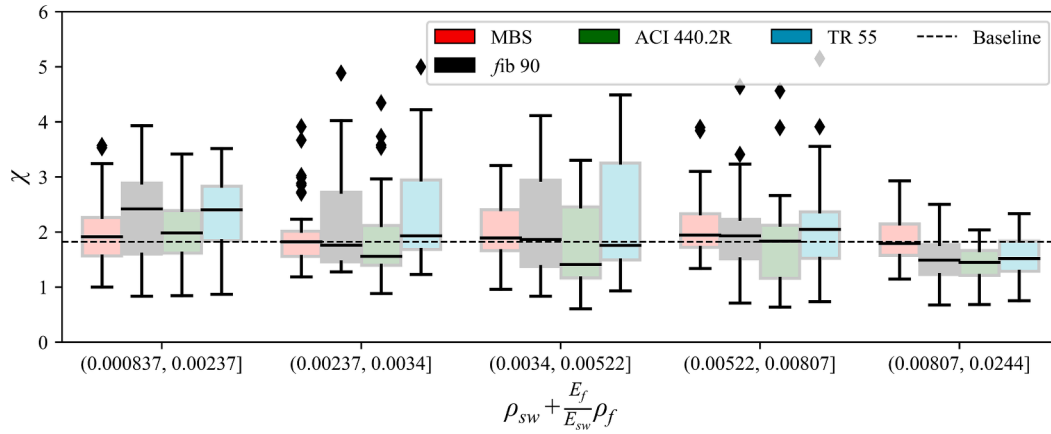


Fig. 13. Prediction performance of models for beams with different equivalent transversal reinforcement.

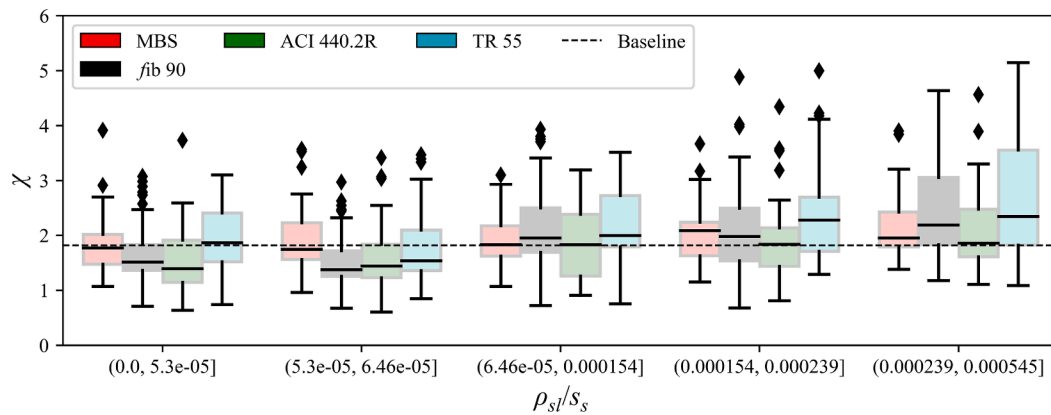


Fig. 14. Prediction performance of models for beams with different ρ_{sl}/s .

Table 4

Cumulative deviations of the median values of χ for each model relative to the baseline across different input parameter ranges.

Input parameter	Models			
	MBS	fib bulletin 90	ACI 440-R2-17	TR55
Type of cross-section	0.097	0.457	0.523	0.431
$\rho_{sw} + \rho_f E_f / E_{sw}$	0.316	1.140	1.218	1.277
ρ_{sl}/s	0.541	1.406	0.870	1.497

longitudinal reinforcement, enhancing its ability to generate less biased predictions for these parameters.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to improve the readability. After using this tool, the authors reviewed and edited the content as needed and takes full responsibility for the content of the publication.

Appendix A. Formulations of design guidelines

Fib bulletin 90

$$V_{Rd} = V_{Rd,sc} + V_{Rd,f} \leq V_{Rd,max} \quad (A1)$$

$$V_{Rd,sc} = \begin{cases} \left[C_{Rd,c} k (100 \rho_{sl} f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} \right] b_w d_s & \text{where } \rho_{sw} = 0 \\ \frac{1}{\gamma_s} 0.9 d_s \frac{A_{sw}}{s_s} f_{swy} & \text{where } \rho_{sw} \neq 0 \end{cases}$$

$$\rho_{sl} = A_{sl} / (b_w d_s) \quad (A2)$$

$$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / (\cot \theta + \tan \theta) \quad (A3)$$

($C_{Rd,c}$, k , k_1 , ν_1 , α_{cw} are introduced in EN 1992-1-1:2004), σ_{cp} is the stress due to axial load.

$$V_{Rd,f} = A_{fvc} h_{fe} f_{fwd} (\cot \theta + \cot \alpha_f) \sin \alpha_f$$

where,

$$A_{fw} = 2 w_f t_f \quad t_f = \begin{cases} t_0 n_f & \text{if } n_f < 4 \\ t_0 (n_f)^{0.85} & \text{if } n_f \geq 4 \end{cases}$$

$$A_{fvc} = \begin{cases} \frac{A_{fw}}{s_f} & \text{if strips of FRP are used.} \\ 2 t_f \sin \alpha_f & \text{if continuous FRP are used.} \end{cases} \quad (A4)$$

$h_{fe} = \min(h_f, h - 0.1 d_s)$ t_0 : thickness of each layer; (n_f in here is the number of FRP layers).

Fully wrapped: $f_{fwd} = f_{fwd,c} = a_t k_R f_{fd}$

U-wrapped: $f_{fwd} = \min(f_{fbwd}, f_{fwd,c})$

$$f_{fbwd} = \begin{cases} \left[1 - \frac{1}{3} \frac{l_e}{(h_{fe} / \sin \alpha_f)} \right] \frac{f_{fbk}}{\gamma_{fb}} & \text{for } x \geq l_e \\ f_{fbwd} = \frac{2}{3} \frac{h_{fe} / \sin \alpha_f}{l_e} \frac{f_{fbk}}{\gamma_{fb}} & \text{for } x \leq l_e \end{cases}$$

$$f_{fbwd} = \begin{cases} \frac{f_{fbk}}{\gamma_{fb}} & \text{for } x \geq l_e; \text{ and } l_e \leq y \leq x \\ \left[1 - \left(1 - \frac{2}{3} \frac{m s_f}{l_e} \right) \frac{m}{n} \right] \frac{f_{fbk}}{\gamma_{fb}} & \text{for } x \geq l_e; \text{ and } y \leq l_e \\ \frac{2}{3} \frac{(n s_f) / [(\cot \theta + \cot \alpha_f) \sin \alpha_f]}{l_e} \frac{f_{fbk}}{\gamma_{fb}} & \text{for } x \leq l_e; \text{ and } y \leq x \end{cases}$$

$$\begin{aligned} x &= h_{fe} / \sin \alpha_f \\ y &= s_f / (\cot \theta + \cot \alpha_f) \sin \alpha_f \\ n &= \text{int} \lfloor h_{fe} (\cot \theta + \cot \alpha_f) / s_f \rfloor \\ m &= \text{int} \lfloor l_e (\cot \theta + \cot \alpha_f) \sin \alpha_f / s_f \rfloor \\ \gamma_{fb} &= 1.5 \end{aligned} \quad \begin{aligned} l_e &= \frac{\pi}{2} \sqrt{\frac{E_f t_f s_{0k}}{\tau_{b1k}}} \\ \tau_{b1k} &= 0.44 \sqrt{f_{cm} f_{ctm}} \\ s_{0k} &= 0.23 \\ f_{fbk} &= \sqrt{\frac{E_f s_{0k} \tau_{b1k}}{t_f}} \end{aligned} \quad k_R = \begin{cases} 0.5 \frac{R_c}{50} \left(2 - \frac{R_c}{50} \right) & R_c < 50 \text{ mm} \\ 0.5 & R_c \geq 50 \text{ mm} \end{cases} \quad (5)$$

$$a_t = 0.8, \quad f_{fd} = \frac{f_{fu}}{\gamma_f} \quad (\gamma_f = 1.25)$$

ACI 440-2R-17

$$V_{Rd} = \phi \left[(0.17 \sqrt{f'_c} + \rho_{sw} f_y) b_w d_s + V_f \right] \quad (A6)$$

$$V_f = \psi_f \left(\frac{A_{fd} f_{fe} d_{fv} (\sin \alpha_f + \cos \alpha_f)}{s_f} \right)$$

where,

$$A_{fv} = 2t_f w_f$$

$$\phi = 0.75, \psi_f = \begin{cases} 0.95 & \text{Fully - wrapped} \\ 0.85 & \text{U - shaped} \end{cases} \quad (A7)$$

$$f_{fe} = E_f \varepsilon_{fe} \quad (A8)$$

TR 55

$$V_{Rd} = V_{Rd,sc} + V_{Rd,f} \leq V_{Rd,max} \quad (A9)$$

$V_{Rd,sc}$ is obtained according to Eq. (A- 2)

$$V_{Rd,f} = \frac{A_{fv}}{s_f} \left(d_{fv} - \frac{n_{ss}}{3} l_{t,max} \cos \alpha \right) E_{fd} \varepsilon_{fse} (\sin \alpha_f + \cos \alpha_f) \quad (A10)$$

where,

$$n_{ss} = \begin{cases} 0 & , \text{fully - wrapped} \\ 1 & , \text{U - wrapped} \end{cases} A_{fv} = 2n t_f w_f l_{t,max} = 0.7 \sqrt{\frac{E_{fd} t_f}{f_{ctk}}}$$

$$E_{fd} = \frac{E_{fk}}{\gamma_{FRP,ME}} \varepsilon_{fse} = \min \left\{ \varepsilon_{fd} / 2, 0.5 \sqrt{\frac{f_{ctk}}{E_{fd} t_f}}, 0.004 \right\} \varepsilon_{fd} = \frac{\varepsilon_{fk}}{\gamma_{FRP,me}}$$

For continuous sheets

$$w_f = \sin \alpha_f, \text{ and } s_f = 1$$

(Assuming CFRP sheet for shear intervention, partial safety factors for young's module, strain at the ultimate, and manufacturing method are obtained from the guideline.

$$\gamma_{FRP,E} = 1.1, \gamma_{FRP,e} = 1.25, \gamma_{FRP,m} = 1.2$$

Appendix B. Summary of experimental and analytical results for the compiled dataset

#	ref	Specimen	$V_{R,f}^{exp}$ (kN)	V_{Rd}^{MBC} (kN)	$V_{Rd}^{fib\ b90}$ (kN)	$V_{Rd}^{ACI440-2R}$ (kN)	V_{Rd}^{TR55} (kN)
1	[17]	M1-a	349.4	160.6	88.9	118.2	99.4
2	[17]	M1-b	324.7	169.9	97.8	134.6	118.9
3	[17]	H1-a	333.8	169.9	90.1	119.9	100.6
4	[17]	H1-b	335.8	188.2	102.8	141.7	123.8
5	[17]	H2-a	338.1	169.9	90.1	119.9	100.6
6	[17]	H2-b	340.4	188.2	102.8	141.7	123.8
7	[36]	S3	101.1	79.4	72.8	59.8	71.3
8	[36]	S5	99.1	85.9	93.7	75.8	90.6
9	[37]	CS1	214.0	119.5	120.3	102.2	109.1
10	[37]	CS2	159.0	107.0	93.6	82.4	88.0
11	[37]	CS3	116.0	61.2	60.3	52.5	54.7
12	[38]	A2	185.0	61.9	46.5	52.3	45.5
13	[38]	A3	187.0	61.9	46.5	52.3	45.5
14	[39]	A-SW3-2	177.0	122.6	150.2	159.6	162.8
15	[39]	A-SO3-2	131.0	57.0	62.3	44.5	56.8
16	[39]	A-SO3-3	133.5	60.6	76.7	53.5	68.3
17	[39]	A-SO3-4	144.5	65.8	100.0	71.5	91.4
18	[39]	A-SO3-5	169.5	73.2	123.9	91.8	118.4
19	[39]	A-SO4-2	127.5	57.2	62.5	44.8	57.1
20	[39]	A-SO4-3	155.0	66.0	100.1	72.0	92.0
21	[39]	B-CW2	214.0	130.1	178.4	176.8	173.6
22	[39]	B-CO2	88.0	50.5	56.6	38.6	52.5
23	[39]	B-CO3	113.0	58.2	88.9	61.4	84.8
24	[39]	B-CF2	119.0	61.6	99.1	83.4	91.1
25	[39]	B-CF3	131.0	68.6	150.3	127.3	138.5
26	[39]	B-CF4	140.0	76.1	99.1	89.2	91.1
27	[39]	C-BT2	155.0	97.0	115.8	90.4	105.4
28	[39]	C-BT3	157.5	103.6	142.3	116.4	140.0
29	[39]	C-BT4	162.0	89.4	74.4	60.8	70.2

(continued on next page)

(continued)

#	ref	Specimen	V_{Rf}^{exp} (kN)	V_{Rd}^{MBC} (kN)	$V_{Rd}^{fib\ b90}$ (kN)	$V_{Rd}^{ACI440-2R}$ (kN)	V_{Rd}^{TR55} (kN)
30	[40]	T4S2-C45	219.1	98.1	64.3	94.7	63.1
31	[41]	T6NS-C45	213.6	139.2	102.4	94.1	101.8
32	[41]	T6S4-C90	272.8	133.8	90.3	113.1	91.1
33	[42]	JS2A	237.1	50.5	76.3	70.1	80.0
34	[42]	JS2B	234.2	50.5	76.3	70.1	80.0
35	[42]	JS4A	243.1	53.8	94.2	87.2	102.9
36	[43]	5	120.1	63.9	88.3	79.2	81.3
37	[43]	6	107.2	65.1	69.9	74.5	—
38	[43]	2	64.5	27.1	61.8	39.3	58.8
39	[43]	3	44.5	28.7	43.1	33.2	—
40	[44]	V10_A	107.0	49.4	54.1	42.1	52.5
41	[44]	V10_B	106.0	49.4	54.1	42.1	52.5
42	[44]	V17_A	103.0	49.4	54.1	42.1	52.5
43	[44]	V11_A	98.0	49.4	54.1	42.1	52.5
44	[44]	V11_B	124.8	49.4	54.1	42.1	52.5
45	[44]	V17_B	92.9	49.4	54.1	42.1	52.5
46	[44]	V12_A	116.0	59.2	54.1	44.0	52.5
47	[44]	V18_A	127.0	59.2	54.1	44.0	52.5
48	[44]	V20_A	140.1	59.2	54.1	44.0	52.5
49	[44]	V12_B	101.7	54.2	54.1	42.1	51.2
50	[44]	V14_B	91.7	54.2	54.1	42.1	51.2
51	[44]	V19_A	118.0	54.2	54.1	42.1	51.2
52	[44]	V19_B	115.1	54.2	54.1	42.1	51.2
53	[44]	V15_B	138.4	54.6	75.3	58.7	72.2
54	[44]	V16_B	112.0	54.6	75.3	58.7	72.2
55	[44]	V16_A	184.0	67.2	75.3	62.6	72.2
56	[44]	V18_B	202.4	67.2	75.3	62.6	72.2
57	[45]	SB3-90	231.5	131.0	181.7	175.5	138.3
58	[45]	SB4-90	217.5	131.0	181.7	175.5	138.3
59	[45]	SB5-90	234.5	131.0	181.7	175.5	138.3
60	[45]	SB6-90	218.0	131.0	181.7	175.5	138.3
61	[45]	SB7-90-0	241.0	144.0	307.1	252.6	176.2
62	[45]	SB8-90-0	227.0	144.0	307.1	252.6	176.2
63	[45]	SB9-45	264.5	135.7	181.7	175.5	138.3
64	[45]	SB10-45	273.5	135.7	181.7	175.5	138.3
65	[45]	SB11-45	254.5	135.7	181.7	175.5	138.3
66	[45]	SB12-45	265.5	135.7	181.7	175.5	138.3
67	[45]	SB13-45-0	256.5	150.7	307.1	252.6	176.2
68	[45]	SB14-45-0	271.0	150.7	307.1	252.6	176.2
69	[46]	US-2	87.5	39.3	47.3	46.3	44.0
70	[46]	US-3	86.3	39.3	47.3	46.3	44.0
71	[46]	US-4	85.0	39.3	47.3	46.3	44.0
72	[46]	UW-2	90.0	41.9	66.0	63.4	68.9
73	[46]	UW-3	88.8	41.9	66.0	63.4	68.9
74	[46]	UW-4	88.8	41.9	66.0	63.4	68.9
75	[47]	C-1	165.0	139.8	146.1	106.0	128.4
76	[47]	C-4	250.0	183.7	158.3	115.2	131.2
77	[48]	case2	212.3	159.8	134.9	129.2	126.7
78	[48]	case3	253.0	168.9	149.7	139.2	137.3
79	[48]	case4	263.0	172.7	157.1	144.2	142.6
80	[48]	case5	293.6	176.2	164.5	149.2	148.0
81	[49]	SB_S0_05L	102.4	76.0	63.0	46.7	58.0
82	[49]	SB_S0_1L	120.0	80.0	81.9	54.8	73.0
83	[49]	SB_S0_2L	121.7	85.4	98.2	67.2	91.5
84	[49]	SB_S1_05L	282.0	147.5	126.4	144.7	121.4
85	[49]	SB_S2_1L	309.4	224.0	199.9	250.9	238.7
86	[50]	SB-S0-1L	59.3	29.8	29.8	16.0	26.3
87	[50]	SB-S0-2L	68.5	32.0	43.7	23.2	37.4
88	[50]	SB-S1-2L	105.1	43.5	49.0	42.9	42.7
89	[51]	UW90	141.5	74.9	71.5	76.2	69.7
90	[51]	2	82.6	52.6	45.5	47.8	46.5
91	[51]	3	82.0	52.0	42.8	45.8	43.6
92	[52]	RS4W	250.0	152.9	143.1	264.9	159.6
93	[52]	RS3W	330.0	161.1	143.1	276.3	159.6
94	[52]	RS2W	300.0	180.4	154.6	302.9	171.2
95	[52]	RS4Wa	250.0	173.1	160.9	282.9	183.2
96	[52]	RS3Wa	330.0	180.7	160.9	294.2	183.2
97	[52]	RS2Wa	300.0	180.7	160.9	294.2	183.2
98	[52]	RS3Ua	225.0	153.8	160.9	215.4	183.2
99	[52]	RS2Ua	280.0	173.0	161.2	241.6	183.5
100	[52]	RS4Ub	225.0	145.7	160.9	204.1	183.2
101	[53]	BT1-1	67.0	42.3	52.8	43.2	50.4
102	[53]	BT1-1I	87.0	42.3	52.8	43.2	50.4
103	[53]	BT1-2I	67.4	37.4	43.0	38.9	41.2
104	[53]	BS1-1	61.0	38.8	48.8	43.2	46.4
105	[53]	BS1-2	51.0	40.8	47.0	38.9	45.3

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#	ref	Specimen	V_{Rf}^{exp} (kN)	V_{Rd}^{MBC} (kN)	$V_{Rd}^{fib\ b90}$ (kN)	$V_{Rd}^{ACI440-2R}$ (kN)	V_{Rd}^{FR55} (kN)
106	[54]	S0-US	377.0	276.5	377.2	390.1	330.5
107	[54]	S0-UA	534.0	276.5	362.1	390.1	330.5
108	[54]	S0-CS	695.0	348.3	460.3	499.6	347.4
109	[54]	S0-CA	915.0	348.3	460.3	499.6	347.4
110	[54]	S5-UA	622.0	398.3	875.3	979.0	843.7
111	[54]	S5-CA	888.0	426.7	973.4	1088.5	860.6
112	[55]	G1-B	72.7	62.0	106.9	89.9	–
113	[55]	G2-B-N8	80.7	62.9	83.2	93.8	–
114	[55]	G2-B-N6	70.0	60.8	56.6	79.0	–
115	[55]	G3-MB-N8-P1	94.0	62.2	80.6	92.5	–
116	[55]	G3-B-N6	74.0	59.0	51.8	75.9	–
117	[56]	RB200C	97.8	36.9	43.4	35.2	45.9
118	[56]	RB150C	101.0	38.2	48.5	39.8	51.9
119	[57]	S3-EB-NA	243.2	118.4	129.1	119.2	104.2
120	[57]	S3-EB	222.0	135.8	117.1	137.7	110.5
121	[58]	SB-U1	65.0	24.1	18.7	21.9	18.4
122	[58]	SB-F1	66.1	27.3	20.7	24.7	18.4
123	[58]	SB-F2	66.7	27.3	20.7	24.7	18.4
124	[58]	MB-F1	236.4	99.8	81.7	96.3	65.2
125	[58]	MB-F2	250.3	99.8	81.7	96.3	65.2
126	[58]	LB-U1	563.4	268.6	206.2	264.0	208.7
127	[58]	LB-U2	559.8	268.6	206.2	264.0	208.7
128	[58]	LB-F1	871.6	305.8	265.8	333.6	208.7
129	[58]	LB-F2	881.2	305.8	265.8	333.6	208.7
130	[59]	RC-8-S90-	850.0	588.2	709.5	643.9	–
131	[59]	RC-12-S90-	765.0	615.5	709.5	685.6	–
132	[61]	ED1-S0-0.5L	102.0	77.1	67.2	48.5	59.7
133	[61]	ED1-S0-1L	120.0	81.4	82.5	57.8	76.0
134	[61]	ED1-S0-2L	122.0	87.3	98.2	71.1	96.1
135	[61]	ED1-S1-0.5L	282.0	117.2	91.2	108.8	83.8
136	[61]	ED1-S2-1L	309.0	167.6	169.5	178.4	163.0
137	[61]	ED2-S0-1L	59.0	26.5	30.9	13.6	24.9
138	[61]	ED2-S0-2L	68.0	28.5	37.9	19.6	35.6
139	[61]	ED2-S1-2L	105.0	42.4	46.7	41.3	44.4
140	[61]	U90S5-a(L)	246.7	135.3	–	158.0	146.3
141	[61]	U90S5-b(L)	235.9	123.1	–	139.9	131.4
142	[61]	U90C5-a(L)	217.0	124.4	167.2	161.5	163.9
143	[61]	U90C5-b(L)	243.3	121.1	161.3	155.8	158.6
144	[61]	U45S5(L)	206.2	148.2	177.6	166.7	156.0
145	[61]	W90S5(L)	276.2	137.5	175.8	175.1	124.4
146	[61]	U90S3-a(L)	207.4	104.8	–	113.1	102.1
147	[61]	U90S3-b(L)	203.6	108.0	–	117.4	106.2
148	[61]	U90S3-c(L)	231.8	104.0	–	112.0	100.9
149	[61]	U90C3-a(L)	208.8	125.4	157.8	157.3	151.8
150	[61]	U90c3-b(L)	201.8	125.4	157.8	157.3	151.8
151	[61]	U45S3-a(L)	189.6	119.3	127.5	125.2	114.7
152	[61]	U45S3-b(L)	211.0	136.4	150.8	148.7	136.0
153	[61]	W90S3-a(L)	311.4	123.7	122.6	135.0	106.2
154	[61]	W90S3-b(L)	284.3	123.8	122.7	135.1	106.3
155	[61]	U90S5-a	218.1	135.3	–	158.0	146.3
156	[61]	U90S5-b	225.6	123.1	–	139.9	131.4
157	[61]	U90C5-a	194.5	124.4	167.2	161.5	163.9
158	[61]	U90C5-b	221.9	121.1	161.3	155.8	158.6
159	[61]	U45S5	221.4	148.2	177.6	166.7	156.0
160	[61]	W90S5	374.3	137.5	175.8	175.1	124.4
161	[61]	U90S3-a	195.6	104.8	–	113.1	102.1
162	[61]	U90S3-b	223.9	108.0	–	117.4	106.2
163	[61]	U90S3-c	179.3	104.0	–	112.0	100.9
164	[61]	U90C3-a	215.4	125.4	157.8	157.3	151.8
165	[61]	U90c3-b	204.0	125.4	157.8	157.3	151.8
166	[61]	U45S3-a	198.8	119.3	127.5	125.2	114.7
167	[61]	U45S3-b	207.6	136.4	150.8	148.7	136.0
168	[61]	W90S3-a	321.4	123.7	122.6	135.0	106.2
169	[61]	W90S3-b	276.6	123.8	122.7	135.1	106.3
170	[63]	RS4WL4	220.0	171.7	158.9	272.6	177.0
171	[63]	RS4UL4	200.0	143.8	158.9	196.4	177.0
172	[63]	RS4WL2.5	165.0	171.7	158.9	272.6	177.0
173	[64]	B-7	68.5	40.4	60.2	36.7	53.3
174	[64]	B-8	85.8	44.2	68.1	50.0	65.1
175	[65]	VC 01	110.0	53.5	37.6	45.1	38.5
176	[65]	VC 03	100.0	55.6	44.0	50.6	45.4
177	[65]	VC 07	103.5	62.8	60.9	74.2	–
178	[66]	S0-0.12R	120.9	81.0	56.3	45.4	55.0
179	[66]	S0-0.17R1	134.5	83.4	62.7	50.2	60.8
180	[66]	S0-0.17R2	102.4	88.7	83.6	65.9	79.8
181	[66]	S0-0.20R1	135.7	84.7	67.1	53.5	64.8

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#	ref	Specimen	V_{Rf}^{exp} (kN)	V_{Rd}^{MBC} (kN)	$V_{Rd}^{fib\ b90}$ (kN)	$V_{Rd}^{ACI440-2R}$ (kN)	V_{Rd}^{FR55} (kN)
182	[66]	S0-0.20R2	131.1	84.7	66.9	53.3	64.6
183	[66]	S0-0.23R	150.6	85.8	71.0	56.5	68.4
184	[66]	S0-0.33R	120.0	88.7	83.6	65.9	79.8
185	[66]	S0-0.66R	121.7	88.7	83.6	65.9	79.8
186	[66]	S1-0.17R1	242.3	138.9	105.9	131.6	104.0
187	[66]	S1-0.17R2	246.7	138.9	105.9	131.6	104.0
188	[66]	S1-0.23R	253.9	140.8	114.2	137.9	111.6
189	[66]	S1-0.33R	250.6	143.1	126.8	147.3	123.0
190	[67]	Beam 2 – CFRP – No anchors	186.5	86.3	74.8	91.1	75.5
191	[68]	JS2A	236.0	54.6	82.7	77.7	86.1
192	[68]	JS2B	233.0	54.6	82.7	77.7	86.1
193	[68]	JS4A	256.0	58.4	102.7	96.9	111.0
194	[70]	C5	135.6	95.6	172.0	158.8	152.6
195	[71]	S00	182.0	111.9	127.2	125.7	120.7
196	[72]	F/295/LP1/4.5	135.0	53.2	63.8	41.5	60.1
197	[72]	F/295/LP1/3.3	122.5	52.2	67.2	44.4	61.0
198	[72]	F/215/LP1/4.6	102.5	42.6	48.3	25.6	41.2
199	[73]	EBL-S0	129.8	102.2	64.3	60.2	–
200	[73]	EBL-S1	260.3	164.9	121.5	156.7	–
201	[73]	EBL-S3	221.9	140.9	88.6	125.2	–
202	[74]	SB_R1	60.2	28.5	43.2	33.0	38.8
203	[74]	SB_R2	65.0	32.1	70.6	54.5	59.9
204	[74]	UW_R1	86.1	29.6	43.7	33.4	39.2
205	[74]	UW_R2	71.3	33.4	71.1	55.0	60.9
206	[74]	FW_R1	72.3	28.5	43.2	33.0	38.8
207	[75]	C1-EB1	120.0	60.6	58.9	56.6	51.9
208	[75]	C2-EB2	107.2	63.4	48.2	65.9	–
209	[75]	C2-EB2R	116.0	63.4	48.2	65.9	–
210	[75]	C2-EB3	120.0	64.1	48.2	72.7	–
211	[76]	S0-U	223.7	103.9	95.5	70.1	–
212	[76]	S8-U	300.8	111.2	95.5	101.4	–
213	[77]	TB2	140.0	54.5	36.8	46.9	–
214	[78]	BSU	159.3	98.1	90.2	84.9	–
215	[78]	WBR1	85.5	79.3	78.5	89.4	101.4
216	[78]	WBR2	178.8	102.3	106.5	132.3	143.6
217	[79]	A-U1-C-17	238.1	127.2	125.9	135.7	119.2
218	[79]	A-U1-C-20	225.0	119.8	116.6	126.8	110.0
219	[79]	A-U1-S-17	247.3	127.2	125.9	135.7	119.2
220	[79]	A-U1-S-20	235.1	119.8	116.6	126.8	110.0
221	[79]	A-U2-C-17	243.0	135.8	163.1	158.4	160.3
222	[79]	A-U2-C-20	229.7	128.6	153.8	149.6	151.1
223	[79]	A-U2-S-17	218.9	135.8	163.1	158.4	160.3
224	[80]	S-2	691.5	487.9	365.8	422.3	338.8
225	[80]	S-3	795.0	541.3	509.8	538.8	387.3
226	[80]	S-4	942.0	577.4	653.8	655.4	424.4
227	[81]	2	285.2	142.0	117.1	99.2	110.0
228	[82]	No. 2	223.0	101.1	69.0	83.7	62.8
229	[83]	B10_M	55.6	32.8	31.9	31.6	39.3
230	[83]	B12_M	71.5	40.0	54.2	47.2	60.6
231	[85]	2S_7M(1)	195.1	114.8	83.6	101.7	86.8
232	[85]	2S_7M(2)	222.1	120.9	109.0	113.6	103.5
233	[85]	4S_4M(1)	250.4	124.8	73.1	110.5	75.1
234	[85]	4S_7M(1)	253.8	128.1	83.6	120.2	86.8
235	[86]	UBF-R6	205.5	95.1	60.7	83.7	56.2
236	[86]	BDF-R6	180.9	96.9	61.5	84.8	57.0
237	[86]	UBF-R8	198.3	115.2	88.9	111.3	84.4
238	[86]	BDF-R8	213.6	114.9	88.9	111.1	84.4
239	[87]	1-R1	215.8	133.8	123.6	171.8	111.4
240	[88]	CS.2	100.5	66.4	60.9	43.4	54.8
241	[88]	DS.2	139.0	75.0	61.5	65.7	55.2
242	[89]	B15	86.9	51.1	92.9	69.5	88.2
243	[89]	B16	80.0	56.7	114.3	92.1	113.4
244	[92]	BS2	124.0	116.2	77.4	116.4	76.6
245	[92]	BS4	126.0	121.2	151.2	149.4	145.1
246	[92]	BS7	117.8	117.6	87.9	101.9	86.4
247	[93]	SB1-3	240.0	108.6	138.1	141.1	127.5
248	[93]	SB1-4	253.0	108.6	138.1	141.1	127.5
249	[93]	SB1-5	246.0	96.6	75.4	93.9	69.1
250	[93]	SB1-6	230.0	96.6	75.4	93.9	69.1
251	[93]	SB1-9	240.0	97.5	85.1	103.6	81.2
252	[93]	SB1-10	243.0	97.5	85.1	103.6	81.2
253	[93]	SB2-3	270.0	96.6	75.4	93.9	69.1
254	[93]	SB3-2	310.0	96.6	75.4	93.9	69.1
255	[94]	US60	111.0	93.5	85.9	97.0	–
256	[94]	USVA	120.0	97.5	87.3	97.7	–
257	[94]	USVA+	135.0	93.5	85.9	97.0	–

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#	ref	Specimen	V_{Rf}^{exp} (kN)	V_{Rd}^{MBC} (kN)	$V_{Rd}^{fib\ b90}$ (kN)	$V_{Rd}^{ACI440-2R}$ (kN)	V_{Rd}^{TR55} (kN)
258	[94]	US45+	126.0	97.5	87.3	97.7	–
259	[94]	UF90	125.0	90.0	90.7	106.8	–
260	[94]	US45++	133.5	125.0	94.7	106.8	–
261	[94]	US45 + A	167.0	93.2	94.7	106.8	–
262	[94]	UF45++B	172.0	93.2	94.7	106.8	–
263	[94]	UF45++C	183.0	93.2	94.7	106.8	–
264	[94]	US45++F	163.5	110.8	94.7	106.8	–
265	[94]	US45++E	150.0	110.8	94.7	106.8	–
266	[94]	US45 + D	164.5	110.8	94.7	106.8	–
267	[94]	WS45++	158.5	145.0	116.1	127.8	–
268	[95]	BT2-1	67.0	35.7	49.4	38.7	47.1
269	[95]	BT2-2	61.0	41.5	59.0	45.8	54.0
270	[95]	BT2-2I	77.0	41.5	59.0	45.8	54.0
271	[95]	BS2-1	54.0	31.6	40.1	34.4	38.4
272	[95]	BS2-2	41.0	38.2	55.4	45.8	50.5
273	[95]	BS2-2I	44.0	38.2	55.4	45.8	50.5
274	[95]	TT1-1	120.6	105.3	107.0	126.1	105.5
275	[95]	TS1-1	94.0	70.3	62.1	83.1	60.5
276	[95]	TS1-2	80.7	69.6	58.2	80.1	57.0
277	[96]	PU1	142.5	59.5	57.7	69.2	59.1
278	[96]	PU2	130.0	58.2	53.7	65.6	54.8
279	[96]	PU3	154.5	64.5	56.6	68.1	56.8
280	[96]	PU4	150.0	63.3	53.9	65.7	54.0
281	[96]	PC1	177.5	68.2	57.7	72.0	59.1
282	[96]	PC2	155.0	66.2	53.7	67.9	54.8
283	[96]	PC3	145.5	75.8	56.6	70.9	57.8
284	[96]	PC4	132.0	73.9	53.9	68.1	54.9

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