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Development of Lightweight Auxetic Composites for Noise Reduction and Vibration Damping

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“Se a terra do Egipto é um presente do Nilo, segundo a frase de Heródoto, a do Minho é, sem dúvida, o resultado das fadigas dos seus habitantes.”

Alberto Sampaio, *O Minho Rural e Industrial*, 1884

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*Dedicado à minha Avó, que ao me levar pela
mão à Escola do Monte, me trouxe até aqui.*

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FCT PhD
PROGRAMMES

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STATEMENT OF INTEGRITY

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Guimarães, 2nd March 2020

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Development of Lightweight Auxetic Composites for Noise Reduction and Vibration Damping

The future of transportation involves the reduction of emissions by the transition from individual to communal types of transport and lowering the consumption of energy. Railway, in particular, stands out from other types of transports since it consumes less energy, requires less space and produces less CO₂. Further emission reduction in railway vehicles may be promoted by their weight decrease. This would not only lower the required energy in cruising regimes, but also facilitate accelerations and reduce braking/propulsion wear. Additionally, the mitigation of railway noise/vibration would increase the attractiveness of this transports on a passenger point-of-view. It may be understood that both issues could be addressed by using lightweight materials with enhanced vibration and noise damping. This study proposes the design of such a material, also considering its structural strength. It is suggested that composite materials with a negative Poisson's ratio (*i.e.* auxetics) and non-stochastic cellular solid configurations are able to display high static and dynamic mechanical properties with lightweight characteristics. A novel additive manufacturing assisted investment casting technique is developed to produce the samples. The cellular solids are scanned using X-ray microcomputed tomography for the dimensional/defect analysis and validation of the manufacturing process. Experimental and numerical analysis are used to analyze the impact of the Poisson's ratio in the stiffness, collapse, vibration damping and sound reflection/transmission of the samples. These results are compared with current available materials to confirm the advantage of their application. It is concluded that the proposed manufacturing process is able to reproduce the intended proposed design. Indeed, the correlation between their static/dynamic mechanical properties, sound absorption/transmission and lightweight characteristics of the samples show a very desirable solution to be used in future railway vehicle fabrication.

Keywords: Auxetic; Cellular Solids; Noise; Vibration.

Desenvolvimento de Compósitos Auxéticos Leves para a Redução de Ruído e Amortecimento de Vibrações

O futuro dos transportes está intimamente ligado à redução de emissões, pela transição de veículos individuais para coletivos e diminuição de consumos energéticos. A ferrovia, em particular, destaca-se dos outros tipos de transporte pelo seu menor consumo energético, ocupação de menor espaço geográfico e pela reduzida emissão de CO₂. Eventualmente, estes impactos podem ser ainda diminuídos com a redução do peso das carruagens. Esta otimização permite, não só, baixar a energia que é necessária para o movimento do veículo, como também, facilitar acelerações e diminuir os desgastes durante os regimes de travagem/propulsão. Adicionalmente, é necessário reduzir as vibrações e o ruído ferroviário para tornar esta alternativa mais atrativa do ponto de vista do passageiro. Facilmente se entende que estes problemas podem ser mitigados com materiais leves e características otimizadas para o amortecimento estrutural e mitigação de ruído. Este estudo propõe o *design* de um material com estas características, considerando que as suas propriedades de resistência estrutural não podem ser negligenciadas. É proposto que materiais compósitos leves com um coeficiente de Poisson negativo (*i.e.* Auxéticos) e com uma configuração de sólido celular não-estocástico podem ser uma solução para obter boas propriedades mecânicas estáticas/dinâmicas e baixa densidade específica. Para a sua fabricação é proposta uma inovadora técnica híbrida que combina a manufatura aditiva com a fundição por modelo perdido. Os sólidos celulares produzidos foram radiografados por microtomografia computadorizada de raios-X para caracterizar o processo de manufatura em termos de defeitos e variações dimensionais. Foram usados ensaios experimentais e análise numérica para avaliar o impacto do coeficiente de Poisson na rigidez, colapso, amortecimento estrutural e absorção/transmissão acústica das amostras. Estes resultados são comparados com materiais disponíveis no mercado para verificar a vantagem do material desenvolvido para aplicação desejada. É concluído que o processo de manufatura desenvolvido é capaz de replicar o *design* desejado. A correlação entre propriedades mecânicas estáticas/dinâmicas, a sua absorção/transmissão acústica e a sua baixa densidade específica revelam que os sólidos celulares desenvolvidos são uma solução promissora para o futuro da construção de veículos ferroviários.

Palavras-Chave: Auxético; Ruído; Sólidos Celulares; Vibração.

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List of Acronyms

ABS - Acrylonitrile butadiene styrene;

CAD – Computer aided design;

EM – Electro machining;

FEA – Finite element analysis;

FRF – Frequency response function;

GP zone – Guinier-Preston zone;

HVAC – Heating, ventilation and air conditioning;

JCAL - Johnson-Champoux-Allard-Lafarge model;

μ-CT - X-ray microcomputed tomography;

MMC – Metal matrix composite;

PLA – Polylactic acid;

SAN - Styrene-Acrylonitrile;

SEBM - Selective electron beam melting;

SEM – Scanning electron microscopy;

SPL – Sound pressure level;

TL – Transmission loss;

TRL – Technology readiness level;

UHMWPE – Ultra-high molecular weight polyethylene;

Latin Symbols – Upper case

Symbol	Description	Unit
A	Grain Area Superficial Area	m ²
A _i	Incident Wave Pressure	Pa
A _r	Reflected Wave Pressure	
A _R	Rib Area	m ²
A _t	Ageing Time	min
A*	Apparent Area	m ²
C ₁₂	Modulus of Dilation	Pa
C _a	Accelerometer Damping Coeff.	N·m·s/ rad
C _c	Cable Damping Coefficient	
C _{eq}	Equivalent Damping Coefficient	
C _s	Sample Damping Coefficient	
C _i	Castability	%
C _p	Thermal Conductivity	J/K
D	Grain Diameter	m
D _s	Sample Depth	
E	Young's Modulus	Pa
E [†]	Tangent Modulus	Pa
E ₀	Base Material Elastic Modulus	Pa
E*	Apparent Modulus	Pa
E*/E ₀	Specific Modulus	-
F	Load Flexure Component	N -
F _{max}	Maximum Load	N
G	Shear Modulus	Pa
G _{Si}	Geometry Selection Index	-
H	Hardness	Pa
HV	Vickers Hardness	HV
I	Moment of Inertia	m ⁴
K	Bulk Modulus	Pa
K _a	Axial Stiffness Accelerometer Stiffness	N/m
K _b	Bending Stiffness	
K _c	Cable Stiffness	
K _{eq}	Equivalent Stiffness	
K _f	Flexure Stiffness	
K _h	Hinging Stiffness	
K _s	Stretching Stiffness Sample Stiffness Coefficient	
K _T	Total Stiffness	
L _d	Weak Pinning Point Length	m
L _h	Layer Height	
L*	Apparent Height	
M	Moment	Nm
M _a	Accelerometer Mass	g
M _{eq}	Equivalent Mass	
M _s	Sample Mass	
M _{Si}	Material Selection Index	-
N	Nozzle diameter	m
P	Grain Perimeter	m

	Rib Perimeter	M
	Vacuum Pressure	Bar
	Sound Pressure Level	Pa
P _a	Printing Area	m ²
P _s	Printing Speed	m/s
P ₁	Pressure of fastest acoustic wave	dB
P ₂	Pressure of slower acoustic wave	
Q ⁻¹	Inverse Quality Factor	-
R	Reflection Coefficient	-
R _h	Relative Humidity	%
R _n	Rib Roundness Grain Roundness	-
T	Transmissibility	dB
	Temperature	°C
T _c	Casting Temperature	
T _M	Mold Temperature	
U _x	Displacement in xx-axis	m
U _y	Displacement in yy-axis	
U _z	Displacement in zz-axis	
V	Volume Volume variation	m ³ -
W	Relative PLA Weight	%
W _s	Sample Width	m
X _a	Axial Horizontal Deformation	
X _b	Bending Horizontal Deformation	
Y _a	Axial Vertical Deformation	
Y _b	Bending Vertical Deformation	
Z	Acoustic Impedance	Pa·s/m
Z _i	Incident Acoustic Impedance	
Z _t	Transmitted Acoustic Impedance	
Z _c	System Acoustic Impedance	
Z ₁	Fast Wave Acoustic Impedance	
Z ₂	Slow Wave Acoustic Impedance	

Latin Symbols – Lower case

Symbol	Description	Unit
a	Particle Length Normalized Acceleration	m m ² /(s·N)
a _i	Precipitate Radius	m
b	Depth Particle Width	
f	Frequency	Hz
f _r	Volume Fraction of Precipitates	-
h	Horizontal Length	m
h _s	Sample Height	m
k	Thermal Conductivity	W/(m·K)

k'	Dynamic Thermal Permeability	$m^2/(s \cdot N)$
$k'o$	Static Thermal Permeability	
l	Vertical Length	m
	Number of Cells in Row	-
m	Number of Cells in Line	
n	Number of Cells in Width	
p_{13}	Correction Factor	
q	Hinging Effect	-
r	Precipitate Radius	m
r_{trans}	Transition Radius	
s	Displacement	
t	Thickness	
	Time	s
u	Particle velocity	m/s
	Displacement in xx-axis	m
v	Displacement in yy-axis	
w	Displacement in zz-axis	
x	Probe Position	
y	Normalized Displacement	-

Greek Symbols

Symbol	Description	Unit
α	Rib Angle	$^\circ$
	Sound Absorption Coefficient	-
	Dynamic Tortuosity	
	Proportion of Solute Release	
α_{inf}	High Frequency Limit Tortuosity	
α_{TRUE}	True Rib Angle	$^\circ$
β	Dynamic Compressibility	Pa
γ	Incident Angle	rad
	Specific Heat	J/K
δ_0	Strain Independent Internal Friction	-
ϵ	Normal Strain	
ϵ_a	Axial Strain	
ϵ_t	Transverse Strain	

ϵ^*	Apparent Strain	
ζ	Damping Ratio	-
	Missing Rib Angle	rad
ζ_s	Sample Damping Ratio	-
η	Dynamic Viscosity	$(N \cdot s)/m^2$
θ	Angle	rad
λ	Wavelength	m
Λ	Viscous Characteristic Length	
Λ'	Thermal Characteristic Length	
ν	Poisson's ratio	-
ρ	Dislocation Density	m^{-2}
	Density	kg/m^3
ρ_0	Bulk Material Density	
ρ^*	Apparent Density	
ρ^*/ρ_0	Specific Density	-
σ	Air Flow Resistivity	$(N \cdot s)/m^4$
	Normal Stress	Pa
σ_i	Initial Strength	
σ_P	Plastic Collapse Stress	
σ_{PPT}	Precipitation Strengthening	
σ_{ss}	Solid Solution Strengthening	
σ_{0ss}	Intrinsic Solid Solution Strength	
σ_Y	Yield Strength	
σ_{xx}	Normal Stress in xx-axis	
σ_{yy}	Normal Stress in yy-axis	
σ_{zz}	Normal Stress in zz-axis	
σ^*	Apparent Stress	
τ	Shear Stress	
Φ	Direction Angle	rad
	Porosity	-
ω	Angular Frequency	rad/s
ω_1	Lower Limit Frequency	
ω_2	Upper Limit Frequency	
ω_r	Resonance Frequency	

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Chapter 1 – Introduction

“Philosophy is written in that great book which ever lies before our eyes - I mean the universe - but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written.”

Galileo Galilei – translated from // *Saggiatore*, 1623

Experiment, postulate and refute hypotheses are basic procedures to understand our surroundings. To do so, it is mandatory to create ingenious ways to perform such experiments and devise conclusions. These are frequently the terms for evolution in the domains of “natural philosophy” and engineering.

This document presents a study on mechanical engineering, showing the current developments of its field, and a research methodology for the production of new knowledge. The concepts to be developed in this work are intended to be submitted as a proof of the candidate suitability to be considered for a Doctor of Philosophy degree.

1.1. Motivation and contextualization

Curiosity and necessity are basic propulsion mechanisms for “artificial” evolution. It is the human nature to defy nature itself, either by sheer fascination of being able to contradict established “natural laws” or only by pure survival instinct. Based on these two established principles, pioneering minds that did not conform their *status quo*, were able to originally develop technology and change the world around them. One can now fly the skies, travel to space and communicate globally. For that we have the inherited “responsibility” of contributing to this technological and, consequently, social advance.

Being able to participate in this evolution is a privilege, and the opportunity to develop new knowledge in the form of technology is the basic motivation for the proposed study. Especially, the current trend of studying, synthesizing metamaterials and engineering composites for the benefit of society, mankind and nature itself.

This mindset is included in the philosophy and policy that contemplates the implications of technology in the future of our society. For instance, the European Union has established one of its goals the reduction of oil and fossil fuel consumption, by decreasing carbon emissions in transports by 60% in 2050 [1]. One key aspect to reach this objective is the transition of 50% road passengers to rail and waterborne transports by the same year [2].

Railway is particularly suitable for this task: it consumes less energy, requires less space and produces less CO₂ than any other form of transportation [3]. Current trends in the railway industry to reduce emissions and economic viability include: (i) lower vehicle weight; (ii) lower wear on braking/propulsion systems; (iii) reduce maintenance costs; and (iv) faster accelerations to reduce journey times [4].

These measures, even though are effective routes to benefit railway transports, do not appeal directly to the passengers. Solutions have to be found to promote the passenger shifting to railway. It is fundamental to determine the factors that are currently dissuading passenger from this transportation, as well as, actions that would increase its attractiveness.

Noise, for instance, is a major reason for public opposition to rail transportation in different European regions. In fact, rail transport is the second principal source of environmental noise in Europe [5]. This is particularly problematic, since Noise is identified by the World Health Organization as the second causer of disease in the European Union, right after air pollution [6]. The noise that is generated by rail transportation, not only affects the neighboring population and residences, but also the passengers

themselves. It is estimated that a reduction in railway noise by at least half of the affected population could generate savings up to 2255M€ during the years 2015-2035 [5]. In conclusion, the design of rail vehicles, either by fuel consumption or noise perspectives, is a social, economic and environmental problem. Population and their political representatives urge the railway industry to become quieter, less prone to vibration and less pollutant.

Considering these common sources of structural vibration/noise in railway transportation have been thoroughly studied, being identified in Table 1.1 [7].

Table 1.1. Sources of noise and vibration in railway transportation.

Type	Description
Rolling noise	Interaction of the wheel with the rail: unevenness in contacting faces.
Track vibration	Vibration with rails acting as waveguides to structural oscillation.
Wheel vibration	Vibration of wheels in resonance.
Aerodynamic noise	Turbulent air boundary layers that surrounds moving vehicles.
Curve squeal	Contacts between rail and self-excited resonant wheel.
Impact noise	Impact between rail and wheel due to discontinuities in track or wheel.
Bridge noise	Vibration of the rails transmitted through bridge radiating noise.
Ground vibration	Transmission of vibration from the rail to the ground.
Equipment noise	Operating equipment noise (<i>e.g.</i> HVAC, converters, motors, etc).

Relatively to the interior vibration and noise, those that affect passengers, it is known that it is mainly originated by exterior sources according to Fig. 1.1(a). Such exterior dynamic excitations usually arise from the sources described in Table 1.1. They are transmitted to the interior of the vehicle by structural vibration and airborne transmission. Structural vibration, however, may also be converted into noise, leading to structure-borne noise transmission.

The noise transmitted to the interior of the vehicle interacts with the noise that is generated within the coach (*e.g.* HVAC and other interior equipment, passengers, etc) and is then reflected in the vehicle walls (Fig. 1.1(b)) [8]. Thus, noise mitigation is directly correlated with the ability of the materials that are used in vehicle manufacturing to display high values of structural and noise damping.

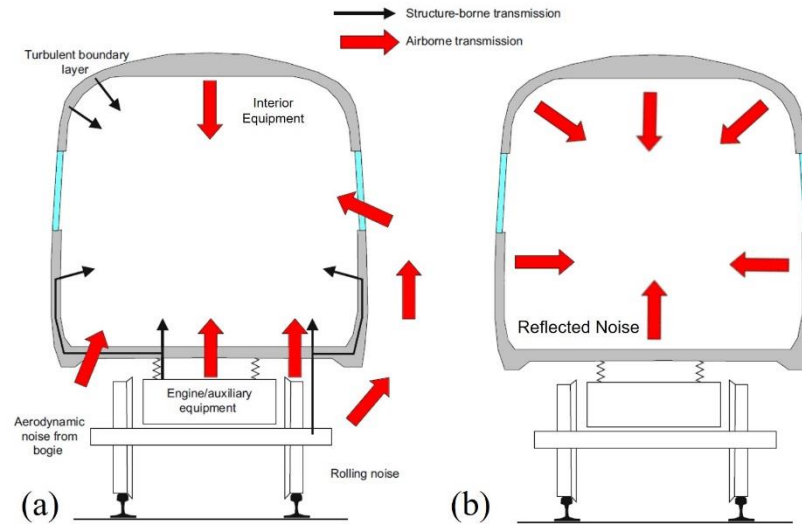


Figure 1.1 Interior noise/vibration (a) transmission and (b) reflection (adapted from [9]).

A combination of the referred vibration/noise damping and lightweight characteristics without compromising static structural strength can be found in metallic cellular solids [10]. This class of materials has already been used in different applications in the transport industry, mostly for crash absorption duties [11] (examples in Fig. 1.2).

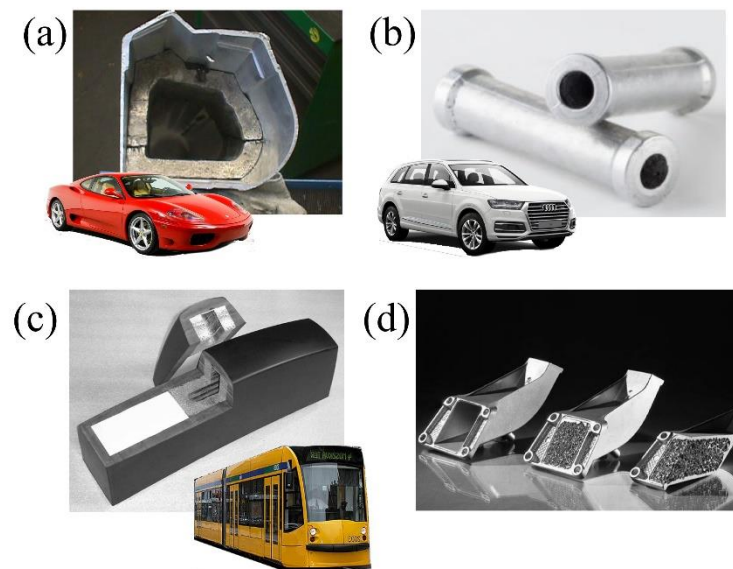


Figure 1.2 Examples of applications of cellular solids in the transportation industry: (a) Ferrari 360 door sill; (b) Audi Q7 crash absorber; (c) Siemens Combino crash absorber; and (d) Prototype of a BMW engine mount.

Recently, cellular solids have also been used to produce a prototype of a High-speed train front using welded cellular sandwich panels in Germany [11]. The panels that were used in the fabrication of

this prototype are advertised as possessing: (i) vibration reduction/damping; (ii) noise-absorption/insulation; and (iii) lightweight with high mechanical properties [12]. Also, there are reports showing the intention of Chinese railway industry to develop similar solutions in the future [13].



Figure 1.3 Prototype of a train front manufactured with welded cellular sandwich panels (adapted from [12,14,15]).

In order to address such issue, one of the focus in the project *iRail - Innovation in Railway Systems and Technologies Doctoral Programme* is the development of novel solutions for vehicle car body-shells. Being enrolled in the referred project, this study proposes the design of structural components in the form of lightweight cellular solids with enhanced vibration/noise damping capacity. Additionally, it also considers the static mechanical performance cannot be neglected, as it must be able to compete with current structural materials.

This work explores a cellular solid manufacturing process recurring to a combination of both classic investment casting and recent additive manufacturing techniques. These composites have their mechanical behavior, being characterized on both static and dynamic domains. Additionally, the benefit of designing cellular solids with auxetic (*i.e.* negative Poisson's ratio) behavior is further analyzed.

Objectively, the design of these cellular solids is devoted to their future use in the fabrication of railway vehicles with enhanced structural performance and reduced environmental/noise pollution.

1.2. Historical note and state-of-the-art: From bulk materials to auxetic cellular solids

Historically, the understanding of material behavior and engineering to optimize their performance for a given application is a classic challenge for mankind [16]. Since the occurrence of intelligent life forms that raw materials are refined to enhance their fundamental properties. This fact has been especially true for metals, which allowed a major acceleration in terms of mechanical technological evolution (from the early Bronze age, roughly 3300 *B.C.*, to nowadays [17]). Firstly by the empowering of armies that generated a competition on the design of novel weapons to supplant their enemies [18], to the relatively more recent industrial and information/telecommunication revolutions, and the expected revolution that may be generated by additive manufacturing [19].

Even though materials have been first used as found in nature, experimenting throughout these years, allowed the understanding that the combination of resources in one structure/material may be extremely advantageous. Such combination process was found to be particularly useful in pure metals to form metallic alloys [20]. Retrospectively, classic examples of combining Copper and Tin to obtain Bronze [21], followed by the mixing of Iron with Carbon to gradually obtain stronger and tougher Steel alloys [18].

This allowed the design of structures and components recurring to less volume of material. Such progress on “new metallic materials” was accompanied by an incremental evolution in manufacturing and processing techniques, such as heat treatments and mechanical processing, to further adapt their use for different applications.

The development of new materials and the increase in audacious projects for their application, has generated the need to study procedures to test and characterize them and, eventually, extrapolate empirical and theoretical rules that describe such behavior. Examples of pioneering works in these fields are found on Da Vinci’s works on the tensile strength of wires (Fig. 1.4(a) [22]). Galileo concluded that the tensile strength (Fig. 1.4(b) [23]), is dependent to the cross-sectional area and independent of its length [16]. Hooke showed that of the load/deformation is proportional in the elastic regime (Fig. 1.4(c) [24]).

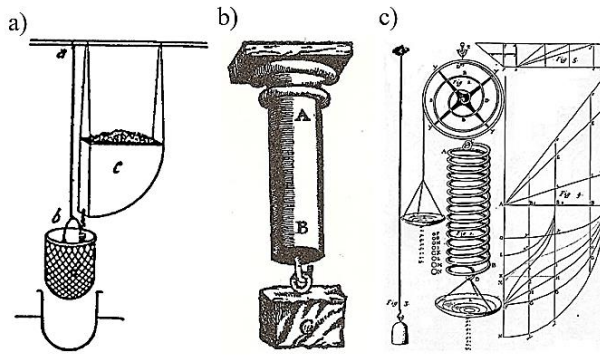


Figure 1.4 Early material/mechanical testing: (a) Da Vinci's wire [22]; (b) Galileo's rod tensile test [23]; and (c) Hooke's spring test [24].

Additionally to the characterization in terms of load and deformation withstanding, these studies allowed the correlation of bulk properties and processing to the importance of the shape (*i.e.* geometry/macrostructure). One classic example of this strong correlation between shape and mechanical properties is presented in the 1849 patent for the production of I-Beams by the Belgian/French company Forges de La Providence [25]. This principle of distributing bulk material into geometries that are an optimized solution for a given application is still a demanding challenge. Although, materials in their own proper class, seem to obey laws that can be described nowadays, there are still materials able to contradict common sense, such as those with a negative value of stiffness [26], compressibility [27], permeability [28] and Poisson's ratio [29].

With the advent of mechanic propulsion the interaction between geometry and structure has been further developed, especially in applications where the reduction of mass is fundamental such as the aeronautic, aerospace, naval, automotive and railway industries. Additionally, researchers showed that the basic principle of combining metallic elements to form alloys could be further explored by the mixture of materials of different nature to form composites [30].

One of the most popular forms of combining these materials is their distribution into void filled bulk materials, producing a composite that is commonly referred as a cellular solid [31]. Cellular solids have been proved to be a very efficient way to perform structural reinforcement duties, while reducing significantly their structural weight. This is especially while using light alloys (*e.g.* Aluminum and Magnesium) and polymeric filling. There are still research opportunities in the studying of base materials and structures to fabricate these composites, *e.g.* by the development of cellular solids with a negative Poisson's ratio to potentially enhance the structural performance and noise/vibration damping in railway vehicles.

Bodies that contract/expand their transverse dimension (ϵ_t) when compressed/tensioned (ϵ_a) possess a negative Poisson's ratio [32] (given the generic definition of Poisson's ratio ν , where $\nu = -\epsilon_t/\epsilon_a$ [33]). These materials have been referred as anti-rubber [34], dilational materials [35], and most commonly as auxetics (from the greek *αυξητικός*, *i.e.* "that tends to increase") [36].

This kind of behavior may seem to contradict common sense (most materials have a generalized Poisson's ratio between 0.25 and 0.35 [37]), it can be observed by the limits of Eqs. 1.1 to 1.3 (Table 1.2) that negative values of Poisson's ratio may be allowed for both three- [38] and two-dimensional [39] classic linear isotropic *Theory of Elasticity*. By the manipulation of the referred equations and interpreting the positive values of the Young's (E), Shear (G) and Bulk (K) moduli, the range of the Poisson's ratio (ν) may be determined, being represented in Table 1.2.

Table 1.2. Thermodynamic limits of the Poisson's ratio for isotropic elastic bodies.

Elastic Domain	First relation	Second relation	Poisson's ratio range
Three-dimensional	$G = \frac{E}{2(1+\nu)}$ (1.1)	$K = \frac{E}{3(1-2\nu)}$ (1.2)	$-1 < 0.5$
Two-dimensional		$K = \frac{E}{2(1-\nu)}$ (1.3)	$-1 < 1$

Fig. 1.5 resumes the influence of an isotropic body Poisson's ratio in its deformation behavior and stability limits for unconstrained and constrained conditions.

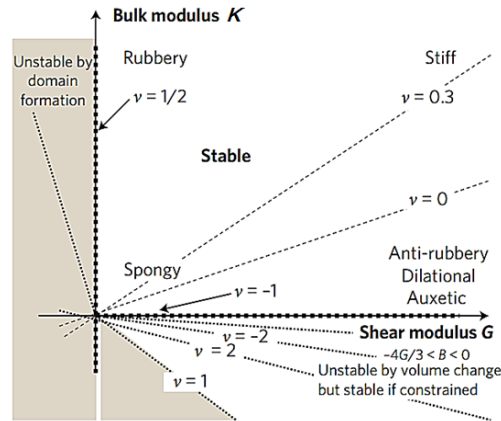


Figure 1.5. Representation of a body deformation stability as a function of Bulk/Shear Moduli and Poisson's ratio for an elastic isotropic body (adapted from [37]).

Due to this counterintuitive behavior, auxetic materials present some characteristics that may be relevant and extremely interesting to be exploited. Compared to "normal" (positive Poisson's ratio) materials, auxetics show enhanced resistance to shear, indentation and fracture, and superior vibration/noise damping [32].

Shear resistance is an important feature in structural applications, especially in components such as beams and plates/panels. Interpreting Eqs. 1.1 and 1.2 (Table 1.2), it may be seen that materials with a Poisson's ratio near 0.5 tend to be isochoric ($K \rightarrow \infty$), while materials in which this elastic constant approximates -1 tend to maintain their shape and be highly compressible ($K \rightarrow 0$). However, rubbery materials ($\nu \rightarrow 0.5$) tend not to be very resistant to shear deformations, while auxetics ($\nu \rightarrow -1$) may have increased shear resistance ($G \rightarrow \infty$). These relations are represented in Fig. 1.6, in which both Shear and Bulk moduli are normalized in terms of Young's modulus. Additionally, it may be seen when $\nu \rightarrow 0.125$, the normalized values of the Shear and Bulk moduli tend to be equal ($G/E = K/E$).

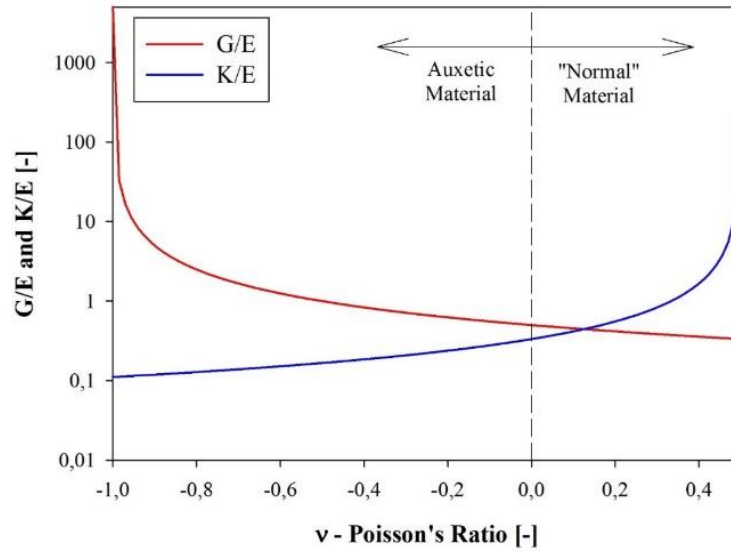


Figure 1.6. Normalized Bulk (K) and Shear (G) moduli dependence of the Poisson's ratio (ν).

The extreme volume variations in auxetic materials, have the practical benefit of enhancing indentation resistance (Fig. 1.7 – adapted from [40]). Given the basic definition of auxetics, such kind of material is able to flow under an indenter and densify [41] (Fig. 1.7 (b)), contradictory to “normal” materials which flow away from a penetrating body (Fig. 1.7 (a)).

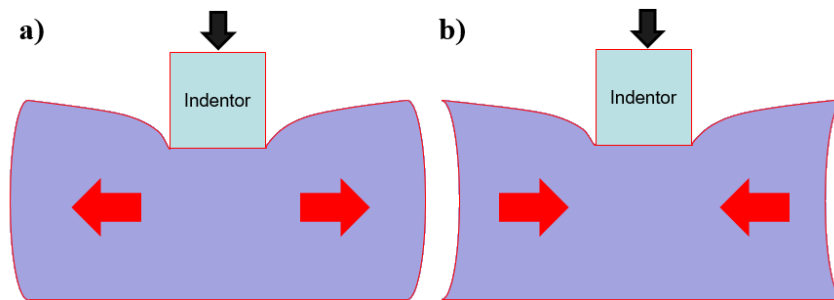


Figure 1.7 Behavior in indentation: (a) “normal” and (b) auxetic materials (adapted from [40]).

The relative increase of indentation resistance is supported by the classical Theory of Elasticity, in which for isotropic solids, is presented in terms of hardness (H) by Eq. 1.4 [41]. The referred equation relates the hardness of a given material, as a function of Young's modulus (E), Poisson's ratio (ν) and a constant that represents the type of indentation contact (γ). The latter may assume the values of 1 or 1/3, respectively, for uniform pressure distribution or Hertzian type of contact. The graphical representation of the referred dependency may be observed in Fig.1.8, where it is clear that there is a significant increase in hardness for lower values of Poisson's ratio.

$$H \propto \left[\frac{E}{(1 - \nu^2)} \right]^\gamma \quad (1.4)$$

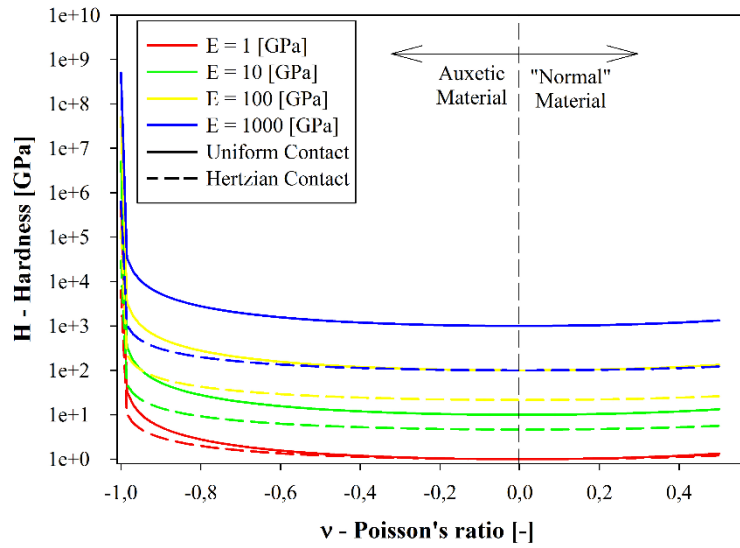


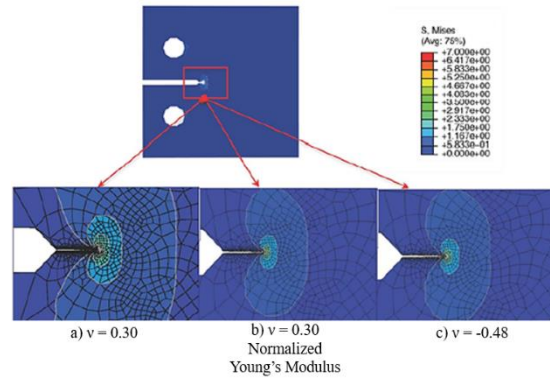
Figure 1.8 Hardness (H) dependence on the Poisson's ratio (ν) and Young's modulus (E).

Experimental and theoretical studies have demonstrated that auxetic materials may be advantageous to enhance indentation resistance. Some of these works are presented in Table 1.3.

This counterintuitive behavior is also predicted to be relevant in terms of fracture resistance. According to finite element analysis (FEA) mode I fracture analysis (Fig. 1.9) it has been predicted that “normal” bulk solid (Fig. 1.9 (a)) has a wider plastic affected zone compared with an auxetic material (Fig. 1.9 (c)) with a normalized Young's modulus [52]. This effect is attributed to local stiffness reduction, justified by the results using artificial reduction of the Young's modulus shown in Fig. 1.9 (b). Additionally, their widening effect in tension may have a relevant role in a “closing” effect around the crack tips [32].

Table 1.3 Published works concerning indentation resistance of auxetics.

Year	Lead Author	Ref.	Description
1993	R.S. Lakes	[42]	Reentrant copper foam is shown to have smaller indentation radius and higher yield strength when compared with conventional foams with the same relative density;
1994	K.L. Alderson	[43]	Negative Poisson's ratio Ultra High Molecular Weight Polyethylene (UHMWPE) is shown to have superior hardness than samples with positive Poisson's ratio;
1998	N. Chan	[44]	Auxetic polyurethane foams present better indentation behavior and increased resilience when compared with conventional polyurethane foam samples;
2000	K.L. Alderson	[45]	Indentation resistance in auxetic UHMWPE is enhanced at low loads when compared with sintered and compression molded samples;
	A. Lowe	[46]	Auxetic polyurethane foams show lower values of localized pressure when indented with human buttocks, in comparison with conventional foams with the same density;
2011	V.L. Coenen	[47]	Enhanced indentation resistance and smaller localized damage areas in auxetic Carbon Fiber laminates, when compared with positive Poisson's ratio samples;
2012	I.I. Argatov	[48]	Friction contact between indenter, elastic medium increases and relative indentation compliance decreases as the values of Poisson's ratio tends to -1;
2015	L.L. Hu	[49]	Indentation resistance changes with cell angles in reentrant structures with similar density;
2016	D. Photiou	[50]	Finite element analysis is used to prove the increase in conic indentation resistance in auxetic materials, due to shear stiffening and miotic response of the elastic solid.
2019	L.I. Hu	[51]	Indentation of auxetic lattices under large plastic deformations. In such regime, indentation resistance in auxetics is not always higher than regular lattices.

**Figure 1.9** Von Mises stress near the crack tip and affected Plastic zone (adapted from [52]).

The auxetic plastic affected zone, although may be smaller, this may not be the case of specific local values of stress or strain. For the diverse fracture modes, the value of Poisson's ratio is a fundamental constant to determine the displacement fields around the crack tip. Eqs. 1.5 to 1.13 in Table 1.4 [53] determine the variation of non-dimensionalized cartesian displacement fields (U_x , U_y and U_z), for a normalized Young's modulus and angle (θ) of the classic fracture model (Fig. 1.10(a)) are shown in Table 1.4 and plotted in Fig. 1.10(b) to (f).

According to the plots presented in Figs. 1.10 (b) to (f), it may be observed that auxetic behavior may be interesting for fracture resistance in particular situations. In tensile scenarios (Mode I – Figs. 1.10 (b) and (c)), it seems that auxetic behavior is beneficial in plane strain conditions. However, in-plane shear

fracture (Mode II – Figs. 1.10 (d) and (e)), auxetic behavior seem to be only hazardous in elevated yy -direction displacements in plane stress conditions. Finally, in out-of-plane shear fracture (Mode III – Fig. 1.10 (f)), auxetic characteristics seem to be advantageous.

Table 1.4 Equations for non-dimensional fracture displacements.

Displacement Field		U_x	U_y
 Mode I	Plane Strain	$2(1 + \nu) \cos(\theta/2) [1 - 2\nu + \sin^2(\theta/2)]$ (1.5)	$2(1 + \nu) \sin(\theta/2) [2(1 - \nu) - \cos^2(\theta/2)]$ (1.6)
	Plane Stress	$2 \cos(\theta/2) [1 - \nu + (1 + \nu) \sin^2(\theta/2)]$ (1.7)	$2 \sin(\theta/2) [2 - (1 + \nu) \cos^2(\theta/2)]$ (1.8)
 Mode II	Plane Strain	$2(1 + \nu) \sin(\theta/2) [2(1 - \nu) - \cos^2(\theta/2)]$ (1.9)	$-2(1 + \nu) \cos(\theta/2) [1 - 2\nu - \sin^2(\theta/2)]$ (1.10)
	Plane Stress	$2 \sin(\theta/2) [2 + (1 + \nu) \cos^2(\theta/2)]$ (1.11)	$-2 \cos(\theta/2) [1 - \nu - (1 + \nu) \sin^2(\theta/2)]$ (1.12)
 Mode III		U_z	
		$4(1 + \nu) \sin(\theta/2)$ (1.13)	

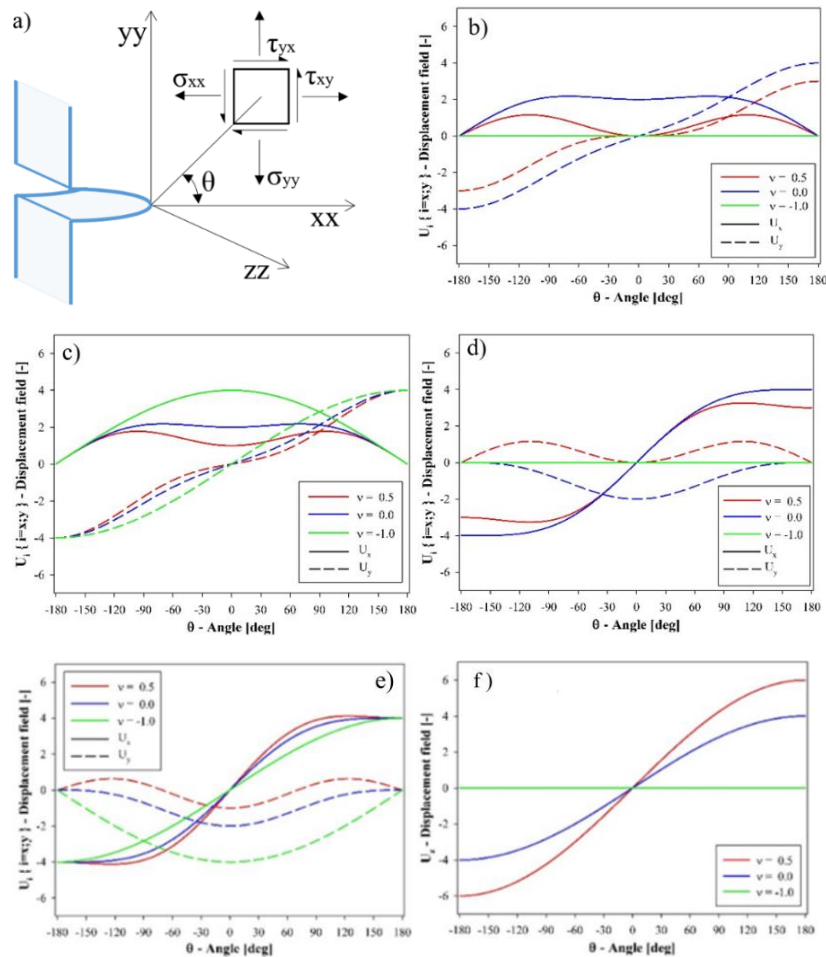


Figure 1.10 Representation of the non-dimensional displacements from the conditions and equations in Table 3: (a) Basic fracture schematics; (b) Mode I - Plane-Strain; (c) Mode I – Plane-Stress; (d) Mode II – Plane Strain; (e) Mode II – Plane Stress; (f) Mode III.

Considering the dynamic behavior and properties, materials with a negative Poisson's ratio are also characterized by a peculiar behavior. It has been shown that auxetics subjected to an incident of a longitudinal wave (A_i) (Fig. 1.11(a)) may exhibit an enhanced capacity to reflect waves (A_r) for a widespan of incident angles (γ) (Fig. 1.11(b)) [54].

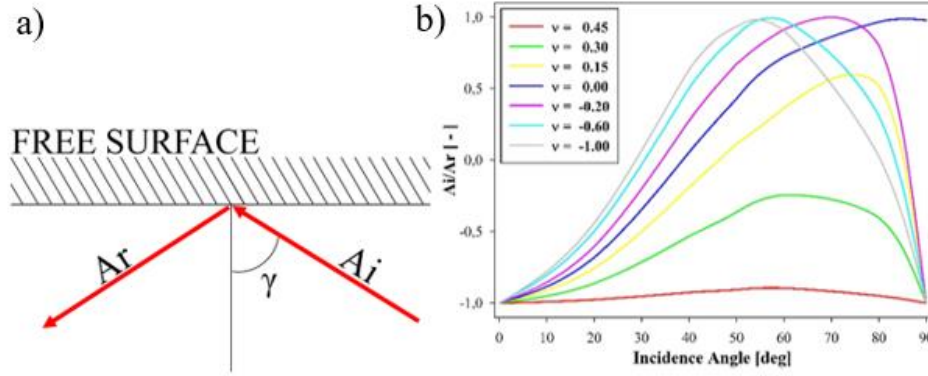


Figure 1.11. Surface reflection: (a) wave reflection scheme; and (b) dependence on the Poisson's ratio and incidence angle (adapted from [54]).

Concerning the propagation of Rayleigh waves, it has been shown that auxetic materials possess enhanced damping properties in comparison with “normal” materials for normalized displacements (y) as a function of wavelength (λ) (Fig. 1.12) [54].

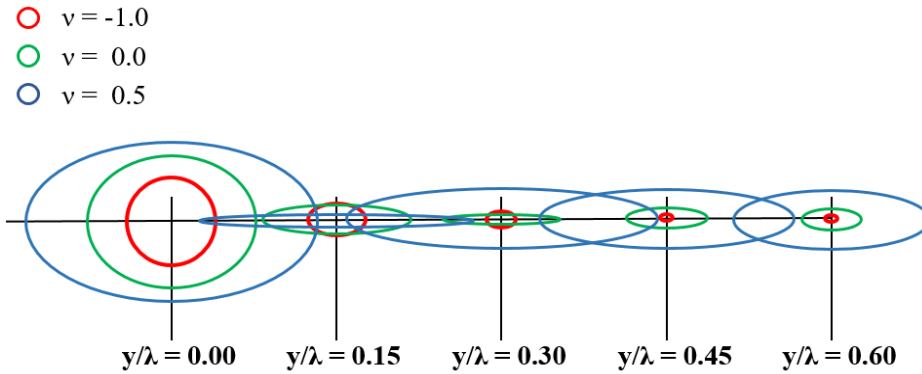


Figure 1.12. Propagation of Rayleigh waves depending on the Poisson's ratio and normalized displacement (adapted [54]).

On the propagation of solitary waves, it is known that extreme values of Poisson's ratio, near -1 or 0.5, generate an increase in wave amplitude (Fig. 1.13(a)) and velocity (Fig. 1.13(b)). Studies show that minimum amplitude and velocity are generated for the Poisson's ratios, respectively, -0.2 and 0. A compromise near these values, on the auxetic domain, is convenient for practical applications [55].

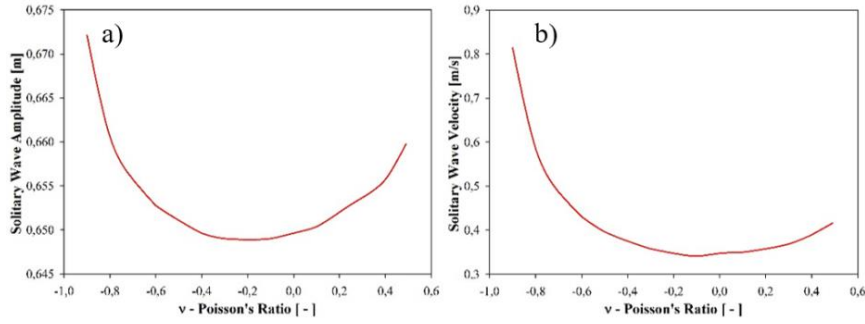


Figure 1.13. Solitary wave: (a) amplitude and (b) velocity dependence on the Poisson's ratio (adapted from [55]).

In terms of lattices with auxetic effect, it has been proven that this kind of structures possess a direction angle (ϕ) dependent band gap at a wide variety of frequencies (ω) (Fig. 1.14 (a)). Band gaps are not observed in regular honeycomb lattices (Fig. 1.14 (b)), being useful in practical applications [56].

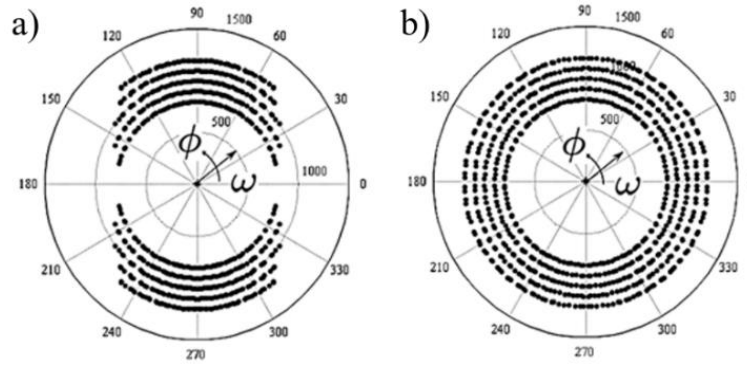


Figure 1.14. Representation of band-gaps in (a) auxetic and (b) "normal" materials (adapted from [56]).

Sandwich panels with auxetic core have also been studied and investigated, being suggested that they may generate enhanced transmission loss, especially at high frequencies [57]. Additionally, it has been shown that auxetic fillings promotes changes of smaller root mean square wave velocity propagation (Fig.1.15) [58].

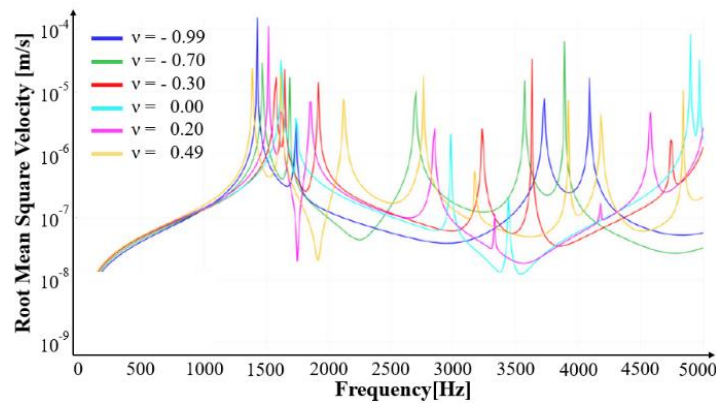


Figure 1.15. Root mean velocity propagation dependence on the Poisson's ratio (adapted from [58]).

Considering the development and study of auxetic materials with enhanced vibration and noise damping, a fundamental evolution on this matter is presented in Table 1.5.

Due to their attractive characteristics, such materials seem to be an interesting opportunity as base material for a novel generation of railway vehicles. There has been a recent effort to use such materials in practical applications, however, this task has been proven harder than it may initially seem. The main reason is that, although auxetic isotropic materials have been proved theoretically, there seems to be a lack of natural auxetics.

Table 1.5. Brief progression of published works concerning the vibration/acoustic propagation in auxetics.

Year	Lead Author	Ref	Description
1988	A. Lipsett	[54]	Study concerning negative Poisson's ratios in dynamic elasticity;
1989	C.P. Chen	[59]	Experimental analysis of the dynamic behavior of auxetic polyester foams;
1991	B. Howell	[60]	Report on the acoustic damping of auxetic polyurethane foams;
1994		[61]	Study concerning the sound absorption of auxetic polyurethane foams;
2000	F. Scarpa	[57]	Influence of cell geometry on the Poisson's ratio and vibration properties of sandwich plates;
2003	M. Ruzzene	[56]	Existence of band-gaps in periodic auxetic lattices;
2010	P. Kolat	[55]	Propagation velocity and amplitude of solitary waves in auxetic plates;
2013	W.I. Azoti	[62]	Natural frequencies and loss factor in sandwich composites with auxetic layers;
2014	F. Agnese	[63]	Use of star shaped inclusions in turbine blades to enhance structural damping;
2014	T.C. Lim	[64]	Elastic wave propagation velocity in auxetic solids;
2015	T. Streck	[58]	Effect of auxetic structure and filler in the dynamic behavior of auxetic sandwich panels.
2017	N.D. Duc	[65]	Analytical deduction of vibration response on sandwich panels with auxetic cores.
2017	X. Chen	[66]	Modeling of the dynamic behavior on thin layered plates including auxetic layers.
2018	L. Ma	[67]	Increase in stiffness and damping capacity in auxetic corrugated sandwich panels.
2018	M. Mazloomi	[68]	Auxetic 2D lattices reduce the radiated sound power level.
2018	H. Reda	[69]	Analysis of acoustic behavior and bandgaps in pre-deformed auxetic scaffolds.
2019	D. Qi	[70]	Acoustic bandgaps in novel hybrid anti-chiral auxetic structures.

The first reference (1882) of experimental evidence of this behavior may be found in the works of Woldemar Voigt when testing Iron Pyrite monocrystals [71]. Historically, it has been shown that naturally occurring auxetic behavior (Table 1.6) is frequently associate to materials and systems with extreme anisotropy. In this last condition, it has been shown that the Poisson's ratio may assume extreme values and are not constrained by the values deduced for isotropic conditions [72]. Counterintuitively to most common assumptions, it has also been shown that roughly 70% of cubic elemental metals possess auxetic behavior, at least, in one preponderant direction [73]. A summary of relevant naturally occurring auxetic materials is presented in Table 1.6.

Table 1.6 Naturally occurring auxetic materials.

Year	Ref.	Poisson's ratio	Material
1882	[71]	$\nu < 0.000$ ($C_{12} = -4.740$)	Iron Pyrite Monocrystals
1892	[74]	$\nu = -0.143$	
1945	[75]	$\nu < 0.000$ ($C_{12} = -4.400$)	
1947	[76]	$\nu < 0$ ($-5.290 < C_{12} < -4.400$)	
1955	[77]	$\nu < 0.000$ ($C_{12} = -4.640$)	
1969	[78]	$-0.100 < \nu < -0.094$	Low porosity dry granite
1970	[79]	$\nu < 0.000$	Cat Skin*
1973	[80]	$-0.266 < \nu < -0.222$	Sandstone**
1974	[81]	$\nu < 0.000$	Unsaturated Clay
1976	[82]	$\nu = -0.300$	Porous Rocks
1981	[83]	$\nu = -0.073$	α - Quartz
1982	[84]	$\nu = -0.070$	Cancellous Bone
1992	[85]	$\nu = -0.175$	α - Cristobalite
	[86]	$-0.133 < \nu < -0.191$	
1995	[87]	$-0.025 < \nu < -0.053$	Carbon Nitride
1999	[88]	$\nu = -0.075$	Monocrystalline Zinc (Basal Plane)
2014	[89]	$\nu = -3.000$	Copy Paper***
		$\nu = -0.300$	Paperboard***
			Cotton Paper***
2015	[90]	$\nu = -1.400$	Human Achilles Tendons
2018	[91]	$\nu = -0.74$	Liquid Crystal Elastomers

* Volume increase in tension.

** Calculated from original data.

*** Thickness (out-of-plane) direction.

Given the absence of naturally occurring isotropic auxetics and the attractive properties that such materials could offer in practical applications, there has been a substantial effort to produce artificial auxetics. The most simple and common example (1985) is the design of mechanisms [92] (Fig. 1.16 (a)) and structures with protruding ribs [93] (Fig. 1.16 (b)), implying that such systems will expand in tension (Fig. 1.16 (c) [94]). Interestingly, there is an evidence of early report [95] (1966) of deforming non-protruding into protruding ribs by plastic deformation (Fig. 1.16 (d)), although there is no evidence of using that effect to obtain auxetic behavior.

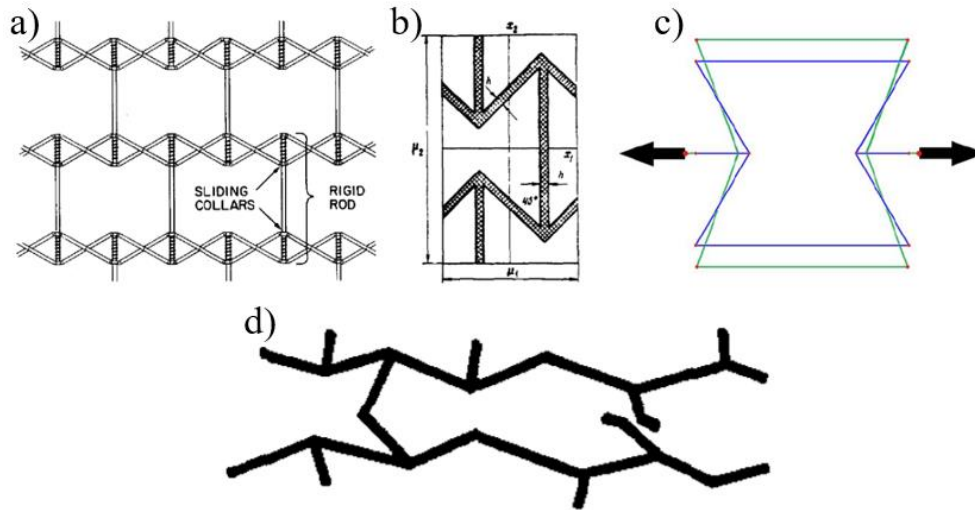


Figure 1.16. Reentrant based auxetic (a) mechanism [92]; (b) structure [93]; and (c) expansion [94].
(d) Possible unidentified reentrant structure [95].

In 1987, the most common three-dimensional isotropic auxetic micromechanical design has been idealized, being shaped in the form of a reentrant tetrakaidecahedron (Fig. 1.17 (a)) [29]. The presented model has been used to propose the manufacturing of bulk polymer auxetics based on the molecular network of (1,4)-reflexyne (Fig. 1.17 (b)) [36].

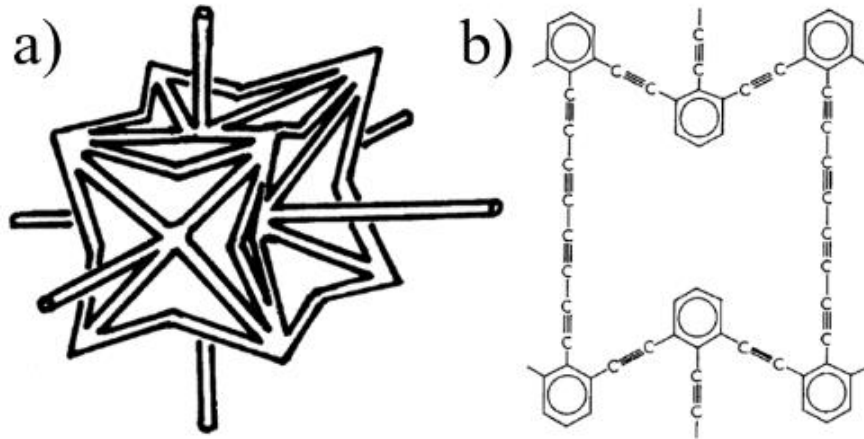


Figure 1.17. Auxetics: (a) Reentrant tetrakaidecahedron model [29] and (b) molecular network of (1,4)-reflexyne [36].

In terms of geometric dependence of the Poisson's ratio presented by these structures, reentrant models (Fig. 1.18) based on rib flexure may be described as a function of unit cell angle (θ), vertical length (l) and horizontal length (h) as presented by Eqs. 1.14 and 1.15 (Table 1.7) [10]. A more detailed deformation behavior may be achieved considering the unit cell thickness (t), depth (b), base material Young's (E) and Shear (G) moduli (Table 1.7). These constants are able to further describe rib flexure

(K_f – Eq. 1.18), stretching (K_s – Eq. 1.19) and hinging (K_h – Eq. 1.20) stiffness, using Eqs. 1.16 and 1.17 [96]. In the case of thick walled cellular lattices the Poisson's ratio may be described by Eq. 1.21 [97].

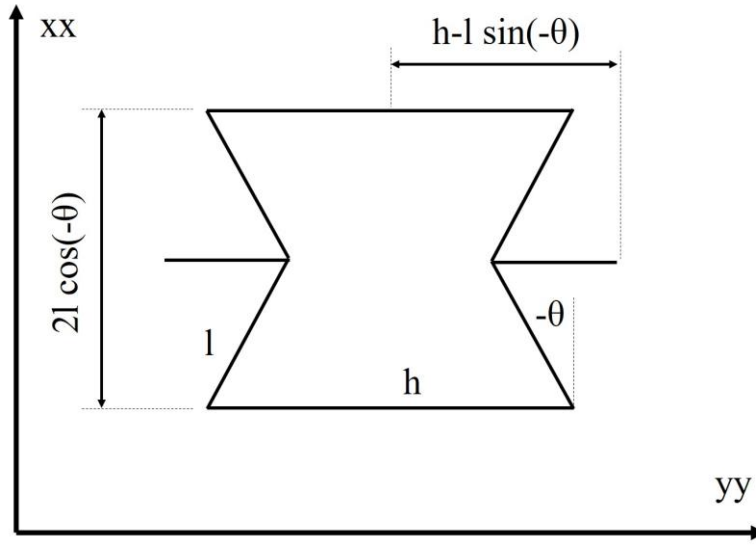


Figure 1.18 Two-dimensional reentrant model [96].

Table 1.7 Poisson's ratio of reentrant structures.

Ref.	Lead Author	Poisson's ratio
[10]	L. Gibson	$v_{xy} = \frac{\sin(\theta) [h + l \sin(\theta)]}{l \cos^2(\theta)} \quad (1.14)$
		$v_{yx} = \frac{l \cos^2(\theta)}{\sin(\theta) [h + l \sin(\theta)]} \quad (1.15)$
[96]	I.G. Masters	$v_{xy} = -\sin(\theta) \left(\frac{h}{l} + \sin(\theta) \right) \left[\frac{\frac{1}{K_s} - \frac{1}{K_f} - \frac{1}{K_h}}{\frac{\cos^2(\theta)}{K_f} + \frac{\cos^2(\theta)}{K_h} + \frac{2h/l + \sin^2(\theta)}{K_s}} \right] \quad (1.16)$
		$v_{yx} = \frac{-\sin(\theta) \cos(\theta) \left[\frac{1}{K_f} + \frac{1}{K_h} - \frac{1}{K_s} \right]}{\left(\frac{h}{l} + \sin(\theta) \right) \left[\frac{\sin^2(\theta)}{K_f \cos(\theta)} + \frac{\sin^2(\theta)}{K_h \cos(\theta)} + \frac{\cos(\theta)}{K_s} \right]} \quad (1.17)$
		$K_f = \frac{Ebt^3}{l^3} \quad (1.18)$ $K_s = \frac{Ebt}{l} \quad (1.19)$ $K_h = \frac{Gbt}{l} \quad (1.20)$
[97]	Malek	$v = \frac{\cos(\theta)}{\left(\frac{h}{l} + \sin(\theta) \right) \sin^2(\theta)} \left[\frac{1 + (1.4 + 1.5v) \left(\frac{t}{l} \right)^2}{1 + (2.4 + 1.5v + \cot^2(\theta)) \left(\frac{t}{l} \right)^2} \right] \quad (1.21)$

Reentrant structures have been analyzed and developed into variations, *e.g.* star-shaped [98] and arrowheads [99]. A fundamental reference to the evolution of this research may be found in Table 1.8.

Table 1.8. Published works on reentrant auxetic structures.

Year	Lead Author	Ref	Description
1985	A. Kolpakov	[93]	Fine-celled structures with protruding ribs shown negative Poisson's ratios;
	R.F. Almgren	[92]	Reentrant structure composed by rods, hinges and springs has a Poisson's ratio = -1;
1990	T.L. Warren	[100]	Linear elastic Poisson's ratio of reentrant structures is dependent on the reticulation angle;
1991	K.E. Evans	[36]	(1,4)-reflexyne groups are presented as a possibility to obtain molecular scale auxetics;
1997	P. Theocaris	[101]	Reentrant (star-shaped) lattices shown an initial rib-angle dependent auxetic behavior;
2004	H. Wan	[102]	Poisson's ratio depends on cell geometry and is not constant in large deformations;
2005	J.N. Grima	[98]	Connected star-shapes are analyzed by the inclusion of non-convex structures;
2009	C. Lira	[103]	Influence of geometry and material in the Poisson's ratio of multi reentrant structures;
2011	W. Miller	[104]	Study on the out-of-plane buckling strength in reentrant honeycombs;
2012	P. Soman	[105]	Combination of reentrant and non-reentrant honeycombs to obtain variable Poisson's ratios;
2013	Y. Sun	[106]	Stiffness of hierarchical honeycombs (ribs composed by auxetic structures);
	L. Yang	[107]	Role of geometry and material in the Poisson's ratio of 3D reentrant structures;
2014	M.S. Rad	[108]	Theoretical and finite element analysis of 3D reentrant structures;
2015	A. Spagnoli	[99]	Analysis of the auxetic behavior of arrowhead reentrant structures;
2016	Z.X. Lu	[109]	Stiff variation of auxetic reentrant structure with connected protruding ribs;
	R. Brighenti	[110]	Analysis of the non-linear behavior of arrowhead reentrant structures;
	T.C. Lim	[111]	Geometrical analysis of novel double-arrowhead reentrant auxetic structures;
	X. Hou	[112]	Study concerning the dynamic crushing behavior of reentrant honeycombs;
2017	M.F. Fu	[113]	Combination of rhombic and reentrant structures to elevate stiffness and buckling strength.
2018	V.H. Carneiro	[114]	Minimum number of cells in 2D auxetic lattices to have bulk mechanical properties.
2019	D. Xiao	[115]	Dynamic compression of 2D auxetic reentrant lattices.
		[116]	Experimental and FEA compression of 2D auxetic lattices.
	Z. Dong	[117]	

Auxetic behavior from reentrant models is generated by the expansive/contractive effect of protruding ribs in tension/compression. This effect, however, has been also shown to be present in diamond shape unit cell structures (Fig. 1.19 (a)) with absent ribs. This model is known as the missing rib model (Fig. 1.19 (b)) [118]. Their value of Poisson's ratio is described by Eq. 1.22.

The deformation behavior of the missing rib model, is geometrically similar to chiral structures (Fig. 1.20). The latter are characterized by their lack of symmetric reflection and by rotating their ribs around a central node, which generates a rotational effect that permit their auxetic behavior (Fig. 1.19 (a)).

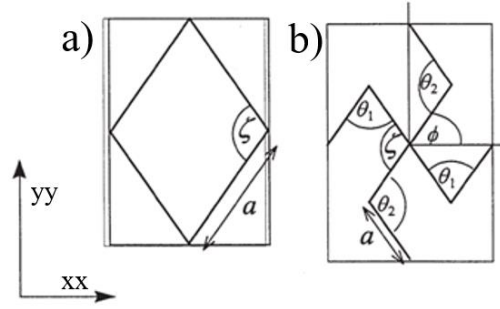


Figure 1.19 Transition from (a) Diamond shape to (b) Missing rib structural models (adapted from [118]).

$$v_{yx} = (v_{xy})^{-1} = -\tan(\phi) \tan(\xi - \phi) \quad (1.22)$$

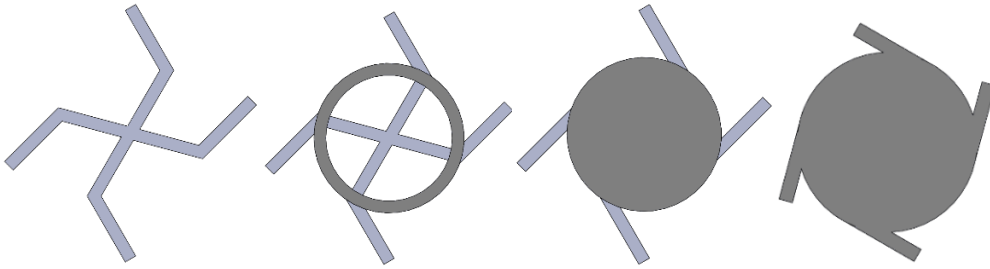


Figure 1.20. Progression from the Missing rib to the Chiral structural model.

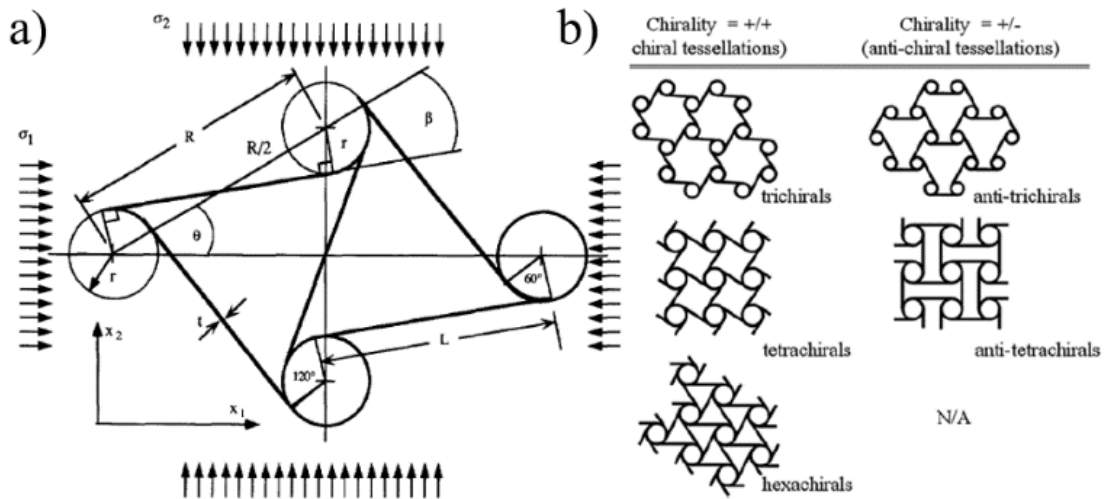


Figure 1.21. Chiral (a) geometry and (b) possible configurations [119].

Even though this kind of arrangement may assume many shapes (Fig. 1.19 (b)), it is theoretically demonstrated and confirmed experimentally that they are characterized by a Poisson's ratio in the vicinity of -1 [119]. Further research was performed to develop this model may be observed in Table 1.9.

Table 1.9 Published works concerning chiral auxetic structures.

Year	Lead Author	Ref.	Description
1991	R.S. Lakes	[120]	Reference to negative Poisson's ratios in non-centrosymmetric (chiral) structures;
1997	D. Prall	[119]	Theoretical and experimental analysis on the deformation of chiral structures;
1998	O. Sigmund	[121]	Chiral structures resultant of topology optimization to find auxetic behavior;
2005	A. Spadoni	[122]	FEA on the out-of-plane buckling of chiral structures;
2007	F. Scarpa	[123]	Analytical, FEA and experimental analysis on buckling of chiral structures;
2008	J.N. Grima	[124]	Combination of chiral cells into "meta-chiral" structures with auxetic behavior;
	M. Hassan	[125]	Research on foldable structures using shape memory materials into chiral geometries;
2010	W. Miller	[126]	Influence of number of ribs per node on out-of-plane buckling of chiral lattices;
	A. Lorato	[127]	Out-of-plane linear elastic mechanical properties of chiral structures;
	A. Alderson	[128]	Analysis of the in-plane elastic properties of tri-, tetra- and hexachiral structures;
	Y.J. Jeřan	[129]	Thermoelastic Cosserat theory used to define the thermal behavior of chiral rods;
2012	J. Dirrenberge	[130]	Study on the auxeticity of chiral structures in the elasto-plastic regime;
2013	Y.J. Chen	[131]	Numerical and experimental elastic behavior of anti-chiral structures;
2014	A. Pozniak	[132]	Poisson's ratio of anti-chiral structures with disordered stochastic nodes;
2016	C.S. Ha	[133]	Analysis of a novel three-dimensional chiral structure.
2018	H. Zhang	[134]	Topological optimization of auxetic chiral lattices using two materials.
	M. Fu	[135]	Novel three-dimensional chiral structure.
	W. Wu	[136]	Deployment of anti-tetrachiral lattices in cylindrical geometry (<i>i.e.</i> stent shape).
2019	S. Jin	[137]	Shear deformation in auxetic chiral lattices.
	N. Novak	[138]	Behavior of composite panels with auxetic chiral core subjected to blast loading.
2020	H. Wang	[139]	Novel peanut-shaped holed lattice with auxetic behavior that is based on chiral lattices.

Recent advances have shown that two-dimensional chiral structures (Fig. 1.22 (a)) [140] may be adapted to a three-dimensional structure by extending the non-centrosymmetric model to the three cartesian planes (Fig. 1.22 (b)) [133]. In fact, the last one is similar to another model, often used to justify auxetic behavior: the Nodule Fibril model (Fig. 1.22 (c)) [141].

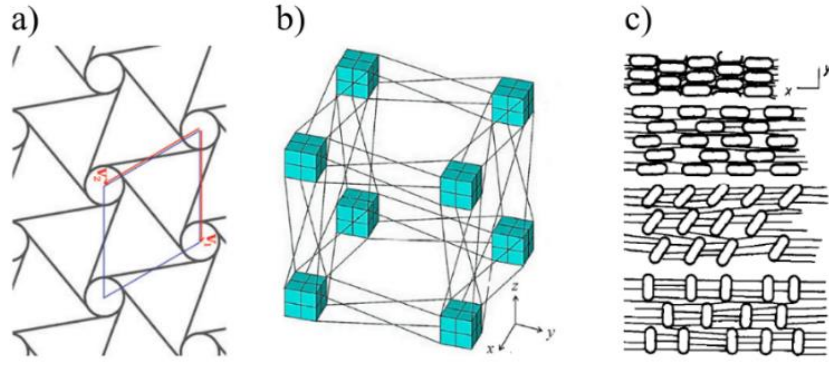


Figure 1.22 Chiral (a) geometry [140] and (b) three-dimensional structure [133]. (c) Comparison with Nodule fibril model [141].

Nodule fibril auxetic structures are characterized by an array of anisotropically shaped particles that are connected by fibrils, as observed in Fig. 1.23 (a) [142, 143]. When these structures are tensioned, the fibrils will transmit the applied load to the particles, due to their non-centrisymmetric distribution will rotate and the structure will expand.

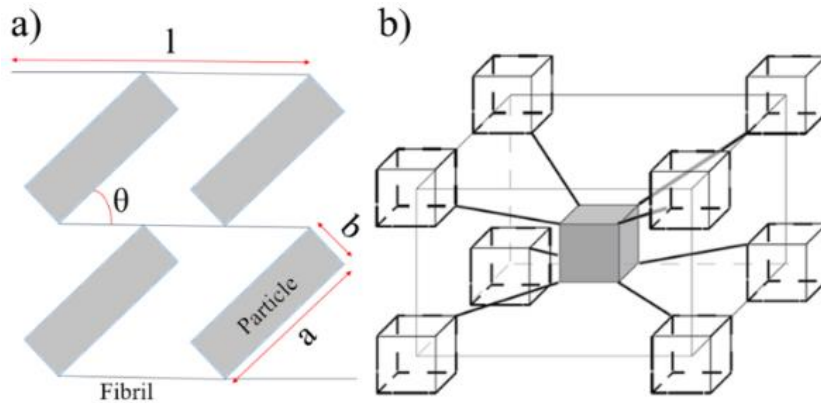


Figure 1.23. Nodule fibril module: (a) basic and (b) three-dimensional structures [144].

The particle length (a), width (b), fibril length (l) and angle (θ) deformation dependent phenomenon may be described by Eq. 1.23 (Table 1.10). Expanding this last concept to describe a possible real application, the fibril hinging, stretching and flexure may also be included in the deformation analysis. The influence of the last is represented by a hinging (K_h – Eq. 1.26) and stretching (K_s – Eq. 1.27) coefficient, which in turn are correlated with the fibril bending moment (ΔM) angle variation ($\Delta\theta$), applied load (ΔF) and displacement (Δs), leading to the Eqs. 1.24 and 1.25 [143]. This concept may be also adapted to a three-dimensional structure (Fig. 1.23 (b)) [144]. A perspective of this evolution and research on this model is represented in Table 1.11.

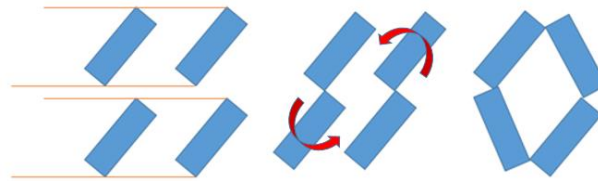
Table 1.10 Poisson's ratio of Nodule fibril models.

Ref.	Lead Author	Poisson 's Ratio
[141]	K.E. Evans	$v_{xy} = \left(\frac{b \sin(\theta) - a \cos(\theta)}{a \sin(\theta) + b \cos(\theta)} \right) \left(\frac{l + b \sin(\theta) - a \cos(\theta)}{a \sin(\theta) + b \cos(\theta)} \right) \quad (1.23)$
[143]	A. Alderson	$v_{xy} = \frac{\left[1 - \left(l^2 K_s / K_h \right) \right] \sin(\theta) \cos(\theta) (a + l \cos(\theta))}{\left[\left(l^2 K_s / K_h \right) \sin^2(\theta) + \cos^2(\theta) \right] (b - l \sin(\theta))} \quad (1.24)$
		$v_{yx} = \frac{\left[1 - \left(l^2 K_s / K_h \right) \right] \sin(\theta) \cos(\theta) (b - l \sin(\theta))}{\left[\left(l^2 K_s / K_h \right) \cos^2(\theta) + \sin^2(\theta) \right] (a + l \cos(\theta))} \quad (1.25)$
		$K_h = \Delta M / \Delta \theta \quad (1.26) \quad K_s = \Delta F / \Delta s \quad (1.27)$

Table 1.11 Published works concerning Nodule fibril auxetic structures.

Year	Lead Author	Ref.	Description
1989	B. Caddock	[141]	Description of a basic auxetic mechanism of particles and fibrils, based on the microstructural modelling of polyurethane foams;
	K.E. Evans	[142]	Analysis and interpretation of the model described in [141]. Deduction of equations to describe the Poisson's ratio of such structures (Table 1.10);
1990	J. Cherfas	[145]	Short communication describing the basic deformation mechanism of the model presented in [141] and its association with its application in <i>Gore-Tex</i> parkas;
1995	A. Alderson	[146]	Detailed analysis of the deformation behavior of the Nodule Fibril model considering the effect of fibril hinging, stretching and flexure;
1997		[143]	Concurrent behavior of the effects considered in [146] and influence in the Poisson's ratio;
1998	C. He	[147]	Description of different polymers compositions and arrangements, based on the Nodule Fibril model geometry, to obtain bulk polymer auxetics;
2011	N. Gaspar	[144]	Transformation of the basic Nodule fibril model to a three-dimensional structure.

Removing the fibrils in the model and reorienting the particles, one can find a model that may be based on the interaction of their contacts, generating a Rotating Geometry auxetic model (Fig. 1.24).

**Figure 1.24** Progression from the Nodule fibril to the Rotating geometry model.

This model may be achieved by the use of geometries, such as squares (Fig. 1.25(a)) and triangles (Fig. 1.25 (b)) [148] hinged around their contacting points. These last are characterized by a Poisson's ratio that equals -1. Recent advances on this model may be obtained by the use of rhombi geometries (Fig. 1.25(c)) [149] and even converted to a three-dimensional structures (Fig. 1.25(d)) [150]. In Table 1.12, can be observed a fundamental research progress on these models.

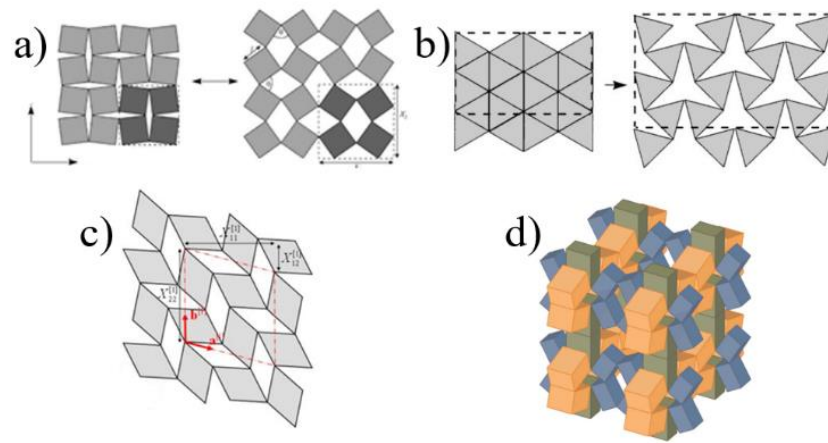


Figure 1.25 Rotating geometries: (a) square, (b) triangular [148], (c) rhombi [149] and (d) three-dimensional [150].

Table 1.12 Published works concerning Rotating rigid auxetic structures.

Year	Lead Autor	Ref.	Description
2000	J.N. Grima	[148]	Analysis on Rotating rigid units using square geometries;
2005		[151]	Use of polygons to obtain auxetic behavior from Rotating rigid units;
2007		[152]	Study on non-rigid polygons (Semi-Rigid Rotating unit) on auxetic behavior;
2008		[153]	Study on Rotating rigid unit is extended to polygons such as rhombi geometries and parallelograms;
		[154]	Study concerning the effect of square stretching on the Poisson's ratio of structures based on square based Rotation rigid units;
	D. Attard	[155]	Description of the different ways to connect rhombi polygons to obtain Rotation rigid models with auxetic behavior;
2009		[156]	Analysis of the different combinations of parallelograms in Rotation rigid units that present auxetic behavior;
2010	J.N. Grima	[157]	Study concerning the influence of shape and density in auxetic triangular- and star-shaped Rotating rigid units;
		[158]	Research on the auxeticity of sheets perforated with diamond and star shapes, based on the Rotation rigid models;
2012	D. Attard	[150]	Adaptation of the basic two-dimensional Rotation rigid unit model to a three-dimensional structure;
2014	E. Chetcuti	[159]	Analysis on the triangle base Rotation semi-rigid unit model to describe the Poisson's ratio in auxetic foams;
2016	D. Attard	[160]	Discussion concerning the use of the Rotation rigid unit model to obtain negative linear compressibility characteristics.
2018	U. Wollner	[161]	Novel 3D auxetic model based on rotating rigid structures with pore configuration emulating rocks.

Rotation rigid models are often used to describe auxetic behavior on Zeolites [162], *e.g.* to justify the negative Poisson's ratio in Natrolite [163] (Fig. 1.26) through a square based model [148].

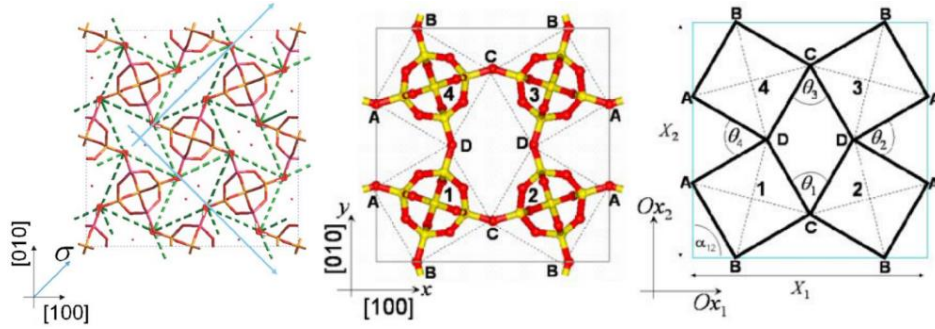


Figure 1.26 Examples of auxetic Zeolites (adapted from [162] and [163]).

Recently, several other models have been developed based on combinations of the referred mechanisms. Generally, they are characterized by the inclusion (in this case, by the removal of material) of geometries and shapes in the bulk body and, frequently, depend on elastic instability to develop auxetic behavior (Fig. 1.27 [164] and [165]). Some examples of these works are summarized in Table 1.13.

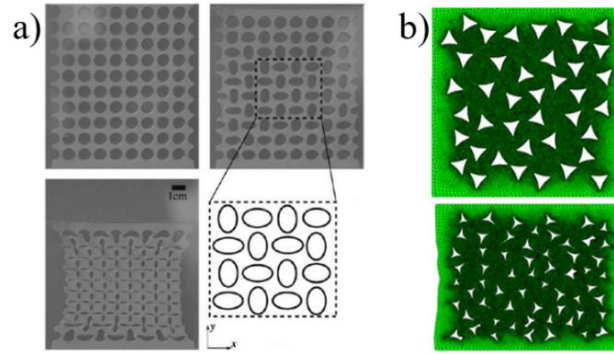


Figure 1.27. Auxetic behavior by the inclusion of geometries in bulk bodies (a) [164] and (b) [165]).

Table 1.13 Published works concerning inclusion based auxetic structures.

Year	Lead Author	Ref	Description
2010	K. Bertoldi	[164]	Auxetic behavior by elastic instability in bulk samples with circular inclusions;
2012	X. Hou	[165]	Auxetic deformation in bulk samples with reentrant triangle inclusions;
2013	H. Hu	[166]	Further development and interpretation of inclusion induced deformation mechanisms;
	S. Babaee	[167]	Three-dimensional auxetic structure based on elastic buckling;
2014	J. Shen	[168]	Auxetic behavior by elastic buckling from spherical inclusions in bulk bodies;
	M. Taylor	[169]	Manufacturing of low porosity auxetic sheets by the inclusion of geometrical shapes that promote a negative Poisson's ratio;
2015	I. Shufrin	[170]	Design of auxetic behavior by the inclusion of hollow spheres in bulk bodies;
	F. Javid	[171]	Novel class of non-porous auxetic by the inclusion of spherical dimples.
	J.N. Grima	[172]	Auxetic behavior in 2D sheets with randomly oriented cuts.
2017	K. Billon	[173]	Perforated plates with hierarchical configuration and auxetic behavior.
2019	L. Mizzi	[174]	Deformation behavior of slit perforated sheets with auxetic behavior.

In terms of transposing the referred models to specific applications, there have been developed manufacturing procedures to produce “artificial” auxetic materials. The most common routine is the conversion of positive Poisson’s ratio to auxetic foams (Fig. 1.26) by the transformation of regular ribs (Fig. 1.28 – microscopies on the left [175]) into protruding ribs [176] (Fig. 1.28 – microscopies in the right [175]). This procedure is based on a thermo/chemicomechanical process and applicable to both polymer and metal based foams. Fig. 1.29 shows the basic procedures to transform regular foams into auxetic.

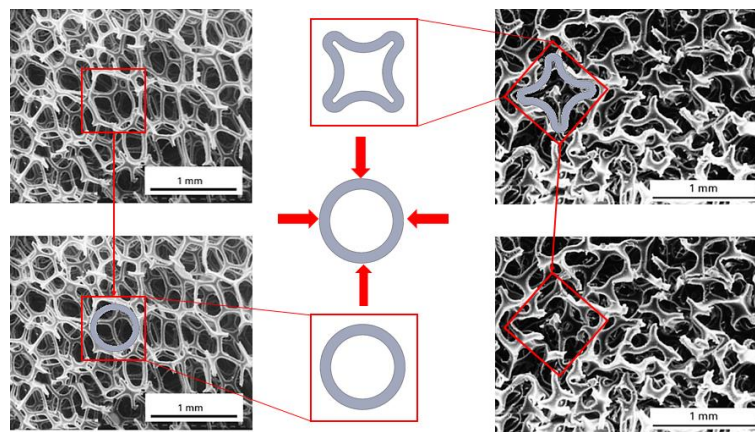


Figure 1.28. Transformation of “normal” into auxetic foam (microscopies adapted from [175]).

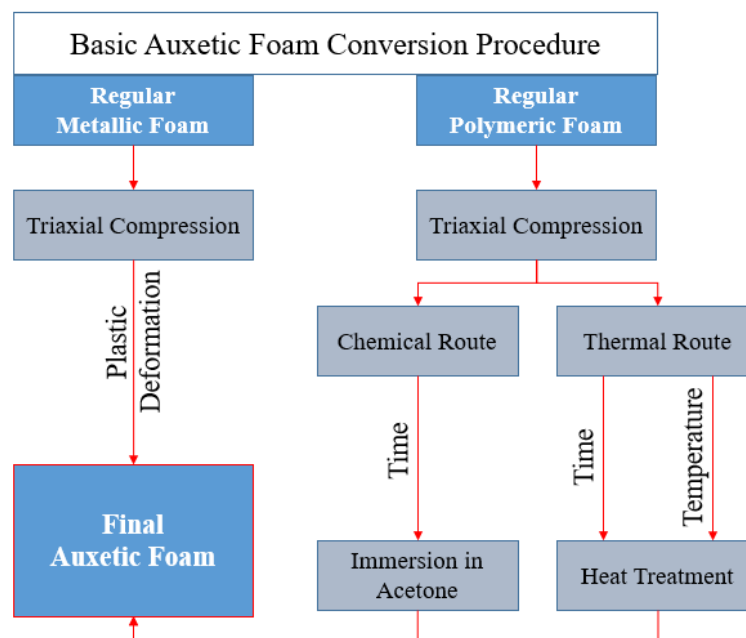


Figure 1.29 Basic foam transformation procedure.

When processing metallic foams to convert them into auxetics, the overall process is fairly simple: it implies a triaxial compression, where the regular tetrakaidecahedron cells are deformed in the plastic

regime to acquire a protruding shape [177,178]. In the case of polymeric foams, this process is more complex. The conversion mechanism is based in shape memory characteristics. Additionally to the triaxial compression, a chemical route (by immersion in acetone) [176][145] or a thermic route [175,179,180] (exposing the samples at temperature higher than the base material glass transition temperature [181]) have to be used. These exposition times and temperatures influence directly the success of the conversion and the value of the Poisson's ratio of converted foams [176]. Additionally, it is currently suggested the inclusion of Styrene-Acrylonitrile (SAN) copolymer particle to improve shape memory characteristics [182,183].

Due to the base foam manufacturing process, cellular foam structures are randomly generated by packed nearly polyhedral cells [184]. Thus, there is a current trend to produce non-stochastic cellular structures with a negative Poisson's ratio [185]. The latter have the advantage of possessing periodic shapes with uniform properties that may be tunable for specific applications by tailoring the foam structure. Such structures may be produced by additive manufacturing techniques, such as 3D polymer printing (Fig. 1.30 (a)) [186] or Selective Electron Beam Melting (SEBM) (Fig. 1.30 (b)) [187].

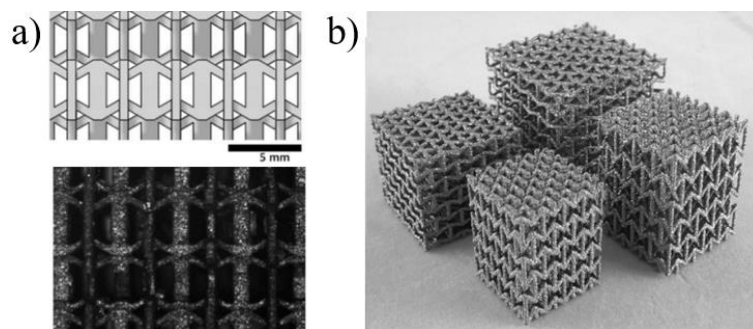


Figure 1.30. Three-dimensional reentrant: (a) polymeric – 3D printed [186]; and (b) metallic – SEBM [187].

Polymeric non-stochastic 3D printed structures have the advantage of being easily produced and fairly economic. However, for the application that this study is aimed and for structures in general, it presents low relative mechanical strength. As for the metallic scaffolds that may be produced by SEBM, they present higher mechanical strength. However, they are characterized by a high cost and low rate of production, difficulty in microstructure control and, generally, usually use base materials (*e.g.* Ti6Al4V and 316L Steel alloys [188]) that are not lightweight.

Another technique currently being used and developed is related with the manufacturing of non-stochastic scaffolds by investment casting (Fig. 1.31) [189,190]. This method has some inherent

advantages, such as: good microstructural control, relative ease in processing and the possibility to easily use lightweight alloys.

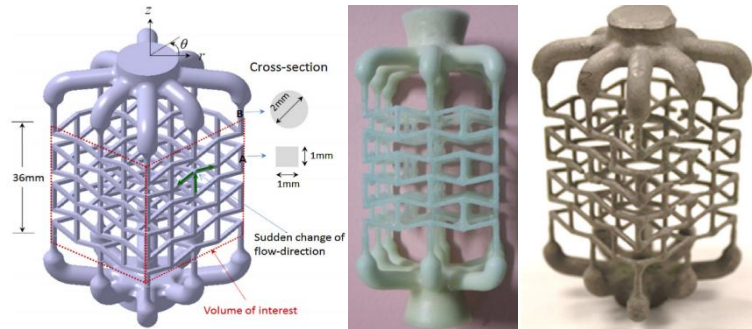


Figure 1.31. Casting of Reentrant structures [190].

A current trend on auxetic material is the use of foam filling of metal structures [191] to enhance mechanical properties (Fig.1.32 [192]) such as toughness and crashworthiness [193]. Even though, most cases present a positive Poisson's ratio structure filled with an auxetic foam, it is suggested in this study that using a negative Poisson's ratio structures filled with expanded foam may be interesting to explore due to the stiffer auxetic phase.

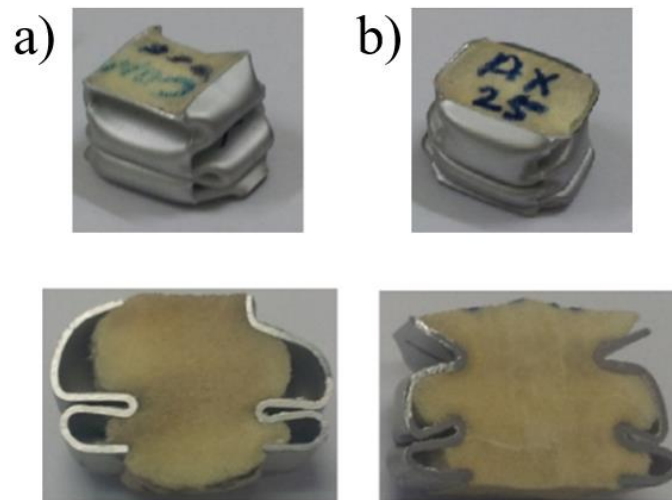


Figure 1.32 Tube filled with (a) "normal"; and (b) auxetic foam [192].

Interpreting the relevant studies performed by researchers in the last years, it may be observed that there is an interesting opportunity to investigate the benefits of using auxetic scaffolding structures filled with polymeric foam for lightweight structural applications with enhanced damping capacity.

1.3. Thesis organization and research questions.

Considering the previous sections, it is established that there is a trend in the design of cellular solids for applications in the transportation industry. Their successful integration in the fabrication of railway vehicles is dependent on the current developments of these materials and their ability to display attractive characteristics for this industry.

The referred trends on the evolution of materials for transportation industries, particularly in railway vehicles, imposes a question: What should be the design and manufacturing processes of cellular solids for the fabrication of lightweight vehicles with high structural vibration/noise damping without compromising structural strength? This is defined as the broad **research question** of this study. It is able to define a boundary of current knowledge, raising awareness of contemporary trends in the scientific community and technological advances. Additionally, its answer would have a potential interest to society.

Given the evidences that are presented in section 1.2, the author suggests the following **hypothesis**: the design of non-stochastic cellular solids with auxetic behavior (*i.e.* negative Poisson's ratio) manufactured by investment casting. If successful, the hypothesis would fulfill the **objective**: to design a lightweight material with high specific structural strength and enhanced vibration/noise damping capacity for future railway vehicle fabrication.

Accordingly, each individual chapter is presented in a detailed research paper format and describes the steps that have been taken to approach the main research question:

- **Chapter 2** is dedicated to the selection and optimization of the base material that comprises the solid phase of the proposed cellular solids suggested in Chapter 1. Such selection and optimization process are performed to optimize the static and dynamic mechanical properties of the base material.
- **Chapter 3** details the design of the lightweight cellular solids and the development of a route for their manufacturing. The parameters of the suggested manufacturing route are optimized to allow the successful fabrication of the design with the base material selected in Chapter 2. Manufactured samples are analyzed in terms of defects and dimensions to correlate their final shape/geometry with the initial design.
- **Chapter 4** shows the static mechanical testing and characterization of the samples that have been designed and manufactured according to Chapter 3. Their static mechanical properties are compared with the specific properties of other materials to estimate if the designed cellular solids are a viable

solution and suitable to the load bearing applications intended in Chapter 1. Their failing mechanisms are also presented by *in-situ* experimental observation techniques and further detailed by FEA.

- **Chapter 5** is devoted to the vibration testing of the samples that have been designed and manufactured according to Chapter 3. The proposed designs are investigated in terms of damping capacity by the calculation of their damping ratios. The experimental results and the sample harmonic behavior are further detailed by FEA. Damping capacity is compared with other current materials to understand if the dynamic properties are interesting for the application defined in Chapter 1.

- **Chapter 6** combines the results of Chapters 4 and 5. The static and dynamic structural behaviors of the samples are compared to identify the design with the best combination of load bearing and vibration damping capacity. The selected design is posteriorly impregnated with polymeric foams to determine the acoustic properties of the cellular solids. Their acoustic reflection and transmission are compared with current solutions to estimate if the designed composites are an interesting solution for the application defined in Chapter 1.

- **Chapter 7** presents the overall conclusions of the study. The initial research question (Chapter 1) is compared with the results and discussion of the previous chapters (2 to 6) and, therefore, determine if the adopted hypothesis is able to fulfill the established objective of this study. Considerations on future works and scientific impacts are also suggested.

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Chapter 2 – Alloy Selection and Processing: Optimization for Static and Dynamic Mechanical Properties

“Give me 30,000 tons of aluminum, and I will win the war”

Soviet leader Joseph Stalin correspondence to U.S. president Franklin Roosevelt, 1941.

2.1 Material and process selection

The fundamental purpose of the composites that are being developed in this study is to promote a system with enhanced structural vibration and noise damping. However, a fundamental constraint is that static structural strength should not be compromised. All these benefits should also be achieved while maintaining a lightweight solution, *i.e.* with low values of relative density in the case of the proposed cellular solid configuration. This chapter is devoted to the **selection and processing optimization of the base material that composes such cellular solid**.

Given the referred prepositions, constraints and the intended fundamental composite configuration (Chapter 1 – 1.3 Objectives and research questions), it is possible to employ a mechanical design approach based on the classic theory that was developed by Ashby [1]. Considering that the fundamental design of a mechanical system may be divided in four distinct areas (Fig. 2.1). The latter are all interdependent, an iterative procedure was used to determine the material and process that is suitable to manufacture the proposed composites.

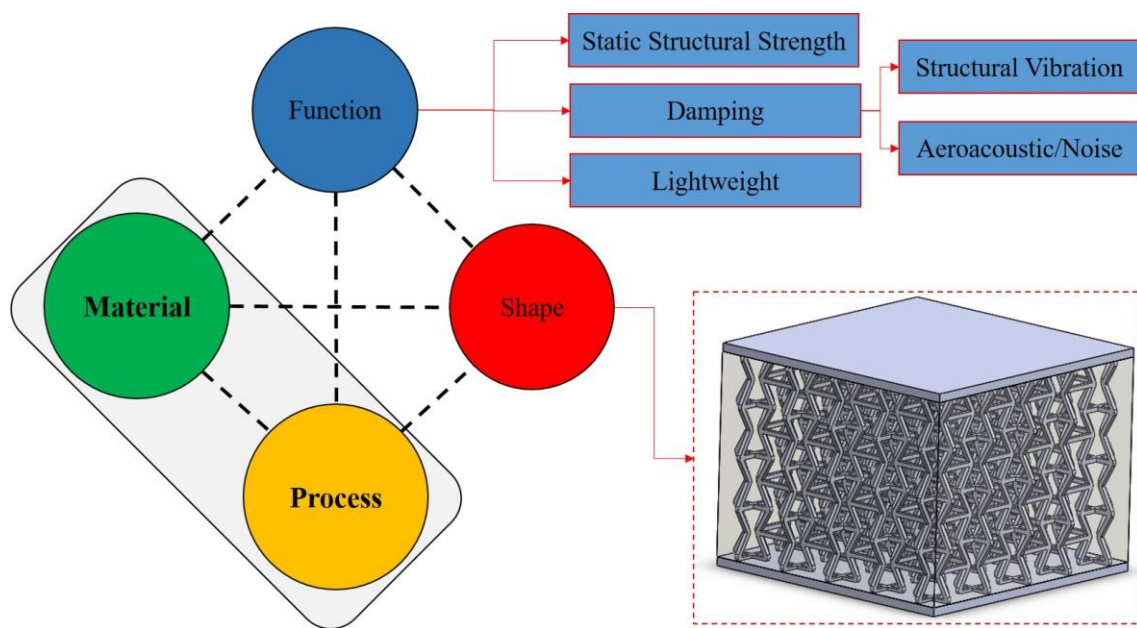


Figure 2.1 Selection of material and process (adapted from [1]).

Considering the different classes of materials, it is known they have inherent contrasting properties. For instance, the high damping properties (*e.g.* loss factor) and low density that characterize polymeric materials may seem a solution for the proposed problem. However, it is also known that in unfilled states, such materials possess lower mechanical strength than metallic and ceramic materials [2]. There is a need to see their fundamental mechanical properties and if they should be maximized or minimized. According to Table 2.1, it was defined that the base material that composes the lightweight cellular solid must possess high damping, yield strength and stiffness (*i.e.* Young's modulus).

Table 2.1 Identification of relevant mechanical properties.

	Dimensionless		Dimensional	
Property	Damping Capacity	Specific Density	Yield Strength	Young's Modulus
Objective	Elevate	Lower	Elevate	Elevate
Relation	Damping Capacity / Specific Density		Yield Strength x Young's Modulus	
Axis (Fig. 2.2)	Vertical		Horizontal	

To facilitate this decision process, the density values were evaluated in terms of specific density. This allows the sub-division of the four magnitudes into two distinct classes: dimensionless (damping capacity and specific density) and dimensional (Yield strength and Young's modulus). According to Table 2.1, the damping capacity was defined as being divided by those of specific density, as these results

would generate the most beneficial combination of these dimensionless magnitudes for the proposed work. A similar methodology was used for the dimensional magnitudes (Table 2.1), however, a multiplication operation was used to correlate the values of Yield strength and Young's modulus due to the intent of maximizing both. It is worth mentioning there is no ponderation between the magnitudes. Considering this method, and using bibliography concerning the mechanical properties of the different classes of materials (compiling the data from [3–28]), it is possible to plot the Ashby diagram in Fig. 2.2.

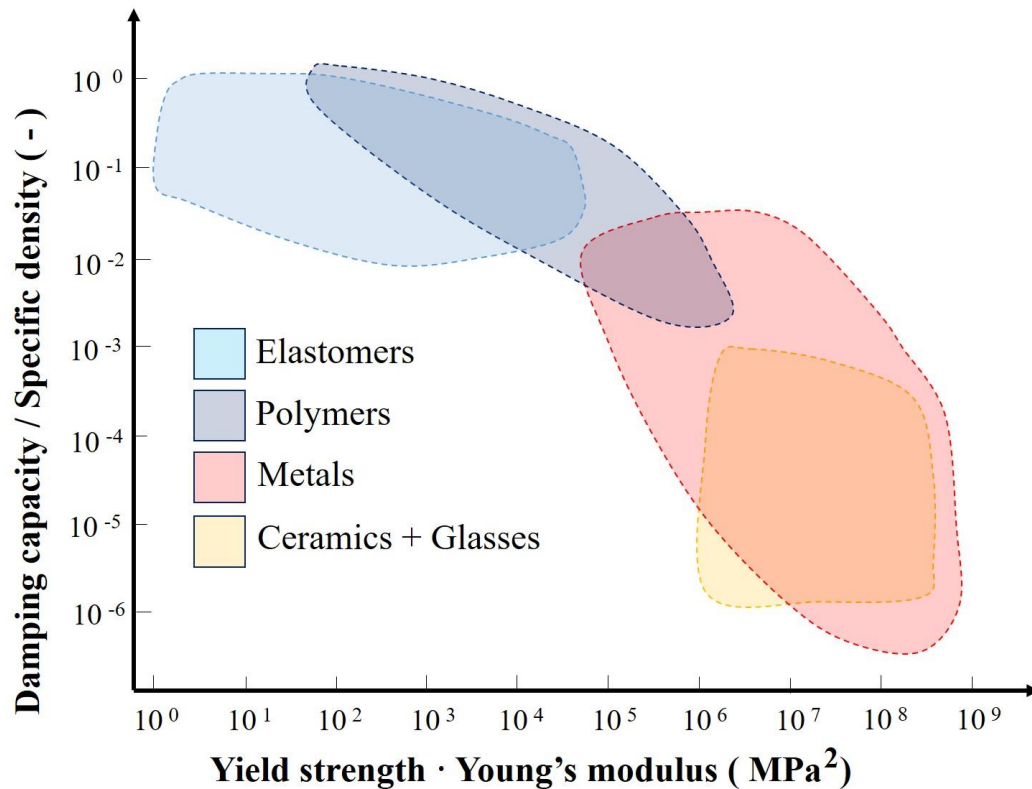


Figure 2.2 Ashby diagram comparing properties of the different material classes (compilation of data from [3–28]).

According to Fig.2.2 it may be visualized that the best compromise, in a bulk material to be used for the established purposes (see section 1.3 – Chapter 1) must be the one who displays the coordinates that have maximum values on both axis. Therefore, it may be inferred that the best compromise in terms of bulk materials are polymer and metal based. In fact, it may be stated that there is even a significant interception between their domains. Thus, a constraint must be imposed to determine a final selection between the two materials classes [29].

It may be seen that the Young's modulus and Yield strength are already characteristics of the material in energetic terms and may be roughly used to describe strain energy states (*e.g.* resilience)

[30]. After some thought, it may be seen that in physical terms, the system also has to be characterized in terms of mass. Since this is directly related to the density values that were already analyzed, an estimation may be drawn by the proportionality to the volume of the system. Comparing the latter, when considering metallic and polymeric materials, it may be observed that metals generally displays higher values of Yield strength (σ_Y). Thus, according to Fig. 2.3, it may be concluded that for a constant load value (F_{max}) a polymeric material would need a larger area (*i.e.* higher volume for constant thickness) to prevent yielding. The ability to support a given load in a lower volume of material is an important feature in most mechanical applications, and thus, this motivates the selection of a metallic material.

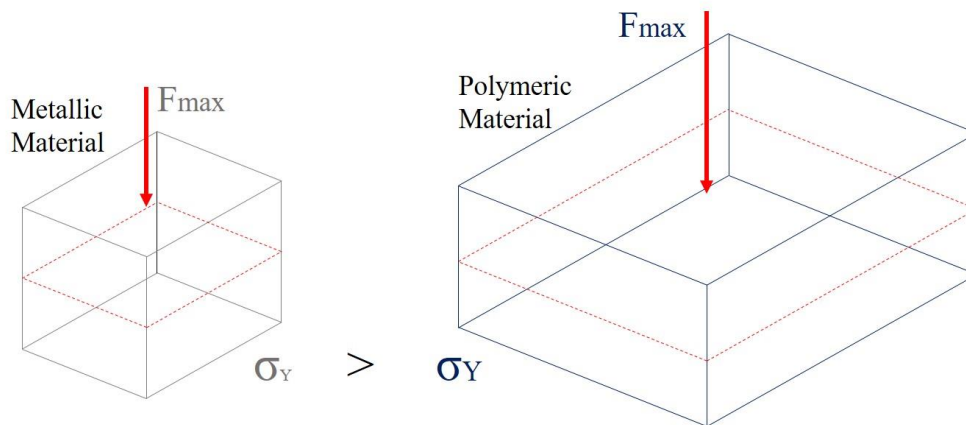


Figure 2.3 Metals vs Polymers: material volume that would be necessary to support the same maximum load (F_{max}).

Having selected the material class, it is fundamental to establish a route to transform the bulk material into the cellular solid final shape. According to Table 2.2, each manufacturing technique may reveal certain abilities to produce specific shapes and are not suited to others [29]. Considering that the intended geometry is mostly correlated with a 3D Hollow structure, it can be concluded that the manufacturing technique must be selected between casting, powder methods, electro-machining or additive manufacturing.

An important feature of the developed composites is the minimum section thickness that is possible to be manufactured by the referred processes. Such approach helps the reduction of the overall composite relative density and, consequently, contribute to its lightweight characteristics. Fig. 2.4 shows the generic range of section thickness for the different manufacturing processes [29] that are able to produce 3D Hollow shapes (according to Table 2.2).

Table 2.2 Shapes for different metal manufacturing processes (adapted from [29]).

	Circular Prismatic	Non-circular Prismatic	Flat Sheet	Dished Sheet	3D Solid	3D Hollow
Sand Casting	●	●			●	●
Die Casting	●	●			●	●
Investment Casting	●	●			●	●
Low Pressure Casting	●	●			●	●
Forging	●	●			●	
Extrusion	●	●				
Sheet Forming	●	●	●	●		
Powder Methods	●	●			●	●
Electro-Machining	●	●	●		●	●
Conventional Machining	●	●	●	●	●	
Additive Manufacturing*	●	●	●	●	●	●

* Added by the author given the current development of this technique.

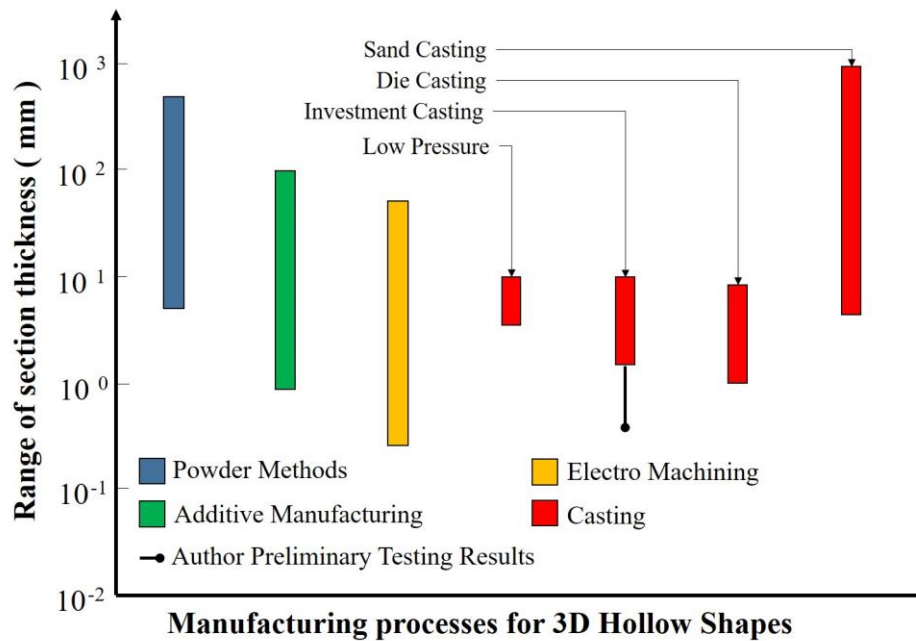
**Figure 2.4** Section thickness for different manufacturing processes (adapted from [29]).

Fig. 2.4 shows that the minimum values of thickness may be obtained by Electro Machining (EM). However, due to its working principles, it may be easily seen that the complex scaffolding of the composite matrix (Fig. 2.1) is not possible to be manufactured by this technique. Even though, some authors discuss

the obtainment complex structures by this technique, these possess an opening in one of the Cartesian planes that is not allowable in this scaffold design [31,32].

Fig.2.5 shows a possible route to obtain an approximate design by EM, in which: the bulk material (Fig.2.5 (a)) is subjected to successive wire cuts (Fig. 2.5 (b)) and drilling (Fig. 2.5 (c)) procedures to manufacture honeycombs (Fig. 2.5 (d)). This may be applied to different Cartesian planes to achieve a quasi-open cell three-dimensional configuration (Fig. 2.5 (e)). It may be observed that, due to the design constraint of not machining one plane, the geometry is not fully open and, thus, platforms are generated in the basis of each cell. In conclusion, the intended design is not fully replicated by the use of EM.

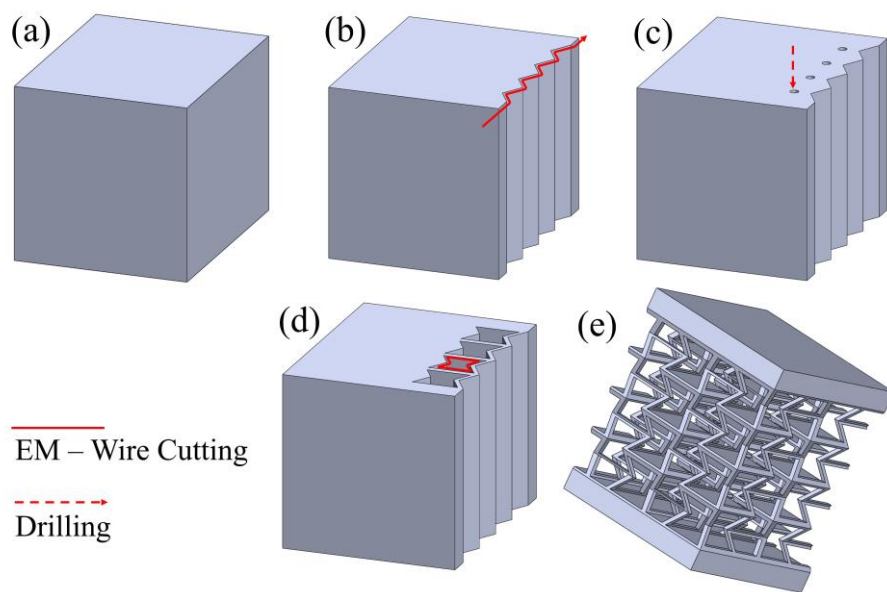


Figure 2.5 Suggested route to manufacture the proposed design by EM: (a) Bulk materials; (b-d) Wire cutting and drilling; (e) Transition for other planes.

Additionally, due to the complexity of the machining routines and the fact that each individual cell must be first drilled for the EM process (Fig.2.5), the overall process is a rather costly and extremely inefficient. Additionally, by exploring different manufacturing techniques, the author was able to produce thin sections ($\approx 0.3 \times 0.3 \text{ mm}^2$) by vacuum investment casting. Thus, it was concluded that investment casting is the most suitable process, being selected to produce the intended cellular solids. In fact, investment casting is a well-known technique to produce complex components with intricate and net-shaped details [33].

Once the process is selected, a new iteration concerning the selection of base material may be performed. Such process is handled on a much more specific approach: (i) given that metallic materials and investment casting were selected, it is necessary to define a specific alloy. (ii) This selection process

was performed by the analysis of the fundamental bulk mechanical and process parameters that are the focus of this study: Yield strength (σ_y), density (ρ), damping ratio (ξ) and castability (C_i).

While, yield strength, density and damping ratio, are related to the ability of the final cellular solid to display high strength, lightweight and damping characteristics, the role of their castability is more complex. Castability is able to directly influence the success to obtain the complex scaffolds by investment casting. Moreover, castability may be defined as “the ability of an alloy to form a dimensionally accurate castings of acceptable soundness and integrity” [34]. *i.e.* the success of the alloy in to be casted with good quality in geometric and metallurgical terms.

Considering these prepositions, a bibliographical search (from the compilation of the data presented in references [3,4,6,10–12,15,17–27,35–37]) was performed to determine the average values of the referred properties for the most common alloys that may be used in investment casting (Fig. 2.6). This average values and the correspondent standard deviations are fundamental tools that allow a direct comparison between the alloy properties. The values of castability were taken directly from the published data, however given that they were presented in different scales, the published values were converted in to a percentage (%) scale.

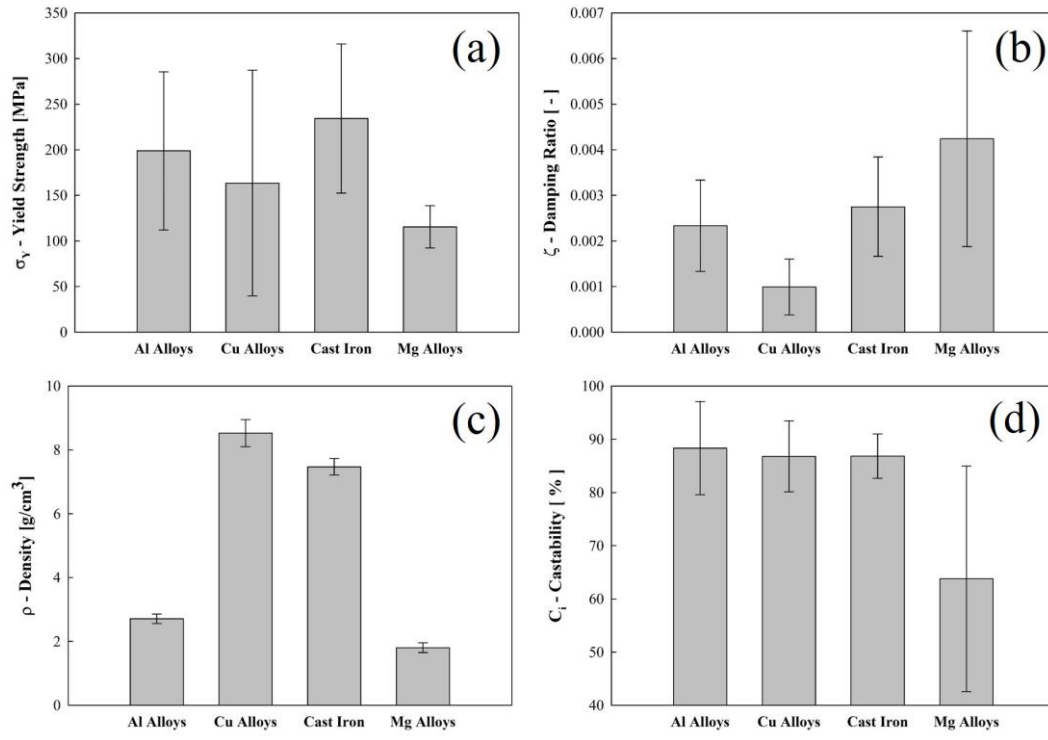


Figure 2.6 Comparison between some castable alloy properties: (a) Yield strength (σ_y); (b) Damping ratio (ξ); (c) Density (ρ); and (d) Castability (C_i).

Observing Fig.2.6, it may be seen that each individual alloy presents elevated characteristics on certain properties and a relatively poor performance on others. It is shown that, for instance, cast iron shows elevated values of yield strength and damping, however it has a high density. Whereas, Mg alloys have a low density and high damping, but have a low relative values of castability.

To overcome the difficulty of this selection process, a protocol was established to determine the alloy that is most suitable for the referred application and process. Considering that each property has the same importance (*i.e.* no weighting criterion was used), a Material Selection index (MSi) was established, according to Eq.2.1. Using this approach, the values that are intended to be enhanced were compared with the maximum values of each variable. For example, the individual values of yield strength (σ_y) compared with the maximum observed yield value ($\sigma_{y_{max}}$) by their ratio ($\sigma_y / \sigma_{y_{max}}$). As for the density, the value for each alloy is compared with the minimum value to generate a relative comparison between them. The result is a dimensionless value that represents the overall role of the selected properties in the alloy performance for this particular study.

$$MS_i = \frac{\sigma_y}{\sigma_{y_{max}}} \cdot \frac{\xi}{\xi_{max}} \cdot \frac{\rho_{min}}{\rho} \cdot \frac{C_i}{C_{i_{max}}} \quad (2.1)$$

The results of this approach are shown in Fig. 2.7, in which it is determined that the metals that display the best compromise for the proposed application are Mg- and Al-based alloys. Comparing the results, it is shown that these light alloys have an overall favorable combination of properties relatively to Cu alloys and Cast Iron.

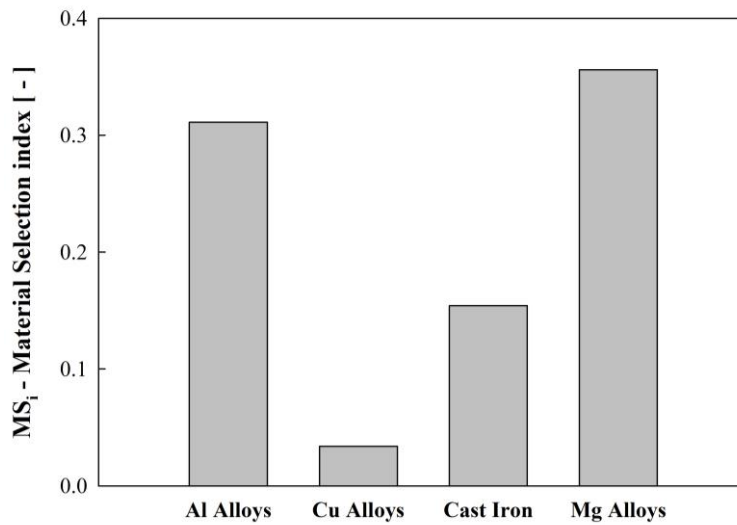


Figure 2.7 Material selection index (MSi) according to the established criteria.

Given the overall interpretation of Figs. 2.6 and 2.7, it is apparent that Mg alloys are the most appropriate solution to be used in this project. However, it is known that the use of Mg alloys in investment casting has inherent problems that may compromise a successful casting [33,38–41]. Fig. 2.8 shows an example of the reactions that occur between Mg alloy melts and mold interfaces (Table 2.3), in which it may be observed a bad surface finishing as a consequence of a violent exothermal reaction [33]. Additionally, this reaction is known to difficult the melt flow and reduce metal fluidity [30]. Given the role that these factors have in the overall success of the manufacturing process, it is concluded that Mg alloys are not applicable to this study. Thus, Al alloys are selected to manufacture the metal matrix scaffold that composes the intended cellular solid.

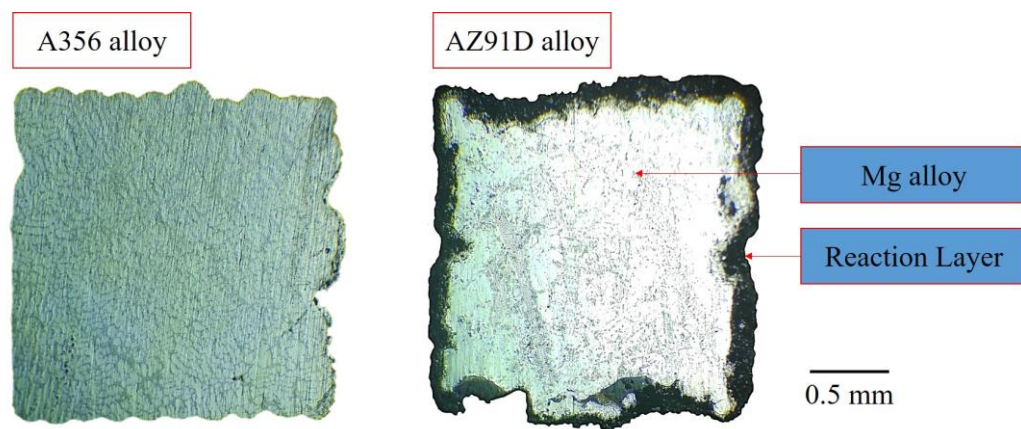


Figure 2.8 Optical Microscopy of sectioned A356 (Al alloy) and AZ91D (Mg alloy) investment casting samples.

Table 2.3 Mold-metal interface reactions in the investment casting of magnesium alloys [40,41].

Reaction	Description
1	$2\text{Mg (l)} + \text{SiO}_2 \text{ (s)} \rightarrow 2\text{MgO (s)} + \text{Si (s)}$
2	$4\text{Mg (l)} + \text{SiO}_2 \text{ (s)} \rightarrow 2\text{MgO (s)} + \text{Mg}_2\text{Si (s)}$
3	$\text{MgO (s)} + \text{SiO}_2 \text{ (s)} \rightarrow \text{MgSiO}_3 \text{ (s)}$
4	$2\text{MgO (s)} + \text{SiO}_2 \text{ (s)} \rightarrow \text{MgSiO}_4 \text{ (s)}$

Fig. 2.9 represents the most common binary and ternary aluminum alloys [10]. It may be observed that the combination of alloying elements has a predominant role in the processing of aluminum alloys and it is known these alloyed with silicon (Al-Si) are preferable for casting. However, it would be interesting, for material tailoring purposes to allow that material to be hardened and change their strength/damping, *i.e.* allow age- or work-hardening to be performed. Given that the thin rib configuration that comprises the metallic cellular solid, it may be easily understood that it would be impossible to

perform work-hardening. Additionally, elements like copper (Cu) and magnesium (Mg) allow the heat treatment of aluminum alloys [42] and are also present in casting alloys such as Al-Si-Mg and Al-Si-Cu. Thus, they are able to be both casted and heat-treated. Finally, Al-Si-Mg alloys are selected to perform this study and produce the metal matrix of the proposed composite due to their common use in investment casting processes.

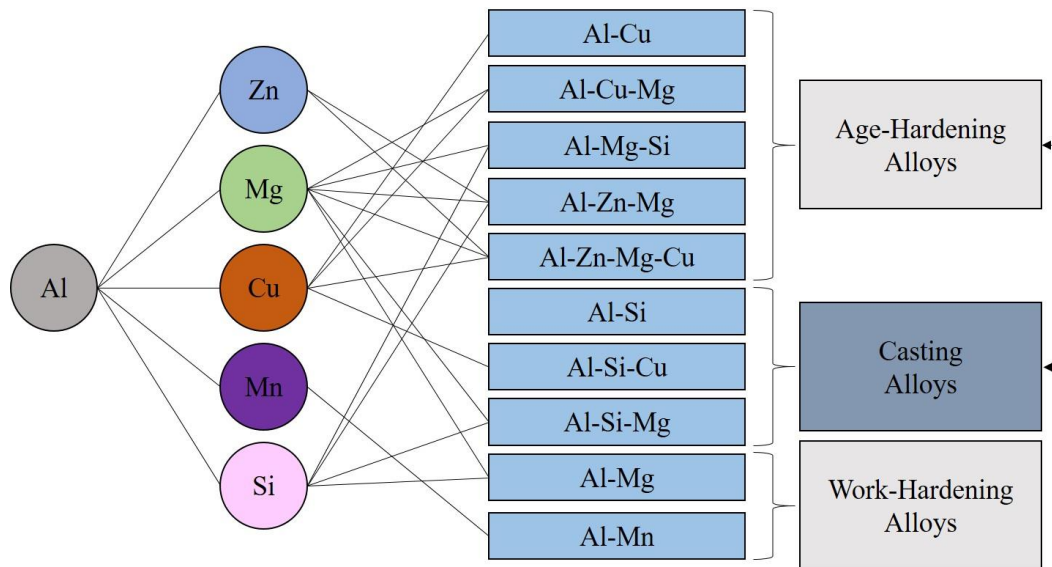


Figure 2.9 Principal aluminum alloys and processes (adapted from [10]).

AlSi7Mg0.3, typically referred as A356, is a classic casting hypoeutectic aluminum alloy. It is usually employed in transportation industries [43] (*e.g.* automotive, railway and aerospace) for its capability to be heat treated, casted in ceramic block to produce thin walled/ complex geometries [44] and overall specific mechanical properties. However, in its as-cast condition, this alloy is characterized by poor mechanical properties (*e.g.* low yield/UTS strength and extension) and, therefore, must be heat treated [45] (generally by T6, *i.e.* solution and quenching followed by artificial ageing).

As this alloy comprises a good part of the overall casted products in bulk [45–49] or composite [50–53] forms, there is a continuous research dedicated to its development and characterization. There has been significant research concerning the A356 α -Al primary dendrites [43,44,54,55], eutectic Si [56–59], main secondary phases (β - Al₅Fe₂Si, and π - Al₃FeMg₃Si₆) [60–63] and heat treatments, such as solution [59,64,65] and ageing [66–70].

Additionally, there are numerous studies concerning the effects of melt treatment by chemical or physical processes at a microstructural level. Most chemical processes are based on common grain refinement using Titanium Diboride (TiB₂) and modification with Strontium (Sr) [71–73] master alloys, or the more expensive combined treatment by Scandium (Sc) [74,75].

Recently, physical processes based on ultrasonic vibration have been employed to promote the referred refinement and modification of the matrix morphology [46,76,77]. Several studies have also been published to characterize the influence of the referred processing parameters in the overall mechanical properties of this alloy [78–80]. Fundamentally, there is a strong emphasis on its load bearing capability under quasi-static loading, either by simple hardness testing [75,81,82] of the casted alloy and/or posterior tensile testing [83–85]. Recent studies show a concern on the damping properties (mainly using dynamic mechanical analysis) of this alloy [43,86].

Given the importance of this characteristic in the reduction of noise/vibration and its implication in practical problems, such as high resonance response, vibration fatigue and comfort issues on the industrial application of A356 alloys, there seems to still be a lack of data on this matter. Even though, it is also known that there is a general inverse proportionality between static and dynamic (*e.g.* vibration damping) mechanical properties [43,86] this correlation has not been directly studied in this alloy.

From a practical designer point-of-view, this chapter intends to relate the dependence of static and damping properties of investment casted A356 alloys and the ageing heat treatment as a route to tailor specific damping ratios and/or yield strengths. The results are used to determine the processing parameters to be used in the design of products that fulfill the properties imposed by the ISO 3522 and ASTM B686 standards.

2.2 Methods for alloy processing optimization

2.2.1. Al alloy processing

A356 alloy study was received in the form of 12.5 kg ingots with the chemical composition shown in Table 2.4. Melting of 0.3 kg of alloy was carried out in an electrical resistance furnace and maintained in a SiC crucible at 720°C during 30 min for homogenization. Master alloys of grain refiner (0.2%wt – Al₅Ti₁B) and eutectic Si modifier (0.3%wt - Al₁₀Sr) were added and the melt was kept isothermic for 20 min, being stirred every 5 min to prevent particle sedimentation (a total of 5 stirring routines). It is known that the addition of eutectic Si modifier generates an increase in porosity due to the decrease in superficial tension [87,88]. Thus, the regular degassing process was inverted being performed after the addition of the master alloy in the melt. The alloy cooled down being degassed for 5 min at 700 ± 5°C with Argon, until a pouring temperature of 700 ± 3°C was reached. The melt was poured into pre-heated (250 ± 2°C) ceramic blocks with six rectangular openings (25×4×140 mm), corresponding to the primitive

samples to be tested. A total of ten castings were performed to obtain sixty samples for optical microscopy, hardness characterization, vibration analysis and tensile testing.

Table 2.4 Aluminum alloy (A356) chemical composition.

Chemical composition (%wt)								
Si	Mg	Fe	Ti	Cu	Mn	Zn	Res.	Al
7.44	0.30	0.13	0.11	0.07	0.07	0.05	0.12	Bal

From the referred samples, ten were kept as as-cast reference with the objective of determining the properties of the casted alloy. The remaining specimens were subjected to a solution heat treatment (540°C for 480 min) and quenched in mild water, in which ten solution treated samples were kept to determine solution sample properties. Finally, forty solution treated samples were subjected to artificial ageing, being maintained at 160°C for 8, 64, 256 and 512 min. The final artificial ageing time was defined as the peak-age, given that a time of 480–600 [min] is recommended to avoid over-ageing effect on the mechanical properties [65,89]. From the tip of each sample, a 5 mm thick specimen was extracted for microstructural analysis and hardness testing. Samples were afterwards machined to the dimension and shape that is defined for cantilever vibration analysis. Posteriorly, the samples were machined by wire erosion into a dogbone shape for tensile testing.

2.2.2. Microstructural characterization and hardness testing

Optical microscopy (Leica DM2500M) was used to evaluate the microstructural characteristics and morphology of the samples. Image-ProPlus software was employed to quantify the average size (D – Eq. (2.2)) and roundness (R_n – Eq. (2.3)) of the α -Al grain and eutectic Si particles, based on their area (A) and perimeter (P). Measurements were carried out according to ASTM E112 Standard and 50 measurements per optical field (100 $\times$) were performed on 5 different fields. As-cast and solution treated samples were deeply etched and observed by scanning electron microscopy (SEM – FEI Nova 200) to determine the shape/morphology of the eutectic Si.

$$D = 2\sqrt{A/\pi} \quad (2.2)$$

$$R_n = \frac{4\pi A}{P^2} \quad (2.3)$$

Vickers hardness tests were performed using a Shimadzu HMV-2 equipment. The indentations were performed on polished (ground using 1200 μm SiC and polished down to 1 μm) specimens with a diamond square-based pyramid under a load of 1 Kgf for 15 s. The measurements were repeated 10 times from which the mean value and standard deviation were calculated.

2.2.3. Alloy dynamic mechanical properties – Vibration analysis

Machined rectangular samples ($23 \pm 0.05 \times 3 \pm 0.05 \times 140\text{mm}$ – Fig. 2.10(a)) were fixed on a cantilever configuration using a vise, maintaining an overall cantilever length of $120 \pm 0.05\text{ mm}$. An impact hammer (PCB 086) was assembled on a rig composed by a rigid structure that allows rotation, while a pre-tensioned ribbon assures that only one impact ($8.5 \pm 0.15\text{ N}$) is performed (Fig. 2.10(b)). A LMS Scadas Mobile spectral analyzer was used to correlate the input excitation data with the velocities captured by the laser (Polytec OFV-512 Fiber Vibrometer). The corresponding frequency response function (FRF - Fig. 2.10(c)) was calculated and the damping ratios were determined by the half-power bandwidth method from the first eigenfrequency response using Eq. (2.4), where ω_1 and ω_2 are the half-power frequency and ω_R is the resonance frequency.

$$\xi = \frac{\omega_2 - \omega_1}{2\omega_R} \quad (2.4)$$

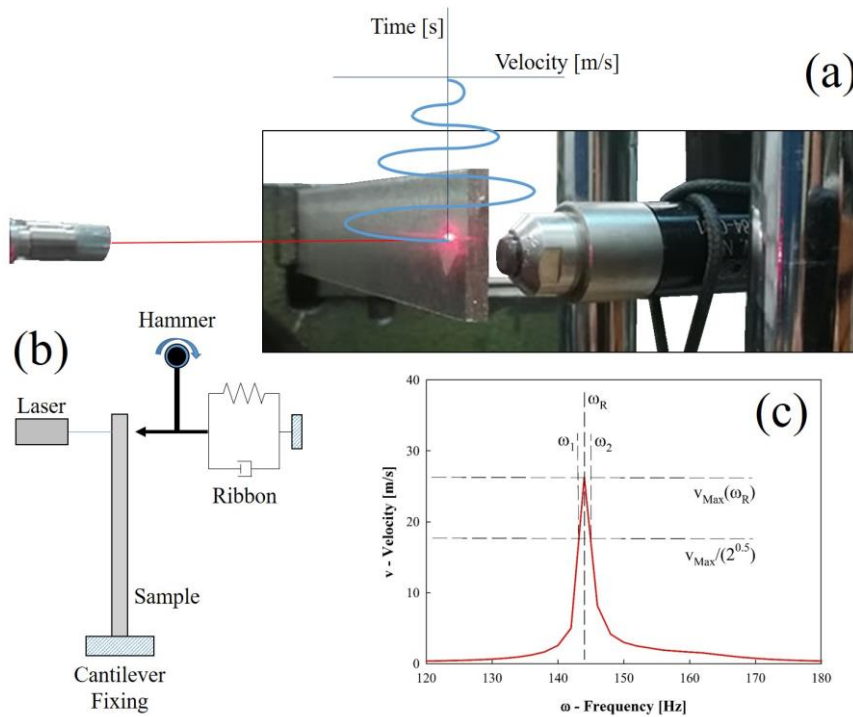


Figure 2.10 Vibration analysis: (a) experimental apparatus; (b) schematic representation and (c) half-power bandwidth method for damping ratio determination.

2.2.4. Alloy static properties – Uniaxial tensile testing

Specimens previously subjected to vibration analysis, had the lateral walls wire eroded to assume a dogbone shape (Fig. 2.11 (a) – Type B1 specimen). Tensile testing was then performed recurring to an INSTRON 8874 universal testing equipment (Fig. 2.11 (b)) according to ISO 6892-1. The samples were tensioned in deformation control with a strain rate of 0.025/min and the test was stopped when an instant load decrease of 60% was monitored, indicating sample fracture (Fig. 2.11 (c)). Instant values of load and strain (recurring to a mechanical strain-gauge – Fig. 2.11 (b)) were monitored to determine the yield strength by the 0.2% permanent strain method.

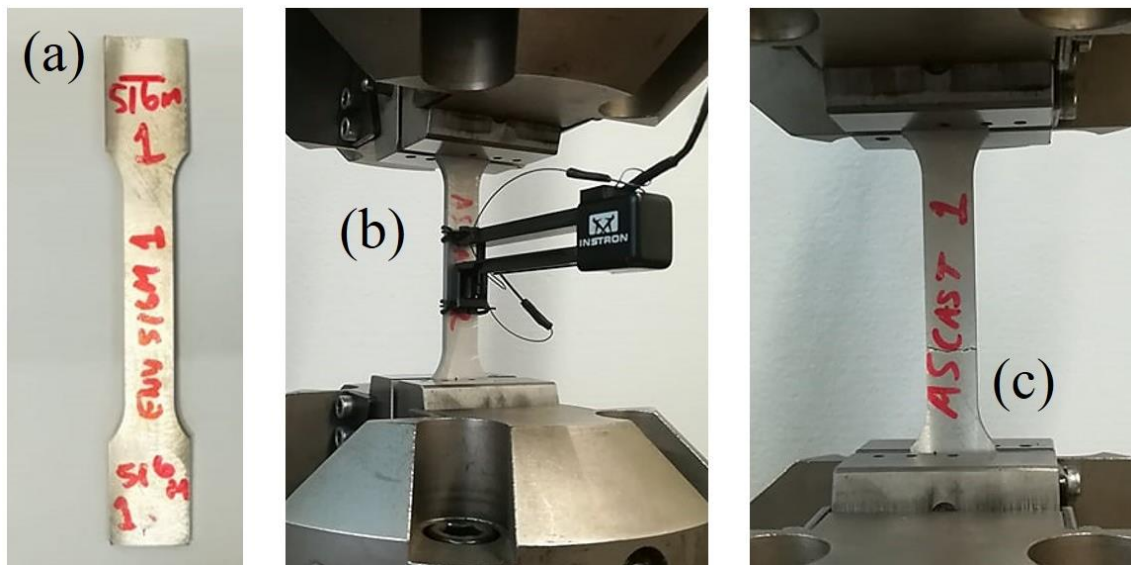


Figure 2.11 Tensile testing: (a) initial; (b) during test and (c) fractured samples.

2.3. Results and Discussion

2.3.1. Alloy processing: Microstructural characterization and hardness

Fig. 2.12 presents the microstructures of the as-cast (Fig. 2.12 (a)), solution treated (Fig. 2.12 (b)) and artificially aged for 8 min (Fig. 2.12 (c)), 64 min (Fig. 2.12 (d)), 256 min (Fig. 2.12 (e)) and 512 min (Fig. 2.12 (f)) samples. The samples in Fig. 2.12 show a characteristic refinement of the α -Al grain resultant from the promoted nucleation by Al₅Ti₁B master alloys.

Furthermore, the results in Fig. 2.13 display an average α -Al grain diameter of 80 μm that, as expected, does not change significantly with the subsequent solution and ageing treatments. According

to these results, although some dendrites may be observed, the high and constant relative roundness of the grains further validate the effectiveness of the performed chemical refinement.

Even though the Si modification may promote eutectic undercooling [90] and increase the probability of hydrogen entrapment (especially for low cooling rates in ceramic block castings), it is apparent that the inversion of Argon degassing is able to contradict this tendency as no significant porosity is observed in all samples.

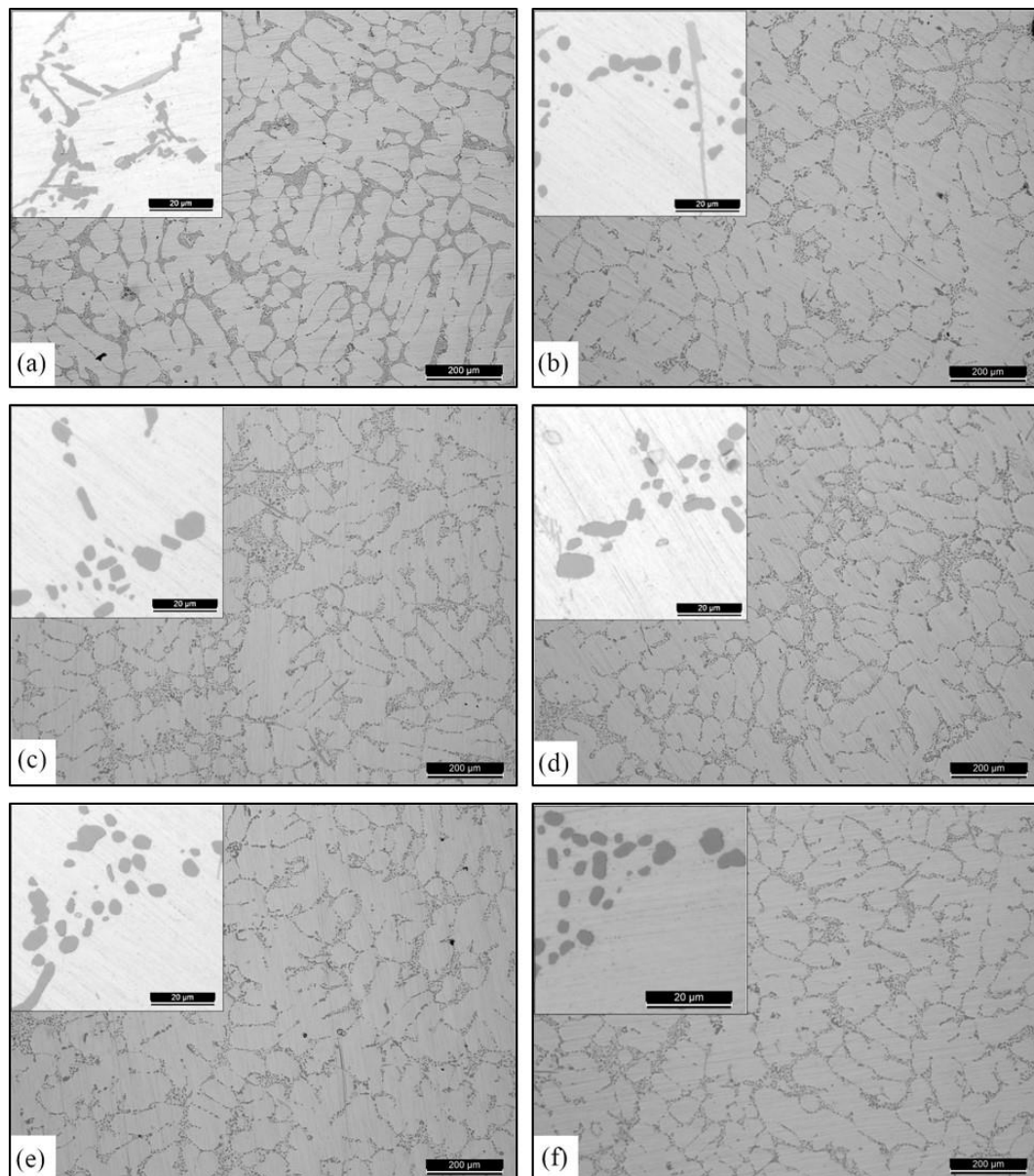


Figure 2.12 Microstructural optical microscopies of the (a) as-cast, (b) solution treated and (c) 8 min, (d) 64 min, (e) 256 min and (f) 512 min artificially aged samples.

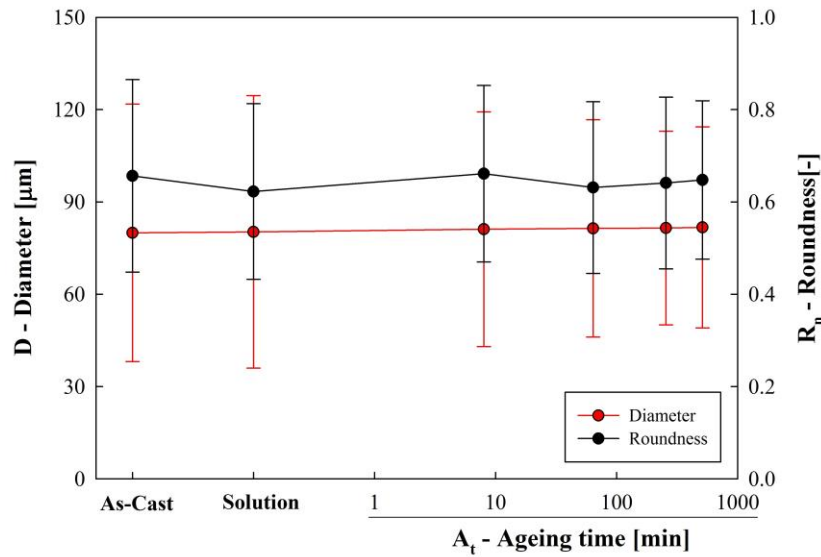


Figure 2.13 Average α -Al grain diameter and roundness for different heat treatments.

Observing Fig. 2.14 (a), it may be stated that the addition of Al₁₀Sr was able to modify the eutectic Si, as it presents a fibrous/coral-shape [73] and no flake-like morphologies may be found. In Fig. 2.14 (b) it may be observed that the solution treatment was able to successfully coarsen and promote the spheroidization of eutectic Si.

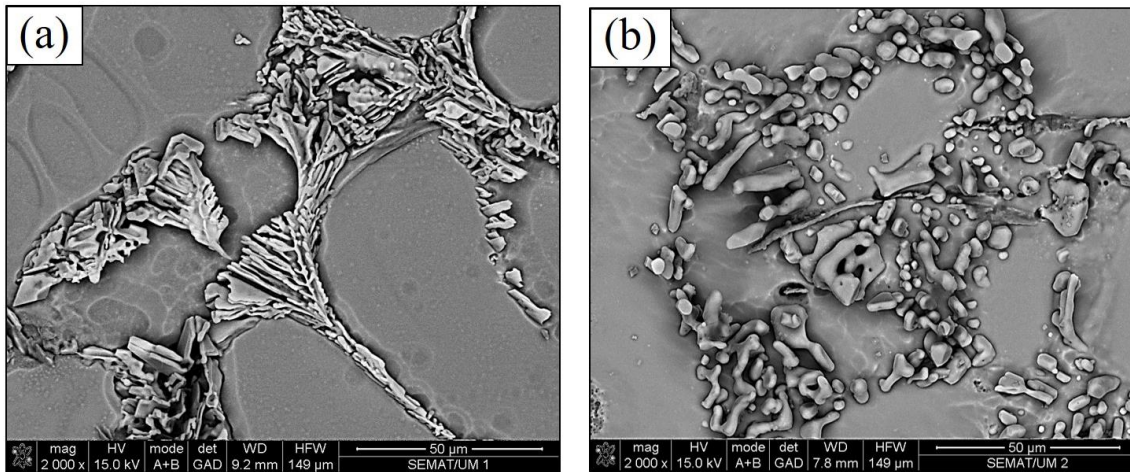


Figure 2.14 SEM of deep-etched with HF (a) as-cast and (b) solution treated samples.

Fig. 2.15 shows the eutectic Si diameter and roundness evolution in as-cast and heat treated samples. As-cast samples (Fig. 2.12 (a)) show smaller modified eutectic Si diameters (2.8 μ m), however, these particles are more angular and show lower roundness (0.4).

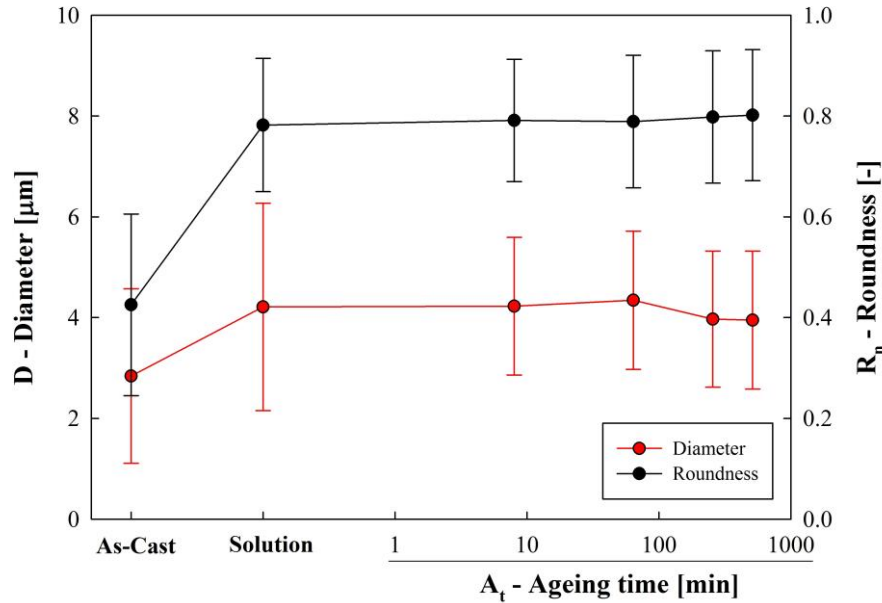


Figure 2.15 Average eutectic Si diameter and roundness for different heat treatments.

It is shown that the solution treatment is able to successfully agglomerate the eutectic Si particles due to Si self-diffusion and inter-diffusion at the Si-Al interface according to an Ostwald mechanism [64]. This promotes an increase in the overall Si diameter (from 3.9 to 4.4 μm) that is maintained during the ageing treatment. Additionally, there is an increase in particle roundness due to thermally activated Si spheroidization (from 0.4 to 0.8) [54,56], in which similar values are reported in [64]. Overall, it may be also observed that the solution treatment time/temperature is appropriate due to the absence of undissolved Mg_2Si in the heat treated samples.

According to Figs. 2.12 and 2.16 it may be observed that the main secondary phases are needle/platelet-shaped β - $\text{Al}_5\text{Fe}_2\text{Si}$ and script-shaped π - $\text{Al}_8\text{FeMg}_3\text{Si}_6$. The precipitation of the latter is further promoted by the low solid solubility of Fe [60] in as-cast samples and the presence of Mg. Although the subsequent heat treatments do not significantly change the basic Fe intermetallic compounds [60], the solution treatment is able to promote a π - β transformation [91,92] (Fig. 2.16). This transformation and the dissolution of Mg_2Si (Fig. 2.16) generates a migration of Mg and Si atoms into the α -Al. Such process allows the solution hardening that is observed in the experimental solution treated results from Fig. 2.17.

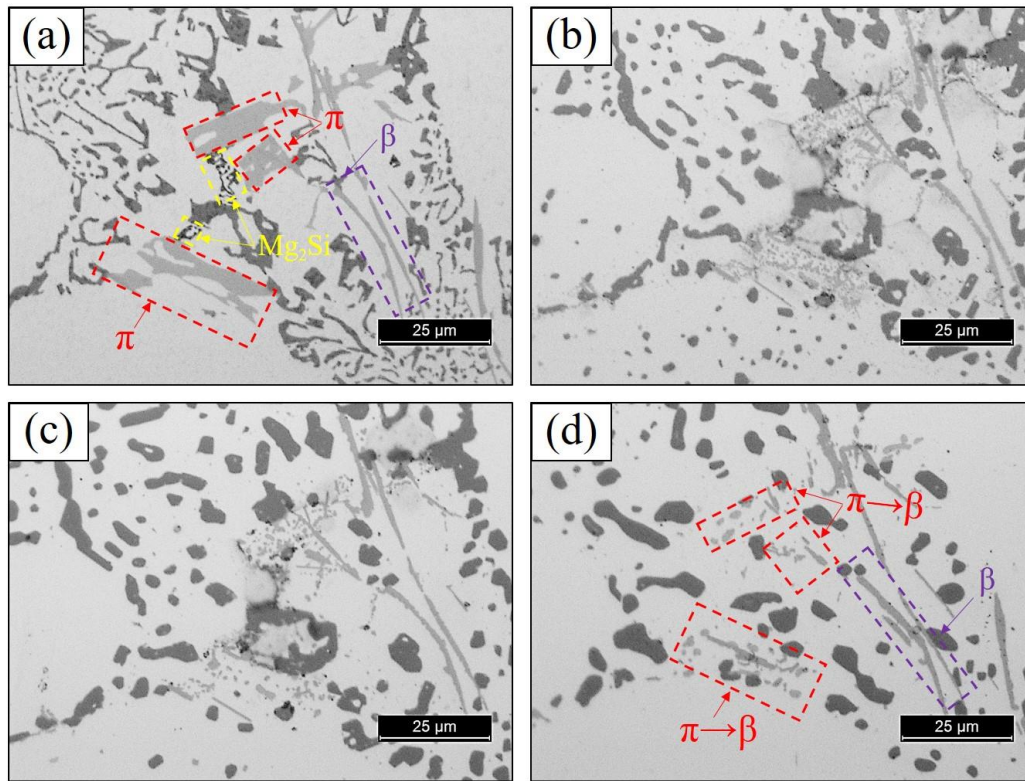


Figure 2.16 Details of π - β and Mg_2Si transformation during solution treatment: (a) as-cast, (b) 2-h, (c) 4-h, and (d) 8-h solution at 540 °C.

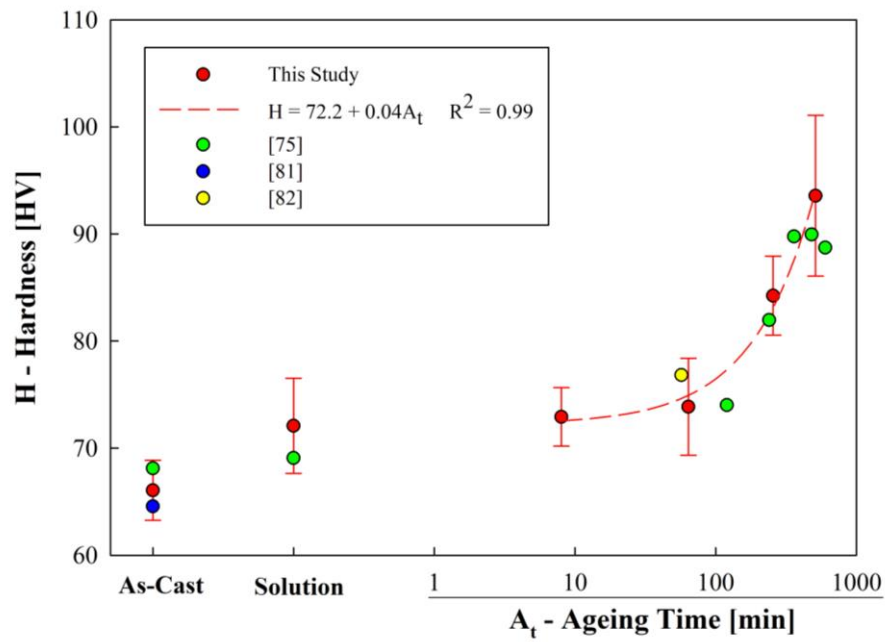
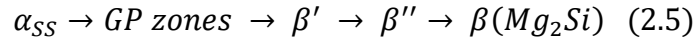


Figure 2.17 Hardness of the samples for different heat treatments [75,81,82].

During the ageing treatment, the supersaturated α -Al (α_{ss}) has its solute Mg and Si atoms clustered and precipitated (Eq. (2.5)) in Guinier- Preston (GP – $Mg_2Si_3Al_6$) zones along the $\langle 100 \rangle$ direction. These primordial precipitates decompose into nano-sized coherent Widmanstätten needle-shaped β'' (Mg_5Si_6) and semi-coherent rod-like β' ($Mg_{1.8}Si$) metastable strengthening phases [89]. Such precipitates are nucleated in the $\langle 100 \rangle$ direction, preferably in dislocation locations. Finally, a dissolved rod/plate-shape equilibrium β (Mg_2Si) FCC phase is precipitated due to minor atomic rearrangement in the matrix [70].



The initial stages of artificial ageing are characterized by low hardness values (Fig 2.17). However, according to Fig. 2.18 and due to the clustering of solute atoms in GP zones and the precipitation of β'' , the overall initial nucleation and growth of shearable precipitates promotes an elevation in hardness [67]. When peak-age is reached, the nucleation and growth of β'' is superseded by the precipitation and coarsening of non-shearable β' and β (Mg_2Si) [93] (Fig.2.18). The dislocation mechanism is changed to an Orowan mechanism and further ageing will imply a reduction in hardness.

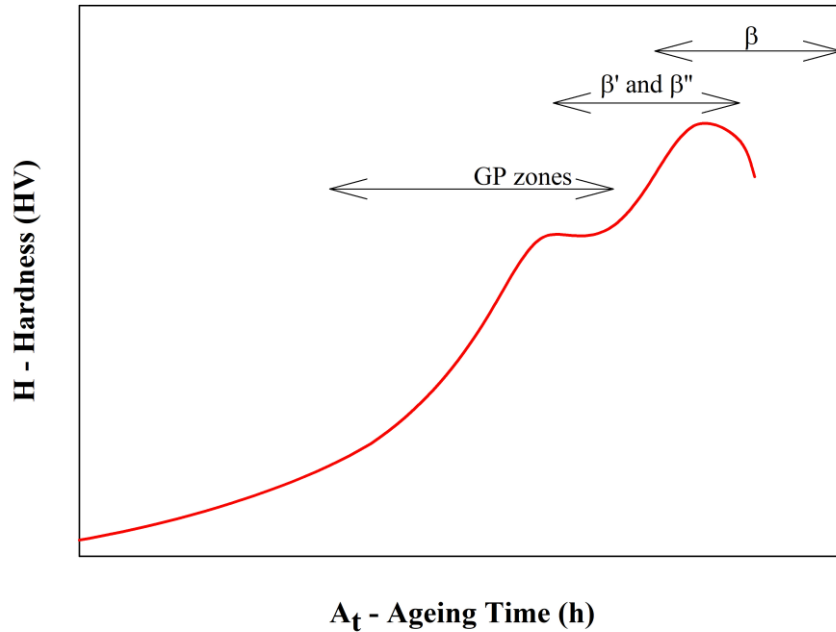


Figure 2.18 Generic progression of hardness and precipitation during artificial ageing.

The increasing hardness in Fig. 2.17 suggests that the methodology was able to avoid a dislocation mechanism transition. Experimental results propose that before dislocation transition (*i.e.* Peak Ageing), the relation between hardness (H) and ageing time (A_t) follows the linear regression (Fig. 2.17) described in Eq. (2.6). Additionally, it may be observed that the experimental hardness values are highly relatable with other studies [75,81,82], suggesting a successful processing of the samples.

$$H = 72.2 + 0.04A_t \quad (2.6)$$

2.3.2. Dynamic mechanical properties: Vibration analysis

Fig.2.19 shows an example of the resulting FRF plots for the different test conditions. It may be observed that the performed heat-treatment is able to successfully change the overall stiffness and damping of the samples. These results, particularly the resonance peaks, were used to estimate the damping ratio of the samples by the half-power bandwidth method.

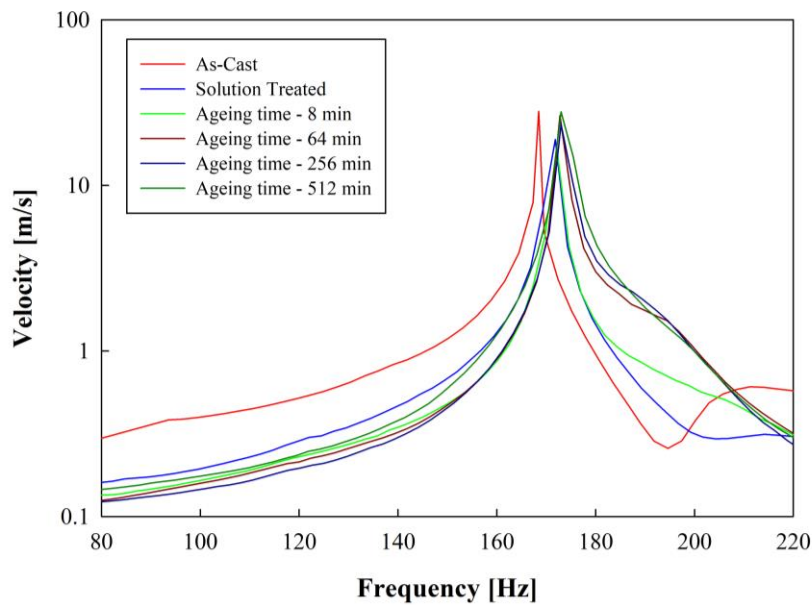


Figure 2.19 Example of FRF curves obtained for the tested specimens.

Fig. 2.20 shows the damping ratios (ζ) that were determined during the vibration testing. It is apparent that the solution treatment and the initial phase of artificial ageing are able to enhance the damping capacity of the A356 alloy relatively to its as-cast state. However, as the ageing treatment progresses, there is a decrease in damping capacity. The experimental results suggest that there is a

linear function that describes the lowering of the damping ratio (ζ) as the ageing time (A_t) progresses, according to Eq. (2.7).

$$\zeta = 0.0042 - 3 \times 10^{-6} A_t \quad (2.7)$$

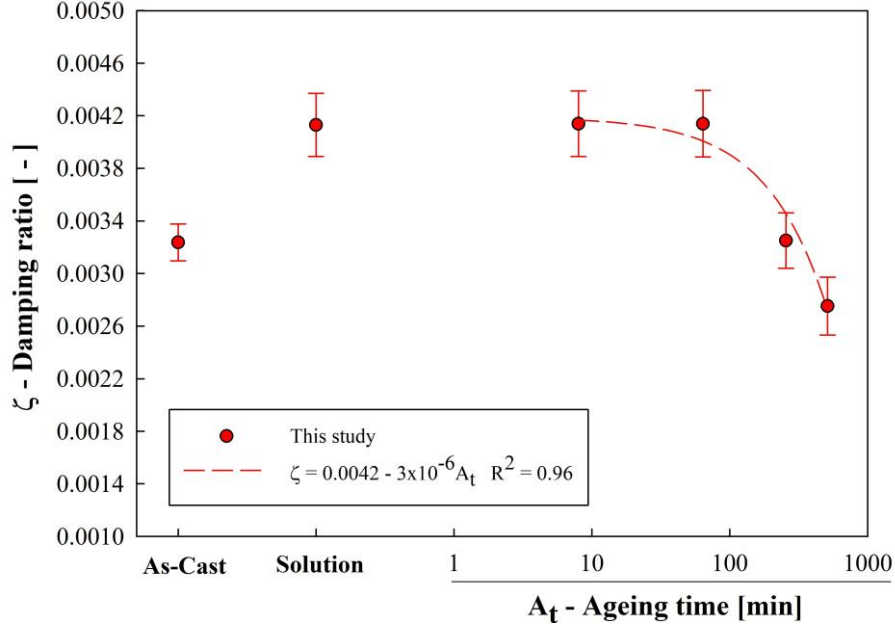


Figure 2.20 Damping ratios for different heat treatments.

Damping is the ability of a material to dissipate energy under cyclic loading, being closely related with the microstructural internal friction and changed by the material processing. The damping mechanisms are known to be originated by thermoelastic, magnetic, viscous and defect effects. However, due to the presented particular processing characteristics and isothermal (room temperature) testing, only the latter is relevant as an internal friction mechanism.

The defect damping is closely related to point (vacancies), line (dislocations), surface (grain boundaries) and bulk (micropores and inclusions) defects [43,94]. Nevertheless, it is known that the internal friction in this particular alloy is regulated by a static hysteretic damping [26] due to dislocation and grain boundary damping [43,94]. Hence, it is suggested that the particular origin of the predominant damping mechanisms are related with the dislocation and pinning in (Figs. 2.21 and 2.22): (i) grain boundaries between α -Al/ α -Al, α -Al/eutectic Si, α -Al/ β -Al₅Fe₂Si and α -Al/ π -Al₈FeMg₃Si₆ pairs; (ii) interactions between dissolved Mg, Si and α -Al in pre-aged states (solution hardening) and α -Al/{ β'' , β' , β (Mg₂Si)} in aged states (precipitation hardening).

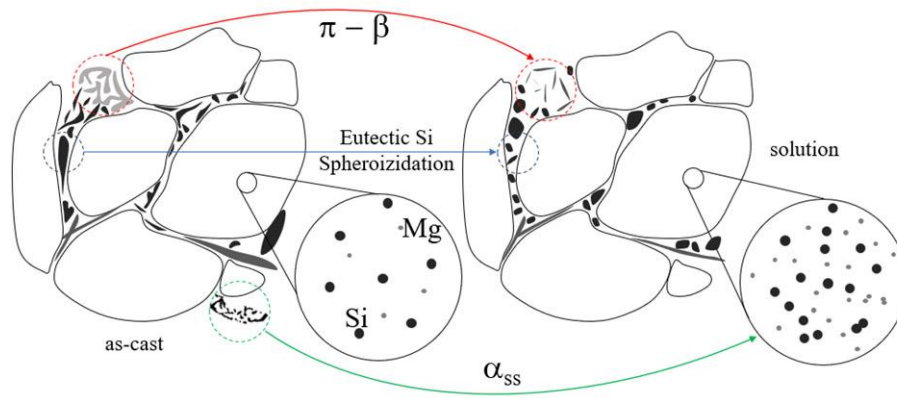


Figure 2.21 Damping changes due to microstructural transformation during solution treatment.

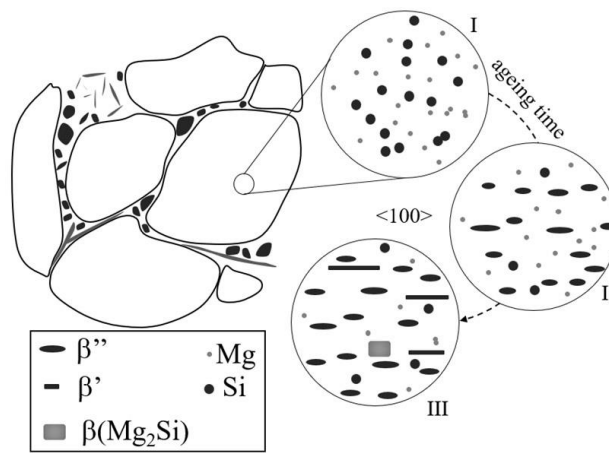


Figure 2.22 Damping changes due to microstructural transformation during ageing treatment.

Fig. 2.21 represents a schematic evolution of the microstructure during solution treatment. Initially, the fundamental constituents of the microstructure are α -Al grains (with small quantities of dissolved Mg and Si), eutectic Si and secondary phases (needle-shaped β -Al₅Fe₂Si, Mg₂Si and script-shaped π -Al₈FeMg₃Si₆ – Figs. 2.12 and 2.16).

During the solution treatment, the eutectic Si is transformed to a spheroidized morphology, migrating and changing the interface balance. It is known that Si migration from the dendrite arms results in the formation of particle-free zones [89] and changes the eutectic Si shape/size. This implies a decrease in the grain boundary pinning effect and an elevation in internal friction due to boundary sliding [86,94]. Schoek [95,96] showed that a spheroid inclusion with a radius (a) in a matrix of volume (V) with a determined Poisson's ratio (ν), generates an inverse quality factor (Q^{-1}) by interface relaxation and anelastic strain when a shear stress (p_{13}) in its respective component ($\bar{p}_{13}$) is applied (Eq. 2.8). Thus, it is suggested that an overall increase in eutectic Si diameter (Fig. 2.15) enhances the interface damping.

$$Q^{-1} = \frac{1}{p_{13}^2} \frac{8(1-\nu)}{3\pi(2-\nu)} \frac{1}{V} \sum a_1^3 (\overline{p_{13}^2})_i \quad (2.8)$$

During solution, however, there is also dissolution of Mg_2Si in the form of Mg and Si elements in the $\alpha\text{-Al}$. Additionally, there is further migration of Mg atoms due to the dissolution of the $\pi\text{-Al}_8\text{FeMg}_3\text{Si}_6$ phase into fine needle shape $\beta\text{-Al}_5\text{Fe}_2\text{Si}$ [91,97] (Fig. 2.16).

This supersaturation of $\alpha\text{-Al}$ enhances pinning and reduces the overall dislocations by solution hardening [93], as it may be observed by the hardness increase in Fig. 2.17. However, due to the scattered dispersions of the atoms, there is no significant reduction in the overall solution treated system damping. Consequently, it may be determined that after solution treatment the maximum value of damping is reached and the most relevant damping mechanism is proportioned by the interface interaction.

Fig. 2.22 represents the overall microstructural transformation in A356 alloys during ageing treatment. As the intermetallic compounds, eutectic Si and grain boundaries remain stable due to the relatively low artificial ageing temperatures, it is suggested that the damping is changed due to precipitation hardening. Alloy hardening until peak-age promotes the inhibition of dislocations, being this shown by the increase of hardness values in Fig. 2.17.

Phase I in Fig. 2.22 represents an initial stage of ageing treatment, where the overall energy levels are not enough to cluster Mg and Si atoms and form GP zones. Studies suggest that GP zones are initially formed after 40 min of ageing treatment [98]. This hypothesis is in accordance with the experimental values of hardness (Fig. 2.17) and damping ratio (Fig. 2.20) for low ageing times. As the ageing treatment progresses, Si and Mg atoms cluster into GP zones, initiating phase II (Fig. 2.22), in which the metastable coherent β'' nucleate and grow in the dislocation lines with a $\langle 100 \rangle$ direction. There is an increase in pinning points and a reduction of the overall distance between them, being the dislocation mechanism governed by shearing [99].

According to the Granato-Lücke theory [100], the strain independent internal friction (δ_0) is proportional to the dislocation density (ρ) and the mean length between weak pinning points (L_d) (Eq. (2.9)) [101]. The independent strain amplitude approach is applied in this study due to low stresses ($\sigma_{\text{Max}} \approx 30$ MPa, by classic Timoshenko beam theory) generated by the hammer strike.

$$\delta_0 \sim \rho L_d^4 \quad (2.9)$$

Therefore, it is suggested that the precipitation of β'' metastable phases is able to lower the damping by the obstruction of dislocations. This hypothesis is supported by the progressive hardness increase during the ageing treatment (Fig. 2.17). As the nucleation and growth of metastable β'' increases its volume fraction, the precipitation hardness tends to peak-age [68]. However, some semi-coherent β' and equilibrium $\beta(\text{Mg}_2\text{Si})$ may simultaneously precipitate from β'' (phase III - Fig. 2.22). The nucleation processes between metastable precipitates are not exclusive and, thus, there is concurrent growth between the meta- and stable-phases [69]. Due to this effect the damping values tend to a minimum, being lower than as-cast samples (Fig. 2.20).

2.3.3. Static mechanical properties: Tensile testing

Fig.2.23 displays an example of stress-strain plots for the tested samples. From the average slopes in the elastic region, it was possible to determine the Young's modulus of the samples as the treatment progresses (Fig.2.24). It is shown that the increase of stiffness is able to be correlated with the shifting in resonance frequency (Fig.2.19).

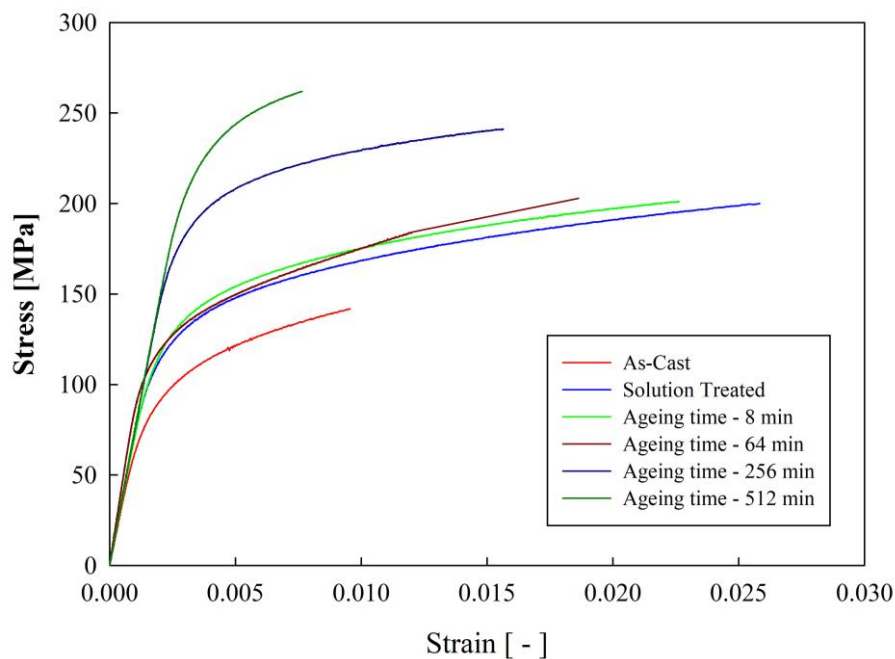


Figure 2.23 Example of tensile stress-strain plots for the different test conditions.

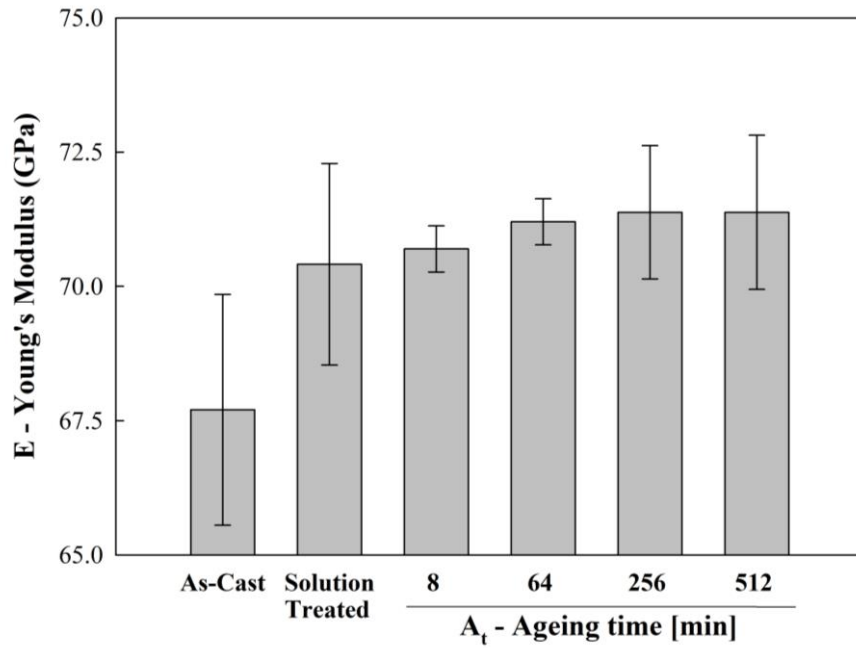


Figure 2.24 Average Young's modulus for the different test conditions.

Fig. 2.25 shows the yield strength (σ_y) of as-cast, solution treated and different artificial aged samples. It may be observed that the solution treatment was able to elevate the alloy yield strength, while the artificial ageing treatment has a very pronounced effect in the enhancement of this variable.

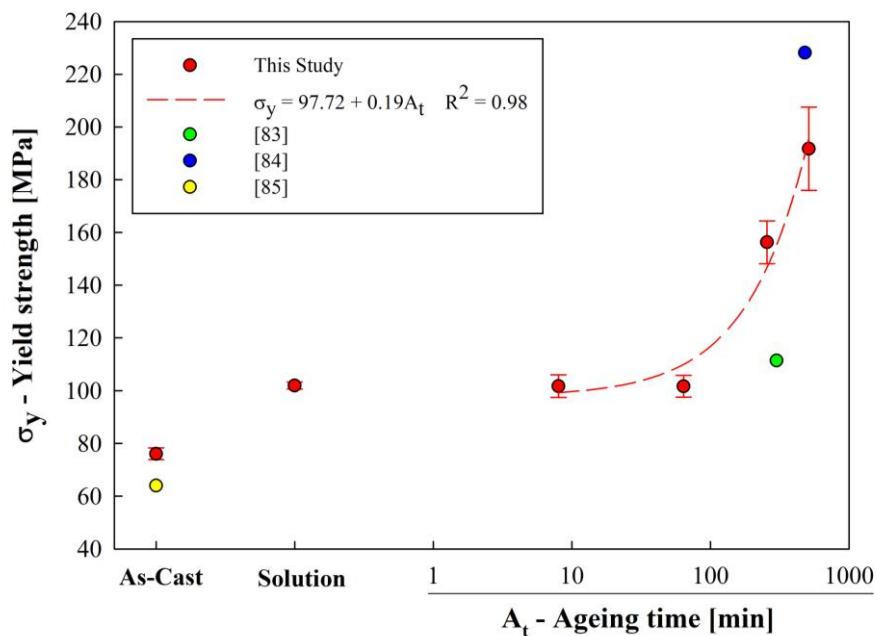


Figure 2.25 Yield strength for different heat treatments [83–85].

According to the experimental results, it is suggested that the dependence between yield strength and ageing time for A356 alloy poured in ceramic block follows the linear function described by Eq. 2.10. Additionally, it may be noticed that this data is fairly correlated and follows the tendency of other published experimental results on the same alloy [83–85].

$$\sigma_Y = 97.72 + 0.19A_t \quad (2.10)$$

The yield strength (σ_Y) of Al-Si-Mg (*e.g.* A356) alloys may be expressed as an addition of solid solution (σ_{ss}) and precipitation (σ_{PPT}) strengthening components to an initial pure aluminum strength (σ_i), as shown in Eq. (2.11) [102].

$$\sigma_Y = \sigma_i + \sigma_{ss} + \sigma_{PPT} \quad (2.11)$$

The solution strengthening term (σ_{ss}) is dependent on the alloy intrinsic solid solution strength (σ_{0ss}), proportion of solute released (α) and relative volume fraction of precipitates (f_r) (Eq. 2.12) [102]. The σ_{ss} term includes the influence of eutectic Si and intermetallic compounds, thus, the overall solution strengthening (σ_{ss}) is increased by solution treatment since: (i) Si spheroidization increases yield strength [56,64]; (ii) $\pi\beta$ transformation is beneficial to reduce microstructural stress concentration [92]; and (iii) precipitation is residual ($f_r \sim 0$) during solution treatment [93]. This increase in yield strength by solution strengthening is observed in the experimental results of Fig. 2.25.

$$\sigma_{ss} = \sigma_{0ss}(1 - \alpha f_r)^{\frac{2}{3}} \quad (2.12)$$

According to the same relation (Eq. (2.12)), due to the nucleation and growth of precipitates during artificial ageing and no variation in σ_{0ss} , the solution strengthening may decrease. However, this effect has a relatively small role when compared with the increase in aged samples precipitation strengthening [102] as shown by the experimental results. Precipitation strengthening (σ_{PPT} – Eq. (2.13)) before peak-age is fundamentally dependent on the volume fraction of precipitates (f_p) and mean precipitate radius (r), for r smaller than the transition radius (r_{trans}). The constant c_1 is related to the Taylor factor, Burgers vector, precipitate shear modulus and interfacial energy [102]. However, according to the relation that is represented in Fig.2.26, when the precipitate reach the transition radius (r_{trans}), the overall dislocation is changed into a bypassing (Orowan) mechanism. The precipitation is then be estimated by Eq.2.14.

$$\sigma_{PPT} = c_1 f_p^{\frac{1}{2}} \langle r \rangle^{\frac{1}{2}} \rightarrow \langle r \rangle < r_{trans} \quad (2.13)$$

$$\sigma_{PPT} = \frac{f r^{\frac{1}{2}}}{r} \rightarrow \langle r \rangle > r_{trans} \quad (2.14)$$

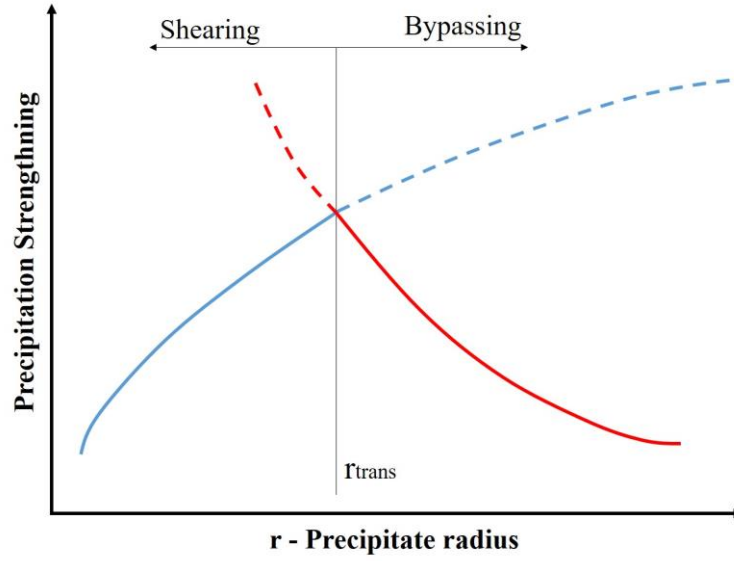


Figure 2.26 Shearing vs Bypassing competing strengthening mechanism.

As the ageing time progresses, GP zones ($r < 2$ nm [67,102]) followed by β'' ($2 < r < 7$ nm [102]) precipitates, are nucleated and grown (Fig. 2.22 – Phase I to II). This increases the mean precipitate radius (r) and the overall yield strength (σ_y) (Eq. (2.25)). Some β' ($7 < r < 15$ nm [66,102]) and β (Mg_2Si) ($15 < r < 500$ nm [102,103]) with r larger than r_{trans} may also nucleate (*i.e.* lowering σ_{PPT} by Orowan dislocation – phase II in Fig. 2.22). However, their presence is residual before peak-age and does not contribute significantly to the mean radius ($\langle r \rangle$) value. Overall, the experimental results (Fig. 2.25) show the described transformations and, consequently, an enhancement of yield strength by precipitation strengthening.

2.4. Tailoring of static and dynamic mechanical properties by artificial ageing

The microstructural changes in A356 alloys during ageing treatment (Section 2.3.1) occur in an inverse proportionality between the experimental damping ratio (ζ - Section 2.3.2) and yield strength (σ_y - Section 2.3.3). These variables are correlated in Fig. 2.27, showing that under-aged samples (8–64 min ageing times) present similar damping ratios and yield strengths, meaning that the precipitation is residual. However, as the ageing time increases, the damping ratios tend to decrease as the yield strength increases on a linear function relation. It is suggested, from a mechanical design point-of-view, that these

variables may be dependently tailored by ageing treatment, employing the relation established in Eq. (2.15).

$$\sigma_y = 367.2 - 64.2 \times 10^3 \zeta \quad (2.15)$$

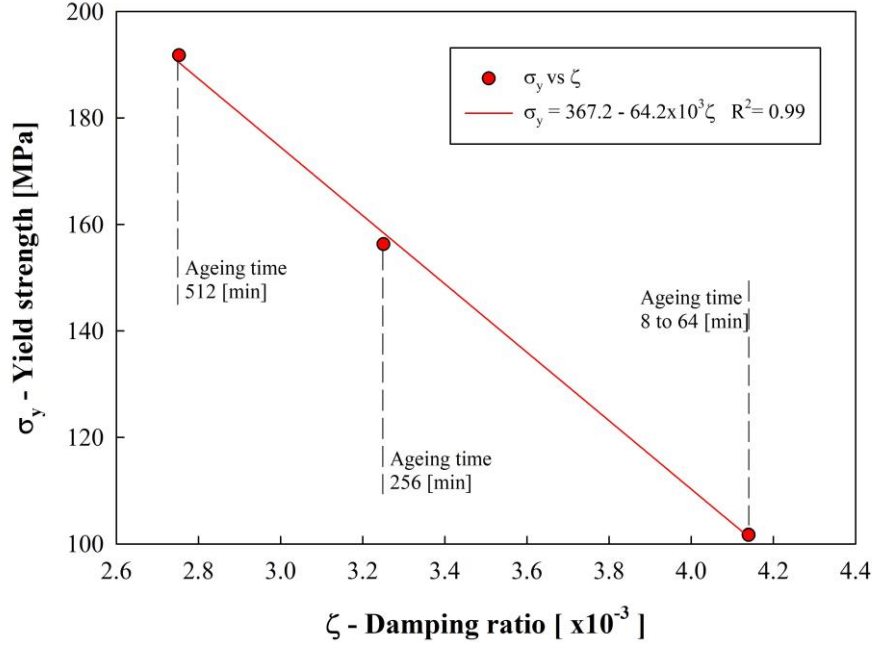


Figure 2.27 Proposed model for yield strength and damping ratio dependence.

Based on these results, an optimal 500 min ageing time was determined to enhance the damping ratio without compromising the static properties imposed by ISO 3522 and ASTM B686 standards. Given the inverse proportionality between the yield strength and damping ratio, it is defined that the selected ageing time is able to a 193 MPa yield strength, imposed as the standard minimum value, and a damping ratio of approximately 0.0027.

2.5. Conclusions on material and process selection

This chapter explores the influence of the processing in the overall microstructural morphology and its transformation in the damping ratio and yield strength of investment casted A356 alloy. This alloy processing is designed to display a compromise between static and damping properties, when applied to the proposed cellular solids. According to the results, a dependence between yield strength (static behavior) and damping (dynamic behavior) is analyzed, and the following conclusions are drawn:

(i) Materials and process selection were performed to determine the appropriate base material and manufacturing technique that are going to be employed to produce the proposed cellular solids. It is shown that an investment casting using an A356 alloy is the most suitable process-material combination for such purpose;

(ii) Solution treatment is able to enhance both yield strength and damping ratio. Damping is raised to a maximum value due to the increase in grain boundary dislocation promoted by eutectic Si spheroidization. The increase in yield strength is stimulated by eutectic Si spheroidization/agglomeration, lowering of stress concentration by $\pi\beta$ intermetallic transformation and solution strengthening due to Mg and Si dissolution in the α -Al;

(iii) Artificial ageing generates a decrease in damping, however, it is able to significantly elevate the yield strength until peak-age. Precipitation hardening, by the nucleation of GP zones and later by the precipitation and increase in volume fraction of β'' , reduces the lengths between pinning points and dislocations as ageing time progresses. This dislocation reduction is able to successfully elevate the yield strength;

(iv) During artificial ageing, the increase in yield strength is inversely proportional to the decrease in the damping ratio. It is suggested that this relation follows a linear tendency. The formulated function is a useful tool in the structural/mechanical design when both static and dynamic characteristics must be evaluated;

(v) An artificial ageing time of 500 min was defined as the most suitable compromise to obtain static properties (193 MPa) that are able to fulfill ISO 3522 and ASTM B686 and maximize the damping ratio (0.0027).

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Chapter 3 – Design and Manufacturing of Cellular Solids

“Well, I think the curves of the four pillars of the monument, as the calculations have provided them ...
Give it a great sense of force and beauty”

Gustave Eiffel in interview to the newspaper *Le Temps*, 1887.

3.1 Introduction to the design and manufacturing of cellular solids

Cellular structures, as a form of composite materials, are a classic route to obtain high values of static and dynamic specific mechanical properties [1,2]. This characteristic has always been advantageous in many fields and applications, from diverse areas such as packaging, the transportation industry (*e.g.*, railway [3–5], aeronautic [6–8], etc), and medical implants [9–11]). This class of materials, when using a metallic matrix, are classically manufactured by foam blowing agents/gas injection [12–17] and casting with space holders with leachable [18–22] or non-leachable [23–25] particles. Other techniques may also be used such as powder metallurgy [26,27] or wire weaving [28–30]. These techniques are able to be produced in an array of specific densities in both open-cell [26,28,31,32] and closed-cell [15,33,34] configurations, both being commonly found in stochastic nature.

Recent studies have explored additive manufacturing (*e.g.*, selective metal laser sintering or electron beam melting) [35–39] and metal casting [40,41] to produce non-stochastic cellular materials. These have the advantage of allowing *in situ* geometric and dimensional control, thus, they are able to display tailored properties [42]. However, the referred processes are known to have inherent problems: the difficult microstructural control in additively manufactured metals [43–45] and the casting [46] of thin-ribbed cellular materials. Vacuum assisted high pressure die [47–49] and low pressure casting

[50,51] are able to produce thin-walled and/or high detailed structures, however, they are not suited for the manufacturing of intricate three-dimensional complex geometries.

This chapter shows an additive manufacturing assisted investment casting technique to manufacture different designs of cellular solids with diverse Poisson's ratios. Although there are already studies that use similar techniques (*e.g.*, 3D printing of sand molds [52–56], wax and PLA/ABS [57,58]), the proposed study recurs to vacuum as an auxiliary technique for the filling of thin-rib and high aspect ratio complex three-dimensional metallic scaffolds. Additionally, vacuum is also used to promote the sanity of the resultant samples, as it is known to reduce oxide inclusion [59,60] and porosity [61,62].

The manufactured samples are also inspected in terms of final dimensions by their comparison to the designed CAD models. For such purpose, the final samples are subjected to X-ray microcomputed tomography (μ -CT). This technique is known as a non-destructive technique that allows a three-dimensional scanning of these structures, being able to execute metrological and defect characterizations. Such technique has successfully been used in to characterize cellular solids in similar cellular solids [63–67]. In this study, μ -CT is employed to characterize the true rib and sample dimensions and defects.

3.2 Methodology for the design and manufacturing of cellular solids

3.2.1. Cellular solid design

Non-stochastic cellular solids may be represented by an assembly of elementary cellular units (Fig. 3.1 (a)). Such units may be assembled on a single planar two-dimensional matrix, forming a lattice (Fig. 3.1 (b)). Otherwise, they can also assume a three-dimensional matrix configuration, when elements are added to a third cartesian direction (Fig. 3.1 (c)).

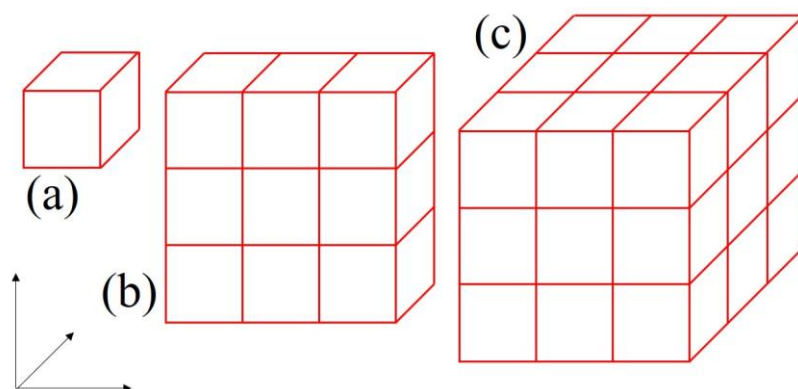


Figure 3.1 Non-stochastic cellular solids: (a) unitary cell - unidimensional (m); (b) two-dimensional ($m \times n$); and (c) three-dimensional ($l \times m \times n$).

Auxetic re-entrant cells were reported in Chapter 1 as having a beneficial increase in static and dynamic mechanical properties. Considering that such configuration facilitates the casting process relatively to other auxetic geometries (*e.g.* 3D Chiral, Rotating Geometries, etc), this was the selected model to perform the study. However, the minimum amount of cells (*i.e.* $l \times m \times n$) that must be used to eliminate any cell size/distribution effect is still a matter of discussion in the field [68].

Fig. 3.2 (a) shows the frequency of cell per row in published works (compilation of data [69–74,68,75–116]) on re-entrant auxetics. It may be observed that 2D cells have been used with a higher number of cells per row. When 3D cells are used, it is not common to use more than ten cells per row.

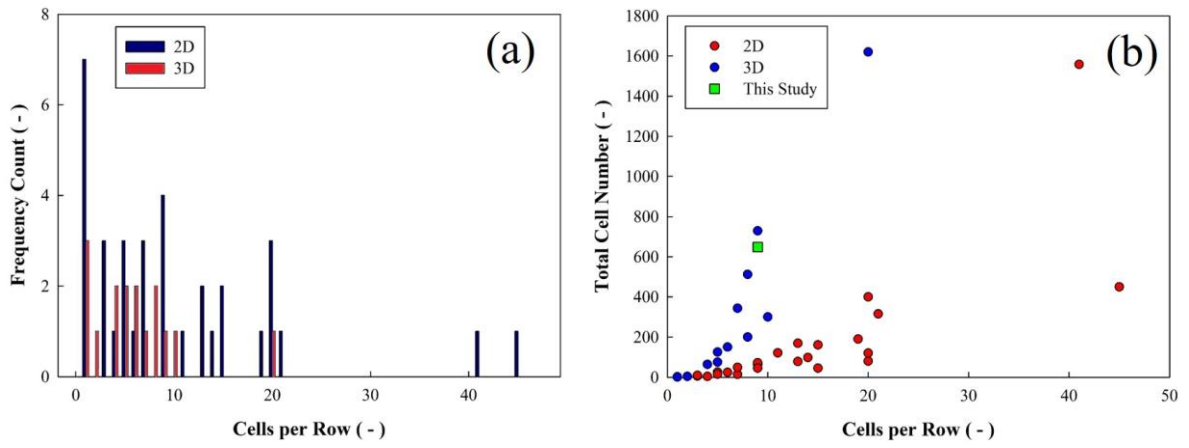


Figure 3.2 Auxetic lattice cell counting (compilation of data on [69–74,68,75–116]): (a) frequency count; and (b) total cell number.

However, since these cells are assembled in a three-dimensional matrix, they may display higher total cell number (Fig. 3.2 (b)). It may be observed that most published studies use less than 400 cells, either in 2D or 3D configurations, according to Fig. 3.2 (b). This may be attributed to the considerable difficulty in the manufacturing and numerical simulation of cellular solids with high cell numbers.

A lattice geometry was designed with 1D struts and defined as an array of repeating unit cells. Individual cells (Fig. 3.3 (a)) are defined by two horizontal and four vertical ribs with a square cross-section. The rib horizontal and vertical lengths, and thickness (h , l and t respectively) were varied to optimize the manufacturing process, while the vertical rib angle (α) was varied to produce cells with different Poisson's ratios (ν).

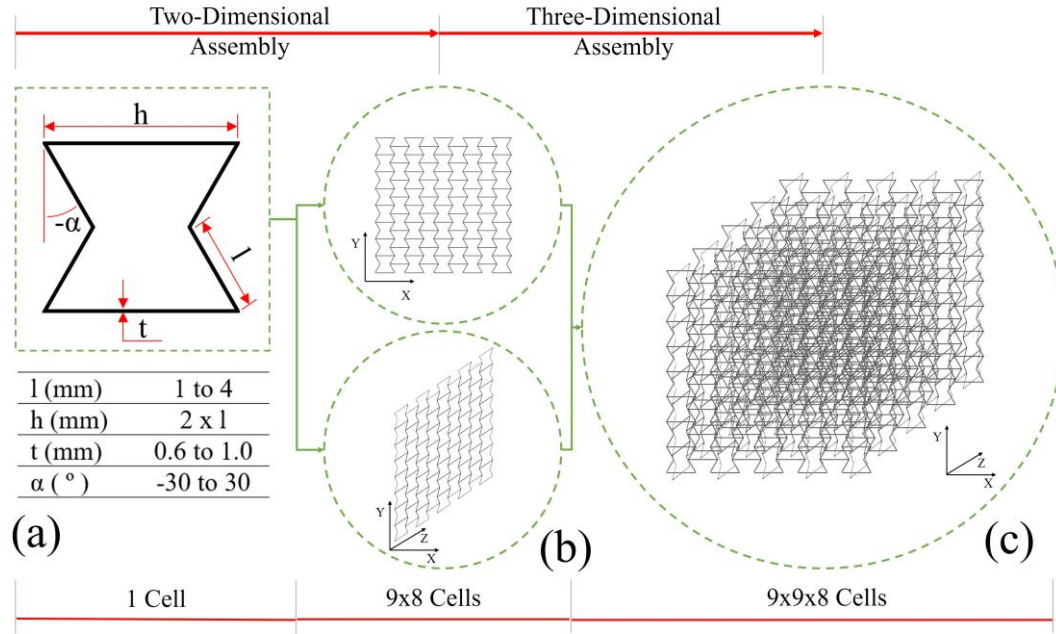


Figure 3.3 Lattice design process: (a) unitary cell; (b) two- and (c) three-dimensional lattice.

Initially, the individual cells may be assembled into a two-dimensional lattice (Fig. 3.3 (b)), combining their common ribs. These lattices are repeated in the three Cartesian axis to produce a three-dimensional (Fig. 3.3 (c)) lattice matrix. To guarantee that size effects do not interfere with the mechanical properties of the samples, they were designed using a basic three-dimensional unitary cell assembly, in which the cells are assembled in a matrix (9x9x8) configuration. This assures that they comply with the cells per row and a higher total cell number that are used in other publications (Fig. 3.2). Sample CADs may be observed in Fig. 3.4. Skins (*i.e.* plates) on the top and bottom faces enclose the cellular core.

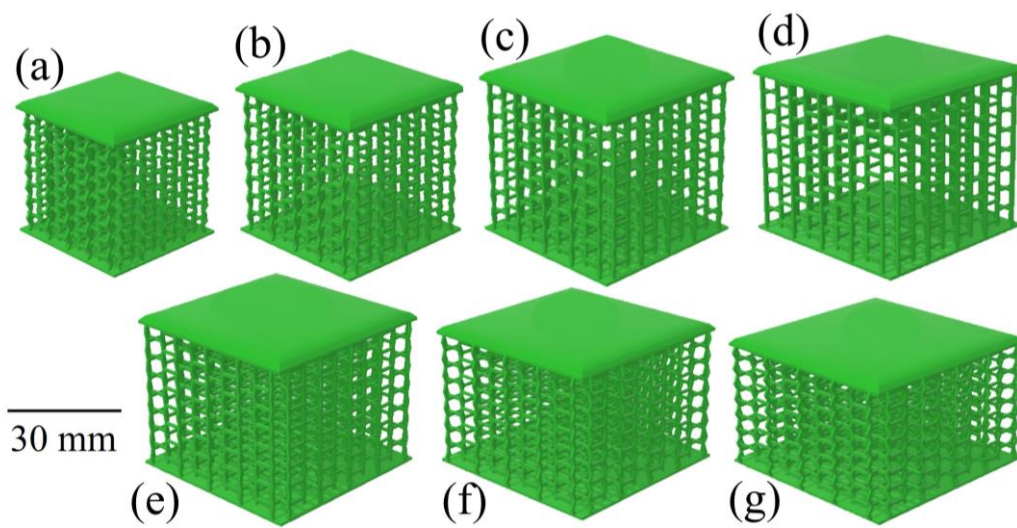


Figure 3.4 CAD models: (a) $\alpha=-30^\circ$, (b) $\alpha=-20^\circ$, (c) $\alpha=-10^\circ$, (d) $\alpha=0^\circ$, (e) $\alpha=10^\circ$, (f) $\alpha=20^\circ$ and (g) $\alpha=30^\circ$ samples.

3.2.2. Model 3D-printing

The lattice investment models were 3D-printed in PLA (Fig. 3.5) by fused filament fabrication. A BCN3D Sigma printer was used to produce these prototypes that were subsequently used to produce the ceramic casting mold. The design of the samples (*e.g.* rib angles, span and orientation) allows 3d-printing without requiring support materials, which benefits the simplicity and speed of the process.

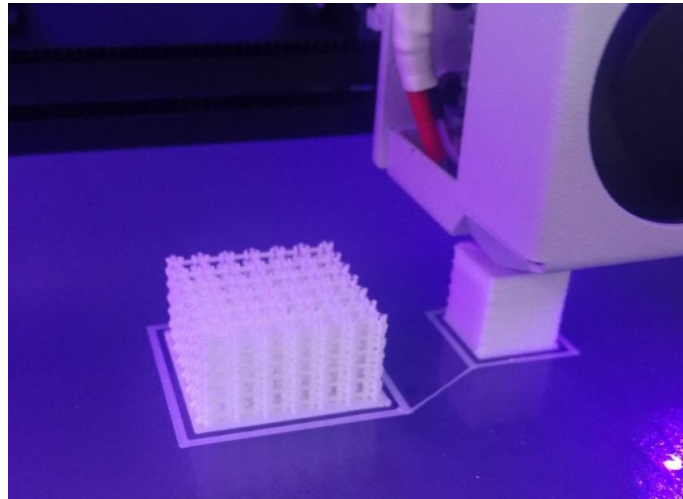


Figure 3.5 3d-printing of investment PLA model.

Printing parameters are shown in Table 3.1. The extrusion/bed temperatures (T) and printing speed (P_s) were selected in accordance with equipment manufacturer guidelines and kept constant. Nozzle diameters (N) and layer heights (L_h) were optimized to provide a successful print (*i.e.* all details are printed) whilst minimizing printing time.

Table 3.1 3d-printing (fused filament fabrication) parameters.

Material		PLA
N - Nozzle diameter (mm)		0.3; 0.4; 0.6; 1
L_h - Layer height (mm)		0.05; 0.10; 0.15; 0.20; 0.30; 0.40
P_s - Printing speed (mm/s)		10
T - Temperature (°C)	Extrusion	210
	Bed	40

3.2.3. Casting mold fabrication

The 3d-printed polymer investment models were subsequently infiltrated with liquid plaster (30%wt distilled water and 70%wt GoldStar Omega+ plaster) to produce ceramic molds. The molds were subjected to the thermal cycle represented in as Fig.3.6.

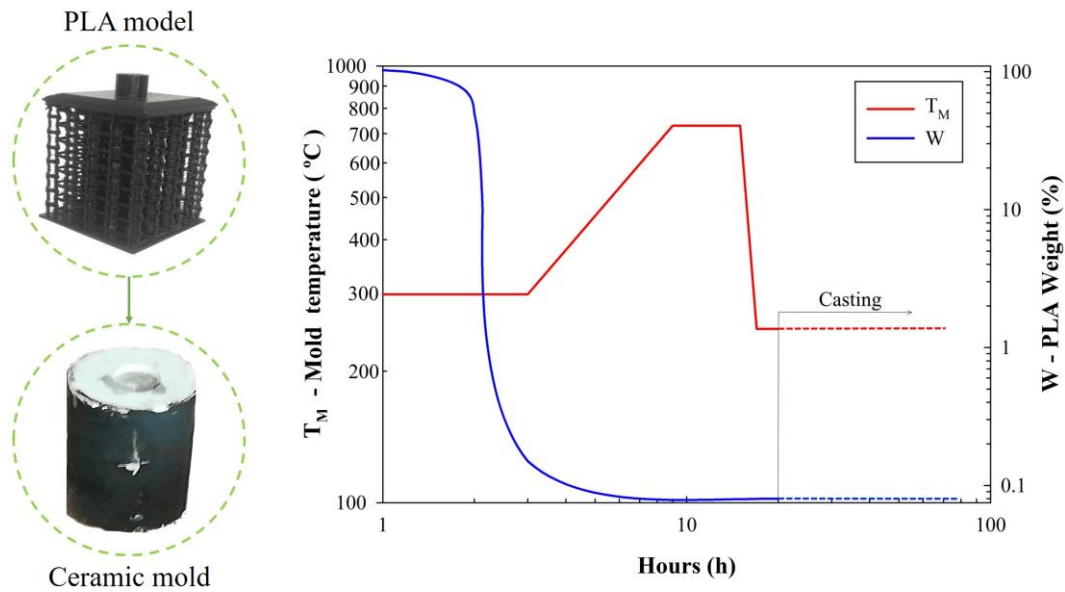


Figure 3.6 Thermal cycle and PLA weight during ceramic mold fabrication.

This thermal cycle allows the complete cure of the plaster, while assuring a good accuracy on the investment model shape and dimension. During the first stage of the curing cycle (isothermal at 300°C for 3h) the PLA model begins to evaporate. At the end of the second stage (ramp from 300°C to 730°C in 5h), only trace residues of the PLA model may be found ($\approx 0.12\text{wt}\%$). Thus, during the third and fourth stages (respectively, isothermal at 730°C for 6h and cooling to 250 °C), the PLA sample is eliminated and all details are embossed in the ceramic mold.

3.2.4. Cellular solid casting

A356 alloy (24g) was cut from a primary fusion ingot (Fig. 3.7 (a)). After being cleaned and dried, the alloy was inserted in a SiC crucible, along with added Al₅Ti₁B (0.05g – 0.2%wt) and Al₁₀Sr (0.07g – 0.3%wt) master alloys, respectively added to promote grain refinement and eutectic Si modification. The maximum material for the lattice is 21g, corresponding to 87.5% of total crucible capacity. The remaining 3g (12.5% crucible capacity) corresponds to the casting gating system that assures sufficient metalostatic pressure during the liquid metal filling.

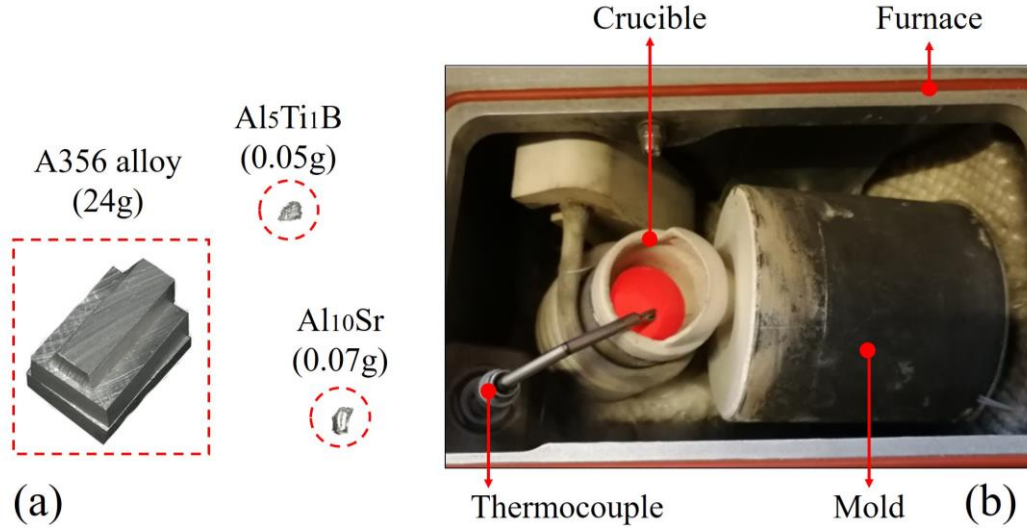


Figure 3.7 Casting process: (a) A356 and Al₅Ti₁B/Al₁₀Sr alloys being (b) cast into the mold.

The alloy was melted (700°C) inside an Indutherm MC15+ induction casting furnace in vacuum ($P=-1$ bar) and kept isothermal at $700\pm 2^\circ\text{C}$ for 3 minutes for homogenization, while the imposed induction magnetic stirring prevented nucleant particle sedimentation. After this period, the melt was cast (Fig. 3.7 (b)) into the ceramic mold ($250\pm 5^\circ\text{C}$). Following a 10 min solidification period, the ceramic mold was submerged in water to separate the plaster from the casted sample. The produced samples were subjected to a T6 heat treatment: solution ($540^\circ\text{C} - 480$ min), water-quench and artificial ageing ($160^\circ\text{C} - 500$ min), according to the alloy optimization that is reported in Chapter 2 (section 2.4). Due to the inverse proportionality between static strength and damping properties, this treatment was tuned to an optimized compromise between maximum yield strength and damping [113] while complying with the ISO 3522 standard.

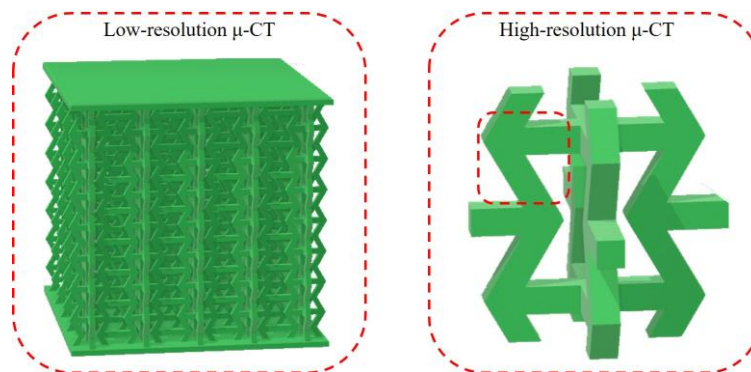
3.2.5. X-ray microcomputed tomography of cellular solids

X-ray microcomputed tomography (μCT) imaging was performed using a Zeiss Versa XRM-520 μCT scanner controlled using XRM scout-and-scan control system. Samples were mounted on a polystyrene wedge to minimize cone-beam artefacts (one sample of each geometry was imaged). The imaging settings were optimized for each sample and are summarized in Table 3.2. Data was reconstructed using XRM reconstruction software, then segmentation and visualization was performed using Avizo standard. Cross-sections through the reconstructed 3D volume were exported as 2D tiff images to allow measurement of rib dimensions. The overall dimensions of the samples and the ribs were measured using the micrographs using Fiji [117].

Table 3.2. X-ray μ -CT testing parameters.

Sample	Low resolution							High resolution	
	$\alpha=-30^\circ$		$\alpha=-20^\circ$	$\alpha=-10^\circ$	$\alpha=0^\circ$	$\alpha=10^\circ$	$\alpha=20^\circ$	$\alpha=30^\circ$	$\alpha=-30^\circ$
	Cast	PLA							Cell
Voxel (μm)	20.9	28.4	26.5	27.2	28.4	56.7	6.3	2.3	
Exposure time (s)	4	6	4			1	6	60	
Source-Sample (mm)	130	192	165	190	192	50	120		
Sample-Detector (mm)	80	36	45		36	220	60		
Voltage (kV)	140	100			140				
Power (W)	10	9			10				
Objective				0.39x					4x
Projections	2401			3001		2401			
Filter				LE3					

The whole samples were scanned in a low-resolution mode (Fig. 3.8) to measure the overall rib dimensions and visualization of exterior defects. Posteriorly, high resolution scans were performed in cellular units to detail the exterior defects and quantify porosity. A final high resolution scan was performed in a horizontal and a vertical rib to characterize porosity. Test parameters are detailed in Table 3.2.

**Figure 3.8** Volumes to characterize by μ -CT.

3.3 Results and Discussion

3.3.1. Optimization of investment model 3d-printing parameters

Printing parameters have a key role in the shape of the samples and investment model printing process. Fig. 3.9 shows the 3D-printer ability to produce the samples using diverse nozzles. Smaller nozzles are able to produce ribs with lower thickness, which is desirable to minimize specific density. Rib thicknesses less than 0.6 mm could not be produced with the tested nozzles.

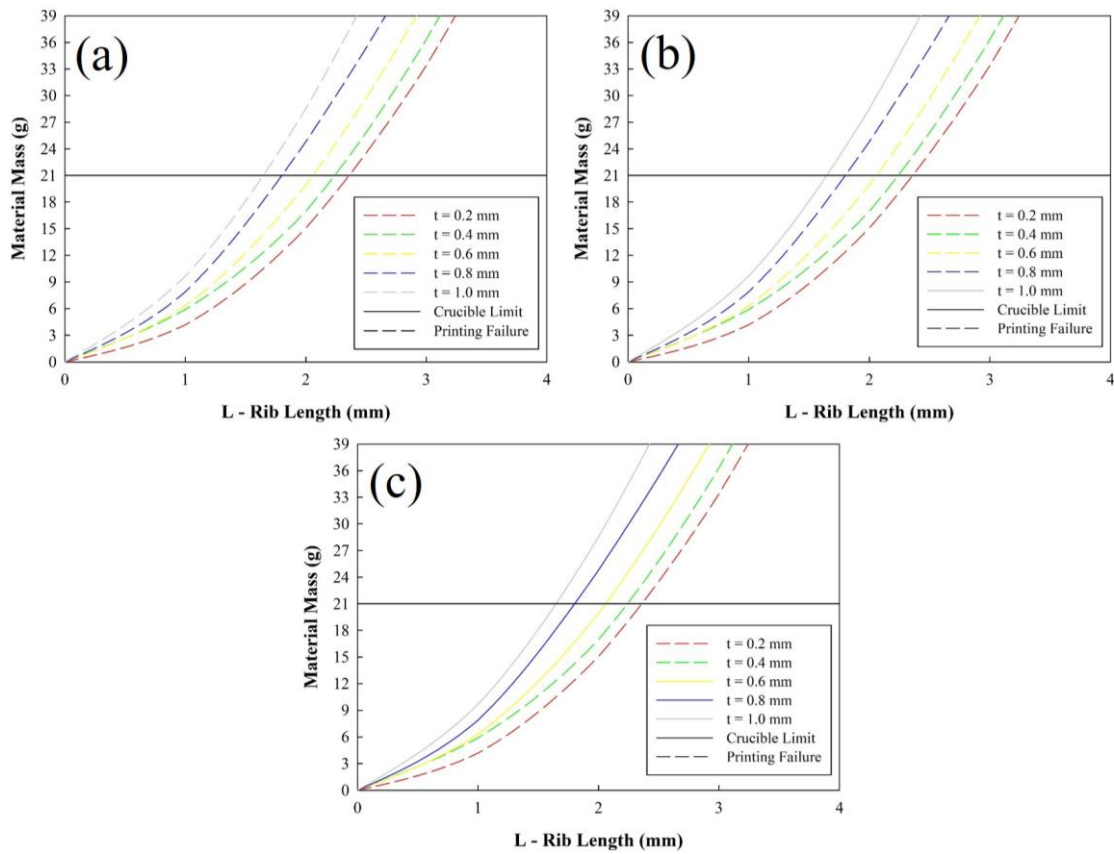


Figure 3.9 Relationship between rib thickness (t) and alloy material mass, for nozzle diameters of: (a) 1 mm, (b) 0.6 mm, (c) 0.4 and 0.3 mm. Note – Solid lines: crucible limit; Dashed line: Printing failure.

These printing failures occurred due to under-extrusion of printing filament (Fig. 3.10 (a)), caused by the wearing of filament in the extrusion gear (Fig. 3.10 (b)). Filament wear is caused by the change in feeding in the extrusion/retraction cycles needed to deploy reduced amounts of PLA in each individual rib [118]. In conclusion, the minimum rib thickness that was achieved in the described process is 0.6 mm, using nozzle diameters lower than 0.4 mm.

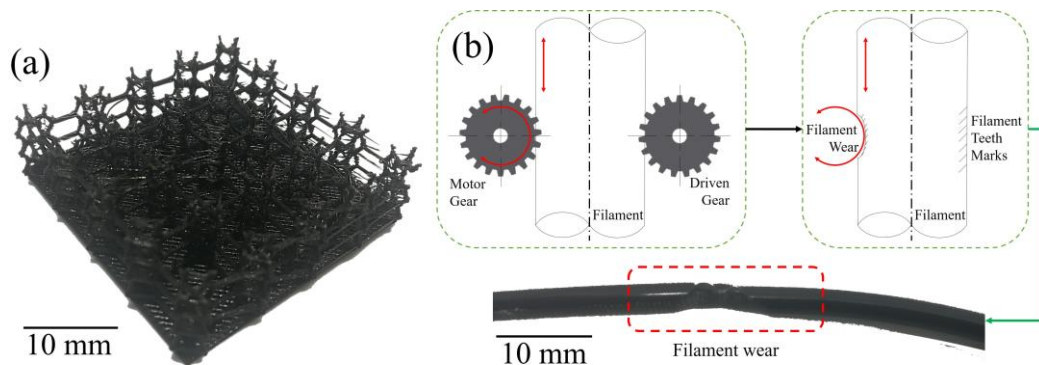


Figure 3.10 Printing failure: (a) whole sample failure by under-extrusion and (b) extrusion failure resulting from filament wear.

Printing success is also dependent on the used layer heights (L_h), according to the results in Table 3.3. As the layer height is increased, the printing eventually fails due to the lack of adherence between consecutive layers. Results demonstrate that whilst increasing layer height reduces printing time, the layer height should not exceed 0.2 mm to assure sufficient printing adherence. The 0.4 mm nozzle, alongside a layer height of 0.2 mm, was therefore selected to minimize the printing time without compromising printing success.

Table 3.3 Relation between nozzle diameter, layer height, printing time and the successful printing of the models ($t=0.6$ mm and $l=2$ mm).

		N – Nozzle diameter (mm)	
		0.3	0.4
		Printing time (h)	
L_h Layer height (mm)	0.05	6.5	5.5
	0.10	3.5	3.0
	0.15	2.5	2.0
	0.20	2.0	1.75
	0.30	-	-
	0.40	-	-
		Successful printing Printing fails	

3.3.2. Optimization of casting parameters

Different ceramic mold and casting temperatures were tested to determine their influence in a successful casting of thin-rib aluminum scaffolds in the presence or absence of vacuum. Fig. 3.11 shows a successful attempt to cast the proposed scaffold using vacuum assistance ($P=-1$ bar) and mold / casting temperatures of, respectively, (T_m) 250°C and (T_c) 700°C.

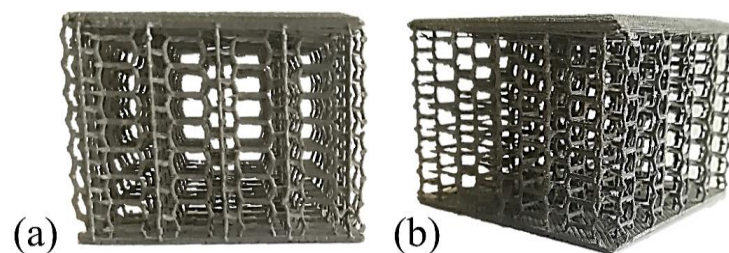


Figure 3.11 Developed manufacturing process as-cast sample ($T_m=250^\circ\text{C}$ and $T_c=700^\circ\text{C}$, using vacuum assistance).

Figure 3.11 shows that the adopted method is able to display a good shape and detail correlation to the CAD model that is displayed in Fig. 3.5 (g). The unitary cells assume a full three-dimensional

honeycomb configuration and are fully connected between themselves, presenting a complete assembly of the scaffold. Using the described technique it is possible to reproduce the intended design, however as shown in Fig. 3.12, the use of different temperature has a fundamental role in the casting success.

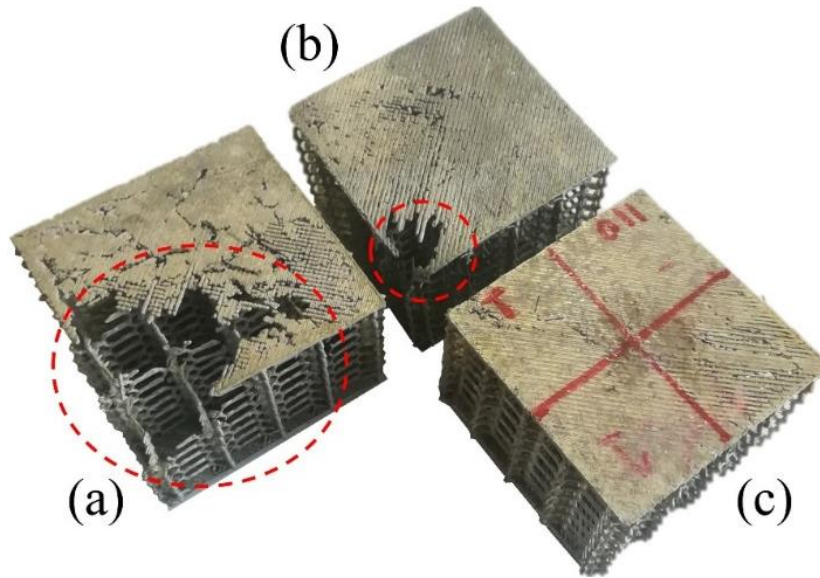


Figure 3.12 Average filling using different temperatures using vacuum assistance: (a) 83% ($\pm 6.3\%$) — $T_M = 200\text{ }^\circ\text{C}$ and $T_C = 680\text{ }^\circ\text{C}$; (b) 92% ($\pm 4.1\%$) — $T_M = 200\text{ }^\circ\text{C}$ and $T_C = 700\text{ }^\circ\text{C}$; and (c) complete filling — $T_M = 250\text{ }^\circ\text{C}$.

According to the results presented in Fig. 3.12 (a), the use of $T_M=200^\circ\text{C}$ and $T_C=680^\circ\text{C}$ with vacuum assistance, is not able to guarantee the successful filling of the model. Further increasing the casting temperature by 20°C ($T_M= 200^\circ\text{C}$ and $T_C=700^\circ\text{C}$ – Fig. 3.12 (b)), it may be stated that the overall filling is increased, however, it is still shown an absence of material in the sample corners. By the increase of mould temperature ($T_M= 250^\circ\text{C}$ and $T_C=700^\circ\text{C}$ – Fig. 3.12 (c)) is possible to guarantee a successful filling of the adopted model, considering that the overall casting process is correctly performed.

Overall, it may be determined by the results in Fig. 3.12 that in near complete filling conditions, the increase of mold and casting temperatures is able to enhance the filling by, respectively, $0.16\%^\circ\text{C}^{-1}$ and $0.75\%^\circ\text{C}^{-1}$. Thus, it is suggested that the influence of casting temperature is more prominent for the filling of complex aluminum thin-ribs in near complete casting conditions.

These suggestions are further supported by the overall comparison of average sample filling showed in Figure 3.13. As expected, high temperatures of both mold/casting and vacuum are extremely useful to allow a complete filling of the model (Fig. 3.13 – (a)). It is also shown that in the absence of vacuum (Fig. 3.13 – (b)), it is not possible to cast the scaffolding structures. Even though, the use of high

mold and casting temperatures are able to enhance the filling of gravity casting samples, overall the results without vacuum suggest a maximum filling of approximately 20%. It could be argued that further increasing the mold and casting temperatures could increase the filling and eventually allow a complete casting in non-vacuum conditions. However, it is also known that the superheating of aluminum alloy melts is hazardous for the final casting products by the decrease in microstructural sanity [60,61]. Thus, minimal mold and casting temperatures should be used, as long as it does not compromise the casting.

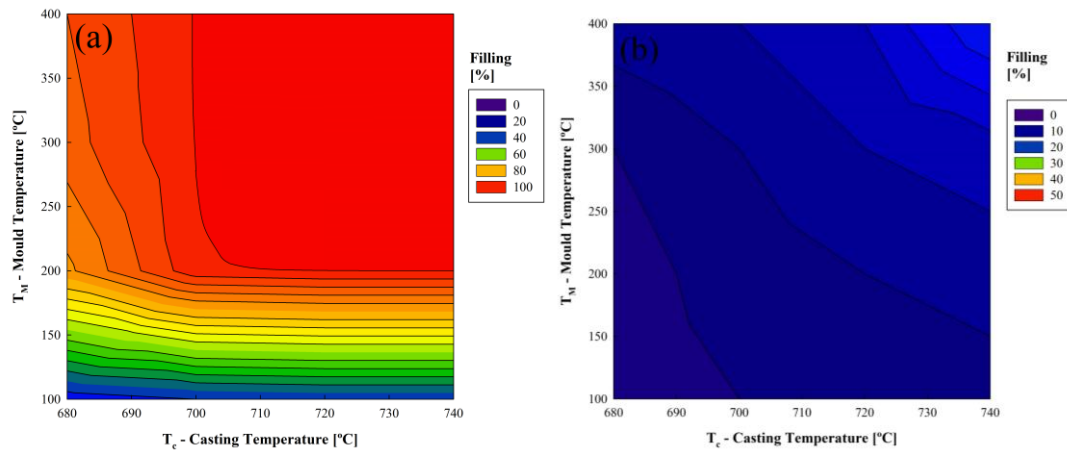


Figure 3.13 Average sample filling using different combinations of mold and casting temperatures: (a) with vacuum; and (b) without vacuum.

Considering these facts and observing Fig. 3.13 (a), it may be stated that the most beneficial combination of mold and casting temperatures for the adopted vacuum assisted investment casting process are, respectively, $T_M=250$ °C and $T_c=700$ °C. Additionally, it is known that the use of vacuum is beneficial for the overall sanity of aluminum cast products. Reducing the referred temperatures is not recommended given that the thin-rib configuration is easily clogged by: (i) premature alloy solidification due to the high cooling rates in low temperature molds; (ii) or near *solidus* alloy in low casting temperatures.

3.3.3. Characterization of the manufacturing process

During the two stage process to manufacture the designed cellular solids, there are significant geometrical differences between the samples. These are variations occur at macro- and microstructural levels, either by dimensional variations or the overall casting process. According to Fig. 3.14 and to analyze such differences, a sample ($\alpha=30^\circ$) on both PLA and aluminum cast configurations were scanned by X-ray μ -CT to analyze their individual cells in detail. This allows the characterization of the different dimensions and details along the defined manufacturing route.

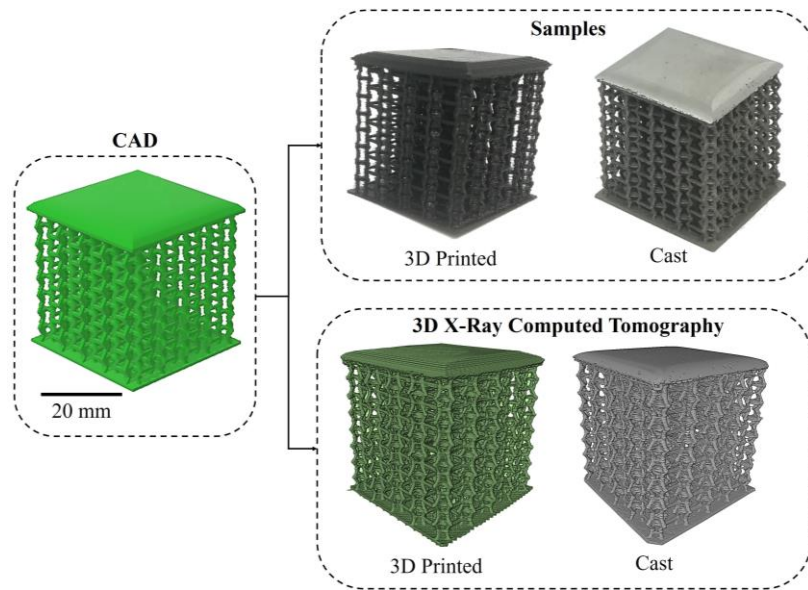


Figure 3.14 Detail of the manufacturing process and characterization of $\alpha=-30^\circ$.

In Fig. 3.15, it may be observed that the printing process leaves printing artifacts in the form of thin PLA threads. These are characterized by thin PLA strings that are promoted by the stretching of the filament once extrusion stops. However, these artifacts are not found in the final cast samples. It is suggested that during the mold fabrication these artifacts are embedded in within the plaster, however, during the casting process, the liquid aluminum is not able to penetrate the vacancies that they generate in the plaster. Thus, they are avoided in the final casted samples.

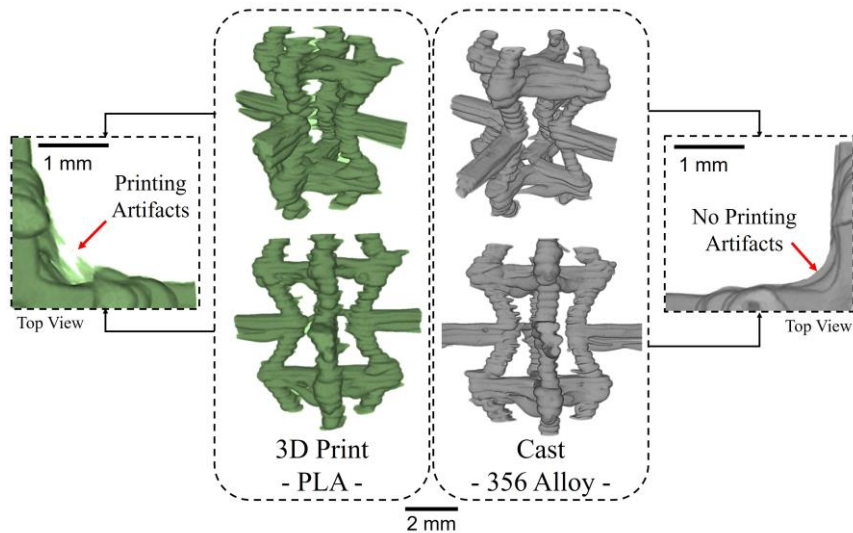


Figure 3.15 Comparison between 3d-printed and casted cells.

According to the methods in section 3.2.5, high-resolution X-ray μ -CT was used to map and quantify the porosity that are originated in the casting process in different rib orientations. According to

Fig. 3.16, it may be observed that the porosity of the samples is distributed uniformly along the ribs. However, a segregation may be found in the horizontal rib.

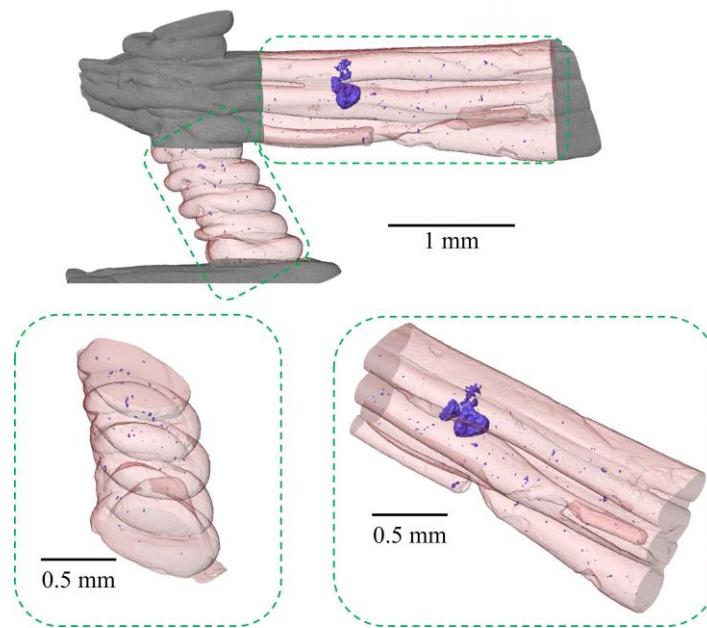


Figure 3.16 Detail of porosity and segregation within the sample ribs.

Fig. 3.17 (a) shows the distribution of pore size in each rib. According to this figure, it may be determined that the pores in the vertical ribs tend to be slightly smaller than in the horizontal ribs. However, this difference in pore size is suggested not to be relevant. It is also proposed that such small pore size does not significantly impact the mechanical behavior of the samples. Cold shut void volume is more significant than pore size (Fig.3.17 (b)). However, by the analysis of the X-ray μ -CT reveal that this defect is only present in 12% of the ribs and is caused by the meeting of two opposite liquid metal stream that meet in horizontal ribs.

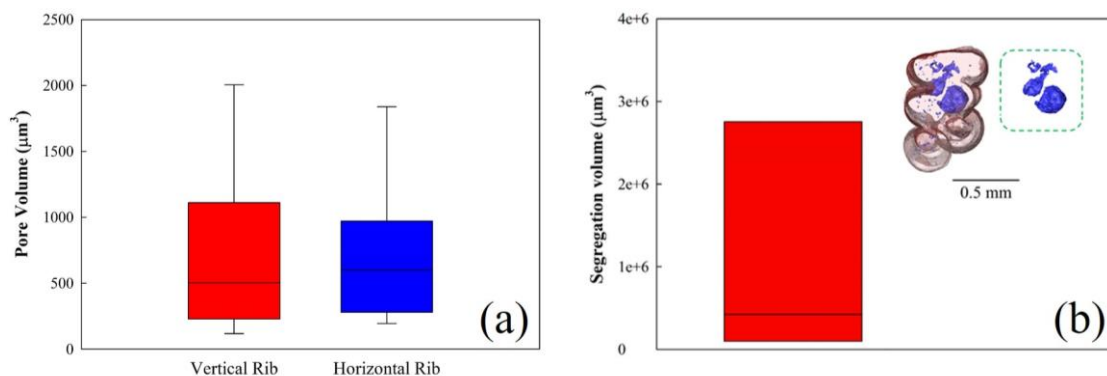


Figure 3.17 Detail of voids in the ribs: (a) pore and (b) cold shut void volumes.

3.3.4. Dimensional characterization of casted samples

Fig. 3.18 shows examples of the final casted samples, in which the details may be observed and compared with the CAD models (Fig.3.4). It is apparent that they are able to fully reproduce the intended shape and details. μ CT imaging was performed in the casted samples (Fig. 3.19) allowing a detailed inspection of the samples, including the internal configuration of each individual periodic cell.

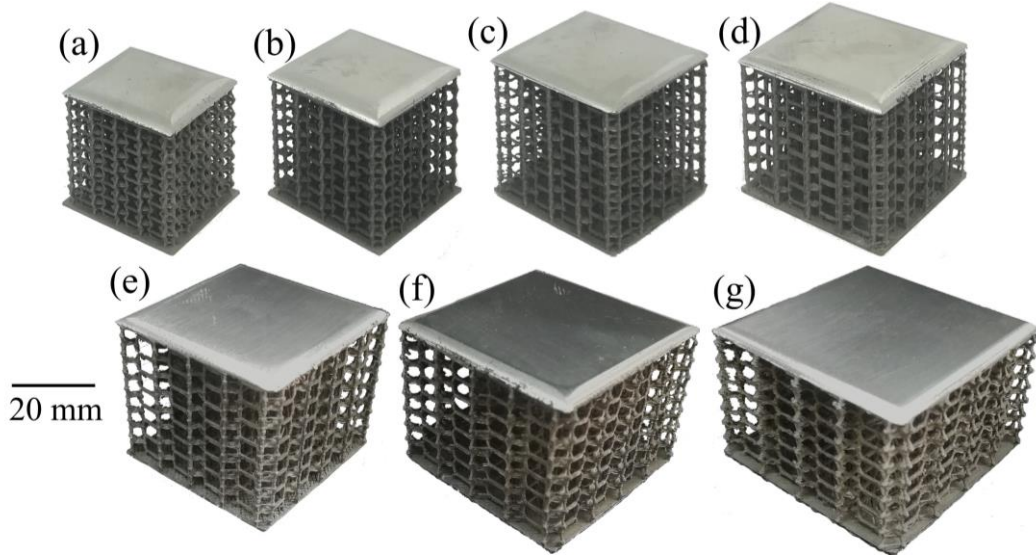


Figure 3.18 Casted lattices: (a) $\alpha=30^\circ$, (b) $\alpha=20^\circ$, (c) $\alpha=10^\circ$, (d) $\alpha=0^\circ$, (e) $\alpha=10^\circ$, (f) $\alpha=20^\circ$ and (g) $\alpha=30^\circ$ samples.

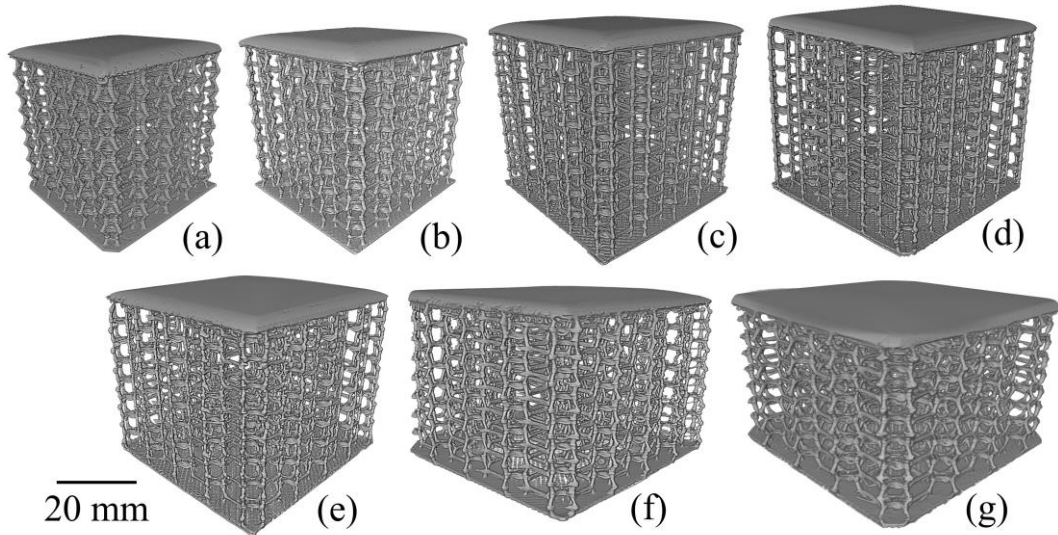


Figure 3.19 X-ray μ -CT casted lattices: (a) $\alpha=30^\circ$, (b) $\alpha=20^\circ$, (c) $\alpha=10^\circ$, (d) $\alpha=0^\circ$, (e) $\alpha=10^\circ$, (f) $\alpha=20^\circ$ and (g) $\alpha=30^\circ$ samples.

μ CT measurements (Fig. 3.20 (a)), demonstrate that the casted sample height (h_s) is higher (+0.2 mm to +0.4 mm) than the CAD model. Rib angle impacts the dimensional variations of the casted sample width and depth (W_s and D_s - Fig. 3.20 (b)) and negative/positive rib angles produce positive/negative dimensional variations (-0.26 mm to 0.54 mm). According to Fig. 3.20 (c), these dimensional variations change the sample specific density (ρ^*/ρ_0). Given that the casted samples display a tendency to have higher dimensions (Figs. 3.20 (a) and (b)) than the CAD models, the sample volume is increased. Thus, there is mostly a decrease in density (-0.008 to 0.001).

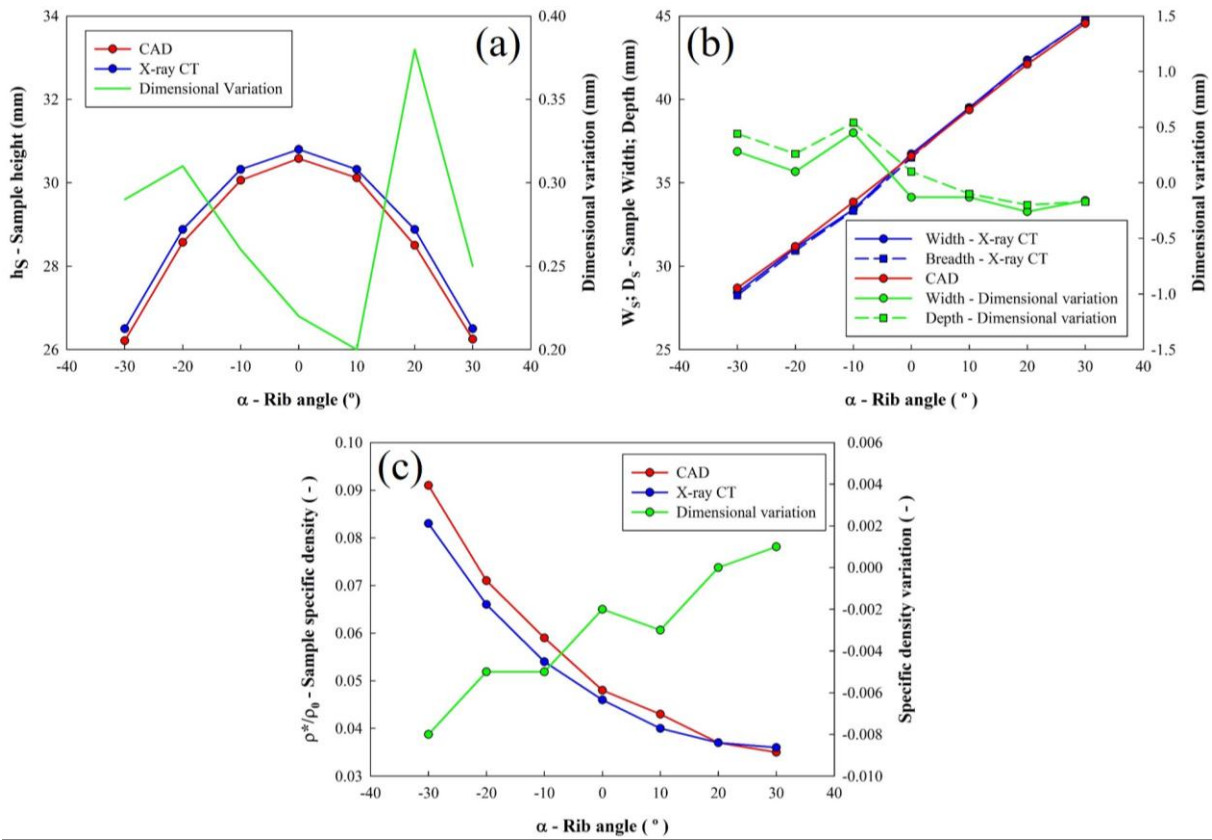


Figure 3.20 Comparison between the dimensions of CAD and casted samples.

Fig. 3.21 shows examples of casted samples periodic cellular units. Samples with negative angles (Fig. 3.21 (a) to (c)) have an inverted honeycomb configuration. This characteristic is related with cellular lattices with negative Poisson's ratios (*i.e.* auxetic behavior) [56]. Fig. 3.21 (d) has a square-shaped cell ($\alpha=0^\circ$), where the ribs may be described by near horizontal and vertical configurations. The samples with positive rib angles (Fig. 3.21 (e) to (g)) have a classic honeycomb (*i.e.* hexagonal) configuration with a positive Poisson's ratio [119].

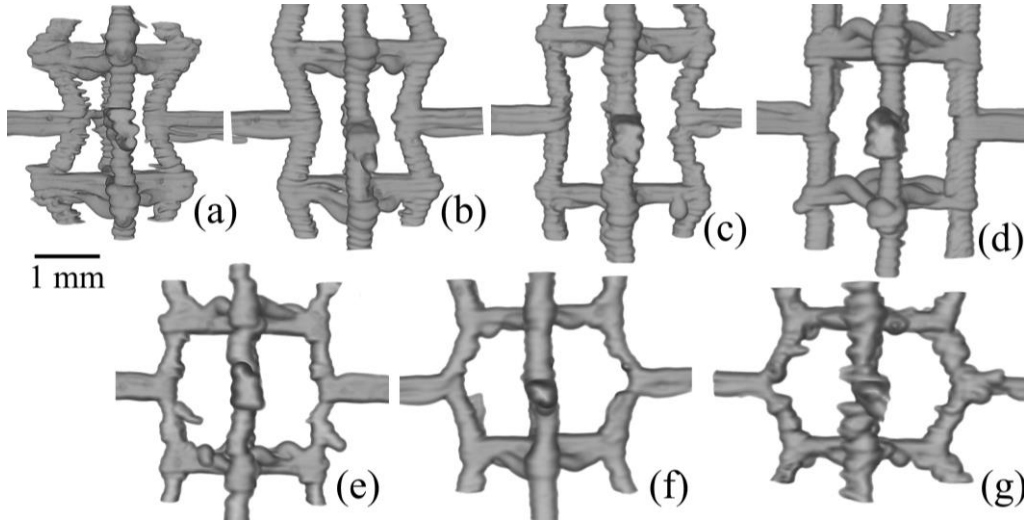


Figure 3.21 X-ray μ -CT of cells: (a) $\alpha=-30^\circ$, (b) $\alpha=-20^\circ$, (c) $\alpha=-10^\circ$, (d) $\alpha=0^\circ$, (e) $\alpha=10^\circ$, (f) $\alpha=20^\circ$ and (g) $\alpha=30^\circ$.

Fig. 3.22 shows that the rib angle has an impact on the overall shape of the cell and in the ability of the studied manufacturing process to replicate the CAD samples. True rib angles (α_{TRUE}) display an angular error (from -1.40° to 0.73° - Fig. 3.22 (a)). However, these errors are small in comparison with the deviations of other linear dimension.

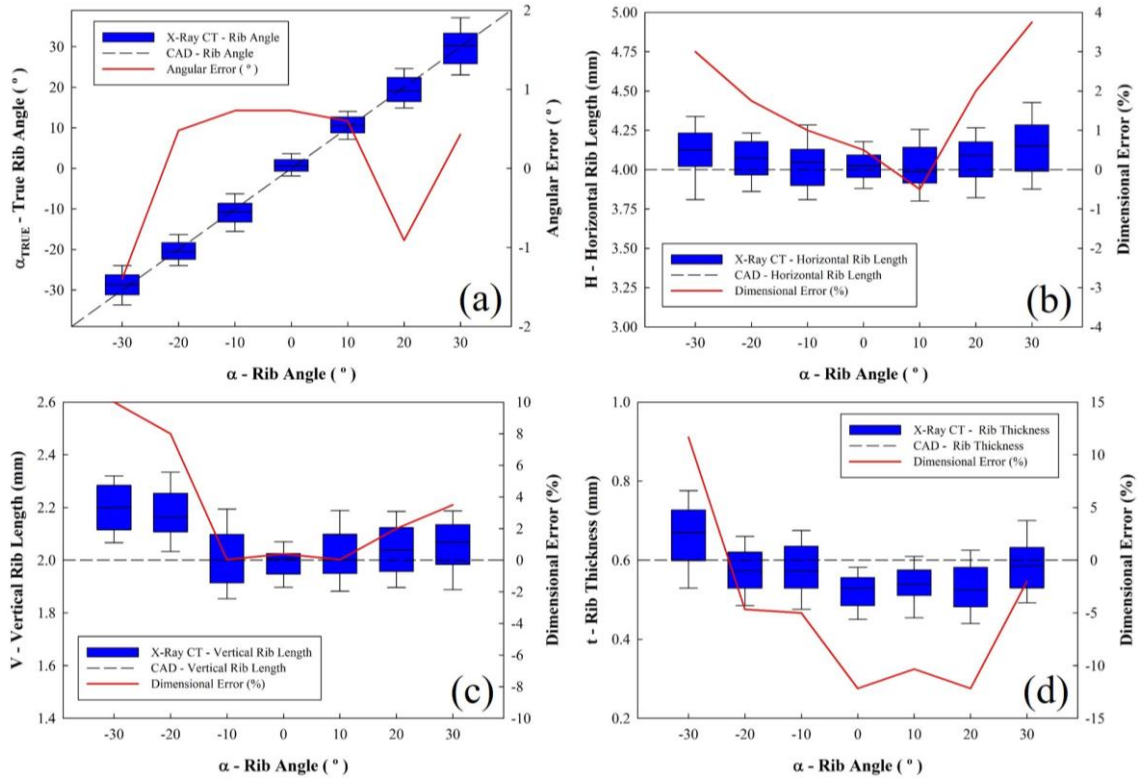


Figure 3.22 X-ray computed tomography dimensional measurements compared against CAD model specifications: (a) rib angle, (b) horizontal rib length, (c) vertical rib length and (d) rib thickness.

Figs. 3.22 (b) and (c), show that the horizontal and vertical rib lengths have a more pronounced dimensional variations. Indicating a prominent role in the dimensional variations of the casted samples (Fig.3.19). Horizontal ribs (h - Fig.3.22 (b)) and vertical ribs (l - Fig. 3.22 (c)) have dimensional range of, -0.50% to 3.75% and 0.04% to $\approx 10\%$ respectively. Such changes are more prominent in samples with wider angles, either negative or positive. This is attributed to the different bending stiffness of the ribs during the printing process.

It is known that in the 3D-printing, thin inclined struts without support (*e.g.* $\alpha = -30^\circ$ or $\alpha = 30^\circ$) are subjected to: (i) bending due to the collision with the rigid nozzle [120,121]; and (ii) coalescence higher surface tension [122]. Ribs with wider angles (*e.g.* $\alpha = -30^\circ$ or $\alpha = 30^\circ$) are subjected to bending while the nozzle is extruding. Ribs with near-zero angles ($\alpha \approx 0^\circ$) are prone to axial loads [121]. Given the matrix configuration of these periodic cellular units, the changes in Figs. 3.22 (b) and (c) are responsible for the increase in the dimensions of samples with wider angles (Fig. 3.22).

Although rib thickness (t – Fig.3.22 (d)) has a relatively low impact on the exterior dimensions of cast samples (Figs. 3.20 (a) and (b)), it changes the sample specific density (Fig. 3.20 (c)). There is a tendency for the samples to have lower values of rib thickness than the CAD models. This is more noticeable when the rib angle is approximates zero ($\alpha \approx 0^\circ$ - Fig. 3.20 (d)). Such change in thickness geometry impacts on rib cross-sectional area.

The rib dimensions that are displayed in Fig. 11 were subjected non-parametric statistical analysis to analyze their significance (Table 3.4) using an one-way ANOVA on Ranks (Kruskal-Wallis test).

Table 3.4. Statistical significance of the rib dimensions.

α	-30	-20	-10	0	10	20	30
-30		••	••	••	••	••	••
-20			••	••	••	••	••
-10				••	••	••	••
0					•	••	••
10						••	••
20							••
30							
h	-30	-20	-10	0	10	20	30
-30		••	••	••	••		
-20							••
-10							••
0							••
10							••
20							•
30							
l	-30	-20	-10	0	10	20	30
-30		••	••	••	••		
-20							••
-10							••
0							••
10							••
20							•
30							
t	-30	-20	-10	0	10	20	30
-30		••	••	••	••	••	••
-20				••	••	••	
-10				••	••	••	
0							••
10							••
20							••
30							

No significant difference.

Significant difference.

• $p < 0.05$

•• $p < 0.01$

In terms of angular dimensions (α_{TRUE} — Fig. 3.22 (a)) each sampling group is supposed to display different rib angles, thus, it is expected that there are significant differences between them. Table 3.4 shows that there is a difference between the groups, although by comparing samples with $\alpha=0^\circ$ and $\alpha=10^\circ$, this difference is less significant ($p<0.05$) than other possible combinations ($p<0.01$). When different groups are compared in terms of rib lengths (h and l), it is desirable that there are no significant differences due to the constant value of these dimensions (*i.e.* $h=4\text{mm}$ and $l=2\text{mm}$). Table 3.4 shows there is a tendency for the different groups not to be significantly different. However, this is not true for the wider rib angles ($\alpha=-30^\circ$ and $\alpha=30^\circ$), in which there is a significant statistical difference (typically $p<0.05$) between these angles and the other groups.

The statistical analysis of rib thickness (t) does not seem to follow a pattern. In fact, according to Table 3.4, most groups show a significant difference with each other. This may be attributed to the overall variations in the 3D-printing process of the PLA investment model. Given that the layer height is constant ($L_h=0.2\text{ mm}$), ribs with wider angles (Fig.3.23 (a)) have less layers than samples with lower rib angles (Fig.3.23 (b)). Since the 3D-printing process follows the sample Cartesian planes, the layer deposition in inclined ribs occurs in a tilted cross-section (*e.g.* Fig.3.23 (a)), generally referred as a staircase surface [123]. These variations, however, are not applicable to the printing process of horizontal ribs (Fig. 3.23 (c)) in which the printing area is more elevated and constant. All these factors generate significant statistical differences in rib thickness.

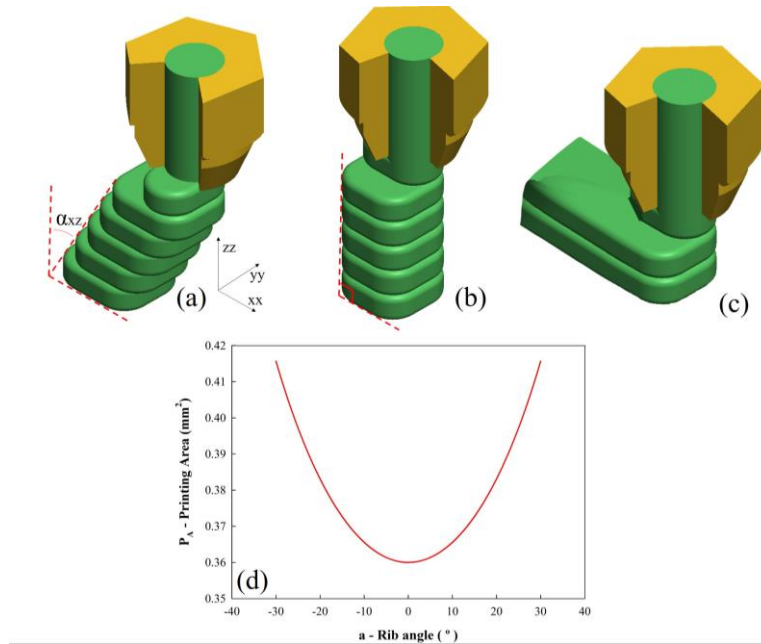


Figure 3.23 Influence of angle in the printing area during the fused filament fabrication process: vertical with (a) $\alpha=30^\circ$, (b) $\alpha=0^\circ$ and (c) horizontal ribs. (d) Printing area plot of for oblique ribs ($t=0.6\text{ mm}$).

Consequently, the printing area (PA - Fig.3.23 (c)) of each layer changes with the rib angle, according to Eq. 3.1. Due to the discrete segmentation process during the G-code programming, this also occurs in metal-based additive manufacturing [57]. It is known that there is a deterioration of strut geometry in inclined angles relatively to the build platform [124–126]. However, in metal additive manufacturing techniques this issue is mainly attributed to the dissipation of heat flux through the neighboring sintering powder in oblique beams [127].

$$P_A = \frac{t^2}{\cos(\alpha)} \quad (3.1)$$

Given that the printer positioning resolution in the XX /YY axes is $12.5 \mu\text{m}$, the reduction in printing area in near-zero rib angle ($\alpha \approx 0^\circ$) increases possible errors (Figs. 3.22 (d) and 3.23 (c)). Due to the equipment resolution and circular shape of the nozzle, the printed model presents with a quasi-circular cross-section instead of the square cross-section in the CAD (Fig 3.24).

Since extrusion cannot be stopped instantly, the constant movement of the extrusion head and filament viscoelastic effects [128], the nozzle leaves protrusion defects in some areas, as observed from μCT imaging (Fig. 3.21 and 3.25).

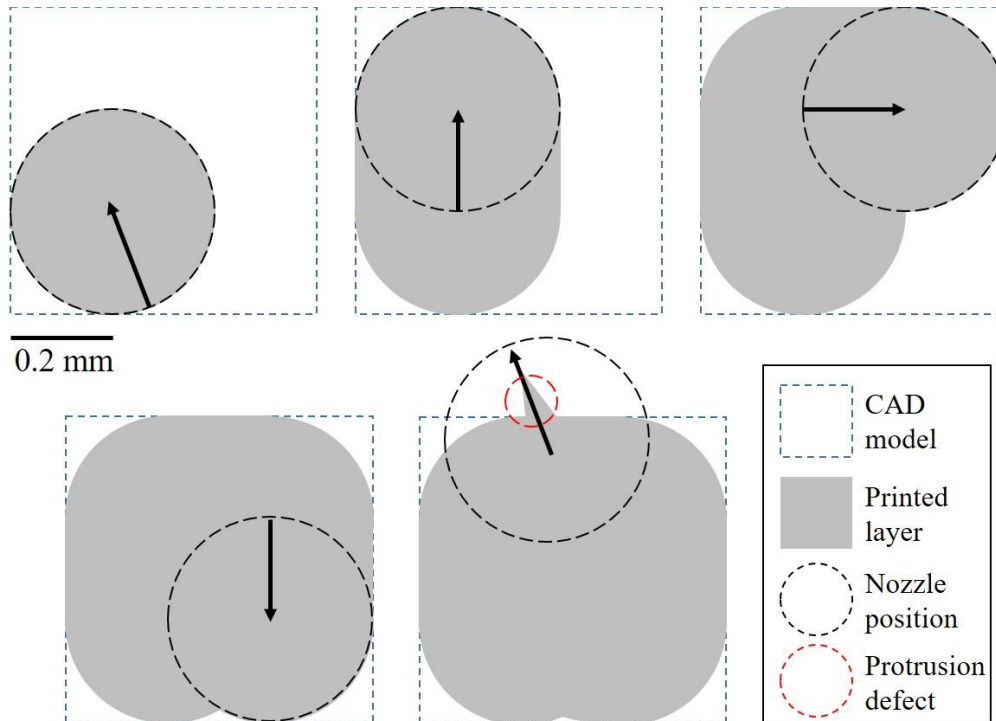


Figure 3.24 Schematic illustrating the deviation in printed rib cross-section from the CAD model, and how this results from the printing process (Top view).

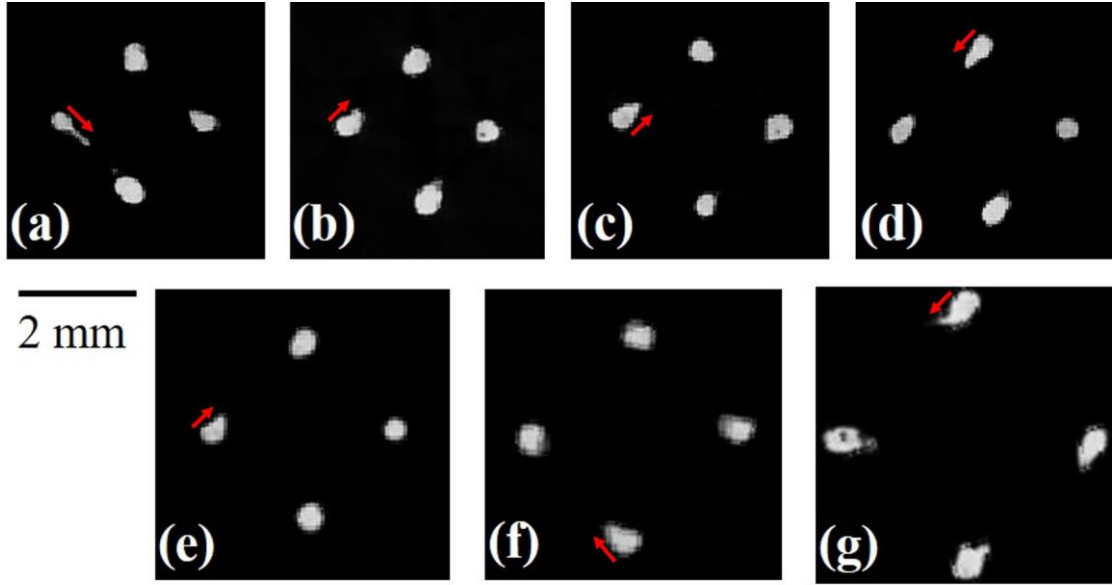


Figure 3.25 Cross-sectional view through the μ CT reconstructed volumes, showing detail of rib cross-section: (a) $\alpha=-30^\circ$, (b) $\alpha=-20^\circ$, (c) $\alpha=-10^\circ$, (d) $\alpha=0^\circ$, (e) $\alpha=10^\circ$, (f) $\alpha=20^\circ$ and (g) $\alpha=30^\circ$ samples.

Fig. 3.26 (a) shows a reduction in the rib areas, especially in near-zero rib angles ($\alpha \approx 0^\circ$), as demonstrated by the model in Figs. 3.23 and 3.24. Furthermore, this changes the rib moment of inertia by: (i) the reduction in rib thickness (t – Fig. 3.22 (d)); and (ii) change in cross-section from a square ($I_{\blacksquare} = t^4/12$) to a circular ($I_{\bullet} = \pi t^4/64$) shape.

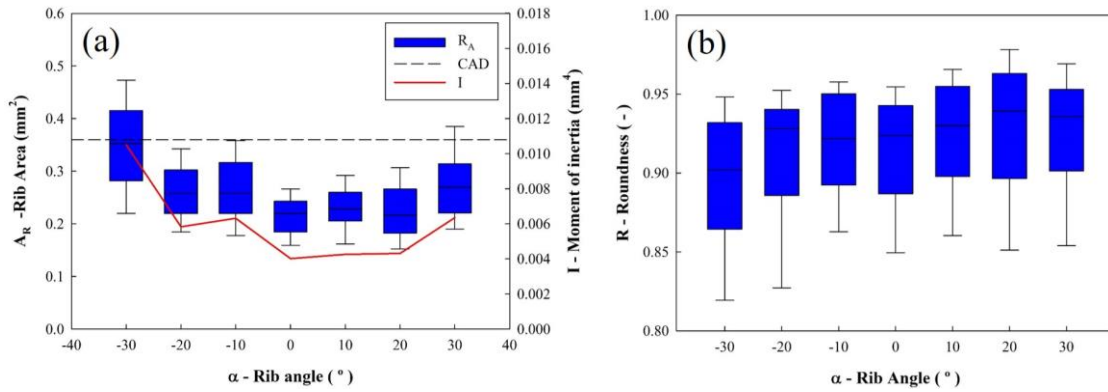


Figure 3.26 Rib cross-section: (a) rib area and moment of inertia; (b) rib roundness ($R=1$, means perfect circle).

The circular shape of the ribs may be observed in the values of roundness (Eq. 3.2) presented in Fig. 3.26 (b), where AR is the rib area and P is the perimeter. Given the high roundness values ($R=0.89$ to 0.93), the cross-sectional shaped may be considered a circular geometry.

$$R = \frac{4\pi A_R}{P^2} \quad (3.2)$$

3.4 Conclusions on cellular solid manufacturing

This chapter details the design and reports a route to producing low-cost non-stochastic cellular lattices by an additive manufacturing assisted investment casting technique. Overall, the process and the involved techniques are detailed to characterize such manufacturing route. The following conclusions are drawn:

(i) Optimization of the fabrication parameters (nozzle diameter and layer height) is essential to produce the thin-ribbed and low specific density PLA investment models that are used to produce ceramic casting molds.

(ii) It is shown that an optimized combination of different mould and casting temperatures must be employed to ensure a complete filling of the samples. A minimum temperature combination of 250°C for the mould and 700°C for the casting must be used to guarantee the complete filling of the samples. It is suggested that in near complete casting parameters, the most important factor is the casting temperature. Additionally, it is shown that the vacuum assistance is needed to ensure the success of the procedure.

(iii) Even though the casting process occurs in vacuum, μ -CT shows that there is still porosity in the samples. However, the volume of these pores is extremely low and it is proposed that this does not significantly affect the mechanical properties of the samples. Additionally, horizontal ribs may display segregation due to the meeting of opposing liquid aluminum streams.

(iv) μ CT demonstrated that linear rib dimensions such as horizontal and vertical rib lengths may deviate from CAD models. Final dimensions may be up to 10% than original CAD dimensions, and these deviations are more pronounced as rib angle is increased. This corresponds with an observed increase in the sample volume, which is beneficial to further reduce the specific density of the casted samples.

(v) The ribs of casted samples present a circular shape and the process is unable to produce square cross-sections as defined in the CAD model, due to the shape and size of the extrusion nozzle. Additionally, the final rib thickness of manufactured samples is lower than specified in CAD models when the rib angle is zero (*i.e.* square honeycombs). These deviations impact on the rib cross-sectional area and moment of inertia. Given that the main deformation mechanism of this kind of composites is rib flexure, this shape and dimensional deviations may impact on structural strength.

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Chapter 4 – Effect of the Poisson's ratio on the Static Structural Behavior of the Designed Cellular Solids

“Our virtues and our failings are inseparable, like force and matter. When they separate, man is no more”

Nicola Tesla, *The Problem of Increasing Human Energy*, 1900.

4.1 Introduction to the static structural properties of cellular solids

The control of mechanical performance in these non-stochastic cellular solids is frequently performed by the monitoring of their Poisson's ratio. There are evidences that negative Poisson's ratios may have a beneficial impact in the overall static mechanical properties (*e.g.* hardness [1–3] and shear/fracture resistance [4–7]).

Given that this behavior is very rare in nature (mostly observed in extremely anisotropic systems [8–10], since the *mid*-1980's [11,12] there has a significant effort to manufacture and characterize cellular solids with this characteristic. Due to this negative elastic constant, these materials expand/contract their cross section when in tension/compression and are called Auxetics [13]. Recent studies have also been devoted and show benefits in the manufacturing of cellular solids with zero Poisson's ratio [14–16].

This chapter studies the effect of the Poisson's ratio on the static mechanical behavior of low density non-stochastic metallic cellular solids that are manufactured using a combination of additive

manufacturing and investment casting. The produced samples are tailored to display positive, zero and negative Poisson's ratios, by changing the rib angles, and are subjected to uniaxial compression testing.

The transition from the negative to positive Poisson's ratio generates a change in the overall collapse mechanism of the samples. An analytical model is proposed to describe the elastic behavior in these composites. Finite element analysis is used to analyze and describe the Poisson's ratio dependent collapse mechanism and the overall mechanical behavior of the samples.

4.2 Static elastic properties – Analytical approach

Even though there have been developments in the manufacturing of these composites and their mechanical properties [17,18], there still no unified theory that describes their elastic behavior.

In the early 1980's Gibson *et al* [19,20] determined Eqs. 1 and 2 (Table 4.1) to calculate the fundamental elastic properties of honeycombs. It is known that the latter have a good correlation with flexure dominated deformation, *i.e.* honeycombs with reduced wall thickness and low specific densities.

Table 4.1 Theoretical models for the elastic properties of honeycomb lattices.

Author	Ref.	E^* Apparent Modulus	ν Poisson's ratio
Gibson (1982)	[19,20]	$\left(\frac{t}{l}\right)^3 \frac{E_s \cos(\alpha)}{\left(\frac{h}{l} + \sin(\alpha)\right) \sin^2(\alpha)} \quad (1)$	$\frac{\cos^2(\alpha)}{\left(\frac{h}{l} + \sin(\alpha)\right) \sin(\alpha)} \quad (2)$
Masters (1996)	[21]	$\left(b \left(\frac{h}{l} + \sin(\alpha)\right) \left[\frac{\sin^2(\alpha)}{K_f \cos(\alpha)} + \frac{\sin^2(\alpha)}{K_h \cos(\alpha)} + \frac{\cos(\alpha)}{K_s} \right]\right)^{-1} \quad (3)$ OBS: $K_f = \frac{E_s b t^3}{l^3} \quad (5) \quad K_h = \frac{G_s b t}{l} \quad (6) \quad K_s = \frac{E_s b t}{l} \quad (7)$	$\frac{-\sin(\alpha) \cos(\alpha) \left[\frac{1}{K_f} + \frac{1}{K_h} - \frac{1}{K_s} \right]}{\left(\frac{h}{l} + \sin(\alpha)\right) \left[\frac{\sin^2(\alpha)}{K_f \cos(\alpha)} + \frac{\sin^2(\alpha)}{K_h \cos(\alpha)} + \frac{\cos(\alpha)}{K_s} \right]} \quad (4)$
Malek (2015)	[22]	$E_s \left(\frac{t}{l_b}\right)^3 \frac{\cos(\alpha)}{\left(\frac{h}{l} + \sin(\alpha)\right) \sin^2(\alpha)} \left[\frac{1}{1 + (2.4 + 1.5\nu + \cot^2(\alpha)) \left(\frac{t}{l_b}\right)^2} \right] \quad (8)$ OBS: $l_b = l - \frac{t}{2 \cos(\alpha)} \quad (10)$	$\frac{\cos(\alpha)}{\left(\frac{h}{l} + \sin(\alpha)\right) \sin^2(\alpha)} \left[\frac{1 + (1.4 + 1.5\nu) \left(\frac{t}{l_b}\right)^2}{1 + (2.4 + 1.5\nu + \cot^2(\alpha)) \left(\frac{t}{l_b}\right)^2} \right] \quad (9)$
Hedayati (2016)	[23]	$\left(\frac{t}{l}\right)^3 \frac{l \sin(\alpha)}{h(\cos(\alpha) + 1)(\sin^2(\alpha)) \left(\frac{t}{l}\right)^2 + \cos^2(\alpha)} \quad (11)$	$\frac{l \sin^2(\alpha) \cos(\alpha) (l^2 - t^2)}{(t^2 \sin^2(\alpha) + l^2 \cos^2(\alpha))(h + l \cos(\alpha))} \quad (12)$

To address these limitations, in the 1990's Masters *et al* [21] included the effect of rib/wall stretching and hinging, interpreting these effects of honeycomb cell geometry in the overall cell stiffness. The fundamental elastic properties are determined by Eqs. 3 and 4 (Table 4.1), being supported by the calculation of the stretching, hinging and flexure stiffness constants (respectively K_s , K_h and K_f – Eqs. 5 to 7 in Table 4.1).

Recently, Malek *et al* [22] determined Eqs. 8 and 9 (Table 4.1) to compensate the effect of thick walled honeycombs and further approximated the values of the theoretical models to the experimental results. A similar approach was performed by Hedayati *et al* [23], in which Eqs. 11 and 12 (Table 4.1) are deduced to model the elastic properties of thick honeycombs.

These models are frequently adopted in two-dimensional honeycombs. However, due to the recent approaches in the manufacturing of three-dimensional honeycombs, there is a need to further analyze and adapt them to this context. Each cell in two- or three-dimensions may be divided into an Euler-Bernoulli beam system, as displayed in Fig. 4.1. It may be observed that the deformation behavior is dependent on the geometry of the cell, *i.e.* rib horizontal length (h), vertical length (l), thickness (t) and angle (α). In two dimensional approaches, the width (b) of the lattice can be also considered to influence the elastic properties. However, this aspect may be simplified in three-dimensional cells if the width is equal to the rib thickness ($b=t$).

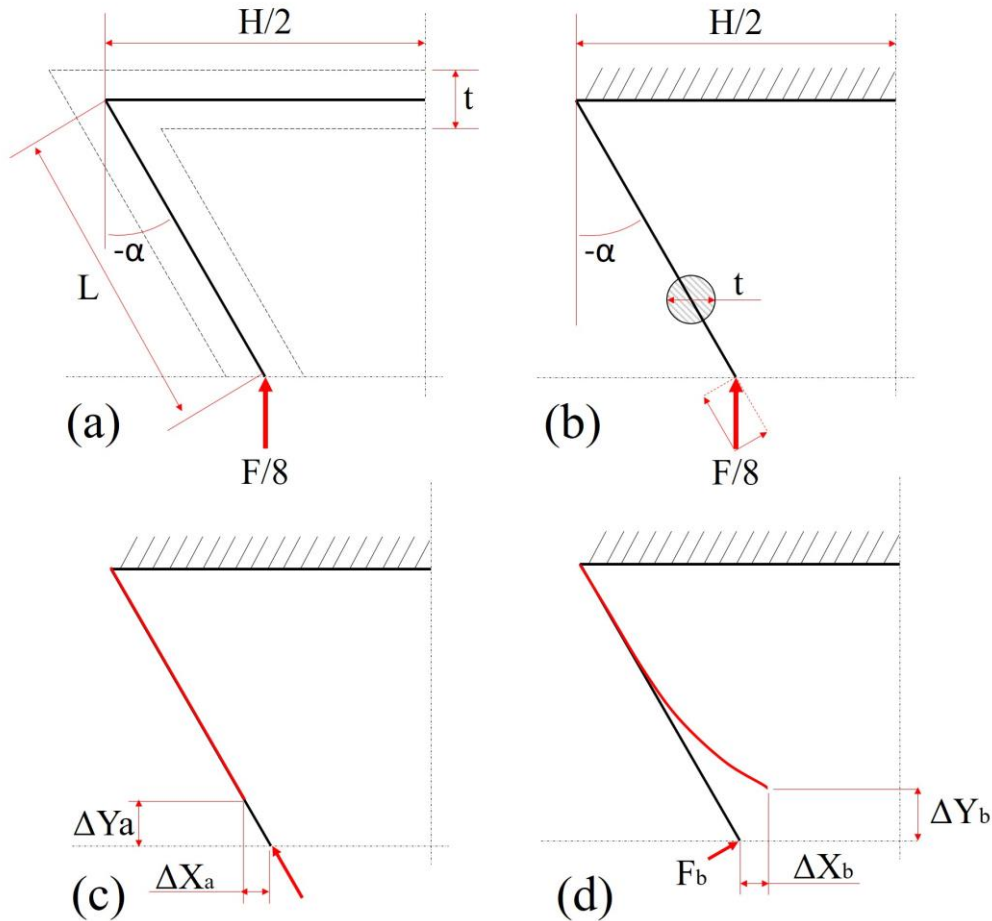


Figure 4.1 Honeycomb deformation mechanism: (a) initial dimensions and loading; (b) load distribution; (c) axial deformation; and (d) flexural deformation.

If a load (F) is applied in the structural system (Fig. 4.1 (b), axial (F_a – Fig. 4.1 (c)) and flexural (F_b – Fig. 4.1 (d)) components are introduced and applied in the ribs. Overall, the vertical (ΔY_a and ΔY_b) and horizontal (ΔX_a and ΔX_b) deformations in the ribs are proportional to the loading by axial and flexural stiffness coefficients (respectively, K_a and K_b – Eqs. 13 and 14). Given that these stiffness coefficients are not dimensionless magnitudes, they are dependent not only on the geometry of the cell, but also on the fundamental elastic modulus of the base material (E_0).

$$K_a = \frac{\pi t^2 E_0}{2 (\cos(\alpha))^2 H} \quad (13)$$

$$K_b = \frac{3\pi t^4 E_0}{8 (\sin(\alpha))^2 H^3} \quad (14)$$

Fig. 4.2 represents the plots of Eqs.13 and 14. As it would be expected, when $\alpha=0^\circ$, the axial stiffness coefficient (K_a) tends to a maximum as the cell tends to a square configuration and there are no flexure solicitations. Thus, for the same angle, the flexure stiffness coefficient tends to a minimum.

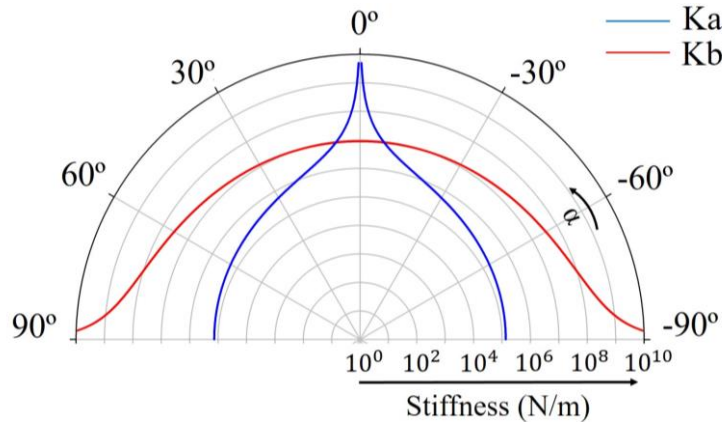


Figure 4.2 Plot of the proposed model for axial (K_a) and flexural (K_b) stiffness coefficients (Note: $t=0.6\text{mm}$; $H=4\text{mm}$; and $E_0=70\text{ GPa}$).

According to Masters *et al* [21] honeycomb deformation model (Table 4.1), it is fundamental to consider a rib hinging effect to determine their elastic properties. Considering the referred model, this hinging effect may be compensated by the inclusion of a hinging stiffness coefficient (K_h – Eq. 6), that is dependent on the cell with (b), thickness (l), length (l) and base material shear modulus (G_0).

The present study considers that this hinging effect is also dependent on the rib angle (α). According to this hypothesis, in situations where the rib angle is negative (Fig. 4.3 (a)), zero (Fig. 4.3 (b)) and positive (Fig. 4.3 (c)), the hinging effect is substantially different. Fig. 4.3 shows that in negative rib

angles, there is a double closing effect, while in positive values, there seems to be a single closing effects. Thus, honeycombs with negative rib angles (*i.e.* auxetic honeycombs) display must display a higher hinging stiffness coefficient.

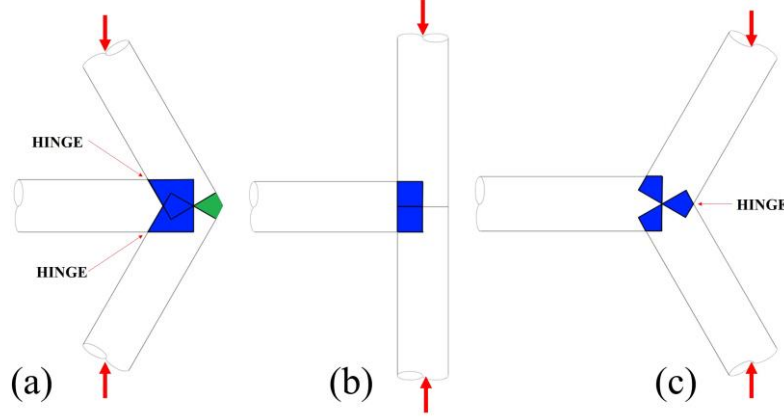


Figure 4.3 Hinging deformation mechanism in honeycombs with: (a) negative, (b) zero and (c) positive rib angles.

To compensate for this rib angle dependent hinging stiffness variation, a hinging coefficient (q – Eq. 15, Fig.4.4) is introduced in the reformulated hinging stiffness coefficient (K_h – Eq. 16).

$$q = \tan(\alpha) \quad (15)$$

$$K_h = \frac{2 t^2 G_0}{H q} \quad (16)$$

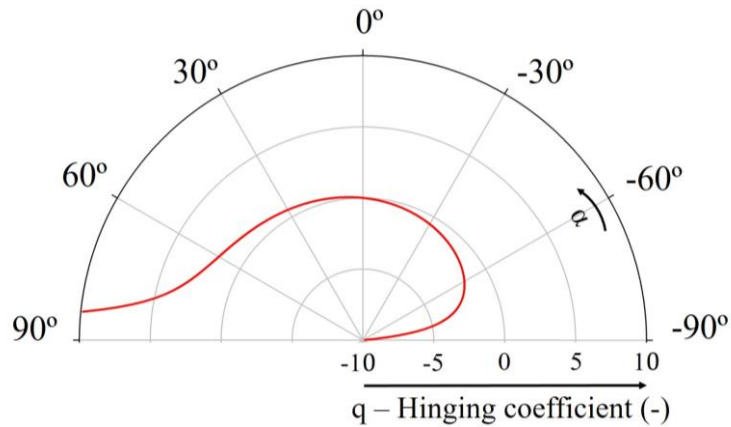


Figure 4.4 Plot of the proposed model for correction hinging coefficient (q).

All the stiffness coefficients (K_a , K_b and K_h) may be correlated by a simple addition operation of springs in series to calculate a total stiffness coefficient (K_T), according to Eq. 17. Fig. 4.5 represents the plot of K_T , showing that the maximum stiffness value occurs near $\alpha=0^\circ$.

$$K_T = \frac{1}{\frac{1}{K_a} + \frac{1}{K_b} + \frac{1}{K_H}} \quad (17)$$

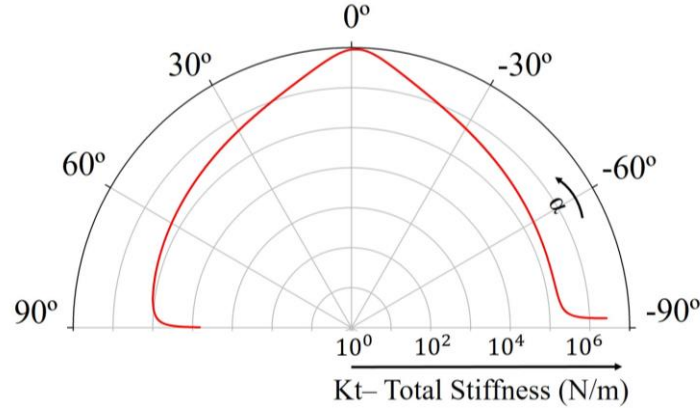


Figure 4.5 Plot of the proposed model for total stiffness coefficient (K_t). (Note: $t=0.6\text{mm}$; $H=4\text{mm}$; $E_0=70\text{GPa}$).

The apparent elastic modulus of the modeled honeycombs (E^*) may be determined using Hooke's law (Eq. 18). In which A^* (Eq. 19 – Fig.4.6) and L^* (Eq. 20 – Fig.4.7) are, respectively, the initial honeycomb apparent resistant area and height. Fig. 4.5 shows that, for the experimentally tested rib angles ($\alpha=-30^\circ$ to 30°), the total stiffness coefficient (K_T) is fairly symmetric for an axis that passes through $\alpha=0^\circ$. Thus, overall, the different values of apparent modulus for symmetrical rib angles are unbalanced by different apparent areas (A^*).

$$E^* = \frac{K_T L^*}{A^*} \quad (18)$$

$$A = [H(1 + \sin(\alpha))]^2 \quad (19)$$

$$L = H \cos(\alpha) \quad (20)$$

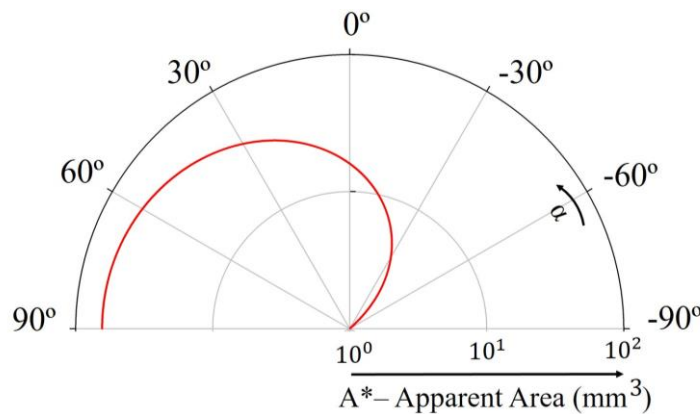


Figure 4.6 Plot of the initial apparent area (A^*). (Note: $t=0.6\text{mm}$; $H=4\text{mm}$; $E_0=70\text{GPa}$).

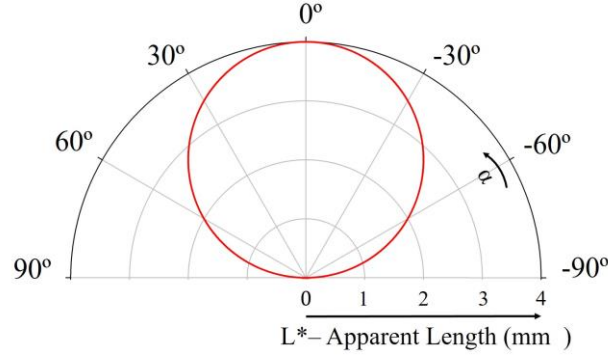


Figure 4.7. Plot of the initial apparent length (L^*). (Note: $t=0.6\text{mm}$; $H=4\text{mm}$; $E_0=70\text{GPa}$).

Fig. 4.8 shows the plot of Eq.18. According to the analytical model, the maximum values of apparent modulus are expected in the vicinity of $\alpha=0^\circ$. This is due to the high values of the axial stiffness coefficient in this angle range (Fig.4.2) and is supported by other analytical honeycomb theories [19–23].

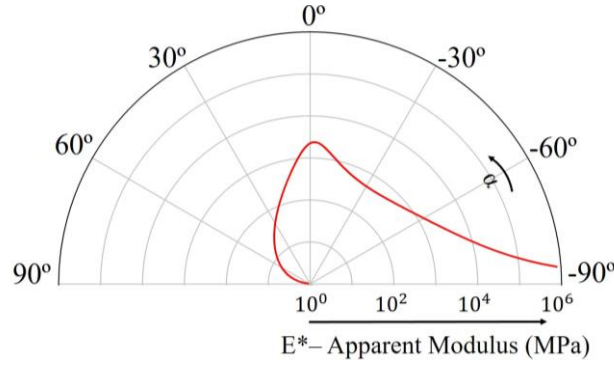


Figure 4.8 Plot of the proposed model for apparent modulus (E^*). (Note: $t=0.6\text{mm}$; $H=4\text{mm}$; $E_0=70\text{GPa}$).

From Fig. 4.1, it is possible to observe that the Poisson's ratio (ν) of the honeycombs is generated by the referred axial and horizontal deformations (ΔY_a , ΔY_b , ΔX_a and ΔX_b). It is suggested that the imposed deformations may be determined by Eqs. 21 to 24.

$$\Delta Y_{axial} = \frac{2 F (\cos(\alpha))^2 H}{\pi t^2 E_0} \quad (21)$$

$$\Delta Y_{bending} = \frac{8 F (\sin(\alpha))^2 H^3}{3\pi t^4 E_0} \quad (22)$$

$$\Delta X_{axial} = \frac{2 F \cos(\alpha) \sin(\alpha) H}{\pi t^2 E_0} \quad (23)$$

$$\Delta X_{bending} = \frac{8 F \cos(\alpha) \sin(\alpha) H^3}{3\pi t^4 E_0} \quad (24)$$

According to the suggested model, the Poisson's ratio (ν) of the modeled honeycombs may be determined by Eq. 25, in which p (Eq. 26) is a correction factor to compensate hinging effect in the ribs. Fig. 4.9 displays the plot of the correction factor (p) due to the hinging effect shown in Fig.4.3, in which it is shown that for negative values it tends to reduce the axial deformation by bending (ΔY_b), while the opposite happens to positive rib angles.

$$\nu = -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{\Delta X_{axial} + \Delta X_{bending}}{A^*} \frac{L^*}{\Delta Y_{axial} + \frac{2\Delta Y_{bending}}{p}} \quad (25)$$

$$p = \cos\left(\alpha - \frac{\pi}{2}\right) + 1 \quad (26)$$

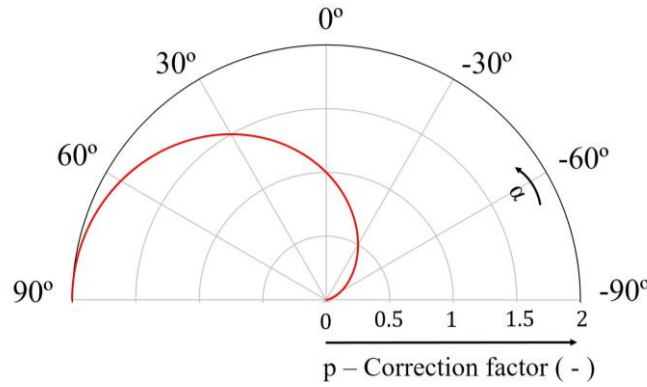


Fig.4.9 Plot of the proposed model for correction hinging coefficient (p). (Note: $t=0.6\text{mm}$; $H=4\text{mm}$; $E_0=70\text{GPa}$).

Fig.4.10 shows the plot of Eq.25, in which it may be observed that negative rib angles generate negative values of Poisson's ratio (*i.e.* auxetic behavior), while positive rib angles imply positive values of this elastic constant. In transition regimes, the plot assumes an asymptote shape that is characteristic in other published honeycomb deformation models.

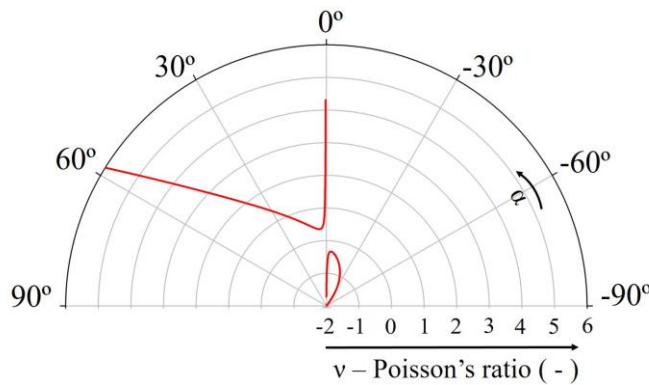


Figure 4.10 Plot of the proposed model for Poisson's ratio (ν). (Note: $t=0.6\text{mm}$; $H=4\text{mm}$; $E_0=70\text{GPa}$).

4.3 Methodology for static structural characterization

4.3.1. Experimental quasi-static uniaxial compression

Samples were subjected to uniaxial compression testing using an INSTRON 8874 universal testing equipment. Five specimens for each rib angle were placed between two steel clamps (Fig. 4.11 (a)) and pre-stressed with a 50N load. A 0.01 mm/s displacement was imposed in the upper clamp to compress the samples while the loads were measured by the equipment load-cell. Axial deformations were measured by placing a mechanical strain gauge in one of the sample faces. The Poisson's ratio of the samples was measured using an additional transverse mechanical strain gauge (Fig. 4.11 (b)). Additional samples (three for each tested rib angle) were also tested without strain gauges and video recorded to monitor their global and local deformation behavior.

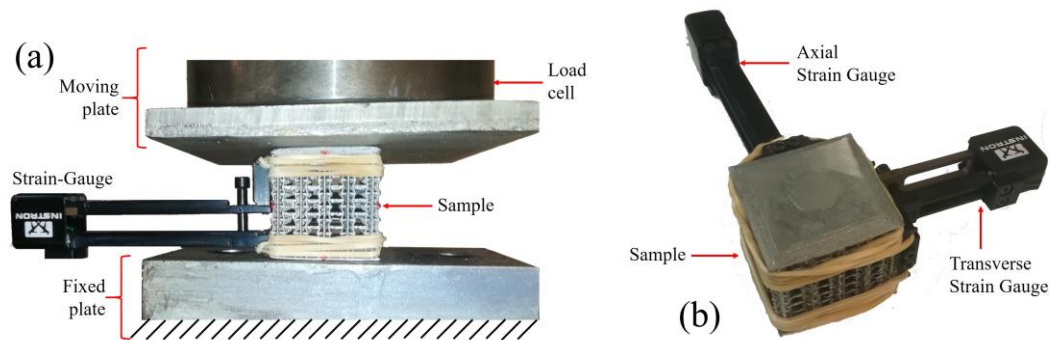


Figure 4.11 Compression testing: (a) Experimental apparatus and (b) Poisson's ratio calculation with axial and transverse strain gauges.

4.3.2. Finite element analysis – Uniaxial compression simulation

Finite element analysis (FEA) was performed in parametrized models to determine the fundamental mechanical behavior and collapse mechanism with more detail that may be obtained in the experimental compression tests. Initially, a model that represents the manufactured lattices was designed to study them in structural simulations. According to Fig. 4.12, an eighth (1/8) of a samples was modeled in ANSYS 17. The lattice ribs were modeled as unidimensional beam elements (*BEAM188*), with an upper top two-dimensional rigid plate (*SHELL181*). Symmetry was imposed in the YX, ZX and YZ mid-sample planes to simplify the simulations. A compression displacement of 1 mm is imposed in the top plate to simulate the experimental compression testing. This reduces the overall simulation processing time/complexity and helps with convergence, while representing the deformation behavior that is observed in the experimental tests.

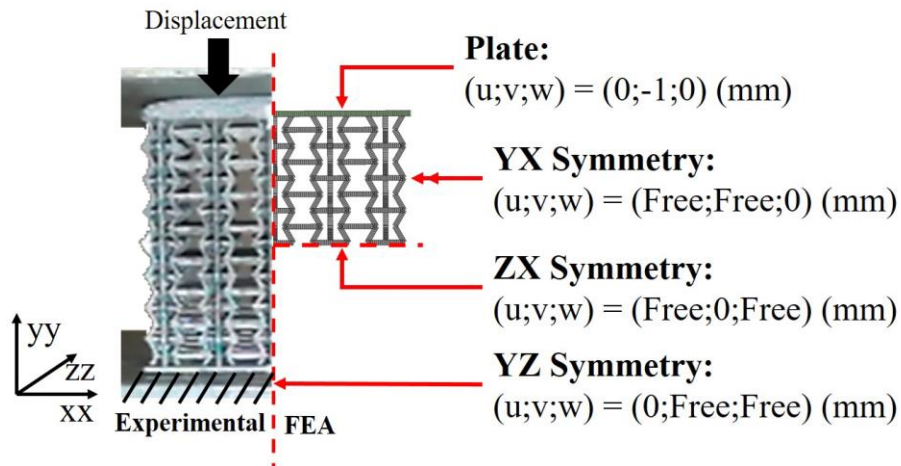


Figure 4.12 Representation of FEA model with boundary conditons.

According to Fig. 4.13, it may be observed that an additional two dimensional fixed rigid plate is added to the bottom of the model. This plate was found to facilitate the conversion of the simulation allowing the contacting ribs to slide in the ZX symmetry plane. Thus, while the contact between the ribs and the top plate was imposed as Bonded, the contact with the lower plate is modeled as Frictionless. The overall mesh for the adopted Beam (*BEAM188*) and Plate (*SHELL181*) elements has a 0.2 mm size. It was observed that this size is able to generate a good convergence of the results, while maintaining acceptable discretization of the model and low processing time.

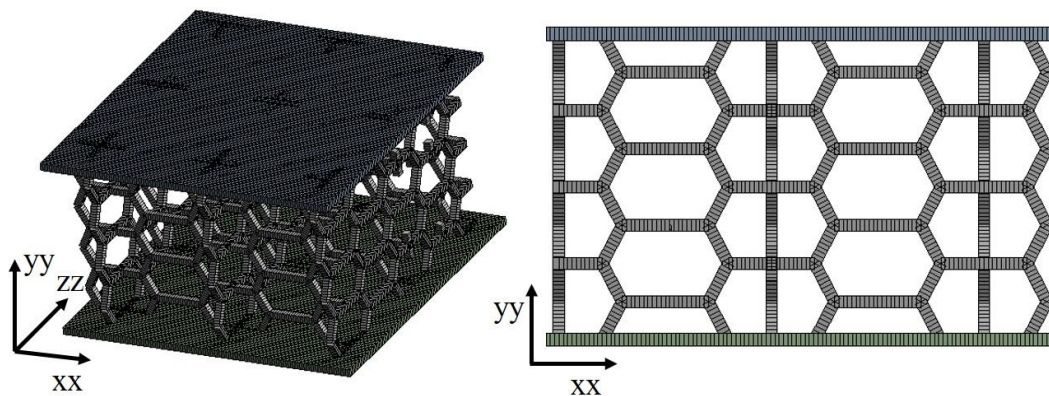


Figure 4.13 Representation of model meshing.

The model was designed with the linear dimensions (according to Chapter 3 – Fig.3.3), given that these dimensions of the casted samples do not present significant changes relatively to the CAD models. However, according to the conclusions of the referred chapter, this is not true for the rib thickness and shape. Thus, the beam moment of inertia in the simulations were corrected to a circular shape and parametrized in terms of rib thickness in accordance to the values of Fig.14.

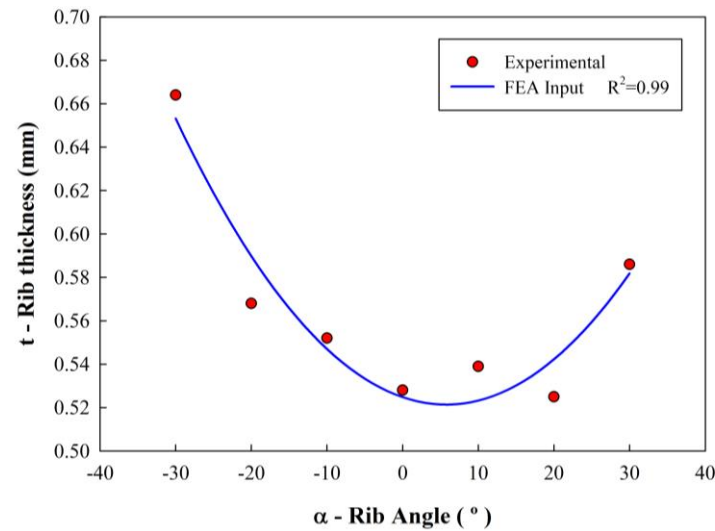


Figure 4.14 FEA input values for rib thickness.

Material properties input was based in the optimization of heat treated A356 alloy that is established in Chapter 2. The stress-strain behavior of the alloy is presented in Fig.4.15.

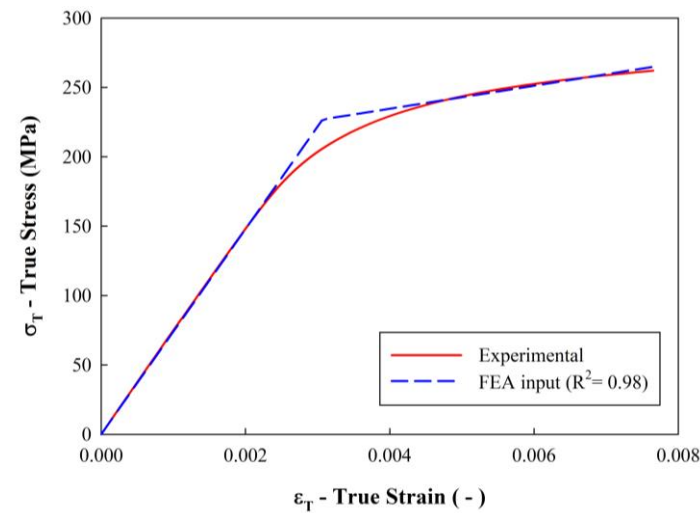


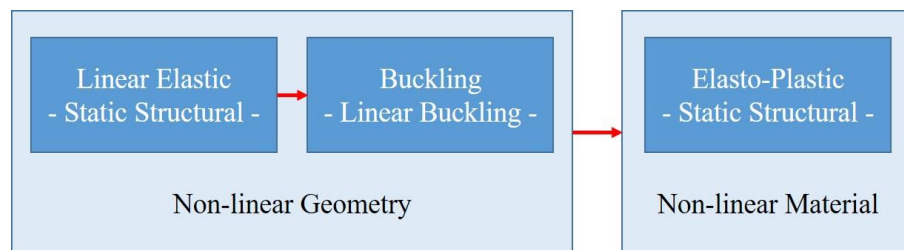
Figure 4.15 Plot of FEA input stress-strain material model.

According to the referred figure, the alloy behavior was modeled using a bilinear isotropic strain hardening formulation. In both elastic and plastic regimes, the FEA input has a considerable approximation to the experimental curve. Basic mechanical properties (E , ν and E_T) are shown in Table 4.2. Their difference consists in the approach for the transition period in the elasto-plastic regime. While the real material has a smooth transition between the two domains that is initiated by the yield strength (σ_y), in the FEA routine this transition is instantaneous. The difference, however, is not significant due to the overall high correlation of both functions (Fig.4.15) and the use of *BEAM188* elements, where there is no stress/strain variation through the rib thickness.

Table 4.2 FEA input for material mechanical properties.

Property	Experimental	FEA input
E - Young's Modulus (GPa)	71.8	
ν - Poisson's ratio (-)	0.3	
σ_y - Yield strength (MPa)	193	226
E^* - Tangent modulus (GPa)	0.3	

These inputs were introduced in a coupled analysis using ANSYS 17 (Fig.4.16). Given that rib buckling in the samples may have a relevant role in the collapse mechanism, an initial linear elastic static structural analysis was coupled to a buckling analysis. This determines if there are relevant buckling deformations by the determination of the load value for the first eigenvalue. Such approach is intended to determine if there is non-linear geometric behavior in the samples and, if so, to monitor how this will affect the sample.

**Figure 4.16** Graphical representation of the adopted coupled structural numerical analysis.

The results of this analysis are posteriorly coupled with the final static structural simulation that considers material non-linearity. This assures that the samples are able to fully mimic the deformations that are observed in the experimental testing.

4.3.3. 4D computed microtomography of the samples in compression

X-ray microcomputed tomography (4D μ CT) imaging was performed using a High-Flux Nikon XTEK bay μ CT scanner, while samples with different Poisson's ratio are compressed. Three samples ($\alpha=\{-30^\circ; 0^\circ; 30^\circ\}$) were mounted on compression rig that allows the scanning with X-rays while they are compressed to visualize their collapse mechanism. All scans were performed using a voltage of 160 kV, intensity of 262 μ A and projections were taken through 360° of rotation. The remaining imaging settings were optimized for each sample and are summarized in Table 4.3. Data was reconstructed using Nikon Metris CTPro software, then segmentation/ visualization was performed using Avizo standard.

Reconstructed 3D volume were exported as 2D tiff images to allow measurement of rib dimensions. The overall dimensions of the samples and the ribs were measured using the micrographs using Fiji.

Table 4.3 4D X-ray μ -CT testing parameters.

Parameter	Samples ($\alpha=\{-30^\circ;0^\circ;30^\circ\}$)
Voxel (μm)	45
Exposure time (ms)	205
Source-sample (mm)	222
Sample-Detector (mm)	989
Power (W)	42
Projections	3141
Filter	1.6 mm Al plate

4.4 Results and Discussion

4.4.1. Experimental quasi-static uniaxial compression

Fig. 4.17 displays the apparent stress-strain curves of the designed samples. It may be seen that the change in rib angle creates a significant difference in the compression behavior of the tested samples. It is apparent that negative rib angles promote an increase in apparent modulus (E^*) and plastic collapse stress (σ_P). However, positive rib angles seem to collapse more uniformly.

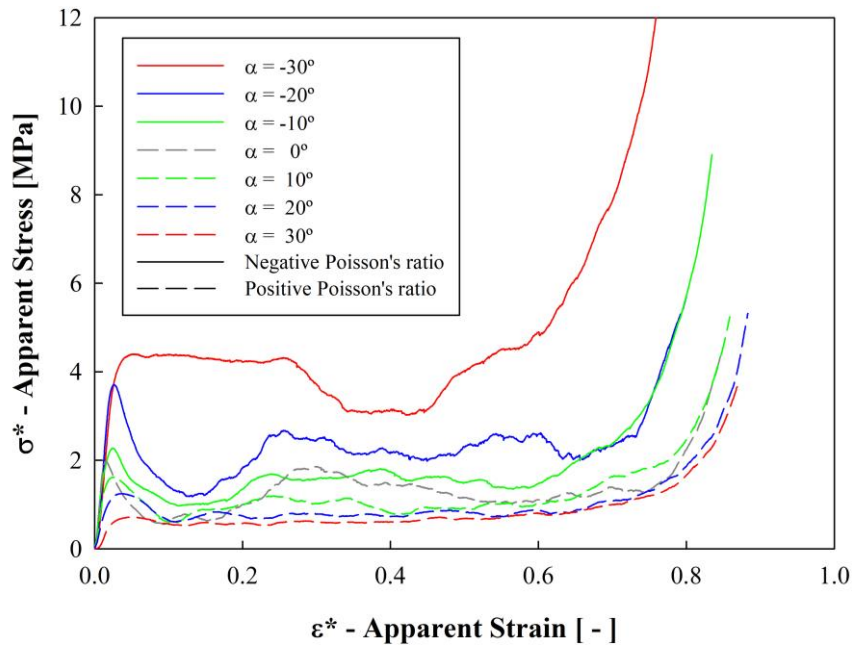


Figure 4.17 Average apparent stress-strain curves for the teste samples.

All tested samples shown a typical behavior of a cellular material: (i) a clear stage of linear elastic deformation followed by a plastic collapse (Fig. 4.18 (a)); (ii) collapse plateau with severe plastic deformation (Fig. 4.18 (b)); (iii) and sample densification (Fig. 4.18 (c)).

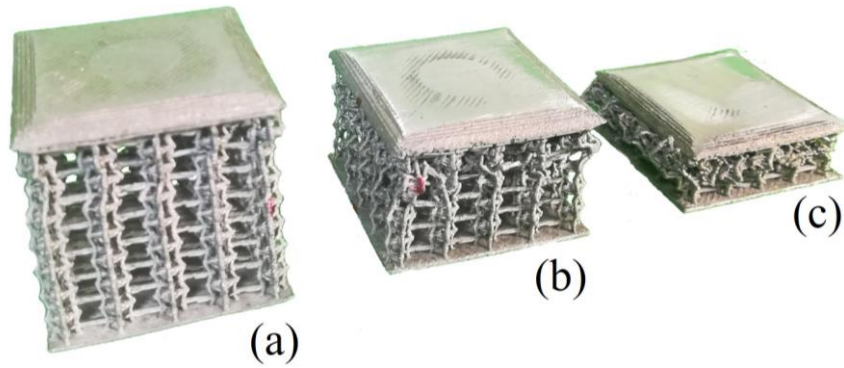


Figure 4.18 Sample ($\alpha = -30^\circ$) in compression testing: (a) After plastic collapse stress ($\varepsilon = 0.06$), (b) during compression plateau ($\varepsilon = 0.50$) and (c) during densification ($\varepsilon = 0.80$).

During the linear elastic stage, the axial and transverse deformations were monitored using mechanical strain gauges (Fig. 4.11 (b)). Correlating both deformations, the values of Poisson's ratio of the samples were calculated and may be observed in Fig. 4.19. As predicted, the negative rib angles generate a negative Poisson's ratio (*i.e.* auxetic behavior) in the samples. As the rib angle is increased to positive values, there is an inversion of this deformation behavior into positive Poisson's ratios (*i.e.* non auxetic behavior). When the rib angle is zero ($\alpha = 0^\circ$), the Poisson's ratio is approximately zero, implying the transition angle may occur in negative rib angles.

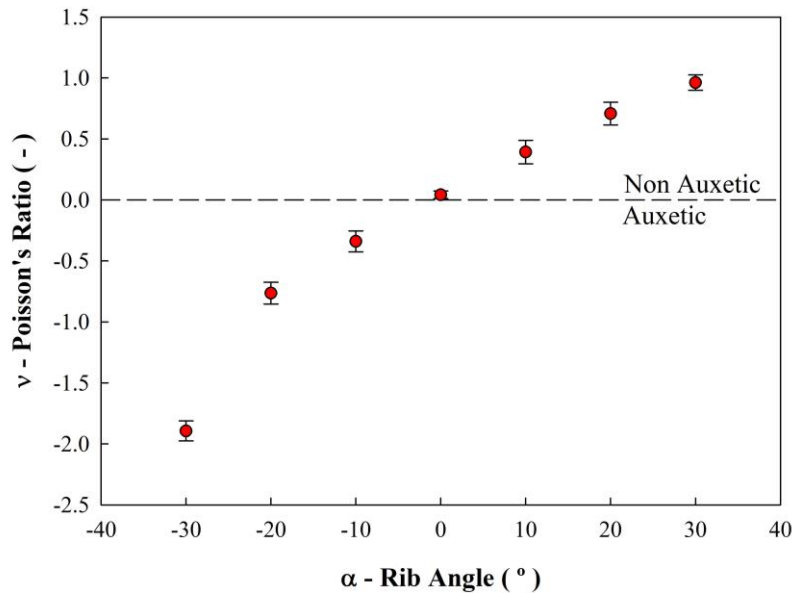


Figure 4.19 Average Poisson's ratio (ν) of the tested samples in uniaxial compression.

Fig. 4.20 shows the apparent modulus (E^*) in the initial linear elastic domain. It may be observed that there is a tendency for negative rib angles to display a higher stiffness. This may be attributed to the auxetic behavior of the samples and their densification due to resistant area reduction in compression.

Given that the deformation behavior of the samples with wider angles (positive or negative), are mainly regulated by rib flexure [19], the auxetic effect is able to compensate this reduction in stiffness. Additionally, this stiffness enhancement is able to promote higher values of apparent modulus even in the case of samples with pure axial loading (*i.e.* $\alpha=0^\circ$).

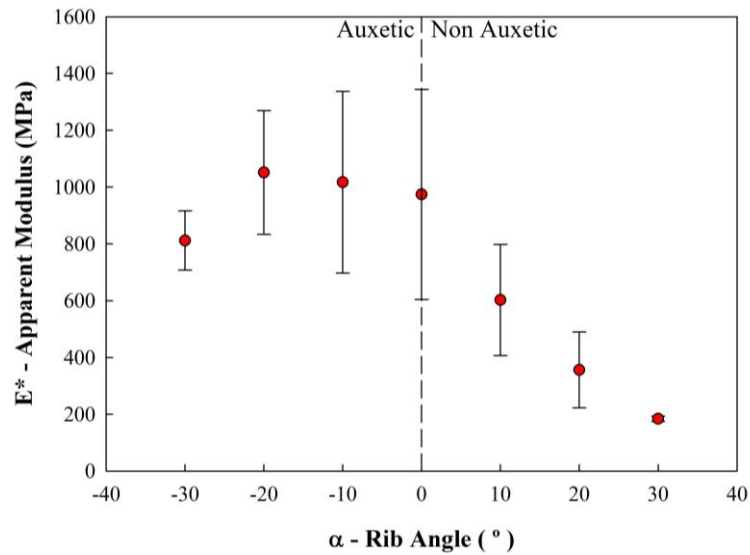


Figure 4.20 Average apparent modulus (E^*) of the tested samples in uniaxial compression.

This elastic domain is ended by the plastic collapse stress (σ_p), where highly localized elasto-plastic deformations are imposed in the sample. In terms of apparent stress-strain behavior, this event is reflected in the sudden drop in proportional loading that may be observed in Fig.4.17. The average values of plastic collapse stress for the different rib angles are displayed in Figure 4.21.

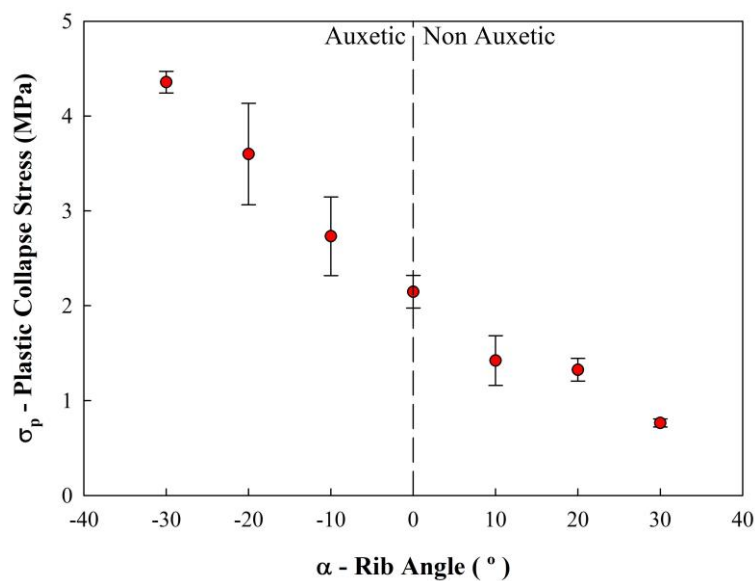


Figure 4.21. Average plastic collapse stress (σ_p) of the tested samples in uniaxial compression.

Again it is suggested that the auxetic behavior due to the adoption of negative rib angles are able to enhance the mechanical behavior of cellular materials. This is seen in the higher plastic collapse stress of the referred samples.

According to the plastic collapse that is portrayed in Fig. 4.21, it may be observed that samples with a negative Poisson's ratio (Fig. 22 (a) and (b)) show a localized collapse. Given that the samples tend to contract in compression, the samples themselves tend to block the progression of the collapse. Thus, this event is concentrated in a specific area in the limits of the sample, being this is justified by the lower stiffness of frontier cells that are not fully connected to other cells.

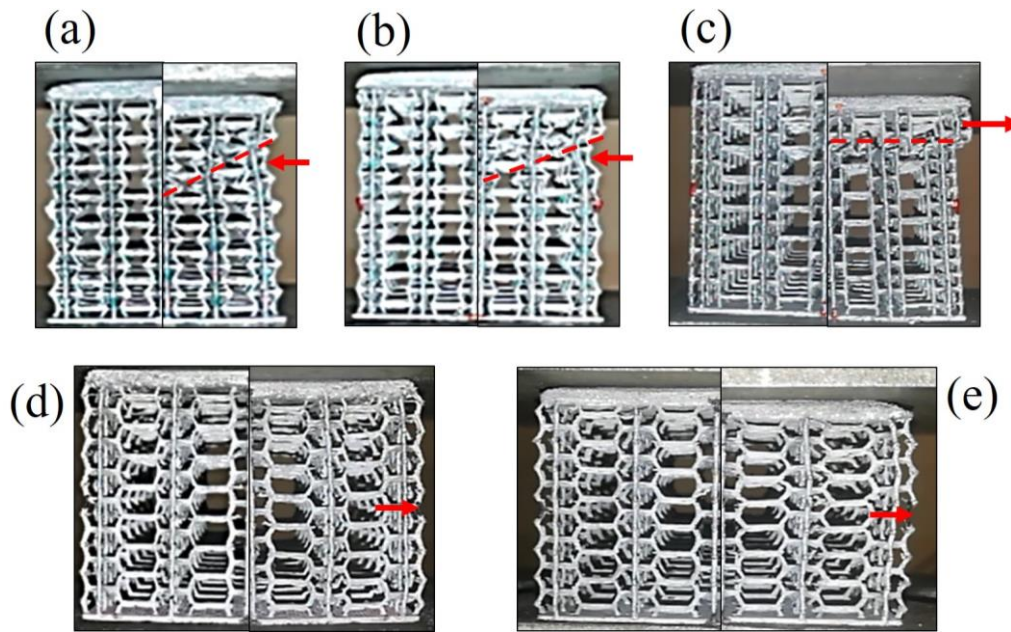


Figure 4.22 Deformation due to plastic collapse ($\epsilon=0.1$) of the samples with (a) -30° , (b) -20° , (c) 0° , (d) 20° and (e) 30° .

Fig. 4.22 (c) represents the transition regime, where the samples display a near zero Poisson's ratio. Thus, in this particular configuration, the cells also fail due to a sudden increase of localized elasto-plastic deformation. However, the pure axial compression loads generate buckling failure.

Samples with a positive rib angle (Fig. 4.22 (d) and (e)) display a deformation behavior that is faded along the sample face. Instead of a highly localized elasto-plastic deformation, these samples show a well distributed deformation. Due to their positive Poisson's ratio, there is no enhanced collapse resistance due to sample densification. Thus, these samples show a clear diagonal collapse that passes through the whole sample. This behavior is further noticeable as the rib angle is increased (*e.g.* transition from Fig. 4.22 (d) to (e)).

The explanation for the deformation behavior of the compressed samples is suggested in Table 4.4. Considering samples with negative Poisson's ratio, there is an e reduction in the cross-sectional area during the linear elastic regime of the compressed samples due to their auxetic behavior.

Table 4.4 Suggested deformation models for the compression of the designed samples.

Poisson's ratio	Linear Elastic	Plastic Collapse	Plateau
Negative			
Zero			
Positive			

Even though the deformation in these samples is dominated by rib flexure, there is a significant densification that enhances the overall stiffness. This is reflected in their high values of apparent modulus (Fig. 4.20). In auxetic samples, the plastic collapse is highly localized, and the angle of collapse is similar to the rib angle (Fig. 4.22 (a) and (b)). Given that this collapse deformation is concentrated in a single area, it is accompanied by a sudden and significant load reduction (Fig. 4.17). This initial collapse occurs in a single row of cells, and thus, this is also verified in the sequential cell row collapse in the compression plateau. These events may be verified by the plateau instability of auxetic samples in Fig. 4.17.

In transition situations (*i.e.* $\alpha=0^\circ$), the Poisson's ratio is near zero. Thus, in the initial linear elastic regime, there is no significant contraction or expansion in the sample faces. This implies that the samples have no densification effect caused by the auxetic behavior and the overall deformation behavior is due to the axial deformation of the ribs. The failure by plastic collapse (Fig. 4.21 (c)) occurs due to the buckling

of the ribs. This event is also highly localized and sequential cell rows during the compression plateau, being verified in the instability of the apparent stress-strain curve in Fig. 4.17.

After the transition, the samples are characterized by a positive Poisson's ratio. Thus, during the compression in the linear elastic regime, their cross-sectional area tends to increase. Without the densification effect that is displayed in the samples with auxetic behavior there is a significant reduction in the apparent modulus (Fig. 4.20). Their plastic collapse is well distributed along the sample faces. This significantly reduces the collapse stress (Fig. 4.21), however, it allows a smooth transition between the elastic and plastic domains.

4.4.2. Comparison with analytical model

Figs. 4.23 and 4.24 represent the plotting of the analytical model for honeycomb elastic properties that is suggested in this study, experimental and published classic honeycomb deformation models [19–23] (Table 4.1). All analytical models are compared with experimental results presented in section 4.4.1 on three-dimensional aluminum honeycombs ($h=4$ mm, $l=2$ mm, $t=0.6$ mm, $E_0=71$ GPa and $G_0=27$ GPa).

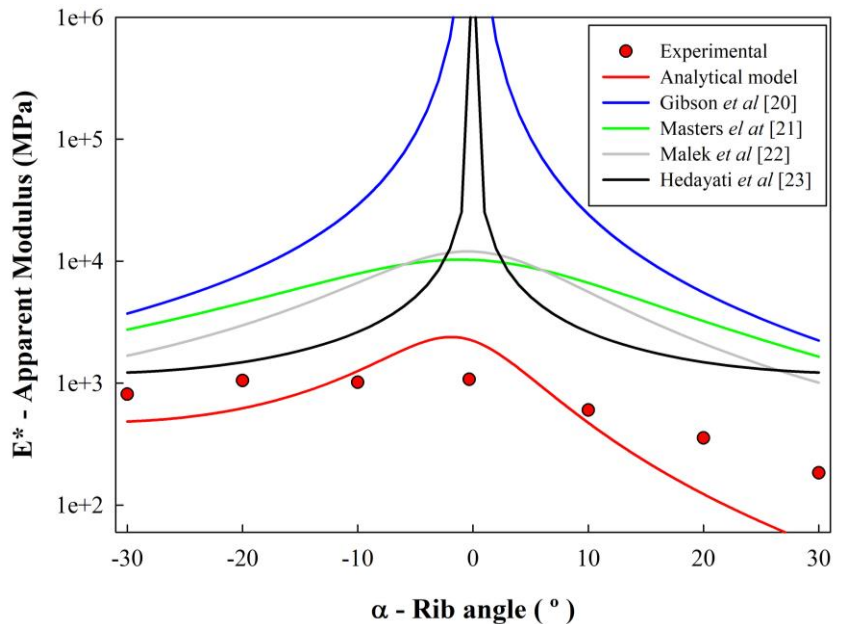


Figure 4.23 Comparison of the suggested analytical apparent modulus (E^*) with experimental results and other published models.

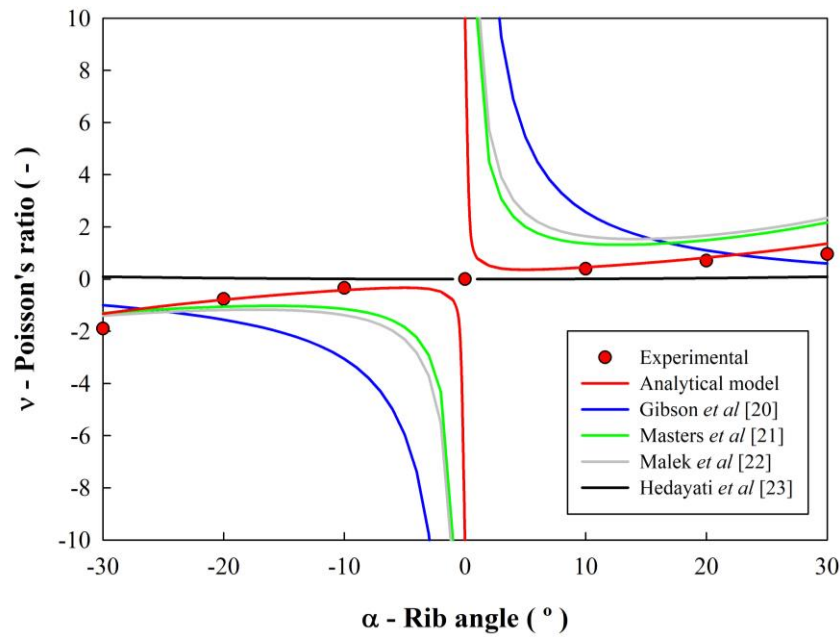


Figure 4.24 Comparison of the suggested analytical Poisson's ratio (ν) with experimental results and other published models.

It may be observed that the suggested model is able to display a good description of the experimental apparent modulus (E^*) and Poisson's ratio (ν) in comparison with other analytical models. While most models are fairly symmetrical between negative and positive rib angles, the suggested model is able to adapt itself to the changes in hinging stiffness.

A final validation of the suggested analytical model may be observed in Table 4.5, in which the plot are fitted with the experimental results. It is shown that the presented model is able to display a significant higher correlation to the experimental data, relatively to the other honeycomb deformation models.

Table 4.5 Correlation of the experimental values with the theoretical models.

Model	E^*	ν
	Apparent modulus correlation (R^2)	Poisson's ratio correlation (R^2)
Analytical (This study)	0.71	0.94
Gibson <i>et al</i> [19,20]	0.20	0.01
Masters <i>et al</i> [21]	0.41	0.64
Malek <i>et al</i> [22]	0.13	0.60
Hedayati <i>et al</i> [23]	0.12	0.01

4.4.3. Finite element analysis – Uniaxial compression simulation

While discrete values of rib angles were analyzed in the experimental approach, the performed FEA analysis was used to obtain a continuous analysis of the influence of rib angles. By the monitoring of the instant values of reaction loading and plate displacement it was possible to determine the influence of rib angle on the apparent modulus and plastic collapse stress (Figs. 4.25 and 4.26).

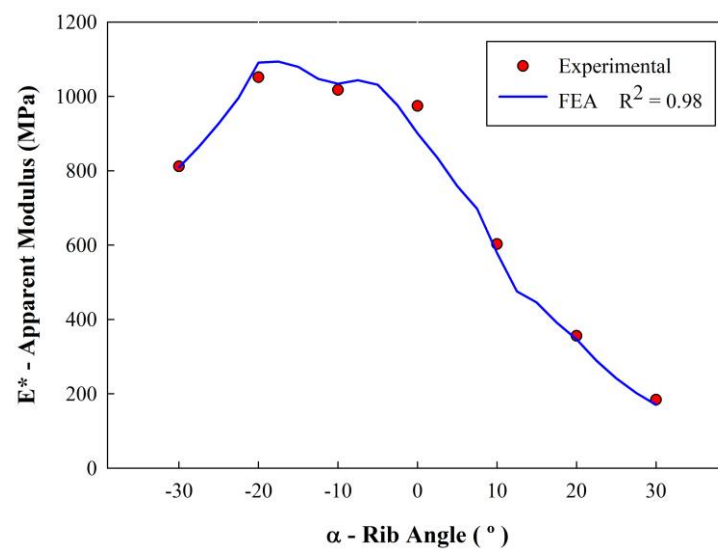


Figure 4.25 FEA results for apparent modulus (E^*).

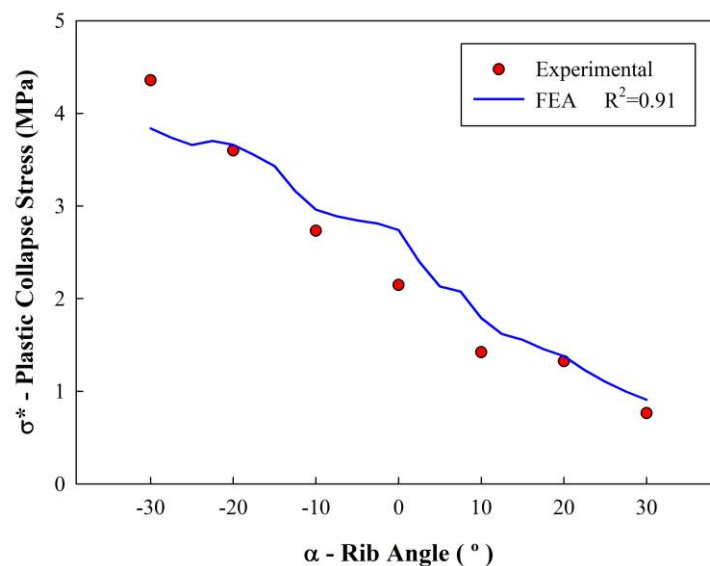


Figure 4.26 FEA results for plastic collapse stress (σ_P).

It may be observed by the high correlation values, that the FEA results are able to fully describe the behavior that is expressed in the experimental results. In conclusion, it is suggested that there is no variation in the tendency displayed by the discrete data of experimental testing. Additionally, Figs. 4.25

and 4.26 are able to validate the numerical analysis. Given the validation of the FEA routines, they may be used to analyze the collapse mechanism of the samples. In section 4.4.1 (Table 4.4), it is hypothesized that the collapse mechanism of the manufactured lattices is dependent on the sample Poisson's ratio. Fig. 4.27 are represents the deformations in the XX axis in samples with different Poisson's ratios.

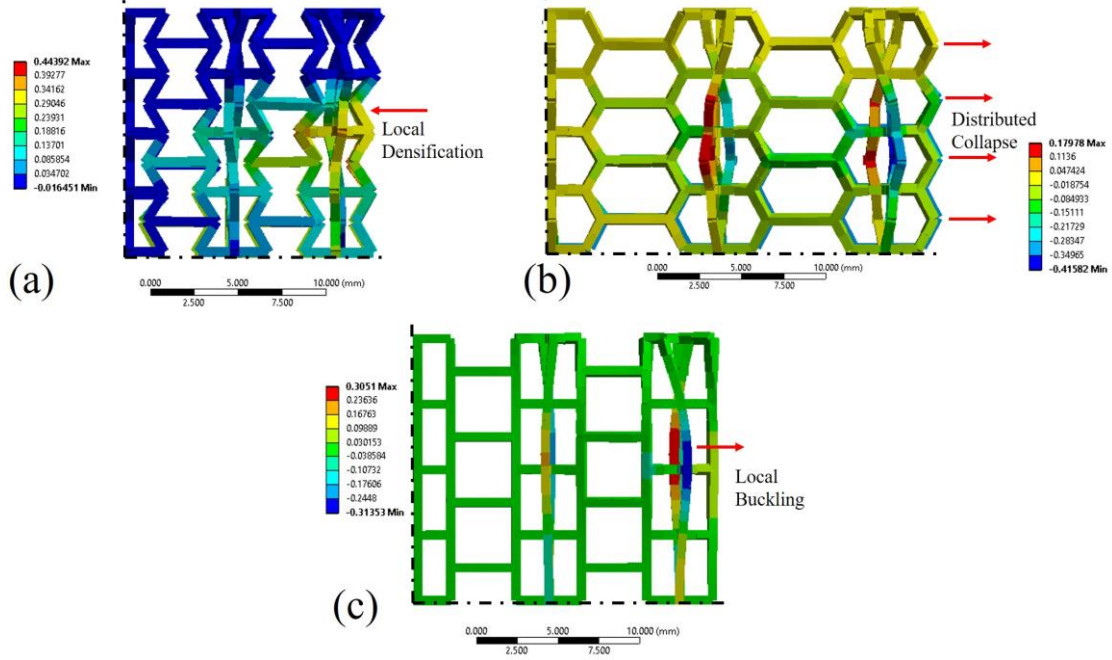


Figure 4.27 Graphical representation of FEA in samples with: (a) negative, $\alpha = -30^\circ$, (b) positive, $\alpha = 30^\circ$ and (c) zero, $\alpha = 0^\circ$ Poisson's ratio.

It is suggested in Fig. 4.27 (a) that samples with a negative Poisson's ratio (*i.e.* auxetic behavior) collapse by the deformation of a single row of cells. Given the negative Poisson's ratio, the collapsing row reduces its width and a local densification effect may be observed. In samples with a positive Poisson's ratio (Fig. 4.27 (b)), given that the sample tends to bulge, such collapse is well distributed along the samples lateral face. This behavior is characteristic of regular materials in compression.

Transition regimes, in which the Poisson's ratio is near zero (Fig. 4.27 (c)), the collapse occurs in a hybrid regime. The buckling occurs in a particular row and is localized, however, there is no densification in the sample. The deformations that are observed in the FEA simulation results are able to validate the hypothesis that is suggested in Table 4.4.

4.4.4. 4D X-ray μ CT – Validation of collapse mechanism

4D X-ray μ -CT was done in three samples in compression, according to the experimental uniaxial compression and FEA described in sections 4.4.1 and 4.4.3. This analysis is intended as an *in-situ*

approach for a final validation of the Poisson's ratio dependent collapse mechanism proposed in section 4.4.1 (Table 4.4). It may be seen that an initially undeformed auxetic scaffold (*i.e.* $\alpha=30^\circ$ - Fig. 4.28 (a)) is compressed, it starts to contract its exterior faces in the elastic regime ($\varepsilon=0.01$ - Fig. 4.28 (b)). After its collapse ($\varepsilon=0.04$ - Fig. 4.28 (c)), one of the rows starts to densify and the deformation is localized according to the model suggested in Table 4.4.

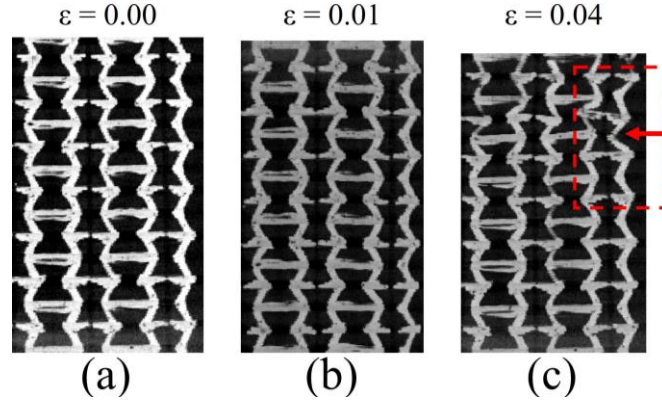


Figure 4.28 4D X-ray μ -CT of the auxetic sample ($\alpha=30^\circ$): (a) undeformed – $\varepsilon=0$; (b) elastic deformation – $\varepsilon=0.01$; (c) post-collapse – $\varepsilon=0.04$.

In samples with hexagonal honeycomb cells (Fig. 4.29 (a)), that are solicited to deformations in the elastic regime (Fig. 4.29 (b)) it may be observed that they bulge as they are compressed, *i.e.* they display a positive Poisson's ratio. The collapse of these samples occurs by a well distributed deformation in the faces of the samples (Fig. 4.29 (c)), in accordance with the model proposed in Table 4.4.

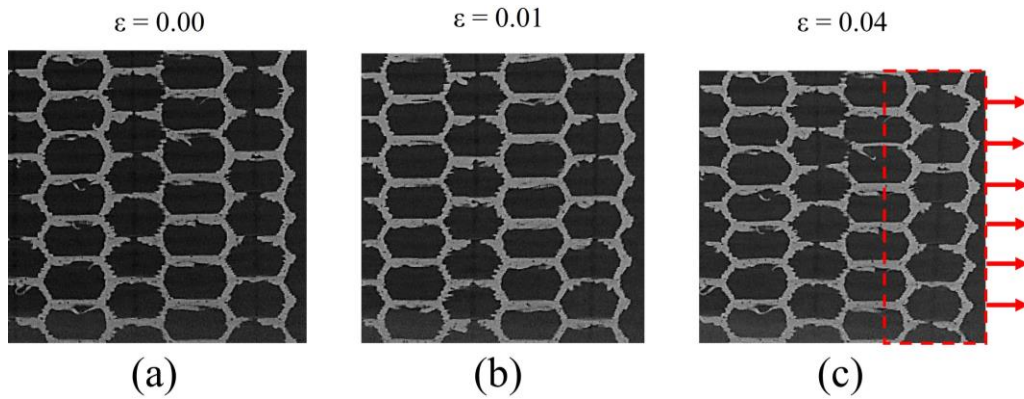


Figure 4.29 4D X-ray μ -CT of the positive Poisson's ratio sample ($\alpha=30^\circ$): (a) undeformed – $\varepsilon=0$; (b) elastic deformation – $\varepsilon=0.01$; (c) post-collapse – $\varepsilon=0.04$.

In samples with a transition Poisson's ratio regime (Fig.30), in which the value of this constant approximates zero ($\alpha=0^\circ$), the collapse mechanism is a hybrid of the results in Figs. 4.28 and 4.29.

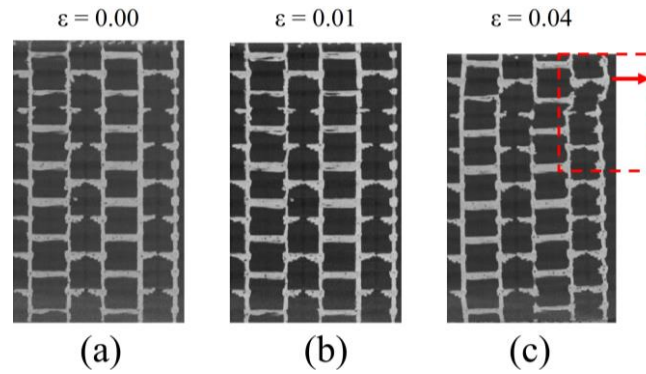


Figure 4.30 4D X-ray μ CT of the near zero Poisson's ratio sample ($\alpha=0^\circ$): (a) undeformed – $\varepsilon=0$; (b) elastic deformation – $\varepsilon=0.01$; (c) post-collapse – $\varepsilon=0.04$.

Using X-ray μ -CT, it may be seen that when an initially unloaded sample (Fig. 4.30 (a)) is compressed in the elastic regime, the deformation is distributed along the faces (Fig. 4.30 (b)). However, it is hardly noticeable due to the near zero Poisson's ratio. Posteriorly to the plastic collapse stress, the sample starts to bulge in localized cell rows, according to Fig. 4.30 (c)). Once again, these results are able to validate the hypotheses that was initially established in Table 4.4.

4.5. Comparison with other materials

To compare the performance of the designed cellular solids, their fundamental static mechanical properties (*i.e.* load and deformation bearing capacity) were compared with other current solutions. For that comparison, the specific modulus and strength (E^*/E_0 and σ^*/σ_0) of published data (Table 4.6) were compared with the experimental results that were determined in this study. Given that only cellular solids are analyzed, the referred properties are also correlated in terms of specific density (ρ^*/ρ_0).

Table 4.6 Materials that were analyzed for comparison.

Material	Method	Poisson's ratio (-)	Ref.
Al alloy	Foaming	Positive	[24–27]
	SLM		[28]
	Cast	Negative	[29]
316 L	SLM	Positive	[30]
Ti-6Al-4V			[31,32]
	EBM	Negative	[33–35]
Ti	Powder metallurgy	Positive	[18]
Mg			[36]
Ni			[37]
Maraging steel	Powder bed fusion	Negative	[38]
Polyurethane	Thermomechanical		[39]
Polymethylmethacrylate	Laser cutting		[40]
			[41]

Fig. 4.31 shows the plot of the specific modulus (E^*/E_0) relatively to the specific density (ρ^*/ρ_0) of the other published data [18,24–27,29–31,33–36,38–41] and the experimental results of this study. Given that it is desirable to obtain cellular solids with high stiffness (*i.e.* E^*/E_0) and low weight (*i.e.* ρ^*/ρ_0), the slopes for each plot are also determined. According to Fig. 4.31 it may be observed that there is a tendency for the cellular solids with a negative Poisson's ratio ($\nu < 0$) to display higher slope values. This implies that these samples are inherently stiffer for the same specific densities, as initially proposed in the study.

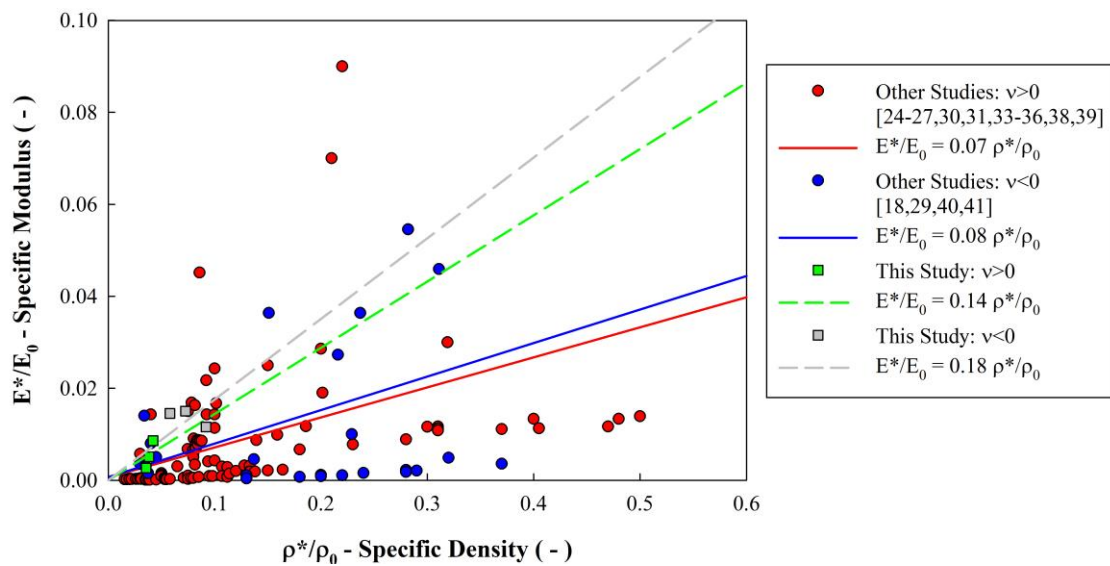


Figure 4.31 Comparison of specific modulus (E^*/E_0) to specific density (ρ^*/ρ_0) of the manufactured samples and other published data.

Comparing the slopes of other published data with the experimental results of this study, it may be also concluded that there is a tendency for the designed samples to display higher stiffness values for the same specific densities.

The same observation is also verified in when the samples are compared to other published data [18,24–29,31–33,35,37–39] in terms of specific strength (Fig. 4.32). It is shown that the designed cellular solids tend to display higher values of specific strength for the same specific density. This is more prominent in samples with a negative Poisson's ratio.

In conclusion, it is proposed that the designed composites are able to display a good relation between the static mechanical properties and specific density. Thus, it is suggested that they may be an interesting solution for high specific mechanical properties in structural applications.

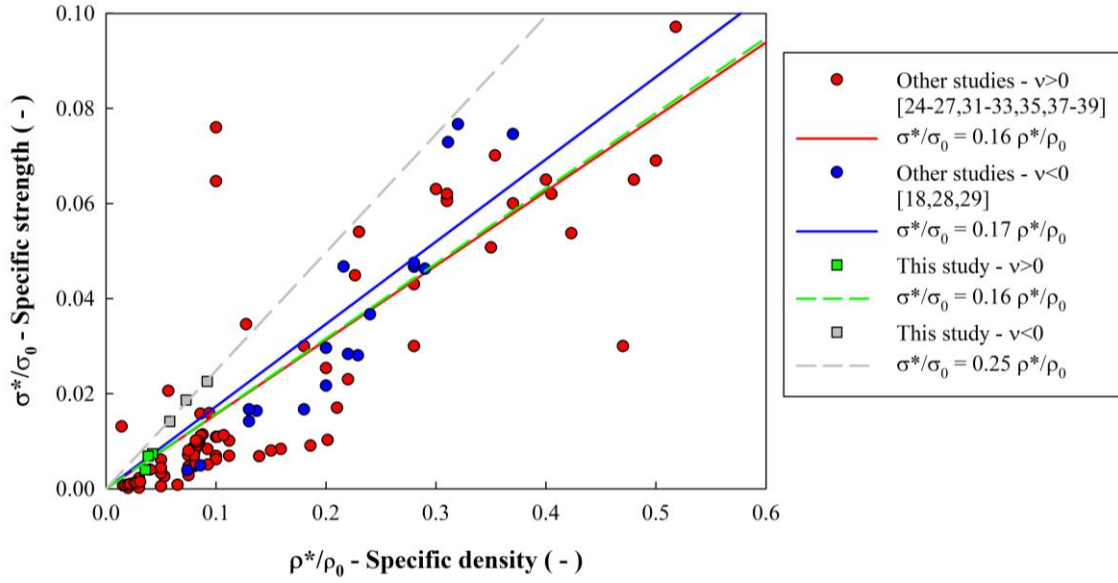


Figure 4.32 Comparison of specific strength (σ^*/σ_0) to specific density (ρ^*/ρ_0) of the manufactured samples and other published data.

4.6. Conclusions on static structural behavior of the cellular solids

This chapter explores the influence of the rib angle and Poisson's ratio in the overall static mechanical properties of three-dimensional metallic cellular solids. Uniaxial compression was performed in samples with different geometries to determine the dependence of apparent modulus and plastic collapse stress to the auxetic and non auxetic deformation behaviors. The following conclusions are drawn:

- (i) A formulation intended to describe the elastic properties of two- and three-dimensional honeycomb lattices is shown to correct the rib angle dependent hinging stiffness. This is fundamental to enhance the correlation between analytical models with experimental results. The suggested model and formulation are a good approximation to determine the elastic properties of these structures based on their initial geometry and base material.
- (ii) A deformation model that depends on the Poisson's ratio was proposed to describe the static mechanical properties of cellular materials. It is shown that the densification of auxetic samples generates a combination of higher values of apparent modulus and plastic collapse stress;
- (iii) Finite element analysis is performed to detail the Poisson's ratio dependent collapse of the samples. The numerical results are validated by the experimental testing and validate the proposed collapse mechanism. The suggested collapse mechanism is further validated by the *in-situ* 4D X-ray μ -CT of samples in compression.

(iv) The static mechanical properties of the developed cellular solids are compared with other published studies. Given that they display an interesting correlation between mechanical strength, stiffness and specific density, the manufactured composites are proposed as an interesting solution for structural applications.

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Chapter 5 – Effect of the Poisson's ratio on the Dynamic Structural Behavior of the Designed Cellular Solids

“...non-equal forces, out of equilibrium, do not lead to equilibrium but in abnormal vibration, the motion caused by them ...”

Plato, *Timaeus*, 360 a.c.

5.1 Introduction to the dynamic structural properties of cellular solids

The design of cellular solids for structural vibration damping purposes is an assignment with variables such as selection of base material, geometry and void filling [1,2]. The behavior of these composites under vibration is directly influenced by their elastic and viscous properties [3,4]. While the first is related with the stiffness of the system [5,6], the latter may be correlated with the solid fraction and the cellular lattice hysteretic damping [7,8].

Given that these materials are known to possess relatively high values of vibration damping, they are desirable in transportation industry [9–12]. The use of such materials allows the decrease of the overall structural weight, saving on fuel costs and emissions, and contributing to passenger comfort by the mitigation of vibrations [13–15]. Additionally, vibration damping is useful in the reliability increase on the mechanical components and reduction of fatigue related issues [16–19].

Generally, the mitigation of vibration in cellular solids is performed using a polymer as the basis for the solid phase [20,21]. There are different studies that report the use of polymeric foams for vibration

control, due to its high viscoelastic damping [22–24]. However, this combination is frequently associated with low static mechanical properties (*i.e.* low elastic modulus and yield strength) [25]. Classically, the dynamic behavior of cellular solids is described in terms of macro-scale properties, such as porosity (*i.e.* fluid phase) and solid phase distribution (*i.e.* geometry) [10]. These variables are correlated with the capacity of the cellular solid to improve damping capacity.

In this chapter the designed cellular solids with different Poisson's ratios have been analyzed to determine the influence of the non-stochastic geometry in their vibration damping. This is performed by changing the rib angle and, consequently using different densities, solid phase distribution and cellular solid Poisson's ratio. Experimental vibration testing is performed to determine the response to vibration and numerical routines are developed to estimate the damping ratios of the developed cellular solids.

5.2 Methodology for dynamic structural properties of cellular solids

5.2.1. Experimental vibration testing

Sample gating from the casting process was used to open a tapered hole for sample fixing during the vibration testing (apparatus in Fig. 5.1 (a)). The testing was performed by exciting the sample using a shaker (LDS V201) while the imposed loads are monitored by a load cell (PCB Piezotronics 221B02). The deformations in the samples were measured by an accelerometer (PCB Piezotronics 356A14), whose position and direction were optimized to capture the global eigenmode with highest predicted accelerations.

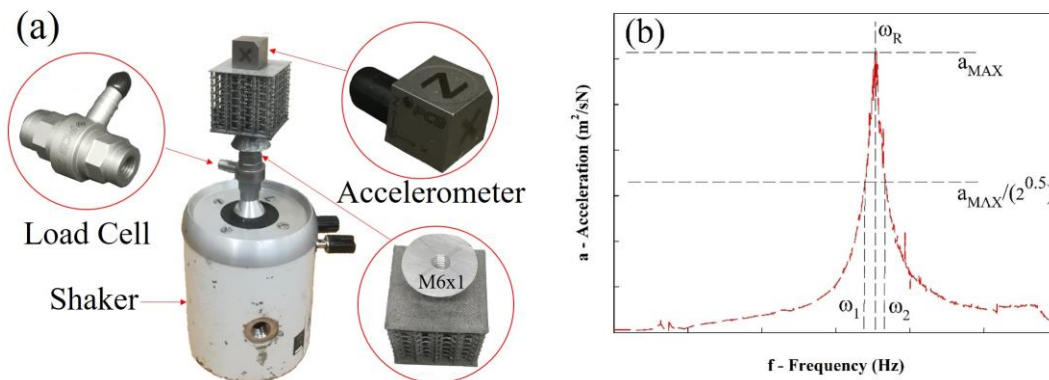


Figure 5.1. Vibration testing: (a) Experimental apparatus and (b) example of damping ratio calculation.

The readings of the accelerometer and load cell were used to calculate the frequency response frequency (FRF) using a LMS Scadas Mobile spectrum analyzer. The corresponding damping ratio (ξ) of

the monitored resonance mode was determined using the half-power bandwidth method ($\xi = (\omega_2 - \omega_1)/(2\omega_n)$, Fig. 5.1 (b)) [26]. A total of five samples for each tested rib angle were analyzed.

5.2.2. Finite element analysis – Harmonic simulation

Fig. 5.1 shows that the experimental approach of section 5.2.1 gives a relative damping response to a vibration, *i.e.*, even though it is useful to compare each individual sample, the results show the vibration response of the global experimental apparatus. Thus, although the apparatus does not influence the stiffness of the samples during the testing, its damping is affected. It may be considered that the glue that is used to fix the accelerometer and the cables themselves increase the overall system damping. In relative terms, the presented experimental analysis is enough to compare the damping of the samples. However, to determine the real values of damping, the experimental approach is not enough. To estimate the true values of damping of the samples, COMSOL Application Builder external routine was programmed using the Eigenfrequency Analysis and Frequency Domain modules (Fig. 5.2) [27].

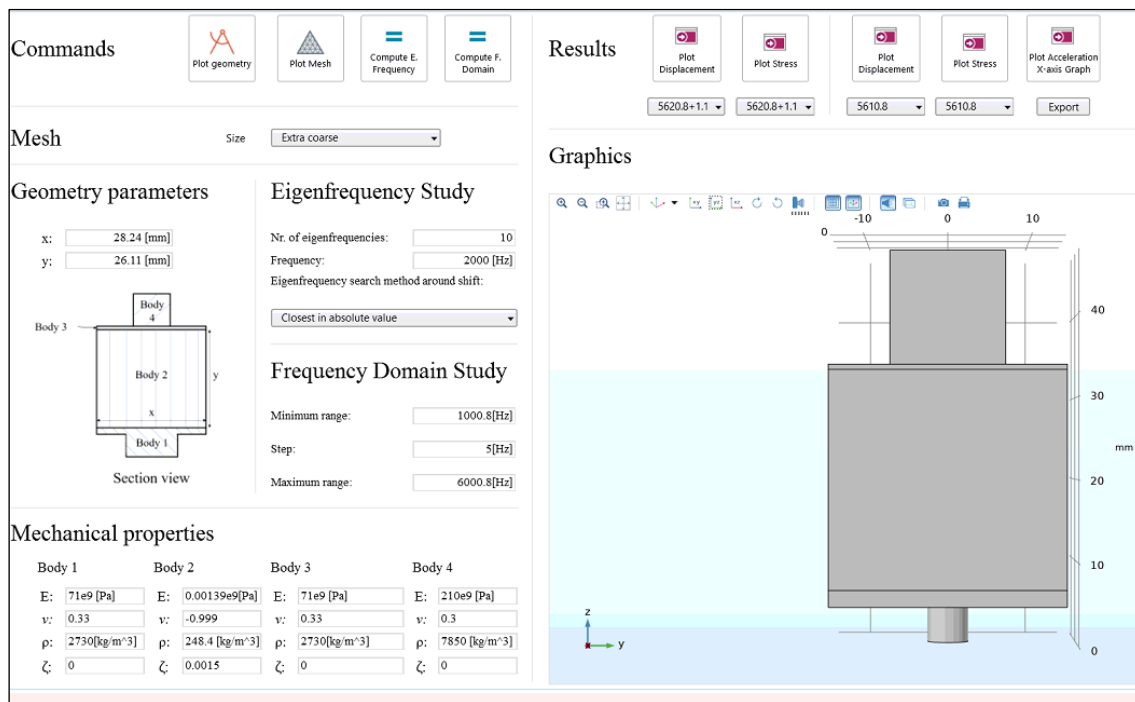


Figure 5.2. Developed Application Builder environment.

The referred modules were used to produce a parametric homogenized model of the system (Fig. 5.1). This model is parametrized for different dimensions, elastic properties and damping ratios in different bodies (Fig. 5.3 (a)) that represent the experimental apparatus. The routine searches for the

acceleration peak and resonance frequency, while considering the input parameters in Table 5.1. The application simulates a cyclic load (1N) with a random frequency (1000 – 6000 Hz) and monitors the accelerations in Body 4. According to the real boundary conditions in the experimental testing, the displacements in Body 1 were limited to the ZZ axis. The overall mesh in Fig. 5.3 (b), a uniform tetrahedral mesh was established with a single 0.60 mm element size.

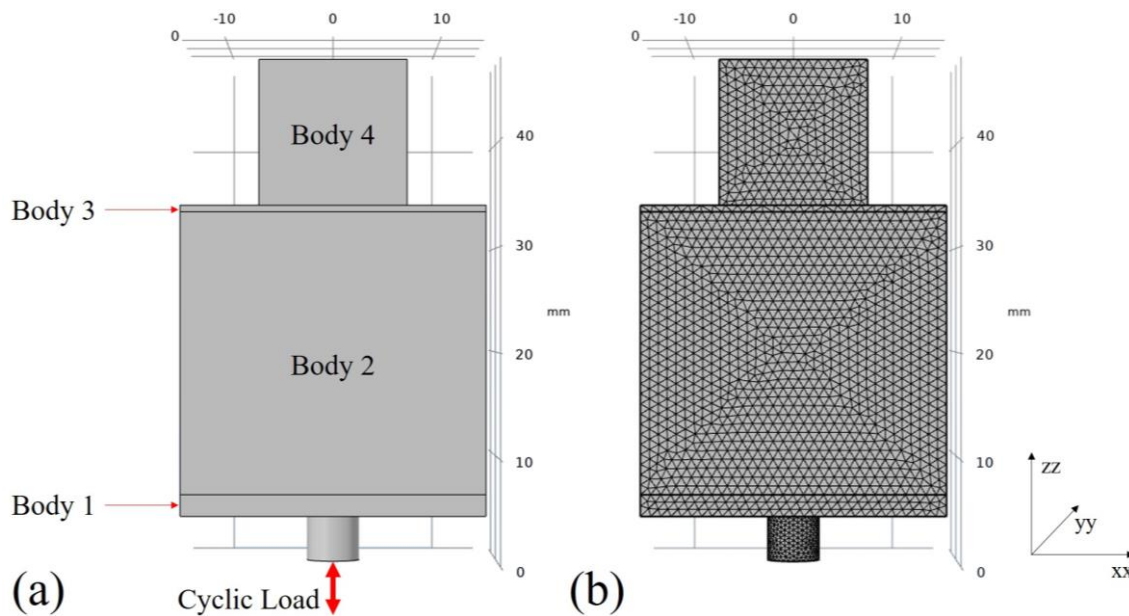


Figure 5.3. FEA model: (a) different bodies and (b) meshing.

Table 5.1. Input parameters for the different bodies.

Body	Description	Elastic Modulus (MPa)	Poisson's ratio (-)	Density (kg/m³)	Damping ratio (-)
1	Top plate (A356)	71000	0.33	2700	0
2	Cellular core	Variable	Table 4.18	Fig. 3.20	Variable
3	Bottom plate (A356)	71000	0.33	2700	0
4	Accelerometer (Ti casing)	110000	0.31	4500	0

The application is also programmed to perform an optimization routine to correlate the numerical with the experimental results. Using a two-step inverse engineering approach, the application is able to: (i) search the true dynamic modulus of the samples (Fig. 5.4 (a)); and (ii) search the true damping ratio of the samples (Fig. 5.4 (b)). Once a 1% difference is found between the experimental and numerical results, the application assumes the referred properties.

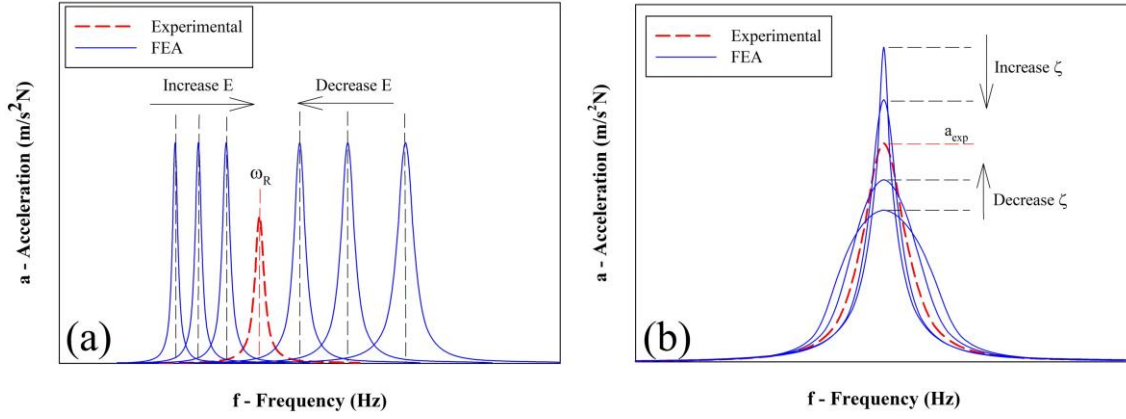


Figure 5.4. Optimization for (a) dynamic modulus and (b) damping ratio.

5.3 Results and Discussion

5.3.1. Experimental vibration testing

Fig. 5.5 shows the frequency response functions (FRF) from the vibration testing. It is evident that the designed geometries have a significant impact in the overall functions in terms of resonance frequencies and damping.

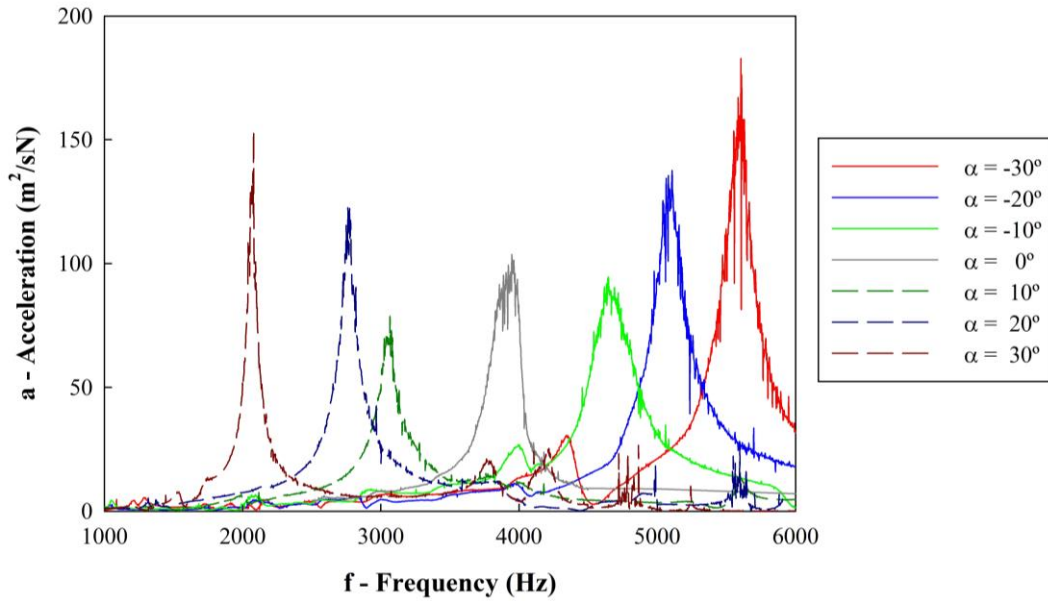


Figure 5.5. Sample frequency response function (FRF) from the vibration testing.

According to previous analysis on the samples, it is known that the adopted vibration testing apparatus (Fig. 5.1) is able to successfully capture the eigenmode with the highest amplitude to facilitate the calculation of the damping ratio. The first mode is composed by torsion (Fig. 6 (a)), while the second mode is composed by flexure (Fig. 5.6 (b)), in which the deformations are completely monitored by the

used accelerometer (XX axis). During the experimental testing, it was found that the second mode is the most useful to determine the damping ratio due to the direction and magnitude of the accelerations.

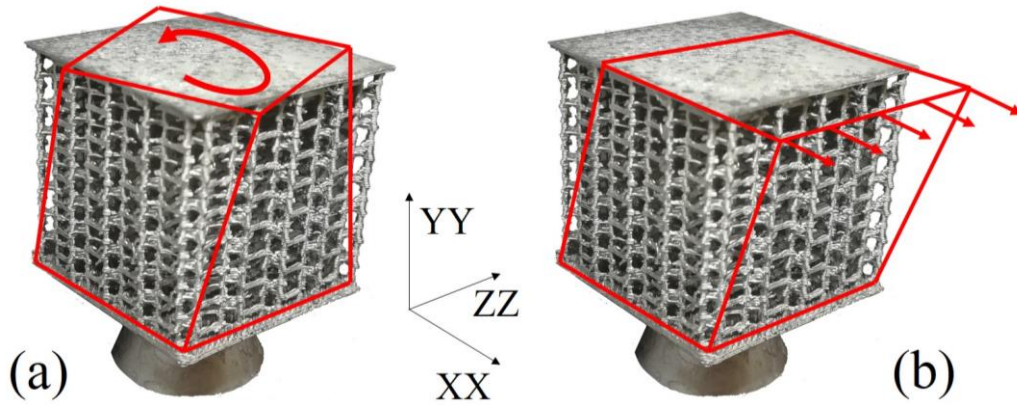


Figure 5.6. Global eigenmodes of the samples: (a) 1st mode (Torsion) and (b) 2nd mode (Flexion).

Fig. 5.7 shows the damping ratios that were determined by the half-power bandwidth method from the vibration testing. Results show that there is an increase in damping when the deformation mechanism tends to change from auxetic to non auxetic (*i.e.* near $\nu \rightarrow 0$). It may be seen that as the rib angles starts to widen, either to negative or positive values, the damping values tend to decrease. However, as the Poisson's ratio (Figure 4.19 – section 4.4.1) of the samples tends to decrease to negative values the damping ratios seem to be slightly lower than the corresponding positive values (*e.g.* comparing -30° to 30°).

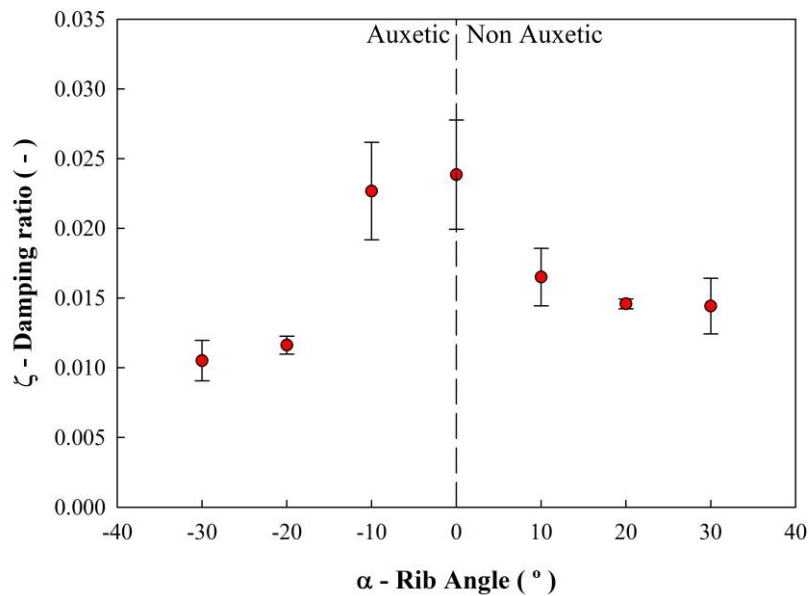
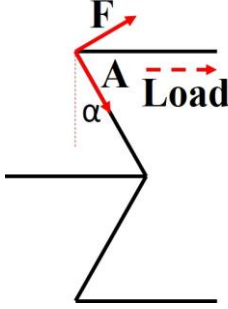
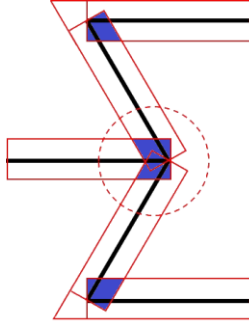
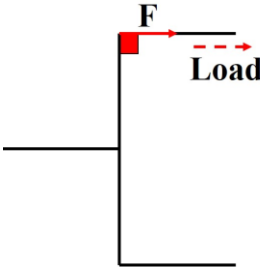
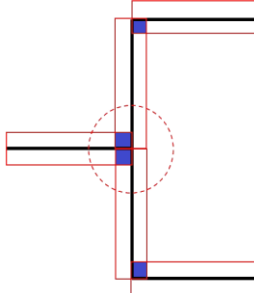
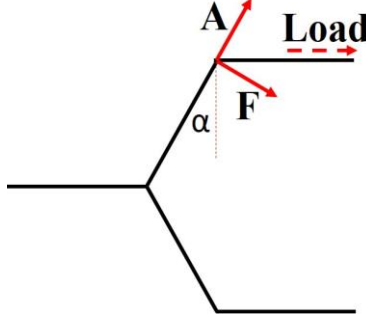
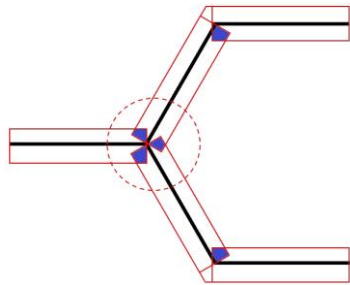


Figure 5.7. Average damping ratios (ξ) from the vibration testing.

The change on the experimental damping ratios is suggested to be attributed to two distinct effects (Table 5.2): (i) the change in deformation mechanism and (ii) the variation in the overall volume variation between the models.

Table 5.2. Suggested models for the variation in damping ratios during the vibration testing.

Poisson's ratio	Deformation Model	Volume Interference
Negative		
Zero		
Positive		

Due to the change in rib angle and the low density of the tested samples ($0.035 < \rho^*/\rho_0 < 0.092$), it is known that the main deformation behavior is composed by flexure. Given that the deformations are very small and there is no self-contact in the samples, the internal friction mechanism is caused by the hysteretic damping during these flexural deformations.

In cases that the rib angle is zero ($\alpha=0^\circ$), the ribs are subjected to flexure and the internal friction is maximized. However, when the angle is widened, either to positive or negative values, an axial

component is introduced (Table 5.2 and Fig. 5.8) and, according to Fig. 5.7, the overall damping is reduced.

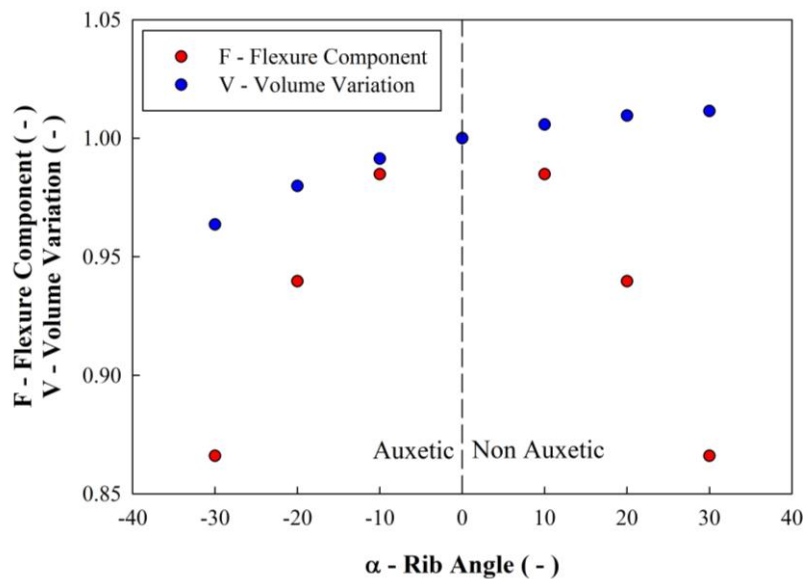


Figure 5.8. Flexural component (F) and volume variation (V) in the samples due to the different rib angles.

Even though, at a design phase, the volume of the samples should be constant, there are effective variations in sample volumes due to the rib interference. Figure 5.8 shows the volume variations in the samples (V/V_{MAX}) is dependent on the rib angle. Thus, this amount of material is able to modify the damping capacity.

5.3.2. Finite element analysis – Harmonic simulation

As reported in section 5.3.1, the experimental results may be used to compare the damping ratio of the samples in relative terms. Although, the cables that connect the accelerometer to the analyzer are in a resting position (*i.e.* not in tension) and the amount of glue is the same for all the samples, it is proposed that they affect the value of damping ratio. Thus, there was established a numerical routine using COMSOL Application Builder to determine the true damping ratio of the manufactured samples.

Fig. 5.9 presents a schematic representation using spring-dashpot models to describe the viscoelastic behavior of the apparatus during the experimental testing. It shows that a dynamic load is applied in the bottom of the sample (in the experimental analysis by a shaker). The sample itself is endowed with a mass (M_s) and a stiffness coefficient (K_s). The hysteretic damping in the sample is

influenced by its damping coefficient (C_s). It is assumed that these constants will change with the different sample designs and Poisson's ratio, as it was verified in the experimental static (section 4.3.1 – Chapter 4) and dynamic (section 5.3.1) results. The vibration response of the sample is monitored by the accelerometer (with a mass, M_a) that is glued to its top. Thus, this interface has its own stiffness and damping coefficients (respectively, K_G and C_G). The cables themselves are connected to the accelerometer and suspended and have their own stiffness and damping coefficients (respectively, K_c and C_c).

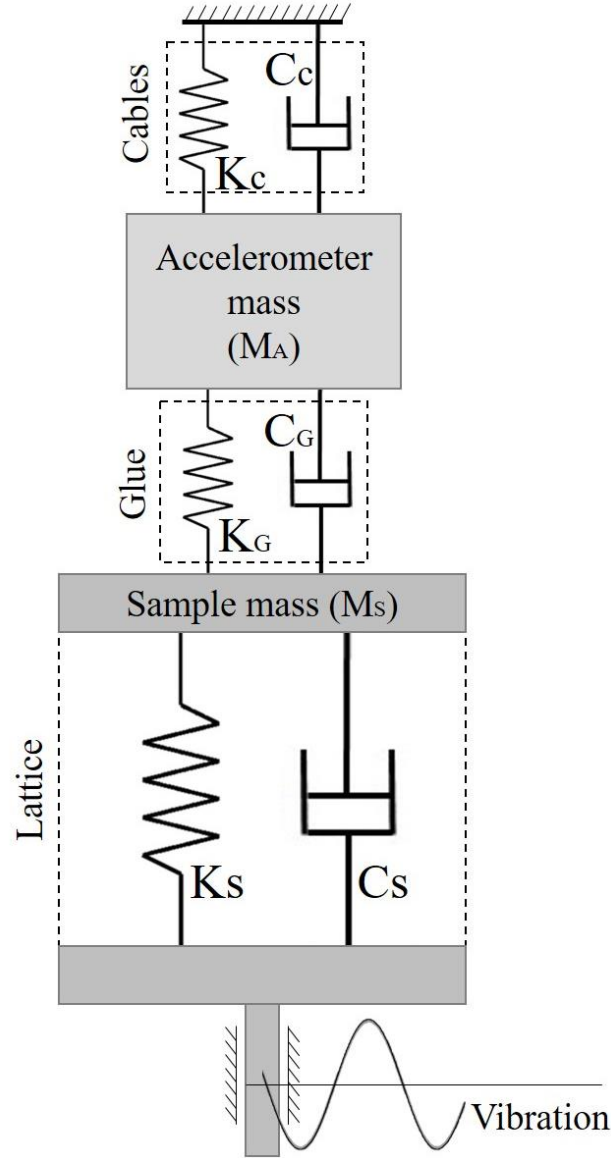


Figure 5.9. Theoretical representation of the vibrating system.

The overall dynamic behavior of the apparatus may be described by the fundamental equation of motion (Eq. 5.1) [28], where M_{eq} , C_{eq} and K_{eq} are, respectively, the equivalent mass, damping and stiffness coefficients of the system.

$$M_{eq}\ddot{x}(t) + C_{eq}\dot{x}(t) + K_{eq}x(t) = F(t) \quad (5.1)$$

Given that all the presented spring-dashpot models are in series (Fig. 5.9), their equivalent coefficients (K_{eq} and C_{eq}) may be determined using Eqs. 5.2 and 5.3 [29].

$$\frac{1}{K_{eq}} = \frac{1}{K_s} + \frac{1}{K_G} + \frac{1}{K_c} \quad (5.2)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_s} + \frac{1}{C_G} + \frac{1}{C_c} \quad (5.3)$$

However, since the processing of the glue and the disposal/connection of the cables was assured as the same in the experimental testing, it may be assumed that their variation is negligible (Eqs. 5.4 and 5.5).

$$\Delta K_G \sim \Delta K_C \sim 0 \quad (5.4)$$

$$\Delta C_G \sim \Delta C_C \sim 0 \quad (5.5)$$

Thus, simplifying Eqs. 5.2 and 5.3, it may be deduced that the overall variation in the system stiffness (ΔK_{eq}) and damping (ΔC_{eq}) coefficients may be simplified to the viscoelastic changes in the sample (Eqs. 5.6 and 5.7).

$$\Delta K_{eq} \sim \Delta K_s \quad (5.6)$$

$$\Delta C_{eq} \sim \Delta C_s \quad (5.7)$$

Considering these simplifications, it may be seen that the damping ratio may be determined by the variation in sample damping coefficient (ΔC_s), mass (ΔM_s) and resonance frequency ($\Delta \omega_n$), according to Eq. 5.8. The latter may be determined using Eq. 5.9 [30].

$$\Delta \zeta_s = \frac{\Delta C_s}{2\Delta M_s \Delta \omega_n} \quad (5.8)$$

$$\Delta \omega_n = \sqrt{\frac{\Delta K_s}{\Delta M_s}} \quad (5.9)$$

As established in section 3.2 (Chapter 3), while discussing the design of the samples, it was established that their geometry would be changed by using different rib angles. This implied that the

overall variation in sample mass (M_s) is minimized to prevent mass attributed changes in the dynamic properties of the samples. Given that the samples are fundamentally described in terms of their rib angle and this influences the exterior plates (*i.e.* skins) and interior cellular solid core masses, Fig. 5.10 presents the influence of rib angle in their relative weight. Additionally, their combined weight is also calculated by the addition of the two components.

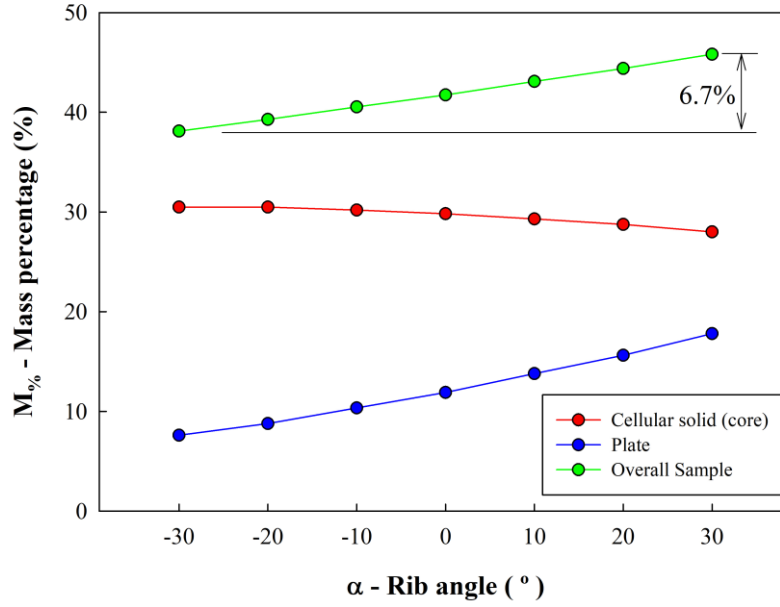


Figure 5.10. Relative weight of the samples in the overall dynamic system (*i.e.* experimental apparatus).

From Fig. 5.10, it may be determined that the maximum variation on sample mass is 6.7%, thus, it is proposed that this change is not the most relevant factor in the overall change in damping of the samples. This reduced importance of mass variation in the samples on the dynamic testing results further validates the adopted FEA methodology, given that the inverse engineering process is focused on the stiffness and damping components of the system (Eqs. 5.6 and 5.8).

Fig. 5.11 shows the results of the simulations for the samples with rib angles. It may be observed that, overall, the FEA routines are able to fairly describe the experimental results in terms of eigenmodes and eigenfrequencies. Although, there are some resonance frequencies that may be observed in the numerical results (*e.g.* high frequency resonance frequencies in Figs. 5.11 (b) and (d)). It is hypothesized that these eigenmodes may be overshadowed by the damping that is introduced by the glue and cables in the experimental approach.

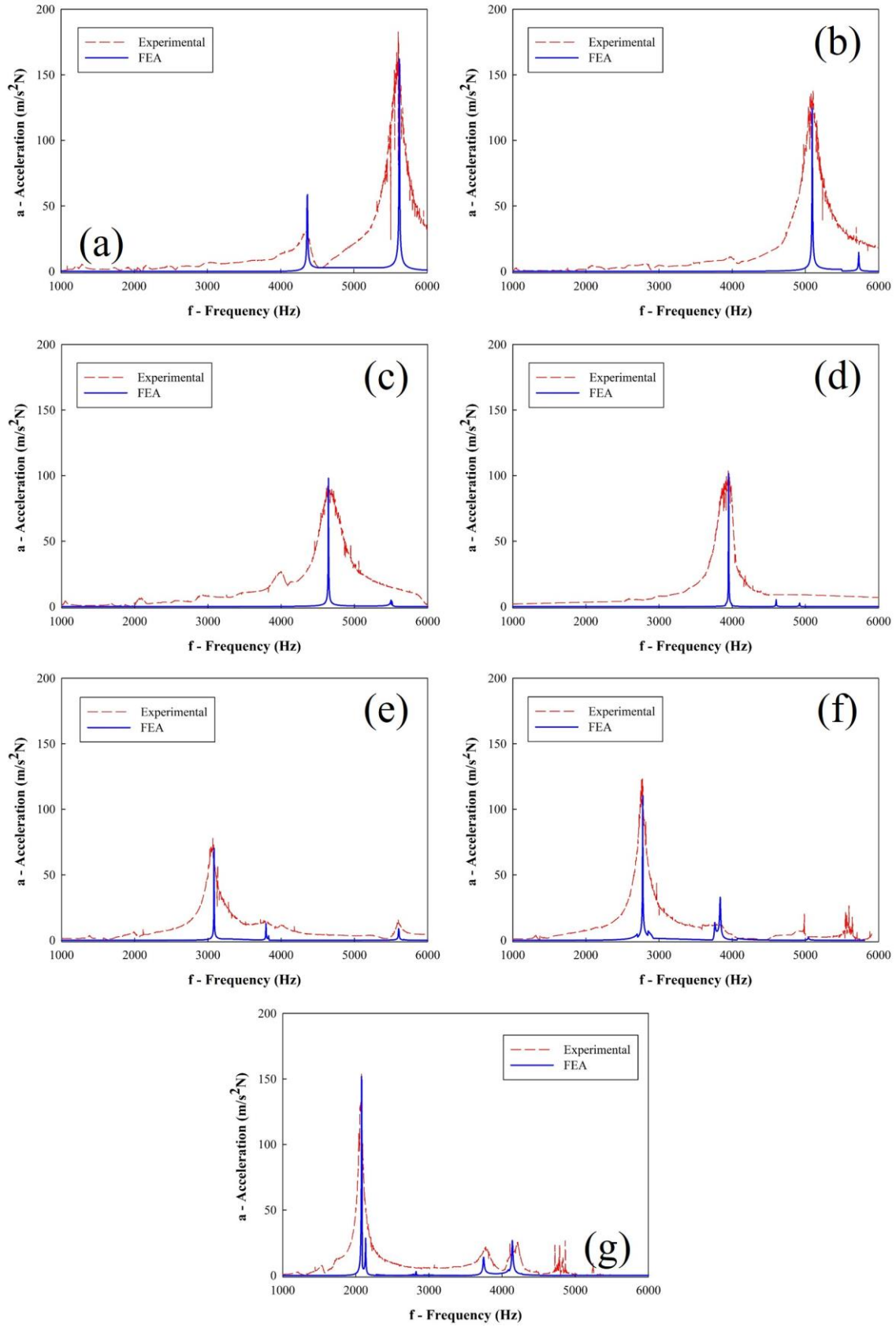


Figure 5.11. FEA results for samples with rib angles: (a) $\alpha=30^\circ$; (b) $\alpha=20^\circ$; (c) $\alpha=10^\circ$; (d) $\alpha=0^\circ$; (e) $\alpha=10^\circ$; (f) $\alpha=20^\circ$; (g) $\alpha=30^\circ$.

As intended in section 5.2.2, the numerical results are able to fully describe the eigenmode with the higher acceleration peaks in terms of resonance frequency and peak amplitude. Thus, it is suggested that the numerical routines are able to successfully determine the dynamic apparent modulus and true damping ratio of the samples.

Analyzing the input data that was used for the inverse engineering process, the resultant static (from section 4.4.1 – Chapter 4) and dynamic apparent modulus (E^*) are presented in Fig. 5.12. It is known that the difference between static and dynamic modulus may present difference higher than one magnitude [31,32]. This difference may be further highlighted in the case of composites, such as cellular solids [33,34]. From the numerical results, it may be seen that the dynamic apparent modulus tends to be lower than the static apparent modulus. However, they display the same tendency as it may be observed in Fig. 5.7.

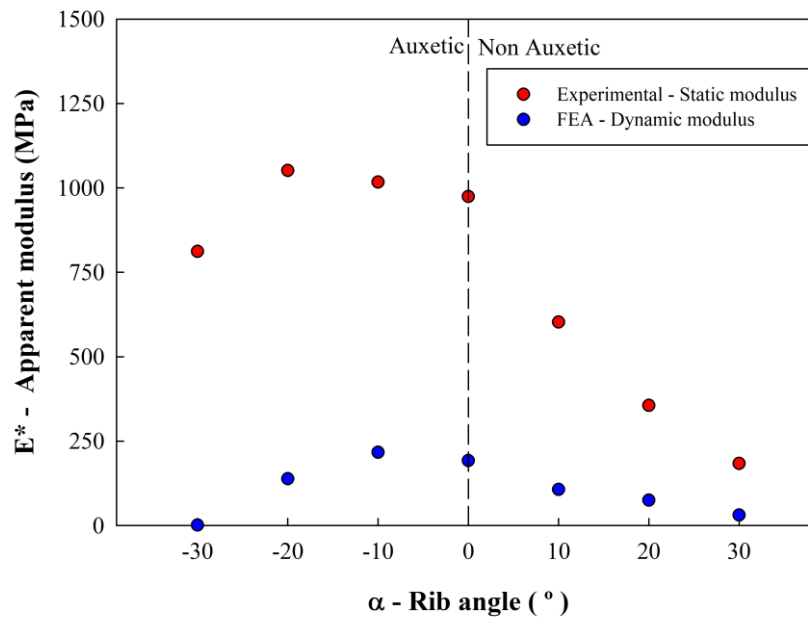


Figure 5.12. Comparison between static experimental and dynamic FEA apparent modulus.

Fig. 5.13 shows a comparison between the damping ratios that were calculated by the experimental and the FEA approaches. As expected, the values of damping of the FEA analysis are lower than those determined by the experimental approach due to the absence of glue and cable interference. Additionally, they follow the same tendency as the experimental results, validating the hypothesis that the higher values in the experimental approach are originated by the other factors that are not related with the samples itself.

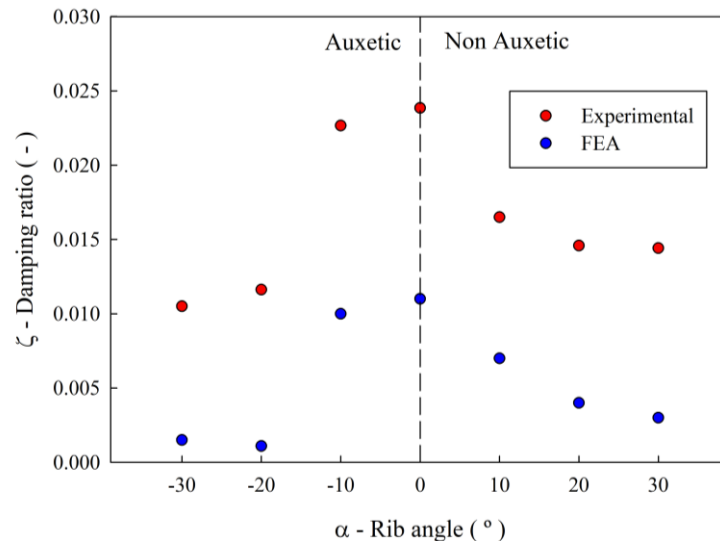


Figure 5.13. Comparison between experimental and FEA damping ratios.

5.3.3. Modeling of dynamic behavior

The dynamic behavior of the developed cellular solids has been proved to be influenced by the rib angle (α), which by itself is able to change the sample Poisson's ratio ($\nu(\alpha)$ - Fig.5.19) and volume ($V(\alpha)$ - Fig. 5.8). All these factor have a relevant role in the variations of hysteretic damping. Thus, it would be useful determine an analytical relation that is able to describe these damping variations. Considering the influence of the referred factors, it is suggested that this change is described by Eq. 5.10. Comparing the analytical model with the experimental results (Fig.5.14), it may be observed that they display a good correlation ($R^2=0.92$).

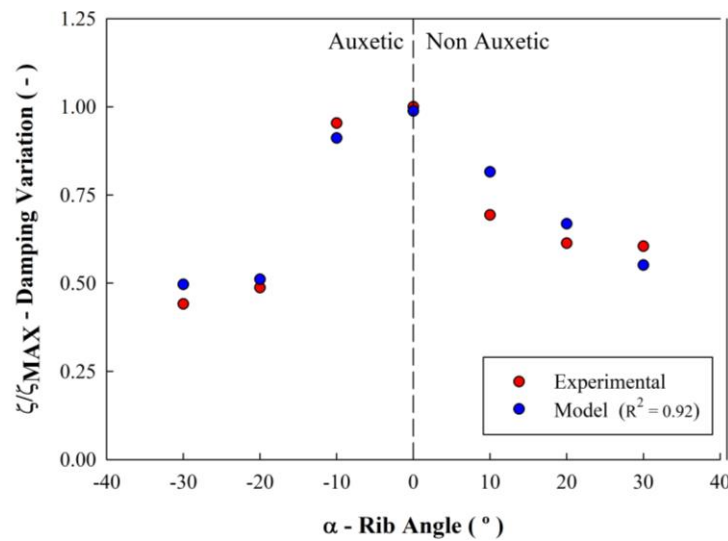


Figure 5.14. Analytical model for relative damping variation (ξ/ξ_{MAX}) due to the variation in rib angle.

$$\frac{\xi}{\xi_{MAX}}(\alpha) = \frac{1}{\cos(\alpha) \cdot \frac{V(\alpha)}{V_{MAX}}} - \sqrt{\left(\frac{v(\alpha)}{i}\right)^2}, \text{ where } \begin{cases} i = 3.5 \rightarrow v(\alpha) \leq 0 \\ i = 2.2 \rightarrow v(\alpha) > 0 \end{cases} \quad (5.10)$$

5.4. Comparison with other materials

To estimate the competitiveness of the cellular solids in terms of dynamic performance, their fundamental dynamic mechanical properties (*i.e.* weight and damping) were compared with other current publications. For such purpose, the specific density and damping ratios (respectively, ρ^*/ρ_0 and ζ) of other published data (Table 5.3) were plotted against the experimental results that were determined by the FEA routines (section 5.3.2).

Table 5.3. Other publication for comparison with this study results.

Material	Method	Ref.
Al	Foaming	[11,35]
	Powder Metallurgy	[36]
Al+SiCp	Foaming	[37,38]

Fig. 5.15 relates the damping ratio and specific density of the analyzed published works [11,35–38] (Table 5.3) and the FEA results. As expected, it may be observed in the published works that aluminum cellular solids tend to display lower values of damping relatively to their specific density when the same cellular solids are impregnated with Carbide Silicon particles (SiCp). This may be further visualized by the slopes of the results.

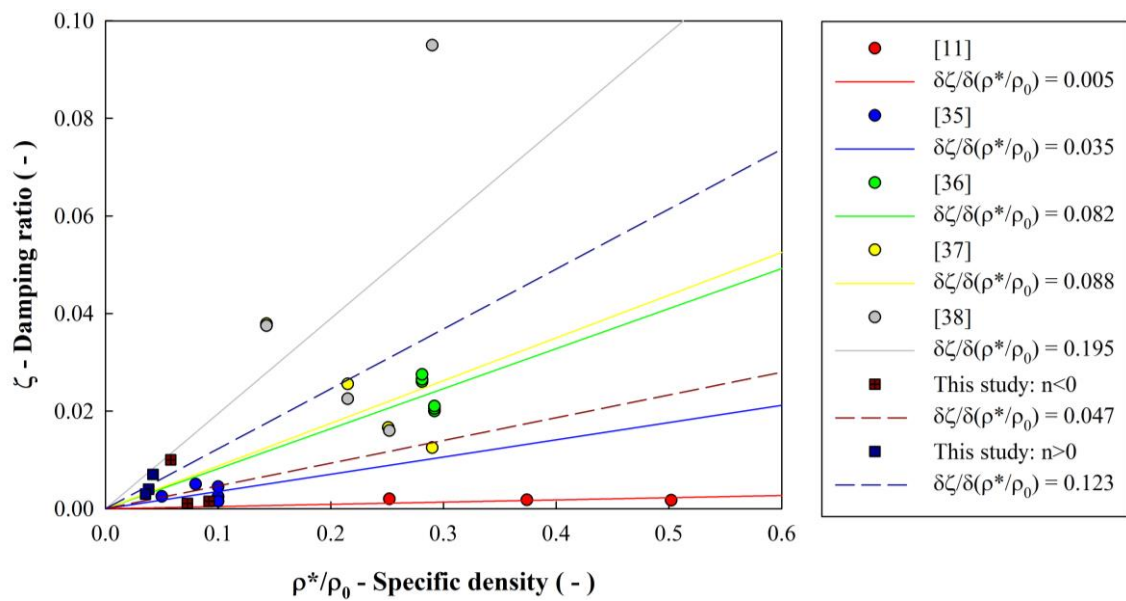


Figure 5.15. Comparison between this study results and other published data.

Comparing the obtained FEA results, it may be stated that the relation between the damping ratio and specific density is more beneficial in the produced samples, due to its higher slope value. Thus, these samples tend to display higher values of damping for lower specific densities. However, even in samples with a negative Poisson's ratio, the calculated slope is still higher than other published data. It is worth mentioning that the slope values of positive Poisson's ratio is able to compete with other published studies of cellular solids impregnated with particles (Al + SiCp).

5.5. Conclusions on the dynamic structural behavior of cellular solids

This chapter studies the influence of the rib angle and Poisson's ratio in the overall vibration damping behavior of metallic cellular solids based in three-dimensional honeycombs. Vibration testing and FEA were performed in samples with different geometries to determine the dependence of the dynamic apparent modulus and damping ratio to the auxetic and non auxetic deformation behaviors. The results are used to describe the Poisson's ratio dependent dynamic mechanical behavior of these cellular solids. The following conclusions are drawn:

- (i) Rib angle influences not only the specific density, geometry and Poisson's ratio of the samples, but is also effectively able to change the damping capacity of the designed cellular solids.
- (ii) Experimental vibration testing of the designed cellular solids may be used to compare their damping capacity in relative terms. FEA routines were implemented to determine the true damping ratio values by an inverse engineering approach.
- (iii) The damping ratios of cellular solids are apparently higher in the transition of the Poisson's ratio from negative to zero. A mathematical model is suggested to correlate the variation in damping to the changes in flexural deformations and volume of material to perform hysteretic damping;
- (iv) Comparing the damping ratios of the designed cellular solids, it is concluded that they are able to display interesting properties when compared to other current solutions. The designed samples are able to show a good compromise between damping capacity and specific density. In fact, they are able to compete with solutions that usually have high damping values, such as metal matrix composites (MMCs).

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Chapter 6 – Acoustic Behavior of the Designed Cellular Solids

“Words cannot express quite a lot of feelings, whereas a noise or tone or drone or sound, an accordion falling down a staircase, can somehow capture an emotion much better”.

John Lydon in interview to the newspaper *The Scotsman*, 2009

6.1 Introduction on the acoustic behavior of cellular solids

There is a current interest in the reduction and damping of unwanted/hazardous noises [1,2], not only in living or working spaces [3], but also in collective transportation systems due to the increase in commuting [4,5]. For instance, several current and future European requirements in transportation systems [6–14] are focused on measures to promote comfort/attractiveness of public transportation in interior and exterior noise reduction [15].

Considering these objectives, cellular solids are known as a possible solution to enhance noise damping when their source cannot be completely eliminated. For instance, cellular solids have been studied and shown to be viable solutions to reduce sound reflection and propagation [16–18]. Thus, they may be designed to have high values of sound absorption coefficient and transmission loss relatively to bulk materials [19–22]. Additionally, they generally have lightweight characteristics due to their low specific density and high specific static/dynamic mechanical properties [23–25]. Given these attractive properties, there is a current trend that is exploring the adoption of cellular solids in applications that classically use bulk materials, especially in the transportation industries.

In applications in which high static loading is also required, these cellular solids have a metallic solid phase [24]. However, it is also known that these metallic cellular solids have lower vibration damping capacity than those with a polymer solid phase [26]. The combination of high values of static and dynamic properties has generated great interest in the scientific community. There have been extensive developments on the design and manufacturing of hybrid configurations using the referred composite materials [27–30]. One such approach is the foaming of polymer based foams within the voids of an architected metallic structure.

This chapter shows an approach to further enhance the damping capacity of the designed cellular solids (Chapter 2 to 4) and they are characterized to determine their noise damping properties. Initially, the designed composites are analyzed in terms of static strength and structural vibration damping to determine the configuration with the best compromise between these properties. Posteriorly, the optimized samples are impregnated with flexible and rigid polyurethane foam and their acoustic properties are analyzed in both unfilled and filled states. The experimental results are compared with current materials in train vehicle construction to determine if the designed cellular solids are an interesting solution for future application railway applications.

6.2 Sample selection for optimized static/dynamic properties

Fig. 6.1 relates the static and dynamic properties of the samples and shows plots that correlate the inherent stiffness of the samples (*i.e.* apparent modulus – Fig. 6.1 (a)) and their ability to support loads in the elastic domain (*i.e.* plastic collapse stress – Fig. 6.1 (b)) with damping.

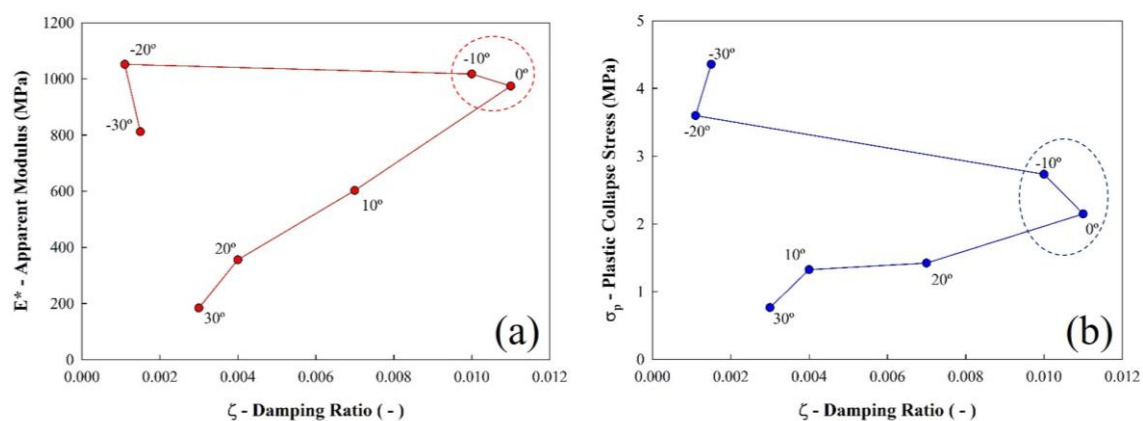


Figure 6.1. Correlation on the damping ratio and (a) apparent modulus and (b) plastic collapse stress.

It is shown that there is no linear correlation between the static and dynamic properties, as usually observed in bulk materials. There is a clear tendency for negative rib angles (auxetics) to exhibit higher

static mechanical properties. However, slightly negative and zero angles display enhanced damping. It is apparent that these angle combinations display the most beneficial combination of static and dynamic properties, as highlighted in Fig. 6.1.

Given that cellular materials are associated with high specific mechanical properties, it is also important to consider the effect of their specific density. To correlate the effect of the static/dynamic properties and density of the tested samples, it was established a criterion that correlates the referred factors. This factor, entitled Geometry selection index (G_{Si} – Eq. 6.1) considers the maximization of apparent modulus (E^*/E_{MAX}), plastic collapse stress ($\sigma_P/\sigma_{P_{MAX}}$) and damping ratio (ξ/ξ_{MAX}), while minimizing the specific density ($[\rho^*/\rho_{0_{MIN}}]/[\rho^*/\rho_0]$).

$$G_{Si}(\alpha) = \frac{E^*(\alpha)}{E^*_{MAX}} \cdot \frac{\sigma_P(\alpha)}{\sigma_{P_{MAX}}} \cdot \frac{\xi(\alpha)}{\xi_{MAX}} \cdot \frac{\rho^*/\rho_{0_{MIN}}}{\rho^*/\rho_0(\alpha)} \quad (6.1)$$

Fig. 6.2 shows the Geometry selection index plot relatively to the rib angle. Considering the static and dynamic mechanical properties together with the influence of sample density, it is shown that there is a clear contrast between the samples with auxetic and non-auxetic behavior. Samples with an auxetic behavior show a beneficial relation between static strength, damping and density.

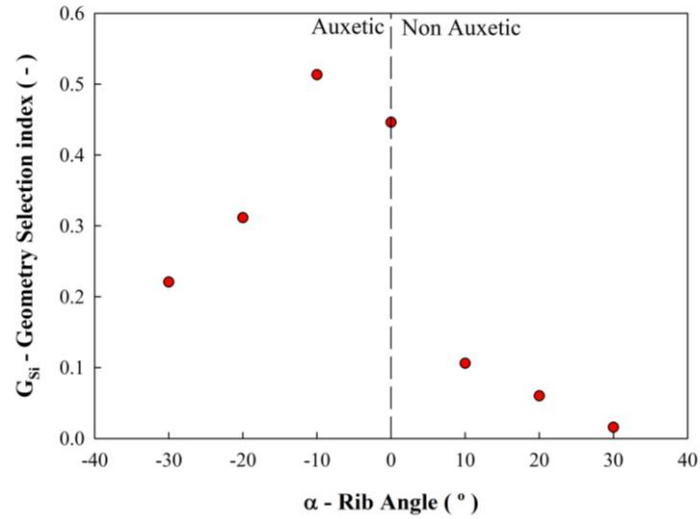


Figure 6.2. Geometry selection index (G_{Si}) for the tested rib angles.

This correlation is maximized in samples with a rib angle ($\alpha = -10^\circ$). When the angle is further increased, there is a tendency for the established criterion to decrease. This is more evident when the deformation mechanism is changed to a positive Poisson's ratio, where the overall properties and density display a much more reduced interaction.

6.3 Methodology for the acoustic testing of cellular solids

6.3.1. Filling of the samples with polymer foams

Polymer based foams are frequently associated with noise damping applications, having been successfully employed in the mitigation of this kind of solicitations in many applications [31–33]. Considering this, the metallic cellular solids that were selected in Section 6.2, where infiltrated with polyurethane foam to determine if this is beneficial for the overall noise mitigation of the designed composites. Given that the mechanical properties of polyurethane foams are related with its chemical composition and density, the samples were infiltrated with a rigid and a flexible foam.

The reagents to manufacture the polyurethane foams (Plastiform foams) were weighted and mixed in a beaker according to the proportions described in Table 6.1. After a homogenized mixture is achieved, the reagents were casted into a vessel holding a sample (Fig. 6.3 (a)). It may be observed by the progression in Figs. 6.3 (b) to (e) that the foaming process is able to penetrate the cellular structure of the sample.

Table 6.1. Proportion of polyurethane foam reagents.

Foam	Proportions		Foam Manufacturer
	Isocyanate	Polyol	
Rigid	1 part	1 part	Cronodur Foam 100
Flexible	1 part	2 parts	Cronodur Foam Flex

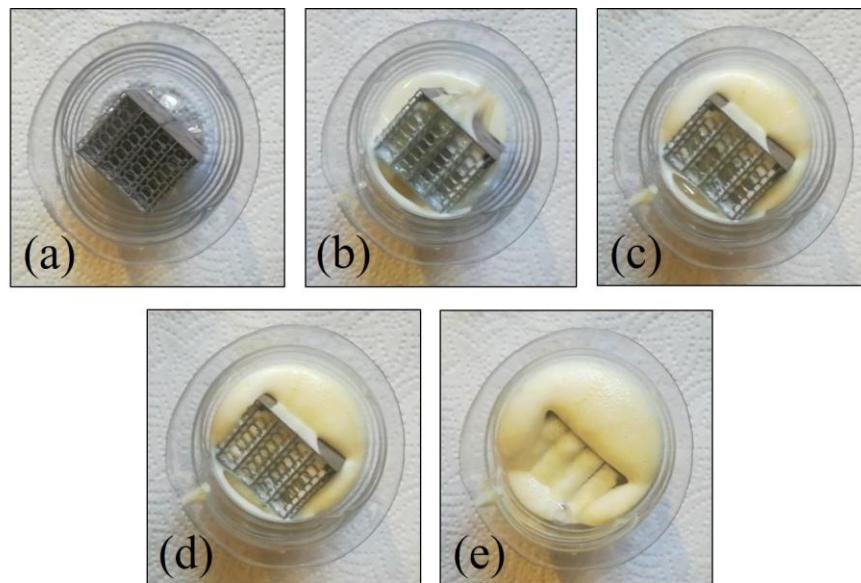


Figure 6.3. Sample foaming: (a) initial sample; (b-e) detail of foam filling the interior of the sample.

The appearance of the final foam filled samples with flexible and rigid foams may be observed, respectively, in Figs. 6.4 (a) and (b). Additionally, the properties of the produced foams may be observed in Table 6.2. Recurring to this table, it is observed that both flexible and rigid foams are characterized by an open-cell configuration.

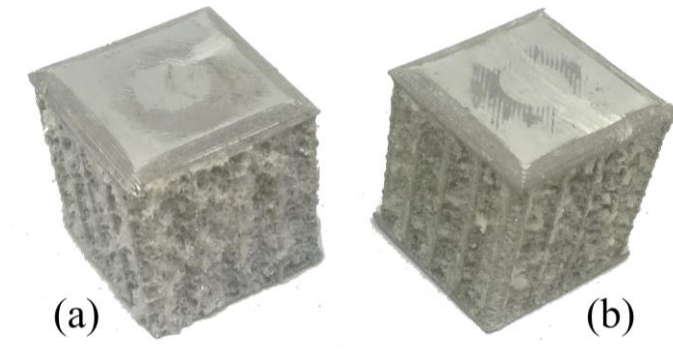


Figure 6.4. Examples of final samples filled with (a) flexible and (b) rigid foams.

Table 6.2. General characteristics of the manufactured foams.

Foam	Flexible	Rigid
Optical Microscopy		
Density (kg/m ³)	44.6	37.2
Porosity (%)	95.9	96.4

6.3.2. Vibration testing – Experimental and FEA

Vibration testing was performed in accordance with the methods detailed in section 5.2.1 (Chapter 5). As suggested in section 5.2.2, the experimental values for the damping ratio are useful for a relative comparison between the samples. However, to determine the true values of damping ratio, the experimental results were compared with the ones obtained by FEA. The latter were calculated with the Application Builder programmed in COMSOL as described in section 5.2.2.

6.3.3. Acoustic testing of the samples

A custom built impedance tube (Fig.6.5) was placed inside an anechoic chamber and used to measure the sound absorption coefficient and the transmission loss of the unfilled and foam filled samples. Due to the geometric characteristics, the tube was designed to be operated in frequencies between 100 Hz and 2000 Hz. Given that the size of the samples ($\alpha=10^\circ$ - Fig.3.20 in Chapter 3) is significantly smaller than the tube diameter (100 mm), a cork board was cut to accommodate the samples during the tests and make an air tight seal (Fig.6.5). Previously to the tests, a calibration procedure was performed with the cork board without the sample to determine the absorption and transmission of the board itself. The equipment software is able to use this calibration to normalize the final tests and estimate the properties of the sample.

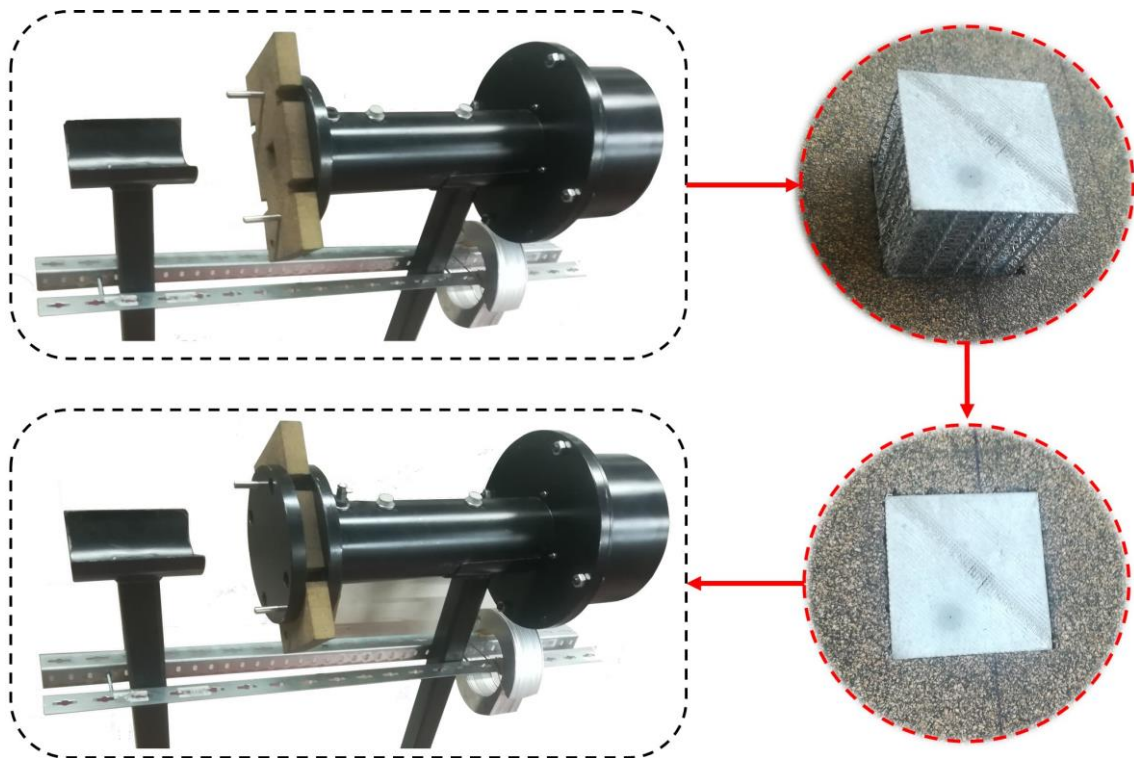


Figure 6.5. Detail of impedance tube and sample assembly for sound absorption testing.

According to the scheme in Fig.6.6, a random sound signal (100 – 2000 Hz) is generated by a Microflown PreAmplifier through a loudspeaker, and travels in the form of a plane wave inside the tube. The referred wave travels along the tube and ends up hitting the sample as an Incident wave (Fig.6.6). In the sound absorption and acoustic impedance tests, the incident plane wave that is reflected (*i.e.* Reflected wave) is measured by the probe that is placed facing the sample.

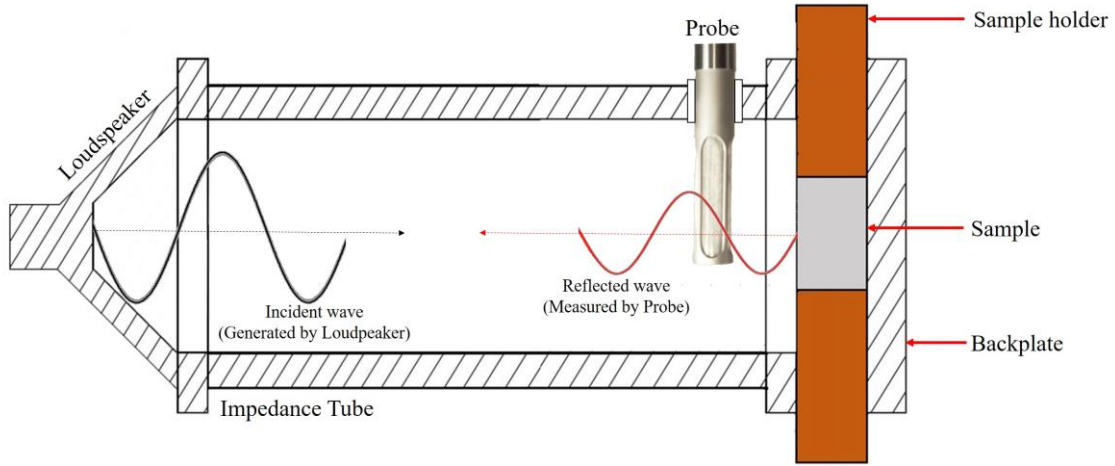


Figure 6.6. Detail of incident and reflected waves during Sound Absorption testing.

The normal sound absorption coefficient (α – Eq. 6.2) was calculated by monitoring the frequency dependent reflection coefficient ($R(\omega)$ - Eq. 6.3) on a fixed position (x). This reflection coefficient is determined by the measurement of the acoustic impedance ($Z(\omega)$) during the test, according to Eq. 6.4, where P is the captures sound pressure level and u the particle velocity.

$$\alpha(\omega) = 1 - |R(\omega)|^2 \quad (6.2)$$

$$R(\omega) = e^{\frac{2i\omega x}{c}} \frac{Z(x, \omega) - \rho c}{Z(x, \omega) + \rho c} \quad (6.3)$$

$$Z = \frac{P}{u} \quad (6.4)$$

It is known that the reflection coefficient may be influenced by the ambient condition, *i.e.* it is dependent on the air density (ρ) and speed of sound (c) according to Eqs. 6.5 and 6.6. These variables are themselves dependent of ambient variables such as temperature (T) and atmospheric pressure (P). To minimize ambient induced errors in the captured data, these variables were monitored during the testing, while average values (Table 6.3) were used to analyze the data.

$$\rho = 1.29 \left(\frac{P}{101.325} \right) \left(\frac{273.15}{273.15 + T} \right) \quad (6.5)$$

$$C = 20.047 \sqrt{273.15 + T} \quad (6.6)$$

Table 6.3. Ambient properties inside anechoic chamber during testing.

T - Temperature (°C)	27 ± 2
P - Atmospheric Pressure (kPa)	104 ± 5
Rh - Relative humidity (%)	45 ± 3

The Transmission loss (TL) of the samples was determined using a variation of the previous process. An additional tube length was added to determine the Sound pressure level after the sample, and an additional measurement was performed using the same probe (Fig.6.7). Transmission loss may be calculated using Eq.6.7, where Z_t and Z_i are, respectively, the frequency dependent transmitted and incident acoustic impedances. The values of incident acoustic impedance (Z_i) were measured recurring to the same method used for the sound absorption coefficient (Eq.6.4). As for the transmitted acoustic impedance (Z_t) was determined in a similar approach, but by changing the probe to the other side of the sample (Fig.6.8) to measure the Transmitted wave.



Figure 6.7. Detail of impedance tube assembly for Transmission loss testing.

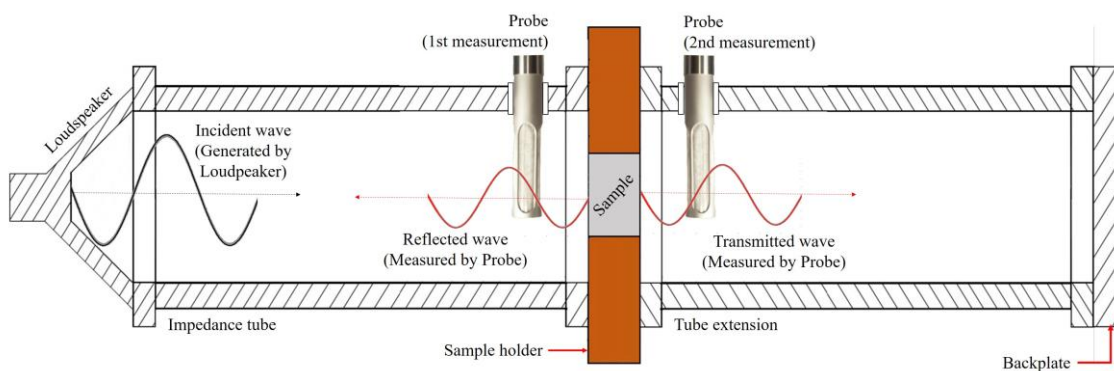


Figure 6.8. Detail of incident, reflected and transmitted waves during Transmission Loss testing.

$$TL(\omega) = 10 \log \left(\frac{Z_t(\omega)}{Z_i(\omega)} \right) \quad (6.7)$$

The usual process for the determination of the Sound absorption coefficient and Transmission loss usually include the use of two distinct microphones to monitor the differential between sound pressures in different tube locations. These methods are described in the standards ISO 10534 and ASTM E2611. In this study, a single probe was used for both methods. The probe that was used in this study, is part of a novel trend that uses a sensor that is able to monitor not only the instant values of pressure, but also the particle velocity. This way, using only one probe, it is possible to calculate the acoustic impedance. This approach has been proved valid and its results are highly correlated with those obtained using the classic standards [34–37].

6.3.4. Acoustic measurements in railway vehicles

To evaluate the acoustic performance of the developed cellular solids and their ability to be used in practical applications, the interior noise of railway vehicles was monitored in an array of different rail carriages. The measurements were performed considering a frequency range between 100Hz and 2000Hz, given that these are the limits imposed in the tests in section 6.2.3.

The software Spectralissime was used as a real-time spectral analyzer in a laptop (Fig.6.9). The latter was used to monitor the minimum and maximum Sound pressure levels (SPL) on a 1/3 octave spectrum using a 1 fps sampling-rate. To simulate the hearing of a common train occupants an A-Weighting filter was applied. The interior noise was recorded using a Behringer C-1U condenser microphone.

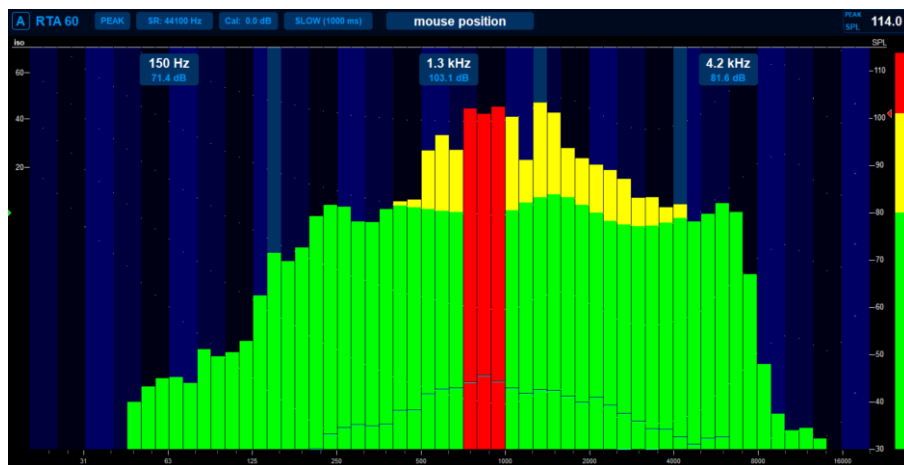


Figure 6.9. Example of measurement using Spectralissime.

Considering the calibration settings of the microphone, it may be observed in Fig.6.10 (a) that it is able to operate without noise interference in significant part of the tested frequency range. Fig. 10 (b)

shows that the directionality of the microphone has a significant impact in the overall sound pressure level reading. Thus, in all readings, the microphone was always placed facing the wall of the carriage to better capture reflected and outdoor transmitted noises. Additionally, the tickets were pre-booked to guarantee the measurement in a seat just above one the bogies and raised to the seat headrest position. The overall technical characteristics of the used microphone are presented in Table 6.4.

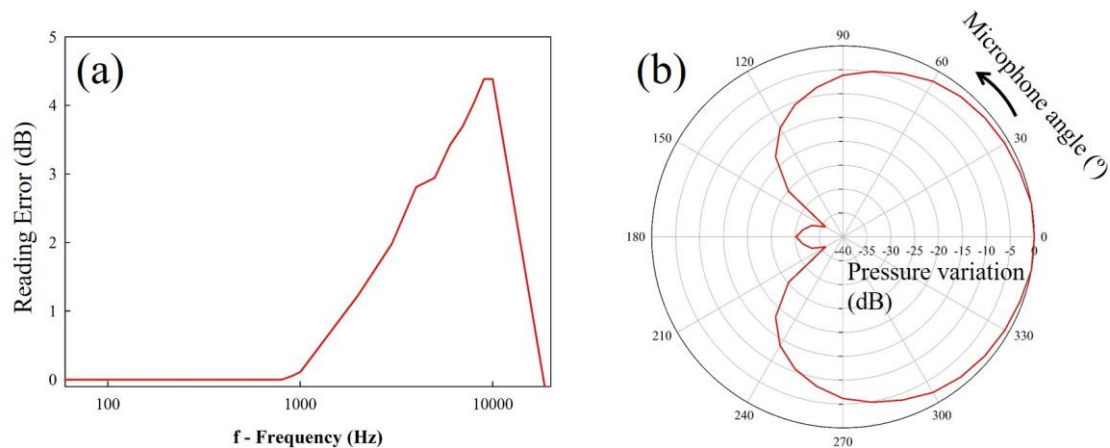


Figure 6.10. Behringer C-1U microphone calibration settings.

Table 6.4. Behringer C-1U microphone characteristics.

Frequency range (Hz)	40 – 20000
Maximum sound pressure level (dB @ 1kHz)	136
Sample rate (kHz @ 16-bit)	48

It was verified that the air conditioning was turned on in all trips. This actually contributes to the overall noise in the interior of the vehicle. Considering that railway user interaction also influences the interior noise due to the 85 Hz to 255 Hz in voiced speech, the tests were performed in hours that avoided rush-hour commuting.

Overall, the measurements were performed in the whole duration of the trips (Table 6.5). Thus, the readings were able to monitor the noise at different speeds when the vehicle is moving in a straight line, curves, rail joints, dipped welds and crossings. To facilitate the differentiation between the different vehicles, they were divided in three classes (according to Table 6.5). High speed class was defined as the one which comprises long-course trips and maximum speeds over 200 km/h. As for Intercity vehicles, they were defined by long-course trips with speeds up to 200 km/h. Finally, Light-rail was used to refer to vehicles that are intended for short-courses and low speeds (up to 100 km/h).

Table 6.5. Detail of experimental noise measurements in railway transports.

Class	Model	Builder	Power	Speed (km/h)	Operator	Location	Route	Date (2019)
High Speed *	X-2000	Kalmar	Electric	220	SJ	Sweden	Stockholm- Gothenburg	17th March
	AP Série 4000	Fiat Ferrovia			CP	Portugal	Braga- Lisbon	21st April
	Serie 103	Siemens		310	RENFE	Spain	Barcelona- Lleida	15th May
	Class 120	Alstom		250			Lleida- Barcelona	16th May
Inter City **	Class 18	Siemens	180	SNCB	Belgium	Zaventem- Brussels	18th March	
	350 Desiro			West Midlands	UK	Liverpool- Coventry	31st May	
	221 Super Voyager	Bombardier	Hybrid	200		Cross Country	Manchester- Coventry	7th June
	185 Desiro	Siemens	Diesel	160		TransPennine Express	Manchester- Man. Airport	1st August
	Light Rail ***	Eurotram	Bombardier	Electric	80	Metro do Porto	Portugal	Airport- Matosinhos
M5000		Metrolink				UK	Man. Airport- Manchester	23rd July
Swing		Pesa	70		Tramwaje Warszawskie	Poland	PI Zawiszy- Centrum	5th September

* Long-distance services and speeds over 200km/h.

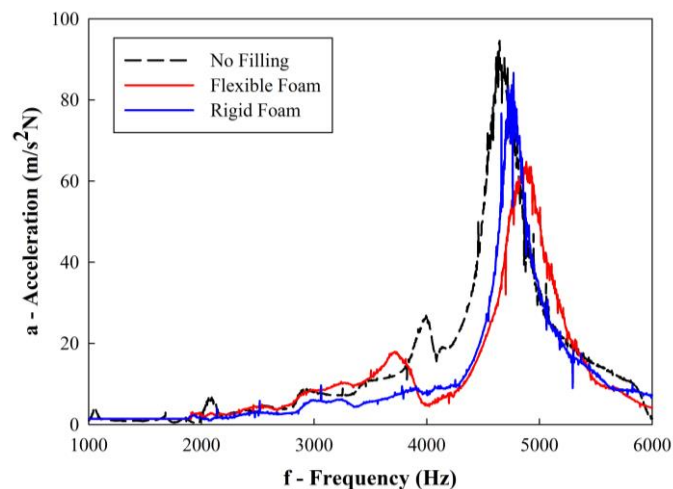
** Long-distance services and speeds between 100 and 200 km/h.

*** Short-distance services and speeds up to 100 km/h.

6.4 Results and Discussion

6.4.1. Vibration testing results

Figure 6.11 shows the frequency response functions (FRF) from the vibration testing. It is evidenced that the inclusion of a polymeric foam filling has a significant impact in the damping of the cellular solids, lowering the acceleration peaks in sample eigenfrequencies.

**Figure 6.11.** Sample frequency response function (FRF) from the vibration testing.

Using the half-power method, it was possible to calculate the damping ratio for each tested sample (Fig. 6.12). It may be concluded that a flexible filling generates more damping capacity than when a rigid filling is used. Polyurethane foams in flexible configurations are known to have higher damping values than in rigid configurations.

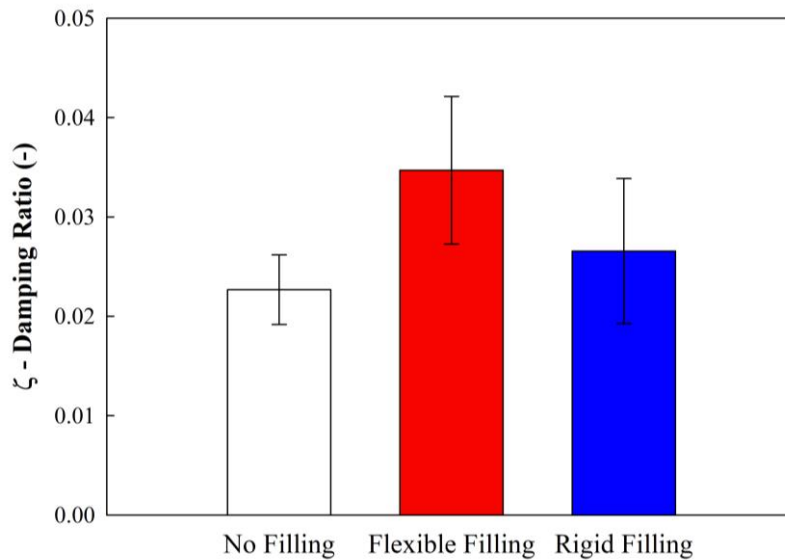


Figure 6.12. Average damping ratios (ζ) from the vibration testing.

This is further supported by the results from the numerical analysis that were performed in accordance with the methods in section 5.2.2 (Chapter 5). Fig.6.13 shows the results of the inverse engineering process by FEA that were used to determine the true damping ratio of the samples. It may be observed that the numerical routines were able to converge with the experimental results.

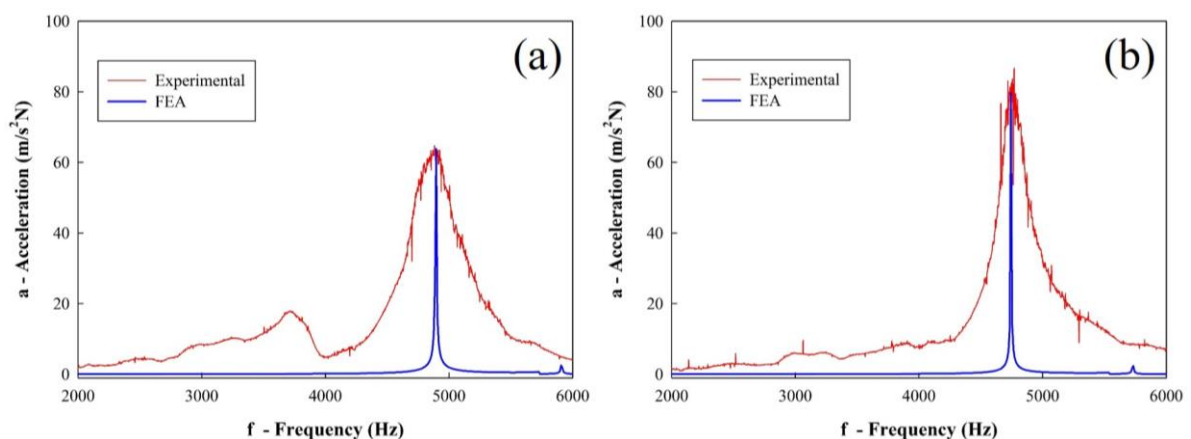


Figure 6.13. FEA results for samples with (a) flexible and (b) rigid foam fillings.

Considering these, it was possible to estimate the true values of damping ratio that are displayed in Fig. 6.14. According to these results, it is shown that they follow the same tendency of the experimental

results: (i) filling the sample increases their damping capacity; and (ii) Flexible foams generate higher values of damping than rigid foams. It may be concluded that the filling of the samples is useful to further promote damping capacity in the designed cellular solids.

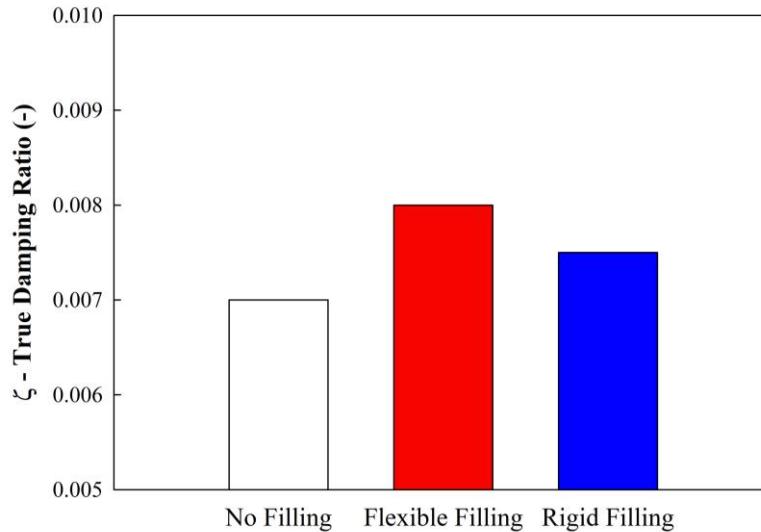


Figure 6.14. Average damping ratios (ξ) from FEA routines.

6.4.2. Acoustic testing results

While the filling of the sample has proven to be beneficial to increase the structural vibration damping capacity, they have also the objective of increasing the acoustic damping. *i.e.* they should promote an overall noise reduction and increase riding comfort.

It is known that the main source of noise is resultant from the exterior of the train cabin (Fig.6.15 (a)). The damping of exterior noise would greatly benefit the riding comfort of the occupants. The damping of these exterior sources may be correlated with the transmission loss in the material that is used to build the vehicle. Thus, on a designer point-of-view, such panels should have high values of transmission loss on a wide range of frequencies. However, it is impossible to completely block the noise transmission and, consequently, there is a part that is conducted to the interior of the cabin. This noise adds up to the noise that is generated in the interior of the cabin (Fig.6.15 (b)), either by its functional equipment (*e.g.* air conditioning, automated doors, etc) or passenger interaction. Thus, the material that is used to build the car body should also display a high absorption coefficient (α) to damp the referred noises.

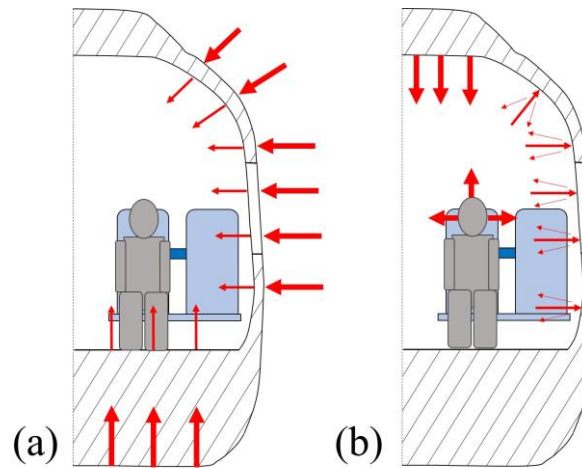


Figure 6.15. Noise progression: (a) external transmission and (b) internal reflection.

Considering the ability of the material to damp the reflected acoustic noise inside the cabin, Fig. 6.16 (a) shows the average resultant frequency dependent sound absorption coefficient of unfilled and foam filled samples. According to Eqs. 6.2 and 6.4, the results in Fig. 6.16 (b) are dependent in the values of acoustic impedance, *i.e.* the opposition that the system imposes to the propagation of acoustic waves.

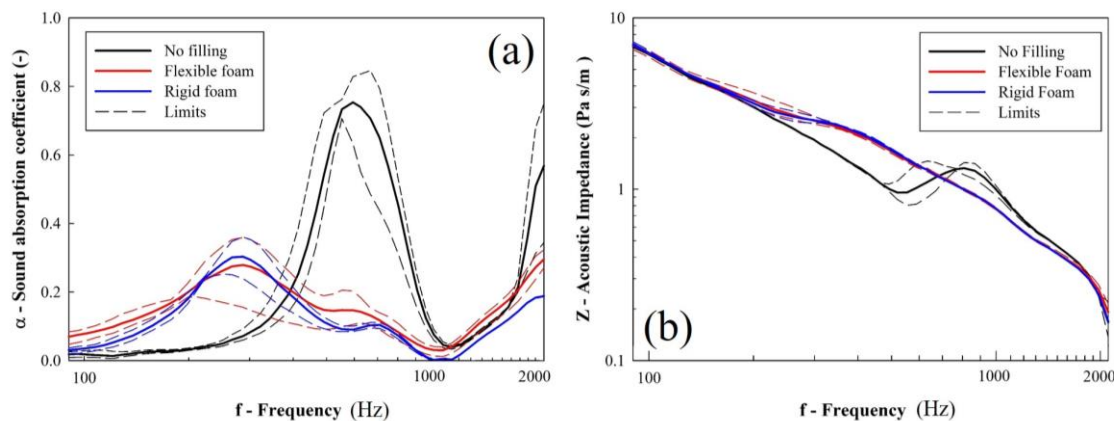


Figure 6.16. Results from acoustic reflection testing: (a) Sound absorption coefficient and (b) acoustic impedance.

The results (Fig. 6.16) show that unfilled samples display a very pronounced absorption peak near 600 Hz. However, when polyurethane foams are used to fill the cellular core, this peak is shifted to a lower frequency (300 Hz). More importantly, even though polymer foams are commonly associated as a solution to enhance noise damping capacity [38,39], the experimental results show a reduction in the absorption peaks.

It is known that the sound propagation in porous media, such as cellular solids, may be described by: (i) rigid or (ii) poroelastic structures [40]. The first, is applicable in rigid porous media and, in this

case, is associated with the acoustic propagation within the designed cellular solid. This propagation may be described by the Johnson-Champoux-Allard-Lafarge (JCAL) model [41,42]. The referred model states that the characteristic acoustic impedance of the system ($\bar{Z}_c(\omega)$ – Eq.6.8) is dependent on the visco-inertial [43] effects by changes in dynamic density ($\bar{\rho}(\omega)$) and visco-thermal effects by variations in the dynamic Bulk modulus ($\bar{K}(\omega)$) [21].

$$\bar{Z}_c(\omega) = \sqrt{\bar{\rho}(\omega)\bar{K}(\omega)} \quad (6.8)$$

The angular frequency (ω) dependent dynamic density ($\bar{\rho}_{eq}(\omega)$) is known to be influenced in four variables: sample porosity (Φ), air flow resistivity (σ), high frequency limit tortuosity (α_∞) and the viscous characteristic length (Λ). These are correlated by Eq.6.9, in which the constants ρ_0 and η are, respectively, the air density and dynamic viscosity [17].

$$\bar{\rho}(\omega) = \frac{\alpha_\infty \rho_0}{\Phi} \left[1 + \frac{\sigma \Phi}{j \omega \alpha_\infty \rho_0} \sqrt{1 + j \frac{4 \alpha_\infty^2 \eta \rho_0 \omega}{\sigma^2 \Lambda^2 \Phi^2}} \right] \quad (6.9)$$

To compensate these effects, the JCAL model corrects the characteristic acoustic impedance (Eq.6.8) with a dynamic Bulk modulus ($\bar{K}(\omega)$ – Eq.6.10). Where the thermal effects are dependent in two variables: the thermal characteristic length (Λ') and the static thermal permeability (k'_0). Eq. 6.10 also includes the contribution of constants, such as: the specific heat (γ), atmospheric pressure (P_0), heat capacity (C_p) and thermal conductivity (k) [21,44].

$$\bar{K}(\omega) = \frac{\gamma P_0 / \Phi}{\gamma - (\gamma - 1) \left[1 - j \frac{\Phi k}{k'_0 C_p \rho_0 \omega} \sqrt{1 + j \frac{4 k'_0 C_p \rho_0 \omega}{k \Lambda'^2 \Phi^2}} \right]^{-1}} \quad (6.10)$$

Eq. 6.9 is related with the visco-inertial effects on the acoustic propagation. However, the heat exchange and thermal dissipation (Eq.6.10) by acoustic excitation in rigid open-cell porous media must also be taken into account to correctly estimate the acoustic propagation [18,43,45].

While in unfilled samples (Fig.6.17 (a)), the visco-inertial propagation of the acoustic waves are performed through the rather open aluminum matrix and the thermal interaction is suggested to be residual due to the low values of superficial area in which the visco-thermal dissipation occurs, this may not be true in foam filled samples (Fig.6.17 (b)). In these samples, the superficial area in the foams are much more significant and so is the visco-thermal dissipation in their boundary layer.

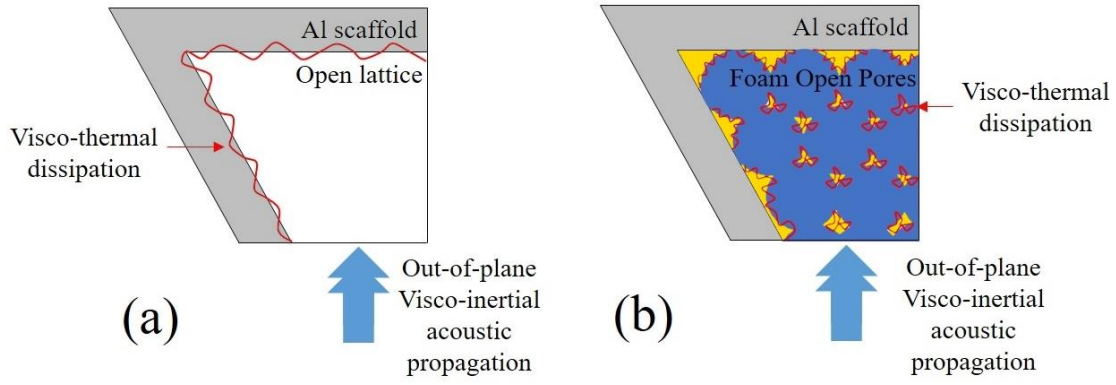


Figure 6.17. JCAL model in (a) unfilled and (b) foam filled samples.

Using the relation in Eqs. 6.8 to 6.10 and a graphical tool made available by Matelys research lab (Vaulx-en-Velin, France) [46] a sensibility analysis was performed to each individual variable to determine their influence and variations in the shape of a typical frequency dependent sound absorption coefficient plot (Figs.6.18 to 6.20). This analysis is used to suggest the transformations in the acoustic impedance and, consequently, sound absorption in the unfilled and flexible/rigid foam filled samples.

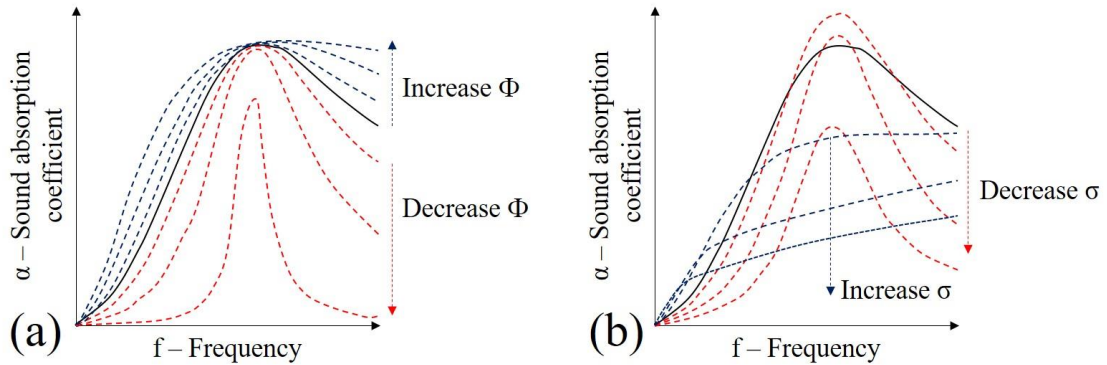


Figure 6.18. Influence of visco-inertial parameters in the sound absorption coefficient of porous media (JCAL model): (a) Φ - porosity; and (b) σ - air flow resistivity.

Sample porosity (Φ – Fig. 6.18 (a)) is defined as the ratio of fluid volume that occupies the interior of a cellular solid [47] and it is, fundamentally, the fraction of the material that is not solid (*i.e.* $\Phi = 1 - \rho^*/\rho_0$). According to Fig. 6.18 (a), it may be observed that as the value of porosity is decreased, so does the sound absorption coefficient. This effect has already been known to occur in other porous media [16,32,48]. This is justified by the closing effect on the cells of the porous media and the difficult absorption during the visco- inertial and thermal process during the propagation [49]. It is worth mentioning that, for the acoustic propagation purpose, only the open-cell porosity is taken in consideration. In fact, this is the case of the adopted foams (see Table 6.2). It is suggested that during

the filling process, the porosity is decreased and, thus, there is a relative reduction in the sound absorption coefficient.

According to Darcy's law [50] (Eq. 6.11), the decrease in porosity may increase the static air flow resistivity (σ) for a given pressure gradient ($\vec{\nabla}P$) and fluid velocity ($\vec{v}$). Observing Fig.6.18 (b), it is suggested that this can lower the sound absorption coefficient, being this fact also reported in other studies [51].

$$\sigma = -\frac{\vec{\nabla}P}{\Phi\vec{v}} \quad (6.11)$$

The high frequency limit of the tortuosity (α_∞) is related to the geometrical disorder of the porous media in which the acoustic propagation occurs [41] and may assume values between 1 and 3 [52]. The lower limit is generally characteristic of parallel streamlines, while the upper limit is used in extremely disordered systems (*e.g.* for a typical polyurethane foam, $\alpha_\infty=1.29$ [45]). It is considered that the tortuosity is increased when the samples are infiltrated by foam. Observing Fig. 6.19 (a), it is suggested that the sound absorption peak may be moved to a lower frequency. This hypothesis may be supported by studies on other porous media [48,53].

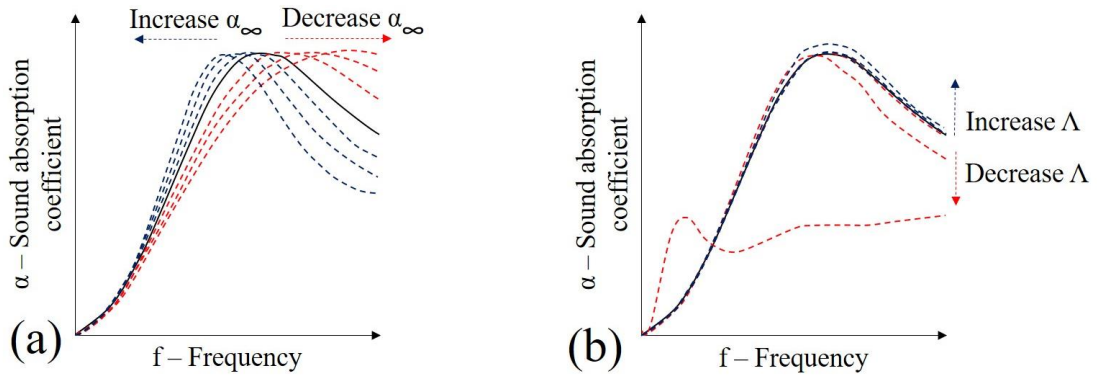


Figure 6.19. Influence of visco-inertial parameters in the sound absorption coefficient of porous media (JCAL model): (a) α_∞ - high frequency limit of the tortuosity; and (b) Λ - viscous characteristic length.

The viscous characteristic length (Λ) describes the viscous effects of the flow [41]. A general description may be observed in Eq.6.12 [54], where this variable is defined as the relation between the pore volume (V) and superficial area (A), in a given fluid velocity (v). It may be understood that this relation is highly dependent on the porosity (Φ) and tortuosity (α_∞) (Eq.6.13, where k_0 is the static viscous permeability). Due to the suggested reduction in porosity and elevation in tortuosity, it is proposed that

the filling of the samples with foam increases the values of viscous characteristic length (Λ). Observing Fig. 6.19(b), it may be observed that this may contribute to the increase in sound absorption coefficient.

$$\Lambda = 2 \frac{\int_V v^2 dV}{\int_A v^2 dA} \quad (6.12)$$

$$\Lambda = \sqrt{\frac{8\alpha_\infty k_0}{\Phi}} \quad (6.13)$$

A similar analysis may be performed to determine the variations in the thermal characteristic length (Λ') that accounts for the thermal dissipation in the periodic unit cell [20]. Given that this variable is also predominantly affected by the ratio of pore volume and superficial area (Eq.6.12) [55] and recurring to Fig.6.20 (a), it may be observed that as the value of this variable increases, there may be an elevation in the sound absorption coefficient.

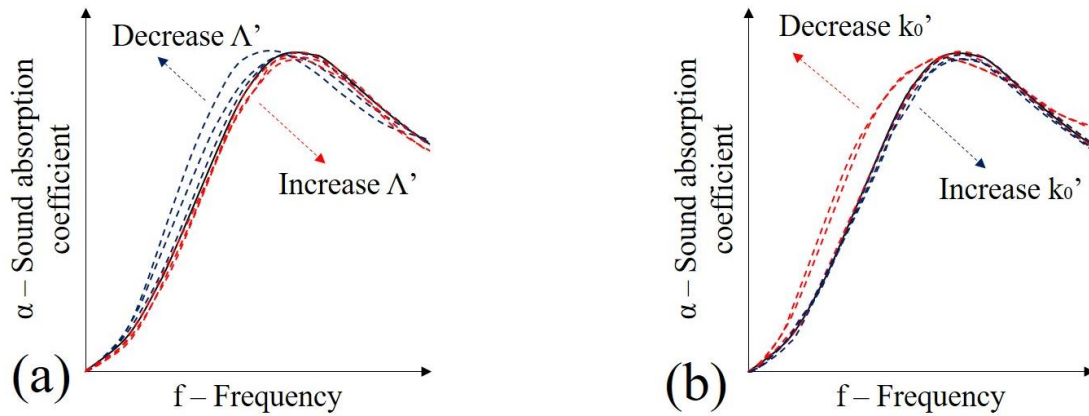


Figure 6.20. Influence of visco-thermal parameters in the sound absorption coefficient of porous media (JCAL model): (a) Λ' - thermal characteristic length; and (b) k'_0 - static thermal permeability.

The frequency dependent dynamic thermal permeability ($k'(\omega)$) may be calculated using Eq. 6.14, being dependent on the differentials of pressure and time ($\partial p / \partial t$), porosity (Φ), air thermal conductivity (k) and excess of temperature inside the cellular solid (τ) [40,42]. The static thermal permeability (k'_0) relates thermal effects between the rigid frame and the fluid at low frequencies [56], actuating in the boundary layer, according to Eq.6.15. Given its dependence on the sample porosity, it is suggested that the foam infiltration process is able to decrease the value of the static thermal permeability. Thus, according to Fig. 6.20(b), it is expected a lowering in sound absorption and peak frequency.

$$k'(\omega) = \Phi \tau \frac{\partial t}{\partial P} \quad (6.14)$$

$$k'_0 = \lim_{\omega \rightarrow 0} k'(\omega) \quad (6.15)$$

If during the acoustic propagation the structure can no longer be considered rigid, for instance in the case of the polyurethane foams that are used to infiltrate the samples, the formulation has to be corrected [43]. According to the poroelastic Biot's theory [57,58], the surface impedance (Z) may be calculated using Eqs.6.16 and 6.17 [59].

$$Z = -j \frac{Z_1^s Z_2^f \mu_2 - Z_2^s Z_1^f \mu_1}{D} \quad (6.16)$$

$$D = (1 - \Phi + \Phi \mu_2) [Z_1^s - (1 - \Phi Z_1^f \mu_1)] \tan(\delta_2 d) \\ + (1 - \Phi + \Phi \mu_1) [Z_2^f \mu_2 (1 - \Phi) - Z_2^s] \tan(\delta_2 d) \quad (6.17)$$

The referred theory considers that the propagation by compression waves is due to three separate waves: fast (P_1) and slower (P_2) compression waves and a shear wave (S) [59]. Considering these facts, the surface impedance (Z) may be divided in four possible impedance combinations according to Eqs.6.18 and 6.19. These combinations are related to the fast (1) and slow (2) waves on the solid (s) and fluid (f) parts: Z_1^f , Z_2^f , Z_1^s and Z_2^s . [60].

$$Z_i^f = \frac{\delta_i (P + Q \mu_i)}{\Phi \omega}, i = \{1; 2\} \quad (6.18)$$

$$Z_i^s = \frac{\delta_i (P + Q \mu_i)}{\omega}, i = \{1; 2\} \quad (6.19)$$

Overall, the constants that need to be determined to perform the calculations in Eqs. 6.16 to 6.19 may be calculated using Eqs. 6.20 to 6.23 [59].

$$\mu_i = \frac{P \delta_i^2 - \omega \rho_{11}}{\omega^2 \rho_{12} - Q \delta_i^2}, i = \{1; 2\} \quad (6.20)$$

$$\delta_1 = \sqrt{\frac{\omega^2}{2(PR - Q^2)} [P \rho_{22} + R \rho_{11} - 2Q \rho_{12} - \sqrt{\Delta}]} \quad (6.21)$$

$$\delta_2 = \sqrt{\frac{\omega^2}{2(PR - Q^2)} [P \rho_{22} + R \rho_{11} - 2Q \rho_{12} + \sqrt{\Delta}]} \quad (6.22)$$

$$\Delta = (P\rho_{22} + R\rho_{11} - 2Q\rho_{12}) - 4(PR - Q^2)(\rho_{11}\rho_{22} - \rho_{12}^2) \quad (6.23)$$

The equivalent mass densities (ρ_{11} , ρ_{22} and ρ_{12}) in the system may be determined by Eqs. 6.24 to 6.26 [61]. The latter are able to relate the poroelastic influence in the initially rigid approach described early by the use of the dynamic density ($\bar{\rho}$ – Eq. 6.9) that is determined in the JCAL model. It may be highlighted that the equivalent mass densities are also influenced by the foam and air densities, respectively, ρ_s and ρ_0 .

$$\rho_{11} = \Phi^2 \bar{\rho} - \Phi \rho_0 + \rho_s \quad (6.24)$$

$$\rho_{22} = \Phi^2 \bar{\rho} \quad (6.25)$$

$$\rho_{12} = \Phi \rho_0 - \Phi^2 \bar{\rho} \quad (6.26)$$

The remaining parameters that need to be used in the solving of Eqs. 6.24 to 6.26 are known as Biot coefficients (P, Q and R) and may be determined by Eqs. 6.27 to 6.29 [40]. Parameters themselves are influenced by the sample porosity (Φ), the shear modulus (G), the variable K_B (Eq.6.30) and the Poisson's ratio (ν). Eq. 6.31 is a variable related with the JCAL model by the inclusion the dynamic Bulk modulus ($\bar{K}$) through Eq. 6.10. The Young's modulus (E) of the sample itself may be directly correlated using the relation between elastic constants from the three-dimensional Hooke's law (Eq.6.32) [62].

$$P = \frac{4G}{3} + K_B + K_e \left[\frac{(1 - \Phi)^2}{\Phi} \right] \quad (6.27)$$

$$Q = K_e(1 - \Phi) \quad (6.28)$$

$$R = \Phi K_e \quad (6.29)$$

$$K_B = \frac{2G(\nu + 1)}{3(1 + 2\nu)} \quad (6.30)$$

$$K_e = \Phi \bar{K} \quad (6.31)$$

$$E = 2G(1 + \nu) \quad (6.32)$$

Considering that the Poisson's ratio of the foams are approximately constant, Fig.6.21 was plotted to determine the influence of the Young's modulus (E) and density (ρ_s) in the sound absorption coefficient of a generic cellular solid. In Fig.6.21 (a), it is suggested that lower values of elastic modulus are useful to increase the sound absorption. However, as the value of this constant increases, the plots tend to approximate the JCAL model (black plot – Figs 6.21). In fact, since as the sample becomes stiffer,

it should approximate the JCAL model that considers the solid portion of the foam as rigid. The same behavior is observed in situations where the density is increased according to Fig.6.21 (b).

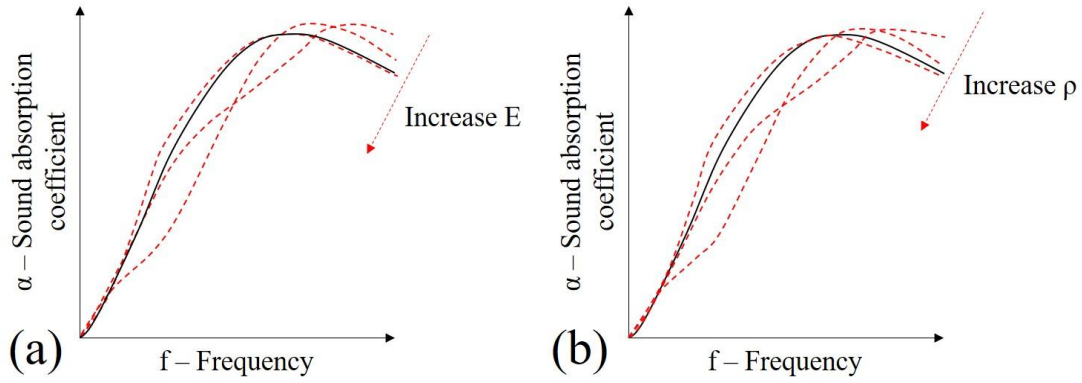


Figure 6.21. Influence of changing poroelasticity parameters in the sound absorption coefficient of a generic porous media: (a) E - elastic modulus and (b) ρ - density.

Fig. 6.22 shows the combination of parameter variations that were analyzed according to what occurs in the samples when foams are introduced in the voids of the initial metallic lattice. In Fig. 6.22 (a), it may be observed that the rigid interaction during the acoustic progressions that is predicted by the JCAL model implies a reduction of the sound absorption coefficient peak. Additionally, there is also a lowering of the sound absorption peak frequency. According to Fig. 6.22 (b), a small increase in the sound absorption coefficient is also expected due to the poroelastic influence.

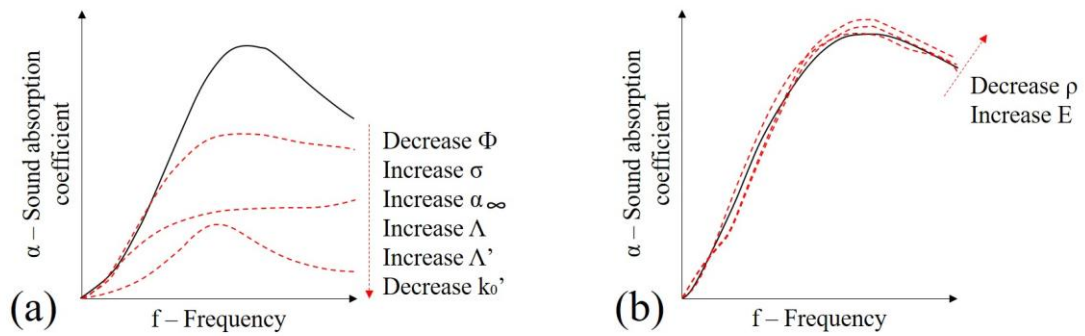


Figure 6.22. Combination of changing parameters in the sound absorption of porous media due to effect of: (a) rigid structure (JCAL model) and (b) poroelasticity (Biot's theory).

Overall, these predictions are able to describe variation of the experimental results (Fig.6.16): (i) the reduction of the peak and its change in frequency that is observed when foam is inserted, are justified by the variations in JCAL parameters described in Fig.6.22 (a); (ii) the small variation between the

samples with flexible and rigid foam samples is due to the change in poroelastic properties shown in Fig.6.22 (b).

Fig.6.23 shows the experimental results concerning the Transmission loss (TL) of the manufactured samples. It is shown that the filling of the samples with foam was able to successfully increase the transmission loss, *i.e.* reduce the propagation of acoustic noise by its damping. More importantly, it may be observed that there are frequencies (*e.g.* 100 - 150 Hz and ≈ 400 Hz) that the unfilled samples are not able to damp the noise.

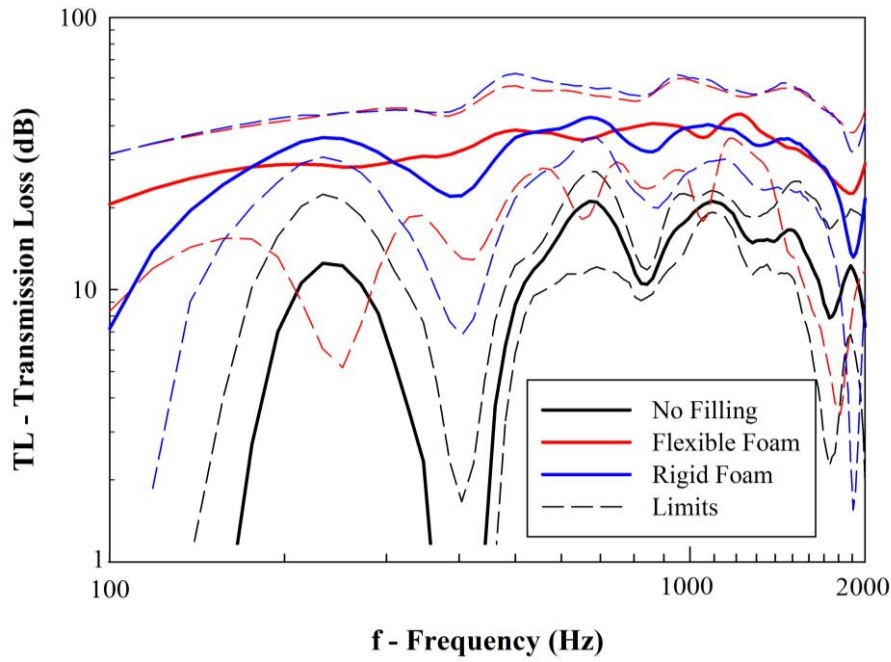


Figure 6.23. Results from acoustic Transmission loss.

The transmissibility (T) of a cellular solid (*i.e.* porous media) may be described by Eq.6.32, in which L is the sample width, $Y(\omega)$ and $k(\omega)$ are angular frequency dependent variables [63].

$$T(\omega) = \frac{2Y(\omega)}{2Y(\omega) \coth(k(\omega)L) + [1 + Y(\omega)^2] \sinh(k(\omega)L)} \quad (6.32)$$

Such constants may be determined by Eqs. 6.33 and 6.34, using the dynamic compressibility $\beta(\omega)$, dynamic tortuosity $\alpha(\omega)$, fluid density (ρ_f) and compressibility modulus of the fluid (K_a) [63].

$$Y(\omega) = \Phi \sqrt{\frac{\beta(\omega)}{\alpha(\omega)}} \quad (6.33)$$

$$k(\omega) = \omega \sqrt{\frac{\rho_f \alpha(\omega) \beta(\omega)}{K_a}} \quad (6.34)$$

It may be observed that the referred variables are described by Eqs.6.35 and 6.36, and that they may be determined using the sample porosity (Φ), high frequency limit of the tortuosity (α_∞), viscous static permeability (k_0), fluid viscosity (η) and Prandtl number (Pr) [41,63].

$$\alpha(\omega) = \alpha_\infty \left(1 + \frac{\eta \Phi}{j\omega \alpha_\infty \rho_f k_0} \right) \quad (6.35)$$

$$\beta(\omega) = \gamma + \frac{(\gamma - 1) \rho_f k_0 Pr}{\eta \Phi j\omega} \quad (6.36)$$

According to Table 6.6 and Eqs. 6.32 to 6.36 and given that the impregnation of the samples with foam reduces the porosity (Φ), there is an increase in the values of tortuosity (*i.e.* $\alpha(\omega)$ and α_∞). Additionally, there is an increase in the values of dynamic compressibility ($\beta(\omega)$).

Table 6.6. Expected variable changes in foam filled samples.

Variable	Φ	α_∞	$\alpha(\omega)$	$\beta(\omega)$	$Y(\omega)$	$k(\omega)$	$T(\omega)$
Change	↓	↑	↑	↑	↓	↑	↓

This elevation in the values of dynamic compressibility reduces the variable $Y(\omega)$, while increasing $k(\omega)$. Thus, according to Eq. 6.32, the foam filled samples are expected to have lower values of Transmissibility (T). According to Eq.6.37 [22] it is also expected that they have an enhanced ability to damp the propagation of acoustic noise and, thus, display higher values of Transmission loss (TL). This is corroborated by the experimental that are presented in Fig.6.23.

$$TL = 10 \log \left(\frac{1}{T} \right) \quad (6.37)$$

Comparing the experimental results for both unfilled and foam filled samples, it may be concluded that: (i) unfilled samples are better for sound absorption, especially in certain frequency ranges (Fig. 6.16); and (ii) foam filled samples, particularly with flexible foams, have more damping capacity and higher values of transmission loss for the tested frequency range (Fig. 6.23).

6.5. Comparison with other materials

Having analyzed the acoustic properties of the designed cellular solids, it is interesting to compare their Sound absorption and Transmission loss with solutions that are currently available for use and reported in other studies. Relatively to the sound absorption coefficient (α), the developed cellular solids are compared with other aluminum foams [64–66] with approximate specific densities in Fig.6.24. Given that unfilled samples showed the most promising results, only this type of samples was selected for comparison.

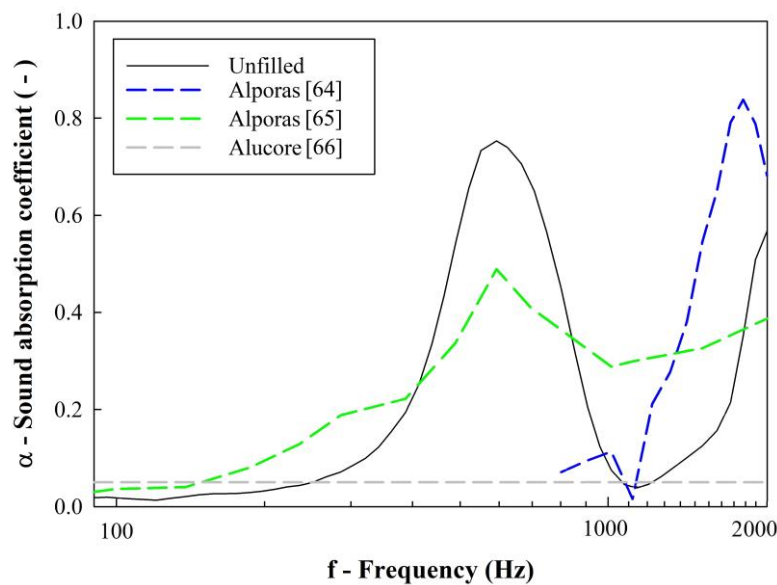


Figure 6.24. Sound absorption coefficient comparison: designed cellular solids vs other Al foams.

Other publications use unfilled low-density aluminum foams and it is observed that their damping peak occurs at higher frequencies than the tested flexible foam sample. However, this is in agreement with the experimental result on unfilled samples. Recurring to Fig. 6.24, it may be observed that the damping peak of unfilled samples has high sound absorption relatively to other published solutions. Additionally, the peak is wider, suggesting that this absorption is distributed along a wider frequency range. Therefore, unfilled samples are suggested to be a good solution for application in which sound absorption is required.

In terms of transmission loss (TL), Fig. 6.25 compares the experimental results with other published works on other materials that are commonly used in train cabin fabrication [67]. Given that unfilled samples show low values of transmission loss, only flexible foam filled samples were used for comparison purposes. According to these results, it may be observed that the designed composites show an overall higher transmission loss than the other current materials.

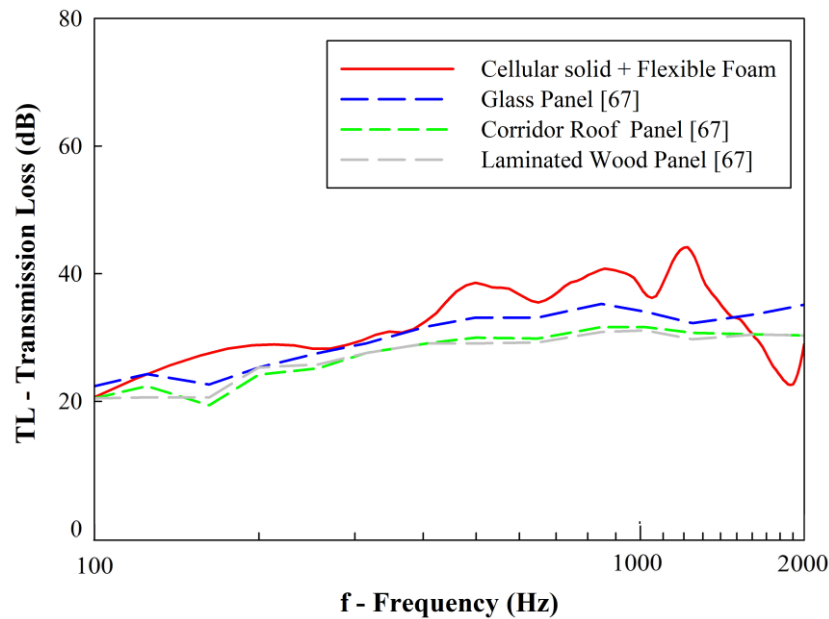


Figure 6.25. Transmission loss comparison: designed cellular solids vs materials used in train vehicle fabrication.

While the developed cellular solids are suggested to enhance the transmission loss relatively to current solutions, Fig.6.26 compares the experimental data with recent published solutions for noise damping in railway vehicles [68].

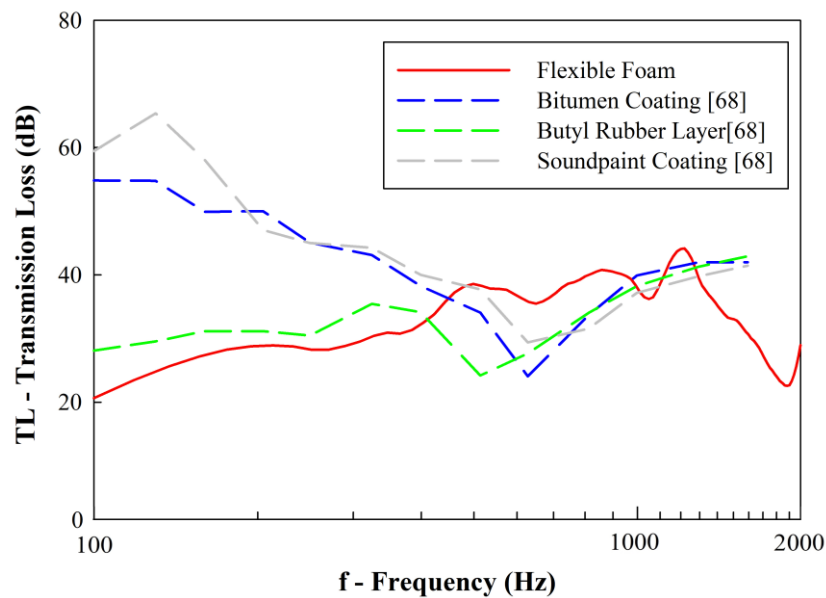


Figure 6.26. Transmission loss comparison: designed cellular solids vs other suggested solutions for noise damping.

From these results, it may be observed that the use of bitumen or Soundpaint (water-based paint for noise damping) coatings increase the values of Transmission loss, especially in low frequencies. Butyl

rubber layers display similar behavior to those shown by the cellular solids filled with foams. Thus, coatings seem an interesting solution to increase the transmission loss. Due to this fact, it is suggested that the coupling of bitumen/soundpaint coating with the manufactured cellular solids may be interesting to enhance the low frequency transmission loss in future studies.

6.6. Comparison with train vehicle acoustics results

To determine the critical frequencies of both sound absorption coefficient and transmission loss in different railway vehicles, the Sound power level was measured in diverse high-speed, inter-city and light-rail vehicles. These measurements were performed to determine the frequencies in which the higher noise values occur, and thus, determine if the manufactured cellular solids would be a solution for their mitigation.

Fig. 6.27 shows the interior noise in High-speed trains. It may be observed that the minimum sound power levels (SPL) remain in the range of 15 dBA to 30 dBA. The exception is the Series 120 that displays minimum values of SPL in the range of 30 dBA to 38 dBA. Relatively to the maximum values of SPL in High-speed vehicles none of the monitored trains display relevant difference in the measured noise. However, some individual peaks in certain frequencies may be observed in each individual train model. Overall, maximum values of SPL in this type of trains was monitored to be in the range of 36 dBA and 57 dBA.

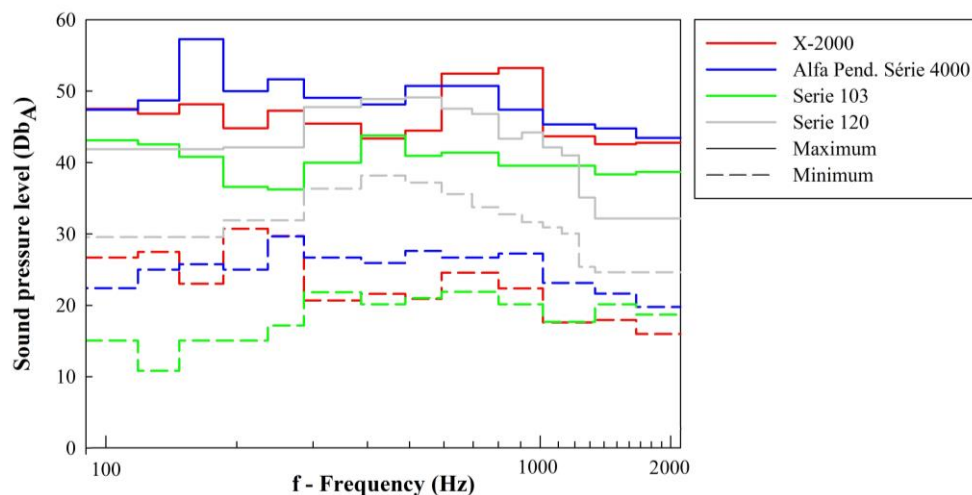


Figure 6.27. Interior noise in High-speed trains.

The SPL values that were monitored in Inter-city trains are shown in Fig. 6.28. According to the latter, it may be observed that there may be found two distinct minimum SPL behaviors: (i) the Class 18 and 185 Desiro trains show low values of minimum SPL (11 to 23 dBA); and (ii) the 350 Desiro and

221 Super Voyager that display higher minimum values of SPL (22 to 38 dBA). In term of maximum SPL, all trains seem to have higher values of SPL in the range of 28 dBA and 40 dBA. The exception is the 185 Desiro that shows two noise peaks (≈ 47 dBA) in frequencies between 250-300 Hz and 500-600 Hz.

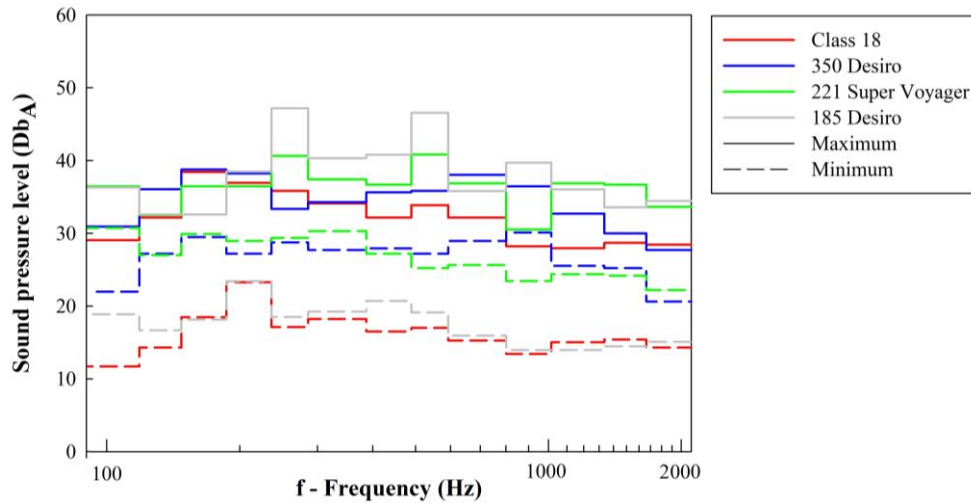


Figure 6.28. Interior noise in Inter-city trains.

Fig. 6.29 shows the SPL inside Light-rail trams. It is shown that the minimum SPL may vary from 16 dBA to 36 dBA and there is a scatter of the results in frequency and vehicle types.

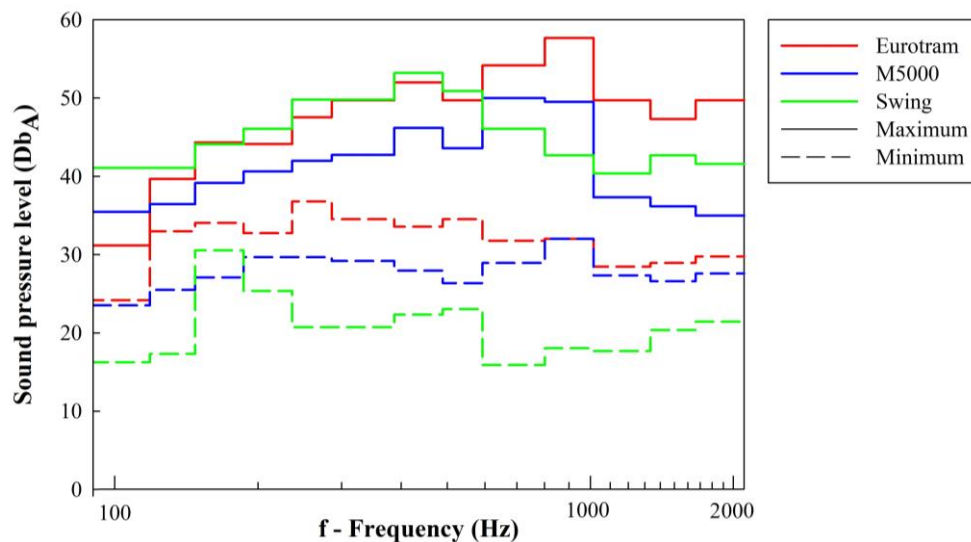


Figure 6.29. Interior noise in Light-rail.

However, Swing vehicles tend to have lower values of SPL. In terms of maximum SPL, there seems to be a common tendency for the frequency dependent SPL in all vehicle types. Eurotram vehicles, though, display higher SPL values at higher frequencies, while the maximum noise in Swing vehicles occurs at lower frequencies.

To estimate the relevant frequencies in which the designed cellular solids could be used to mitigate noise inside the vehicle cabin, the Figs.6.30 to 6.32 were analyzed to determine the frequency ranges with maximum SPL values. Given the low relative values of the overall minimum SPL (between 11 dBA and 36 dBA), this analysis was focused primarily in the maximum noise values. The critical values of frequency were identified by the observation of the common maximum SPL on such frequency domains. According to the referred criterion, Fig.6.30 determined that the critical frequencies in High-speed trains are in the ranges of 160-200 Hz and 600-800 Hz.

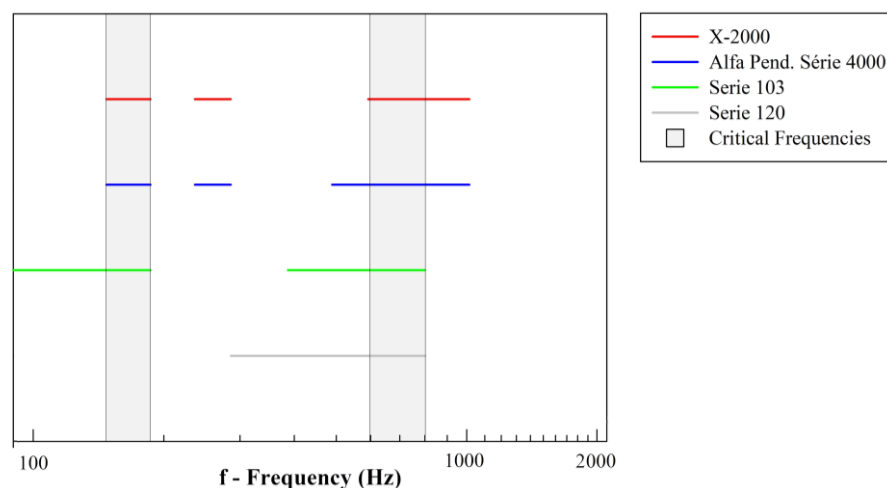


Figure 6.30. Identification of critical frequencies in High-speed trains.

In the case of Inter-city trains (Fig.6.31), such critical frequencies are suggested to be concentrated in the value of 250 Hz and in the range of 500-600 Hz.

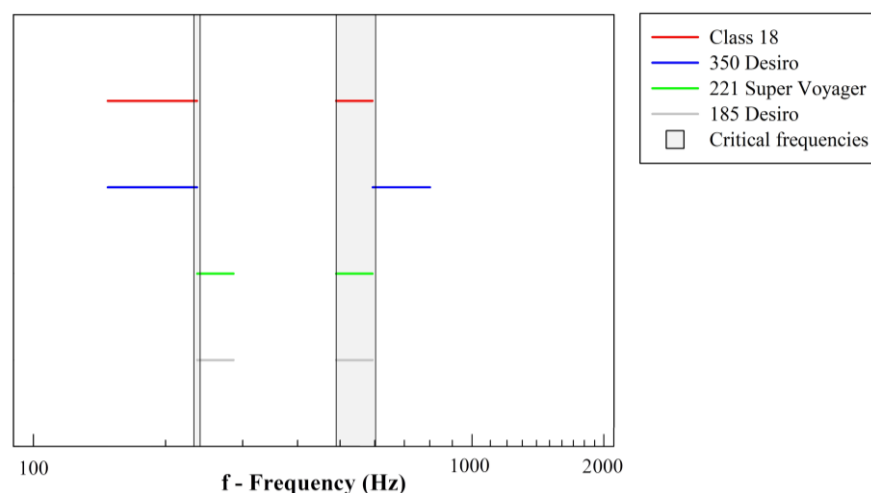


Figure 6.31. Identification of critical frequencies in Inter-city trains.

Critical frequencies in Light-rail (Fig.6.32) seem to be more concentrated in the ranges of 400-500 Hz and the individual frequency of 600 Hz, according to Fig.6.32.

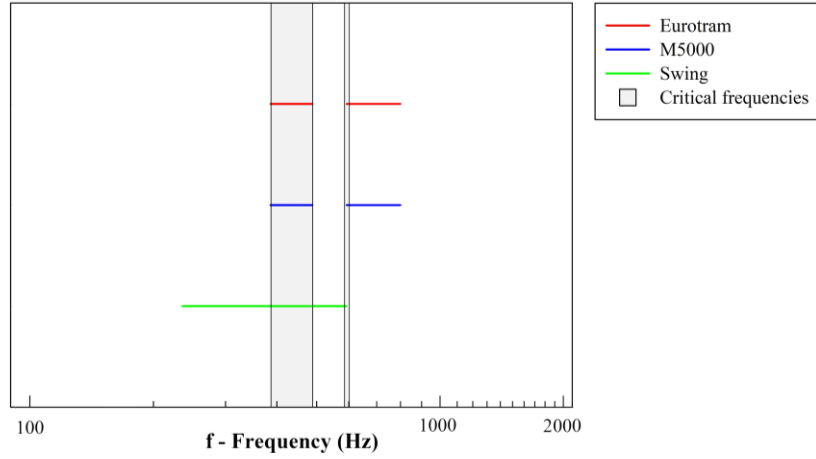


Figure 6.32. Identification of critical frequencies in Light-rail trains.

Once the critical frequencies are known for each train type (Figs. 6.30 to 6.32), it is possible to combine this analysis with the experimental results on Sound absorption and Transmission loss (Fig.6.16 and 6.23). Given that critical frequencies change with each train type, the variation of properties between unfilled and foam filled samples may be used to tailor the cellular solids for specific critical frequencies.

According to Fig.6.33 (a) the sound absorption peak in unfilled samples is located in the frequency range of one of the critical frequency range of High-speed trains. This may be useful to reduce the overall reflection inside the vehicle. In terms of Transmission loss (Fig.6.33 (b)), it may be observed that a using the developed cellular solids with a rigid foam filling may be beneficial to reduce the noise that is propagated to the interior of the train cabin.

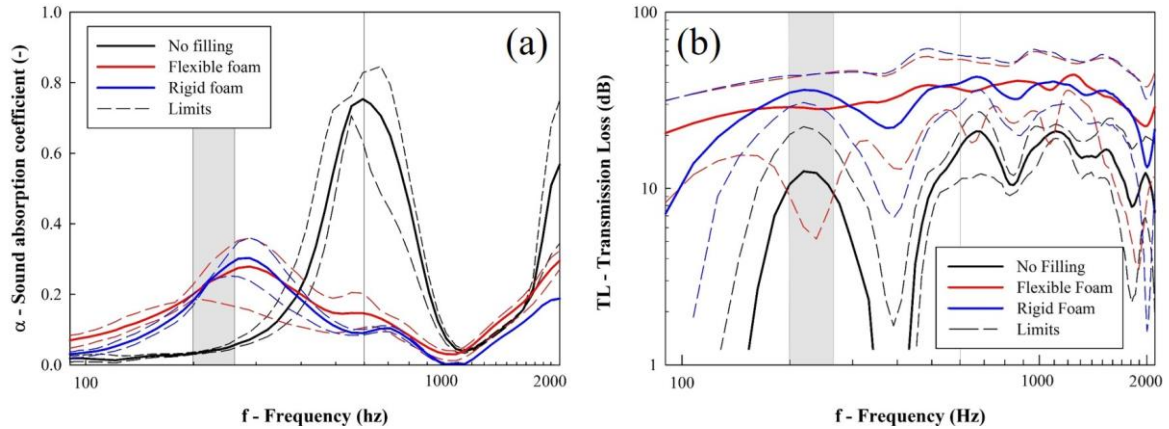


Figure 6.33. Comparison between experimental tests and critical frequencies in High-speed trains: (a) Sound absorption coefficient and (b) Transmission loss.

Comparing the sound absorption coefficient in Inter-city vehicles (Fig.6.34 (a)), it is observed that the first critical frequency may be addressed using foam filled samples, while the second is located in the

peak of unfilled samples. Given the broader frequency spectrum of the second critical range, it is recommended that unfilled samples may be interesting in this particular situation. Analyzing Fig. 6.34 (b), it may be seen that samples with rigid foam show higher values of transmission loss in the critical frequencies. They are more appropriate to mitigate the progression of noise to the cabin interior.

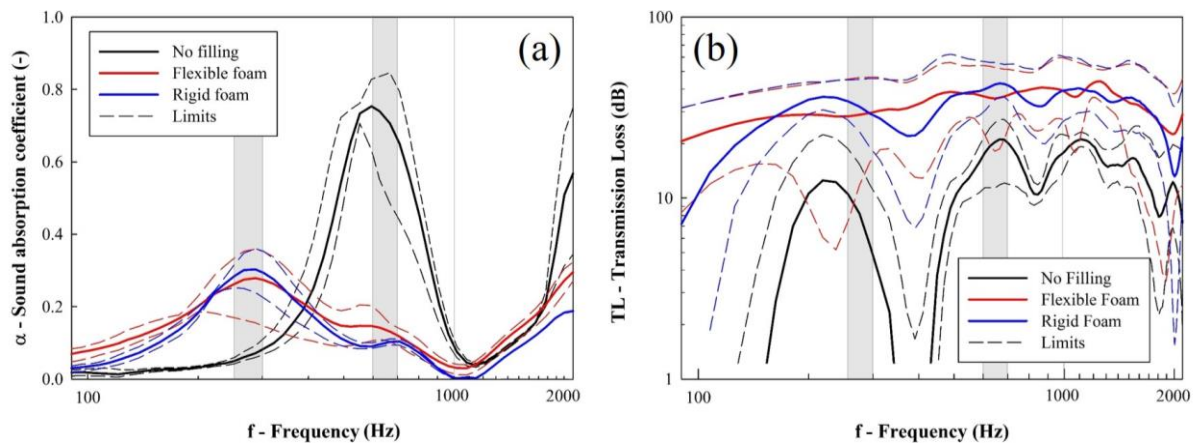


Figure 6.34. Comparison between experimental tests and critical frequencies in Inter-city: (a) Sound absorption coefficient and (b) Transmission loss.

In the case of Light-rail, it may be observed in Fig.6.35 (a) that the critical frequencies are located in the frequency range of the sound absorption peak of unfilled samples. Thus, this configuration seems useful to reduce the sound reflection inside the vehicles. Comparing the values of Transmission loss (Fig. 6.35 (b)) of the samples, it may be observed that the critical frequencies occur in ranges where the higher values of transmission loss are mostly represented by flexible foam filled samples. It is suggested that the latter are the more suitable to reduce the propagation of noise to the interior of the vehicle.

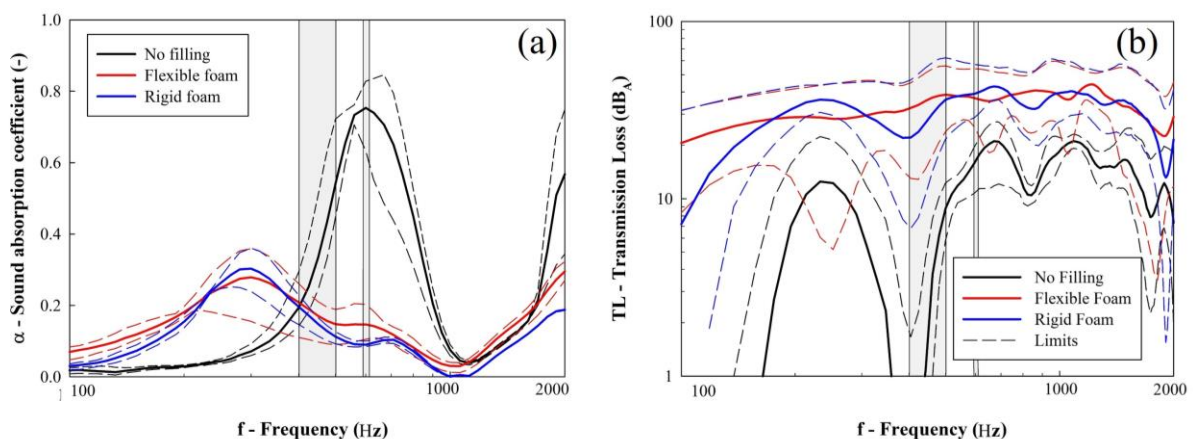


Figure 6.35. Comparison between experimental tests and critical frequencies in Light-rail: (a) Sound absorption coefficient and (b) Transmission loss.

According to the analysis of Figs. 6.33 to 6.35, it may be concluded that unfilled samples are more suitable to reduce the sound reflection inside the vehicle. However, foam filled samples are more promising to mitigate the propagation of noise from exterior sources.

It is suggested that a possible solution for the combination of these properties would be the future development on a hybrid composite. Such cellular solid would have an unfilled part and another that is filled with polyurethane foam, as represented in Fig.6.36. The unfilled portion would face the interior of the cabin due to its higher values of Sound absorption coefficient, while the foam filled portion would face the exterior due to its higher values of Transmission loss. Additionally, the foam filled portion of High-speed and Inter-city trains would be filled with rigid foams, while in Light-rail this filling would use flexible foams.

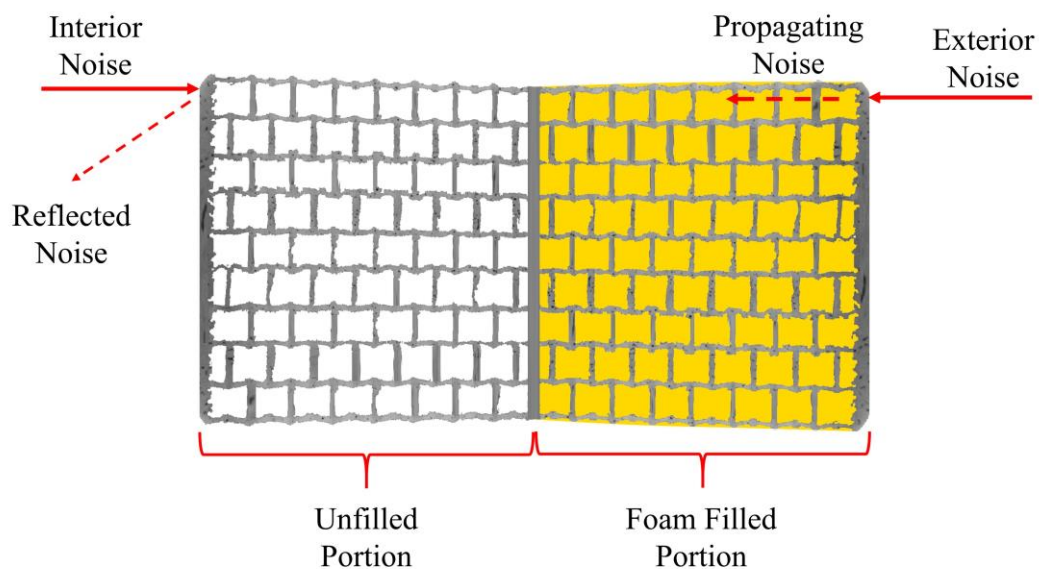


Figure 6.36. Hybrid cellular solid concept: combination of unfilled and foam filled portions.

6.7. Conclusions on the acoustic properties of the cellular solids

This chapter shows the selection of the sample design with the optimized combination of static (Chapter 4) and dynamic (Chapter 5) properties. The selected sample design was used to study the impregnation with polyurethane foam as an attempt to enhance both structural vibration and noise damping. Also, interior noise measurements were performed in different train classes to estimate the applicability of the designed cellular solids in the acoustic damping of noise in critical frequencies. According to the results, the following conclusions were established:

(i) An optimized rib angle ($\alpha=10^\circ$) with auxetic behavior was selected for its beneficial combination of stiffness (*i.e.* Apparent modulus), strength (*i.e.* Plastic collapse stress), specific density and vibration damping. These samples were successfully impregnated with flexible and rigid polyurethane foams to enhance the structural and noise damping;

(ii) Both experimental and inverse engineering FEA approaches show that sample filling with polyurethane foams is able to successfully increase the vibration damping capacity (*i.e.* damping ratio). Samples filled with flexible foam show higher values of damping ratio;

(iii) Acoustic tests using an impedance tube show that the filling of the samples with foam is beneficial to decrease noise propagation (*i.e.* increase Transmission loss). However, unfilled samples display higher values of sound absorption in a wider range of frequencies than foam filled samples. This is mainly attributed to the lowering in porosity, increase in tortuosity and flow resistivity in the foam filled cellular solids;

(iv) Comparing the acoustic properties of the designed cellular solids with current aluminum foams and materials that are used in train vehicle fabrication, they are suggested to be a promising solution to be used in this particular field. Recent approaches in sound dampening coatings may also be found beneficial for such purpose. It is suggested that a future combination between the presented cellular solids and such coating may be a promising route to further decrease noise in railway applications;

(v) The critical frequencies and interior noises of different train classes have been identified. It is shown that the designed cellular solids are able to damp acoustic noise within the identified ranges of critical frequencies. Unfilled samples are suggested to reduce reflection in the cabin interior, while foam filled samples may be used to reduce the exterior noise propagation. A hybrid that combines unfilled and foam filled compositions is suggested to enhance Sound absorption and Transmission loss.

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Chapter 7 – Conclusions

“We should use such discovery as an opportunity to investigate more exactly the properties discovered and to prove or disprove them; in both cases we may learn something useful.”

Leonard Euler – translated from

Novi Commentarii academiae scientiarum Petropolitanae, 1761

7.1. Study conclusions

Future transports are intended to be more environmentally conscious. While policy and measures in energy consumption tend to be stricter and there are current milestones to be achieved in terms of transportation energy consumption, railway seems to have the upper hand relatively to other types of transport. However, there are still opportunities for the optimization of energy consumption in railway transports. For instance, the reduction of structural weight in railway vehicles, could directly impact the energy consumption and mechanical wear of different components. All these benefits would be able to reduce the emissions and running costs. Overall, it is expected that these stimuli may even reduce riding costs for the passenger.

It is known, however, that one of the main public opposition to railway is noise and vibration. These are issues not only for track neighboring residences, but also, to the passengers themselves due to cabin interior noise. Thus, the reduction of unwanted noise/vibration in train vehicles indeed would increase the appeal of this type of transport.

Considering both mentioned challenges, *i.e.* weight reduction and noise/vibration mitigation, an **objective** has been established: the design of a lightweight material with high specific structural strength and enhanced vibration/noise damping capacity for future railway vehicle fabrication.

This study **hypothesis** is related with the development of lightweight composites in the shape of non-stochastic cellular solids with a negative Poisson's ratio manufactured by investment casting. Also, it is considered that these materials could also display high specific static mechanical properties, a fundamental characteristic for the intended purpose.

Given that the cellular solid configuration is an intricate combination of material and shape, the initial steps of this study (detailed in Chapters 2 and 3) focus on the processing of the base material to achieve high static/dynamic mechanical properties and how it can be molded into the intended geometry.

This study suggests that castable and heat treatable A356 alloy comprises the best combination of physical (*i.e.* density), mechanical (*i.e.* stiffness/structural strength and damping) and manufacturing (*i.e.* castability) for the methodology, application and ability to assume the desired cellular solid shape. Additionally, heat treatment has been used as a route to tailor the mechanical properties of the alloy and optimize its static and dynamic mechanical properties.

An additive manufacturing assisted investment casting technique has been developed to process the selected alloy into the cellular solid shape. Different cellular solid designs were proposed, not only to produce samples with negative Poisson's ratio (*i.e.* auxetic behavior), but also samples with near zero and positive values of the referred constant. The suggested manufacturing process was characterized in terms of dimensional and defect analysis by X-ray microcomputed tomography, being validated and considered to be able to reproduce the intended designs.

The analysis of Poisson's ratios allowed their mechanical characterization using this elastic constant as a reference. This allowed to understand if auxetic behavior is able to promote enhanced static/dynamic properties as hypothesized in this study. Chapter 4 and 5, respectively, present the static and dynamic mechanical characterization of the manufactured samples.

In Chapter 4, the static mechanical behavior of the samples is detailed, showing that the Poisson's ratio has a significant impact in their load bearing capability. Analytical, experimental and numerical approaches are used to prove that auxetic behavior enhances the stiffness and collapse strength of this type of cellular solids. The Poisson's ratio dependent failure, *i.e.* collapse mechanism, is

explored using 4D X-ray microcomputed tomography in compressed samples, showing a beneficial densification of auxetic samples that dynamically resist the imposed loads.

The dynamic behavior, by the analysis of the vibration response of the manufactured samples is explored in Chapter 5. Through experimental and numerical techniques it has been shown that auxetic samples near the negative-positive Poisson's ratio transition regimes display higher values of damping ratio. It is suggested that these samples present a more favorable combination of geometry and deformation mechanism under random vibration that facilitates hysteretic damping.

The results of the static and dynamic mechanical characterizations are compared in Chapter 6. It has been determined that non-stochastic configuration with a -10° rib angle and auxetic behavior display the better compromise between structural stiffness/strength, vibration damping and lightweight characteristics. A method for their impregnation with polyurethane is proposed, while foam filled and unfilled samples are tested to determine their sound absorption and transmission. The results show that unfilled samples present higher values of sound absorption for a wider frequency range. ON the other hand, filled samples promote higher transmission loss in the full tested frequencies. Acoustic results are compared with sound pressure levels in different types of train vehicles, and it is found that the developed cellular solids may be useful to reduce sound reflection/transmission in critical frequencies.

After their static/dynamic mechanical and acoustic characterization, the manufactured cellular solids are compared in all referred domains with current available solutions. Results show that they reveal properties that are able to compete with current materials. Moreover, they may also be presented as an integrated solution, as they show high relative values in all referred domains.

Overall, this study identifies auxetic non-stochastic cellular solids manufactured by additive manufacturing assisted investment casting using A356 alloys as a possible route to achieve composite materials with high specific noise/vibration and load bearing characteristics. This solution is proposed as the next step for the manufacturing of a novel class of lightweight railway vehicles being able to fulfill this study objectives, research question, upcoming trends and policy measures on the railway industry.

7.2. Future works

This study has been performed as a proof-of-concept, *i.e.* it is intended to identify if the suggested design and methods, showing properties that have the potential to be a solution for the future of railway vehicle construction. Given that the results and conclusions suggest the fulfillment of this condition, it is

mandatory to define the future steps that should be adopted by the relevant stakeholders for the social benefit from this study.

The validation of the developed cellular solid designs and methods at a laboratorial scale imply that their technological readiness level (TRL) is on a research proved feasibility stage (TRL=4, according for example to ISO 16290). Future works should be given to the increase of ten current TRL. Given that the next TRL level comprises the validation of the proposed solution in a relevant environment. Hence, further research should be focused on the upscaling of the cellular solids. A possible solution is their manufacturing into panels (*e.g.* sandwich), keeping the optimized cellular core and exterior plate configuration.

The results show that the adopted designs and method are already able to fulfill the desired effect, however, it is suggested that static/dynamic mechanical properties may be still enhanced. It is proposed that changes in the manufacturing process could allow the inclusion of reinforcement nanoparticle in the metallic matrix. This would increase both the load bearing and vibration damping capacity without compromising its lightweight characteristics. However, this increment should be combined with a thorough life cycle analysis to guarantee that such reinforced cellular solids comply with future life cycle policies.

7.3. Scientific and academic impact

The presented work is expected to be a proof of the candidate scientific capability to be considered for a Doctor of Philosophy (PhD) degree. Thus, it is fundamental that the contents developed in this research are accepted and disseminated among the scientific community. Table 7.1 presents a compilation of the papers that have been already published, are currently submitted/under review or being prepared for submission in international peer-reviewed journals and books.

The main findings of this study have also been disseminated by their presentation in scientific conferences, on the roles of regular oral presentation and invited speaker. The participation in scientific events also included presentations by poster format. The referred dissemination actions are detailed in Table 7.2.

Moreover, the knowledge that has been gained in this study, namely the recent approaches in design, manufacturing, mechanical testing and numerical methods, are currently being used in the delivering of practical lectures of curricular units (*CAM/CAE Techniques, Computer Assisted Design & Modelling and Drawing & CAE Graphics*) in the Mechanical Engineering Integrated Master Degree of the University of Minho.

Table 7.1. Scientific outputs in written documents.

Paper	Type	Status
Analysis of the geometrical dependence of auxetic behavior in reentrant structures by finite elements (2016)	Journal Paper	Published in <i>Acta Mechanica Sinica</i> (Vol. 32 pp. 295-300)
Mechanical Behavior of Honeycomb Lattices Manufactured by Investment Casting for Scaffolding Applications (2016)	Journal Paper	Published in the <i>Journal of Materials: Design and Applications</i> (Vol. 231 pp. 73-81)
Effect of the cell number on the elastic constants of auxetic reentrant lattices (2017)	Abstract	Published in <i>Cellular Materials: Structural Behaviour, Modelling and Characterisation</i>
Significance of cell number on the bulk elastic properties of auxetic reentrant lattices (2018)	Journal Paper	Published in <i>Science and Technology of Materials</i> (Vol. 30 pp. 8-12)
Heat treatment as a route to tailor the yield-damping properties in A356 alloys (2018)	Journal Paper	Published in <i>Materials Science & Engineering A</i> (Vol. 729 pp. 1-8)
Auxetic lightweight composites – enhanced mechanical properties and vibration damping in transportation structures (2018)	Conference Paper	Published in <i>Proceedings of the 7th Transport Research Arena</i>
	Abstract	Selected and published in <i>Transportation Research Arena Visions 2018</i>
Thin-Rib and High Aspect Ratio Non-Stochastic Scaffolds by Vacuum Assisted Investment Castings (2019)	Journal Paper	Published in the <i>Journal of Manufacturing and Materials Processes</i> (Vol. 3 pp. 34)
Positive, zero and negative Poisson's ratio non-stochastic metallic cellular solids: Dependence between static and dynamic mechanical properties (2019)	Journal Paper	Published in the <i>Composite Structures</i> (Vol. 226 pp. 111239)
Additive manufacturing assisted investment casting: A low-cost method to fabricate non-stochastic metallic cellular lattices	Journal Paper	Submitted to the journal <i>Additive Manufacturing</i>
Effect of architecture on the Poisson's ratio and collapse mechanism of low density non-stochastic cellular solids	Journal Paper	Prepared for submission.
Metallurgical characterization of metallic lattice structures manufactured by fused filament fabrication assisted investment casting	Journal Paper	<i>In progress</i>
Acoustic characterization of non-stochastic metallic cellular lattices with polyurethane foam filling	Journal Paper	<i>In progress</i>

Table 7.2. Dissemination of results and conclusions in scientific events.

Presentation title	Type	Status
Mechanical Behavior of Honeycomb Lattices Manufactured by Investment Casting for Scaffolding Applications	Oral	<i>MDA2016</i> (Porto, 30 th June – 1 st July 2016)
Auxetic behaviour obtained by cellular metallic structures: advantages and applications	Invited speaker	<i>Multifunctional Cellular Metal – International Workshop</i> (Aveiro, 23rd June 2016)
Effect of the cell number on the elastic constants of auxetic reentrant lattices	Poster	<i>MATCEL'2017 & DynMATCEL'2017</i> (Aveiro, 25th – 27th September 2017)
Significance of cell number on the bulk elastic properties of auxetic reentrant lattices	Oral	
Auxetic lightweight composite panels – enhanced mechanical properties and vibration damping in transportation structures	Oral	<i>TRA'2018</i> (Vienna, 16th – 19th April 2018)
Development of auxetic lightweight composite panels for vibration and noise damping	Invited speaker	<i>Workshop iRail – Innovation in railway systems and technologies</i> (Porto, 15th February 2019)
Design and manufacturing of auxetic cellular solids with tunable static and dynamic mechanical properties	Invited speaker	<i>4th International Workshop on Advanced Lightweight Materials and Structures</i> (Aveiro, 10th July 2019)
Dimensional characterization lightweight non-stochastic cellular lattices by μ CT	Poster	<i>1st SAMmeeting</i> (Guimarães, 23rd to 24th October 2019)