

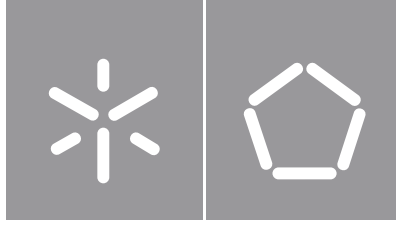
Universidade do Minho
Escola de Engenharia

Development and Validation of Empirical Seismic
Vulnerability Models for Historic Masonry Towers

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**Development and Validation
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Doctoral Thesis
Civil Engineering

Work conducted under the supervision of
Professor Paulo José Brandão Barbosa Lourenço
Doctor Alberto Barontini

March 2025

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Guimarães,

March 2025

Statement of Integrity

I hereby declare having conducted this academic work with integrity. I confirm that I have not used plagiarism or any form of undue use of information or falsification of results along the process leading to its elaboration. I further declare that I have fully acknowledged the Code of Ethical Conduct of the University of Minho.

Resumo

Esta tese tem como objetivo avaliar a vulnerabilidade sísmica de torres históricas em alvenaria e pretende desenvolver métodos empíricos com base em dados recolhidos após sismos. Embora existam modelos empíricos bem estabelecidos na literatura para avaliar a vulnerabilidade sísmica de estruturas, poucos estudos abordam sistematicamente as torres históricas em alvenaria, sendo ainda mais limitados aqueles que se focam na sua validação e na análise comparativa. Assim, o principal objetivo é analisar, validar e melhorar os modelos de vulnerabilidade existentes, com especial foco na avaliação do comportamento distinto e interligado dos vários componentes, o fuste e o campanário, que constituem as torres. São explorados dois conjuntos de dados distintos. O primeiro resulta de uma revisão bibliográfica extensa, reunindo dados de danos causados por sismos anteriores em Itália, compilados numa base de dados em acesso aberto. O segundo aborda algumas limitações do conjunto anterior, menos abrangente, através da recolha e filtragem de informações relativas à sequência sísmica da Itália Central entre 2016-2017, disponível na plataforma Da.D.O., Base de Dados de Danos Observados, disponibilizada pelo Departamento de Proteção Civil Italiano. Este estudo analisa o desempenho de várias medidas de intensidade sísmica, incluindo a intensidade macro-sísmica Mercalli-Cancani-Sieberg (MCS), a Aceleração Máxima do Solo (PGA) e a Aceleração Espectral (SA), de forma a melhorar a precisão das previsões de dano. A estimativa da SA recorre a modelos de movimento do solo existentes, juntamente com novas formulações empíricas desenvolvidas para estimar o período fundamental das torres. Além disso, a investigação conduzida analisa como a vulnerabilidade e o nível de dano são influenciados pelas características da ação sísmica, tais como eventos principais simples ou múltiplos, e fatores relacionados com as torres, tais como geometria, interação com estruturas envolventes, o nível de manutenção e a localização geográfica. A investigação integra metodologias como Matrizes de Probabilidade de Dano (DPMs), funções de vulnerabilidade e de fragilidade, e introduz novos modelos que consideram diferentes classes de torres e cenários sísmicos, com especial ênfase nos efeitos de danos cumulativos.

Palavras-chave: Avaliação empírica de vulnerabilidade; Dados pós-desastres; Danos cumulativos; Património construído; Torres em alvenaria.

Abstract

This thesis aims to assess the seismic vulnerability of historic masonry towers and seeks to advance current empirical methods derived from post-earthquake survey data. Although empirical models for evaluating the seismic vulnerability of structures are well-established in the literature, few studies systematically address historic masonry towers, and even fewer focus on their validation and comparative analysis. Therefore, the main objective of the thesis is to analyse, validate and enhance existing vulnerability models, with a particular focus on assessing the distinct and interconnected behaviour of tower's shaft and belfry components. Two distinct datasets are exploited. The first is derived from an extensive literature review, gathering damage data from past earthquakes in Italy within a dataset shared in open access. The second addresses some limitations of the previous, less comprehensive, by collecting and filtering information about the 2016-2017 Central Italy seismic sequence, hosted in the Da.D.O., Database of Observed Damage, platform released by the Italian Department of Civil Protection. This study examines the role of various seismic intensity measures, including Mercalli-Cancani-Sieberg (MCS) macroseismic intensity, Peak Ground Acceleration (PGA), and Spectral Acceleration (SA), in improving the accuracy of damage predictions. SA estimation resorts to existing ground motion models alongside newly developed empirical formulations to predict the towers' fundamental period. Additionally, this research investigates how vulnerability and damage levels are affected by key features of seismic actions, such as the occurrence of single or multiple main shocks, and factors related to the towers, such as geometry, interaction with surrounding structures, maintenance level, and geographical location. To this end, the research integrates methodologies such as Damage Probability Matrices (DPMs), vulnerability and fragility functions and introduce novel models that account for different tower classes and seismic scenarios, with particular emphasis on cumulative damage effects.

Keywords: Empirical vulnerability assessment; Post-disaster data; Cumulative damage; Built heritage; Masonry towers.

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List of Abbreviations/Acronyms/Symbols

GEM Global Earthquake Database
NSPP National Seismic Prevention Plan
INGV National Institute of Geophysics and Volcanology
DPMs Damage Probability Matrices
SA Spectral Acceleration
MCS Mercalli-Cancani-Sieberg
EMS-98 European Macroseismic Scale
Da.D.O. Database of Observed Damage
PGA Peak Ground Acceleration
GMM Ground Motion Model
MM Modified Mercalli
MSK Medveded-Sponheuer-Karnik
JMA Japan Meteorological Agency
ITACA Italian Accelerometric Archive
GNDT National Group for Earthquake Defence
PGV Peak Ground Velocity
PGD Peak Ground Displacement
SV Spectral Velocity
PSV Pseudo Spectral Velocity
SD Spectral Displacement
BDPF Binomial Density Probability Function
LSE Least Square Estimation
MLE Maximum Likelihood Estimation
ULS Ultimate Limit State
DBMI15 Italian Macroseismic Database
EC8 Eurocode 8

RSS Residual Sum of the Square

R^2 Coefficient of Determination

MSE Mean Squared Error

PCC Pearson Correlation Coefficient

AIC Aikake Information Criterion

H Hazard

V Vulnerability

E Exposure

d Individual Damage Level

ρ Importance Coefficient

i_d Global Damage Index

D Global Damage Level

μ Global Mean Damage Level

V_I Vulnerability Index

Q Ductility Index

V_0 Average vulnerability Index

V_j Corrective vulnerability Parameters

I_v Global Vulnerability Score

q_i Vulnerability Score Parameters

w_i Weight Parameters

p Damage Probability

ϕ Standard Normal Cumulative Distribution Function

θ Median Estimate Parameter

β Standard Deviation Estimate Parameter

M Magnitude

R Site-to-source Distance

M_w Moment Magnitude

H_{tot} Total Height

H_{eff} Effective Height

L_{min} Minimum Side Length

L_{max} Maximum Side Length

H_{tot}/L_{min} Aspect Ratio

s Base Wall Thickness

f_1 First Natural Frequency

T_1 Fundamental Vibration Period

CHAPTER 1

INTRODUCTION

1.1 MOTIVATIONS

Earthquakes are natural hazards that can have catastrophic effects on human lives, built environment, infrastructure, economies and ecosystems, hindering the development of the affected regions. Severe events, such as the 2023 Morocco earthquake ($M_w=6.9$, resulting in 2960 deaths) and the 2023 Syria-Turkey earthquake ($M_w=7.7$, with over 60000 deaths), serve as recent reminders of the destructive power of seismic activity worldwide. In the European context (Figure 1.1), Italy stands out due to its high seismicity, which is the result of its geographical location along the boundary of the Eurasian and African tectonic plates. This geotectonic setting has led to several significant seismic events throughout its history, including the 1976 Friuli earthquake ($M_w=6.5$ and 978 fatalities), the 1980 Irpinia earthquake ($M_w=6.9$ and 4689 fatalities), the 1997 Umbria-Marche earthquake ($M_w=6.0$ and 14 fatalities), the 2002 Molise earthquake ($M_w=5.7$ and 32 fatalities), the 2009 L'Aquila earthquake ($M_w=6.3$ and 295 fatalities), the 2012 Emilia earthquake ($M_w=5.9$ and 24 fatalities) and the 2016 Central Italy earthquakes ($M_w=6.2$ and 327 fatalities).

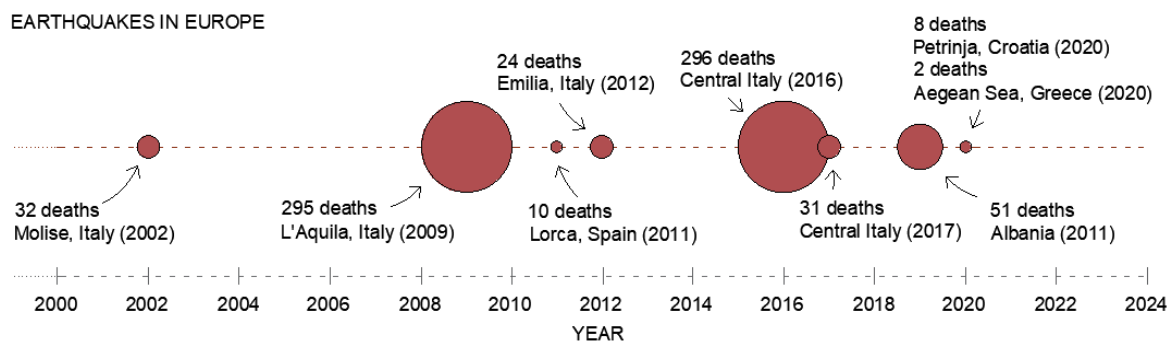


Figure 1.1: European fatalities from earthquakes over the last twenty years (Source: The International Disaster Database, EM-DAT (Delforge et al., 2023) - with major processing by Our World in Data).

Critical factors contributing to human and economic losses include the population density and the vulnerability of the built environment, particularly the ability of existing structural systems to withstand seismic loads. Earthquakes can also induce indirect economic costs such as disruption to tourism sector, reduced productivity, population displacement, and long-term social consequences. Focusing on the Italian scenario, economic losses related to the emergency management and reconstruction process have exceeded EUR 200 billion (Dolce et al., 2021). According to the Global Earthquake Model (GEM), Italy experiences average annual earthquake-related losses of approximately EUR 3 billion, which is the result of direct damage to buildings, infrastructures and fatalities, as well as indirect economic impacts. As shown in Figure 1.2, these losses are expected to be mainly concentrated in the Central, Southern and Eastern Italy, given the overlap of the higher seismic hazard with a considerable vulnerability of the building stock. This vulnerability is compounded by the presence of historical, artistic and monumental heritage, alongside the larger population density that result in increased exposure.

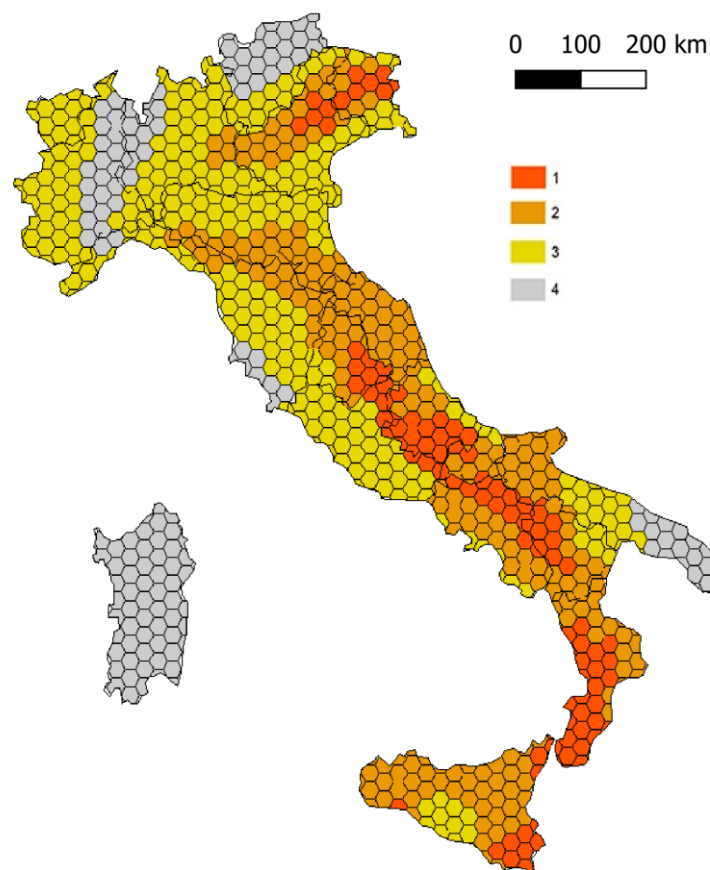


Figure 1.2: Seismic hazard map of Italy classified in 4 areas (re-interpretation of the official map released by the National Institute of Geophysics and Volcanology, INGV)

Among a wide range of building typologies that constitute the Italian built environment, past seismic events have shown that historic masonry heritage is particularly vulnerable to earthquakes. Churches and especially towers (both as integral components of churches and as standalone structures) are among the most prevalent heritage assets due to their significance, number and characteristics. The exact number of these structures remains undetermined and extensive efforts to document them are still ongoing.

The vulnerability of these structures, designed and constructed without reference to modern building codes, is further exacerbated by several critical aspects, including the heterogeneity of the masonry, the complexity of the geometry, the fragility of the historic fabric, the inadequate maintenance, the aging and the harsh environmental conditions resulting from changing climate patterns. Addressing the seismic vulnerability of these structures is not merely a compelling engineering challenge, given their complex seismic behaviour, but also a moral imperative to ensure the survival of these symbols of history and identity for future generations. Indeed, these structures are not only architectural landmarks but also carry immense cultural, social and economic values. Besides attracting several visitors annually, they serve as active centres of cultural and community life. In such a context, understanding the seismic risk and implementing risk reduction strategies become paramount to safeguard and protect these irreplaceable structures, ensuring structural and public safety, and preventing dramatic socio-economic consequences.

In response to these ongoing needs, Italy has invested in seismic monitoring systems and earthquake preparedness programmes to mitigate the risks, improve early detection, build resilience in infrastructure, and raise public awareness. In Italy, national regulations have been established to assess the structural safety of heritage-listed monuments and historic buildings, in compliance with current Building Codes. The Italian Ministry for Infrastructure and Transport and the Italian Ministry of Cultural Heritage developed guidelines for seismic risk assessment and mitigation for heritage structures (DPCM, 2011). These guidelines encourage and provide a multilevel approach to risk evaluation. In the new Building Code (NTC, 2018), these guidelines are explicitly referenced as a reliable source of guidance for evaluating the vulnerability of historic buildings in seismic prone areas. However, achieving a balance between ensuring safety and preserving heritage requires specialised expertise.

Following the 2009 L'Aquila earthquake, a comprehensive National Seismic Prevention Plan (NSPP) was implemented by the Italian Government and coordinated by the Italian Civil Protection Department. The NSPP aimed at reducing the human losses and improving the response in case of emergency, by assessing the seismic risk at urban scale and by reducing it through seismic interventions when

necessary. To support this initiative, a fund of EUR 965 million was allocated and distributed across the regions according to their seismic risk indices, taking into account the probability of building collapse in the event of an earthquake (CDRI, 2024). This approach highlights the importance of developing reliable seismic vulnerability models suitable for large-scale territorial applications, as they play a critical role in driving informed decision-making and optimising resource allocation. This strategy is expected to provide significant benefits and aligns with the multi-level approach outlined in the (DPCM, 2011) guidelines. This is particularly relevant given that, according to the Italian regulations, the conservation of monuments and historic buildings listed as heritage is mandatory, yet this responsibility is a very onerous duty for public authorities, especially when the funding is limited and the number of heritage buildings facing both natural and anthropogenic threats is large.

Post-earthquake reconnaissance and damage surveys have played a crucial role in the initial development of vulnerability models capable of assessing at territorial scale the effects of earthquakes on masonry churches and historic towers (Calvi et al., 2006). Damage Probability Matrices (DPMs), vulnerability and fragility functions are today widely used models by researchers, government authorities and heritage organisations.

These models, based upon the collection of observed damage data from past earthquakes, provide valuable insights into the seismic response and behaviour of given classes of structures. They allow identifying critical buildings, optimising allocation of the economic resources, prioritising inspections and the implementation of timely structural interventions. The empirical models based on the observed data, typically relating the level of structural damage in a given building typology to a seismic intensity measure, allows to conduct reliable condition screening as well as predict the expected damage in future seismic scenarios. A large number of studies addressing seismic vulnerability assessment of masonry churches at the territorial scale can be found in the literature, including but not limited to (Canuti et al., 2021; Ceroni et al., 2022; Cescatti et al., 2020; De Matteis et al., 2016; De Matteis & Zizi, 2019; Hofer et al., 2018; Penna et al., 2019; Ruggieri et al., 2022; Sisti et al., 2023; Sorrentino et al., 2014). However, most of the existing works have treated churches as a whole structural system, often overlooking the specific behaviour of individual macroelements (e.g., façade, nave, lateral naves, transept, dome, apse, roof, lateral chapels and bell tower), as highlighted in the AD-C Survey Form (DPCM, 2011). This limitation prevents critical insights into the localised vulnerabilities that significantly influence the overall seismic performance of these structures. Nonetheless, the macroelements influence in governing the seismic response of such buildings is well-documented, as recently studies have emphasized (Ceroni et al., 2022; Sisti et al., 2023). Among these macroelements, historic

masonry bell towers have demonstrated to be particularly susceptible to damage and are therefore highly vulnerable. For instance, data from the 2009 L'Aquila earthquake and the 2016-2017 Central Italy seismic sequence show the activation of damage mechanisms in 60% and 70% of the towers, respectively (De Matteis et al., 2016; De Matteis & Zizi, 2019). Moreover, in the case of Central Italy, 16% of towers within the investigated dataset reported in (De Matteis & Zizi, 2019) experienced partial or total failure, with higher damage levels compared to other structural macroelements.

The vulnerability of the bell towers is exacerbated by a common lack of maintenance and conservation. They typically exhibit local modes within the range of frequency more affected by ground motion actions, differing from other building components. Additionally, their connection to the rest of the church is often inadequate, suggesting the possibility of a dynamic response distinct from the rest of the church. Due to these reasons, the vulnerability of these structural elements can be more effectively investigated by developing dedicated models, separated from the surrounding structures.

Despite their cultural significance and proven high vulnerability, historic masonry towers have received limited attention in the field of empirical seismic vulnerability assessment at territorial scale. Therefore, the present thesis focuses on the development and validation of vulnerability models specifically for historic masonry towers. These models aim to enable rapid condition screening and damage forecasting, providing a valuable contribution to advancing the knowledge into this field of research.

Although this thesis strongly relies on information collected from the Italian context, including post-earthquake surveys as well as hazard and vulnerability characteristics of the Italian assets, promoting a culture of disaster risk prevention and encouraging risk assessment and mitigation activities specifically for heritage structures is a common objectives shared by several countries as well as international organisations like the International Council on Monuments and Sites (ICOMOS, 2001), the United Nations Educational, Scientific and Cultural Organisation (UNESCO, 2007), the United Nations Office for Disaster Risk Reduction (UNISDR, 2013, 2015), and the International Centre for the Study of the Preservation and Restoration of Cultural Property (ICCROM, 2016). Therefore, the methodologies and results presented in this thesis are expected to be beneficial in other contexts as well.

1.2 OBJECTIVES

The present thesis addresses the theoretical and methodological aspects of assessing the seismic vulnerability of historic masonry towers, with particular emphasis on bell towers, relying on post-earthquake damage data. The adopted methodology integrates multidisciplinary tools, combining earthquake engineering, structural conservation, vulnerability modelling and risk analysis. The primary objective is to investigate the seismic behaviour of this peculiar building typology after earthquake

events, through the application of state-of-the-art empirical vulnerability models, namely DPMs, vulnerability and fragility functions. Additionally, the thesis provides a critical discussion of potential biases that may arise in the data, highlighting issues such as data incompleteness, sample size limitations, and other challenges in data collection and model development.

Few vulnerability models exist for masonry towers and their thorough validation using post-earthquake damage data from specific past seismic events are scarce in the literature. Moreover, these models often overlook specific tower features that may influence their vulnerability. Therefore, in this thesis, two distinct set of data of seismic damage observed in historic masonry towers are analysed and exploited. Existing vulnerability functions are extensively tested against these empirical data and new models are developed, incorporating and accounting for novel factors such as the influence of single main shock or multiple shocks, relevant geometrical and structural features, and different seismic intensity measures. These elements have not been previously addressed in the analysis of historic masonry towers. Similarly, fragility functions are derived, with particular emphasis on the methodology used for their calibration, to prevent the crossing behaviour, which is an aspect often neglected in similar studies. Additionally, efficient empirical formulas are proposed to predict the first natural frequency of historic masonry towers. These formulas are used, within the context of the thesis, to support the estimation of the spectral acceleration (SA) at the tower locations. The general framework is illustrated in Figure 1.3, while the description is provided hereafter.

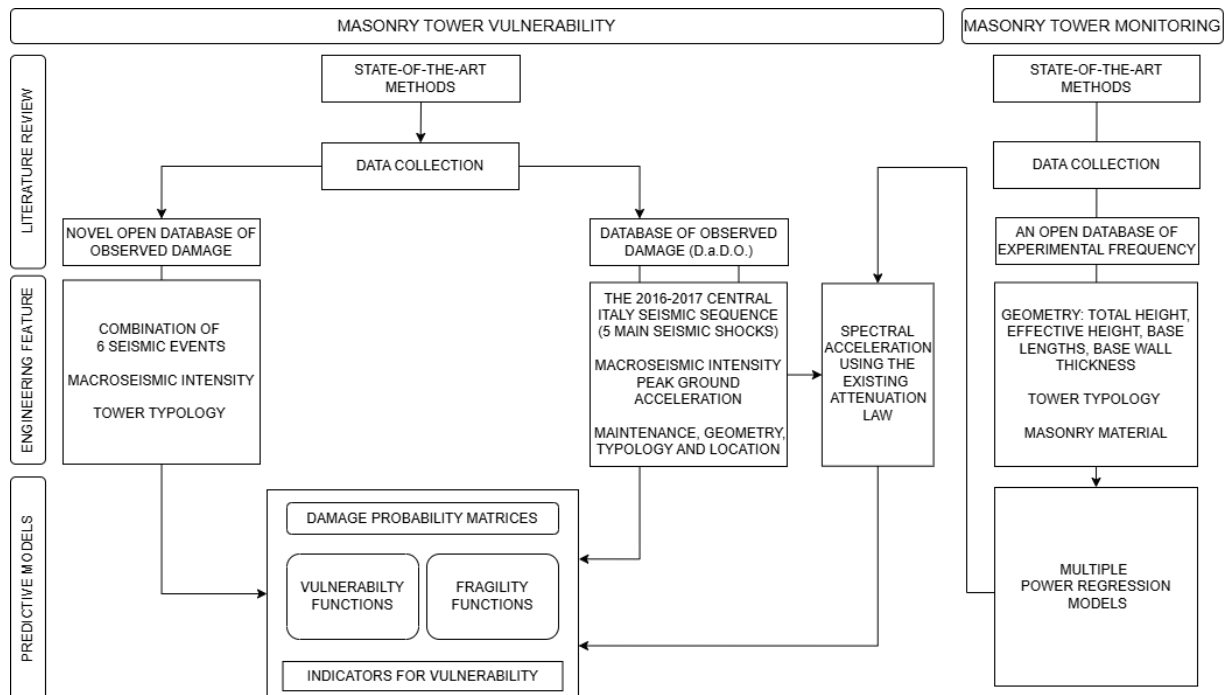


Figure 1.3: General framework adopted in the present thesis.

Initially, an open database of historic masonry bell towers struck by past seismic events in Italy is compiled through an extensive literature review. The collected data include the macroseismic Mercalli-Cancani-Sieberg (MCS) intensity to characterise the earthquakes and the level of damage according to the European Macroseismic Scale, EMS-98 (Grünthal, 1998), for the two main components of the bell towers, namely shaft and belfry. The data, provided in open access, are exploited to assess the robustness of existing seismic vulnerability models for historic masonry towers. Data incompleteness, as identified through a comparison with similar existing datasets, is considered when training a new vulnerability model, and the limitations posed by the sample size are tackled by implementing a validation strategy against randomly sampled subsets. Afterwards, an extended distinct dataset is considered, particularly extracting the information about the historic masonry bell towers from a database that comprises more than 3000 masonry churches damaged after the 2016-2017 Central Italy seismic sequence. This database is hosted on the online platform Da.D.O., Database of Observed Damage (Dolce et al., 2019), released by the Italian Civil Protection Department. The richness of this dataset enables an exploration of the cumulative damage due to repeated events, an evaluation of the model performance when using different intensity measures, including the Peak Ground Acceleration (PGA) and the MCS intensity, and an analysis of the impact of specific tower key features such as geometry, plan configuration, maintenance state and location of the investigated towers on their vulnerability. In the final stage of the research, starting from the same dataset of the Central Italy, novel vulnerability models are derived using SA as an alternative seismic intensity measure. To this end, a well-established Ground Motion Model (GMM) specific to Italy is used to estimate the SA as a function of earthquake magnitude, epicentral distance, soil classification and fundamental period of the structure. In this context, empirical formulations for predicting the frequency of historic masonry towers are developed, based upon the largest database ever used for this purpose. These data are also shared openly in a data repository for future research. The main purpose of these predictive formulations is to provide more accurate tools for estimating the natural frequency of historic masonry towers, offering an alternative to the reference formulas currently reported in national codes. Finally, relying on various performance metrics, a comparative assessment is carried out between the developed vulnerability models, as a function of the SA, and their counterparts derived using PGA, to highlight potential improvements in the accuracy and reliability due to the intensity measure adopted.

1.3 OUTLINE

The thesis has been organised into six Chapters as listed below:

- **Chapter 1** provides an overview of the motivations and objectives of this research, outlining the significance of assessing the seismic vulnerability of historic masonry towers and the goals of the study.
- **Chapter 2** provides an introduction to seismic risk, including a review of state-of-the-art methodologies for assessing seismic vulnerability, with a specific focus on historic masonry towers. It offers a comprehensive overview of the methodologies commonly employed in large-scale assessments, as applied in this research. Additionally, this Chapter discusses the principles outlined in the Italian Guidelines for seismic safety assessment and mitigation of heritage structures, particularly with regard to towers.
- **Chapter 3** compiles damage data from an extensive literature review on Italian masonry bell towers affected by past seismic events, creating an open-access database. This database is used as a testbed to assess existing territorial scale vulnerability models. The data is initially analysed through DPMs, with a comparison to analogous DPMs available in the literature. The observed mean damage is then computed for various macroseismic intensities, allowing an evaluation of the predictive capabilities of existing vulnerability functions. Development of new vulnerability functions is performed, and cross-validation strategies are employed to assess the effects of sample size on the reliability and the predictive performance of these models. Finally, fragility functions are derived using cumulative probability represented by the binomial distribution, and their validity is assessed by comparing them to observed probabilities.
- **Chapter 4** analyses damage data from historic masonry towers following the 2016-2017 Central Italy seismic sequence. The data, sourced from the Da.D.O. platform (Dolce et al. 2019), are used to develop vulnerability models, including DPMs, vulnerability functions, and fragility functions, with PGA employed as the seismic intensity measure. The models are initially developed and compared under different scenarios (towers affected by a single main shock or multiple shocks of the sequence) to investigate the impact of cumulative damage. Key tower features, such as location, geometry, configuration, and state of maintenance, are considered to assess their influence on vulnerability. Fragility functions are derived by fitting a lognormal cumulative distribution function to the observed probabilities of damage exceedance using the Maximum Likelihood Estimation (MLE) method.
- **Chapter 5** proposes novel vulnerability models based on the data analysed in Chapter 4, using SA as an alternative seismic intensity measure to PGA. Since SA is not included in the Da.D.O. database, an existing GMM is applied to estimate it. This requires knowledge of the

fundamental period of the structures. While several empirical relationships for predicting the fundamental period of historic masonry towers are available in the literature, this Chapter introduces novel empirical laws derived from an open dataset compiled through an extensive literature review. The newly developed vulnerability and fragility functions are compared to those based on PGA, using various performance metrics to highlight the impact of intensity measure selection on the accuracy and effectiveness of these models.

- **Chapter 6** summarises the main innovative aspects and outcomes of this work, offering conclusions and recommendations for future research in the field of seismic vulnerability assessment of historic masonry towers.

CHAPTER 2

SEISMIC RISK ASSESSMENT: STATE OF THE ART

ABSTRACT

This Chapter presents an overview of seismic risk and systematically reviews the methodologies for assessing the vulnerability of masonry structures, with a specific focus on slender buildings and towers. It highlights current, advanced techniques of increasing complexity and detail, prioritising those most suitable for large-scale territorial applications. While various well-established classifications are presented, these methodologies are primarily organised by distinguishing between empirical and analytical approaches, reflecting the emphasis on the former in the subsequent Chapters. Both solutions are critically evaluated, addressing their advantages and limitations, and discussing relevant illustrative case studies.

2.1 SEISMIC RISK

Seismic risk, R , may be defined as the characterisation of the potential consequences of an earthquake, commonly represented as an estimate of the expected losses, such as damage, fatalities and economic losses, resulting from the event. Risk assessment is typically concerned with a large asset stock rather than a single building, often categorising specific types (e.g., residential dwellings, public buildings, heritage, infrastructures, critical facilities) and it is a function of how the seismic hazard, H , combines with the exposure, E , of the considered assets and their vulnerability, V , thus being a spatial and temporal convolution of these three specific models, as in Figure 2.1.

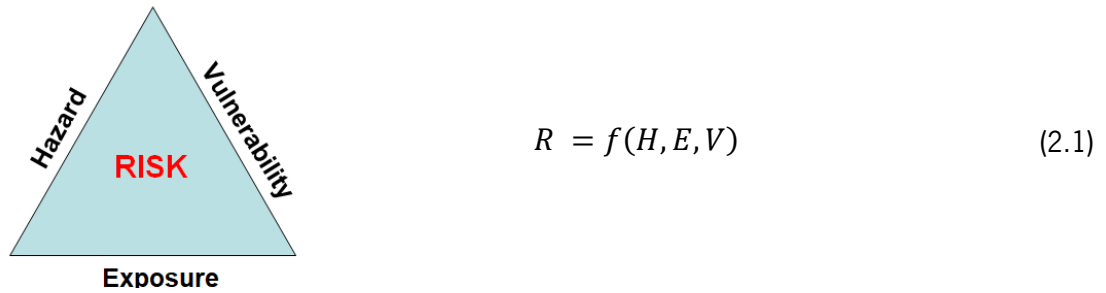


Figure 2.1: Seismic risk components.

More recently, the capacity as qualitative parameter has been introduced in the above equation by (UNISDR, 2015) for a more comprehensive definition of the risk. Developing robust models which can accurately describe and predict the characteristics of such entities and their dynamic interactions in order to assess the overall seismic risk, is a challenging multidisciplinary task. It requires the collaboration among various experts, including seismologists, hazard modellers, data analysts and structural engineers.

In general terms, the seismic hazard H refers to the probability of a seismic event which has the potential to cause fatalities, injuries, building damage, services downtime and socio-economic interruptions within a specific territorial region. The main parameters characterising the seismic hazard are the frequency of occurrence (the return period) and the force (the severity) of the expected ground shaking in each area. Hazard assessment involves evaluating the seismicity, through appropriate and suitable seismic intensity measures. Several intensity measures are today available in the literature to characterise strong ground motions affecting the built environment. Typical seismic intensity measures are the macroseismic intensity, magnitude, Peak Ground Acceleration (PGA), Peak Ground Velocity (PGV), Peak Ground Displacement (PGD), Spectral Acceleration (SA), Spectral Velocity (SV), Spectral Displacement (SD), Arias intensity, among others. Today, hazard maps used for design purposes are mostly given in terms of PGA, presenting the spatial distribution of the expected severity for a given

return period. Therefore, the seismic hazard can be generally defined as the probability of occurrence of an event exceeding a certain value of PGA within a given time span. Several strategies can be undertaken for assessing the seismic hazard, ranging from deterministic (Silva, 2016) to probabilistic approaches (Dolce et al., 2021). In this field, significant advances have been made and the seismic hazard can be nowadays computed by means of hazard curves at any site in Europe (Pagani et al., 2014).

The exposure, E , refers to the location and qualitative attributes (e.g., occupants, building type, properties, and artefacts, collections, paintings and sculptures, if present) of the assets at risk, describing the possible consequences of a hazardous event in economic, cultural and social terms. Typically, data related to the location, typology classification, construction age, cost of the building stock and occupancy class, with potential number of occupants, are required for the development of robust exposure models. This information, stored in a building inventory when available, of great interest for planning decision and for safeguarding human life, allowing to extrapolate qualitative estimates of the people affected and economic losses caused by the event (Dolce et al., 2021; Lagomarsino & Podestà, 2004b).

The vulnerability, V , refers to the susceptibility of buildings to damage when exposed to a seismic event. The damage depends on the response of buildings to the ground shaking and their capacity. The more a building is vulnerable, the greater is the expected damage. The damage mechanism type and extent can be related primarily to key physical features and characteristics of the investigated building stock and their surrounding that govern their response to the seismic action. These features include but are not limited to the type of the structural system, geometry regularity, materials and soil conditions. Other relevant characteristics, such as construction age and state of maintenance, including existing damage, may influence the susceptibility. In risk assessment, the damage can be modelled in different ways. Suitable discrete damage scales (e.g., the European Macroseismic Scale, EMS-98) are typically adopted in empirical studies, especially in post-earthquake investigations, to conduct the statistical analysis of the event and its consequences and generate the distributions of observed damage. Differently, engineering demand parameters, such as drift or crack width, are mainly used in numerical investigations, to realistically reproduce possible damage scenarios.

Among these three components of the seismic risk, existing hazard and exposure can be hardly modified and often evolve slowly over time. Distinct levels of accuracy in their assessment can be ensured with different model refinements, depending on the objectives of the assessment, expertise and economic budget. Current efforts are primarily focused on enhancing vulnerability models for

seismic risk assessment and mitigation, since reducing the vulnerability of buildings is a more viable approach to decreasing the seismic risk.

Therefore, understanding the seismic risk and developing effective models for evaluating it and its components ensure consistent structural condition screening, that is paramount to support the emergency response and disaster planning, safeguard critical buildings and, at the same time, minimise human and economic potential losses (Ferreira et al., 2021). The structural condition screening, in seismic risk assessment, is enabled by the identification and prediction of adequate loss metrics (i.e., damage, casualties, costs) measured in correlation with the increasing levels of one (or more) seismic intensity measures. Existing models designed for this purpose are comprehensively described in the following Sections.

2.2 ASSESSMENT METHODOLOGIES: AN OVERVIEW

Although various criteria have been discussed in literature to classify risk analysis and especially vulnerability assessment methods, as for instance in (Calvi et al., 2006; Vicente et al., 2011), hereafter the following taxonomy is used to introduce and discuss existing strategies for masonry structures at territorial scale: (i) analytical models; (ii) empirical models; (iii) heuristic models; (iv) hybrid models.

Analytical models are based on the idealisation of the investigated structural typology through numerical simulations, defined to quantitatively estimate the structural response to expected local seismic intensities (Shabani et al., 2021; Zizi et al., 2021). As the level of details of the involved models may vary significantly depending on the scope, these methods can be further subdivided to account for the accuracy, complexity, computational resources needed and input data required, addressing all different scales from large territorial studies to building scale assessments. Empirical models are directly based on experimental or observational data, typically collected after a seismic event, resorting to a data-driven evaluation of the structural performance through statistical methods (Rota et al., 2008). The adoption of real observational data makes them extremely reliable, although, at the same time, limits their applicability to contexts for which statistically relevant data of past events are available, and their scalability and generalizability to other contexts may be questionable (Rossetto & Ioannou, 2018). Heuristic methods rely on expertise and technical judgment, qualitative criteria, and rule-based decision-making to estimate the vulnerability of a building typology and/or to identify the main factors affecting its seismic performance (Giuliani et al., 2021; Romão et al., 2016). These methods are particularly useful when quantitative observational data or computational simulations are not readily available nor easy to achieve. Focusing on expert insights and opinions rather than detailed structural analysis, they have been extensively used for rapid territorial scale assessments within simple frameworks. Finally,

hybrid models result from possible combinations among the models previously described (Cardinali et al., 2022; Kappos, 2016; Kappos et al., 2006; Sandoli et al., 2023). Within this latter class, various combinations of approaches are included as for instance the use of observed data to calibrate numerical models, enhancing their reliability, and then exploit them to expand the investigation to unknown seismic scenarios.

Among these methodologies, the empirical models and the analytical models have been extensively adopted for assessing the seismic risk and vulnerability of historic masonry towers and slender structures, in some cases hybridised with expert decision regarding the weight of influential factors. Due to the relevance for the scopes of the present thesis, these methodologies are shortly presented hereafter, and more details are provided in the following Section, alongside with their practical and potential applications.

Examples of common tools adopted by empirical models are the Damage Probability Matrices (DPMs), vulnerability functions and fragility functions, as summarised in Figure 2.2. While these models statistically analyse post-earthquake damage data to establish specific relationships between building typology, its features and observed damage relative to seismic intensity, vulnerability index methods, reported in Figure 2.2, often rely on expert judgement to enhance the models. This approach allows for the inclusion of information that may not be readily available from past events.

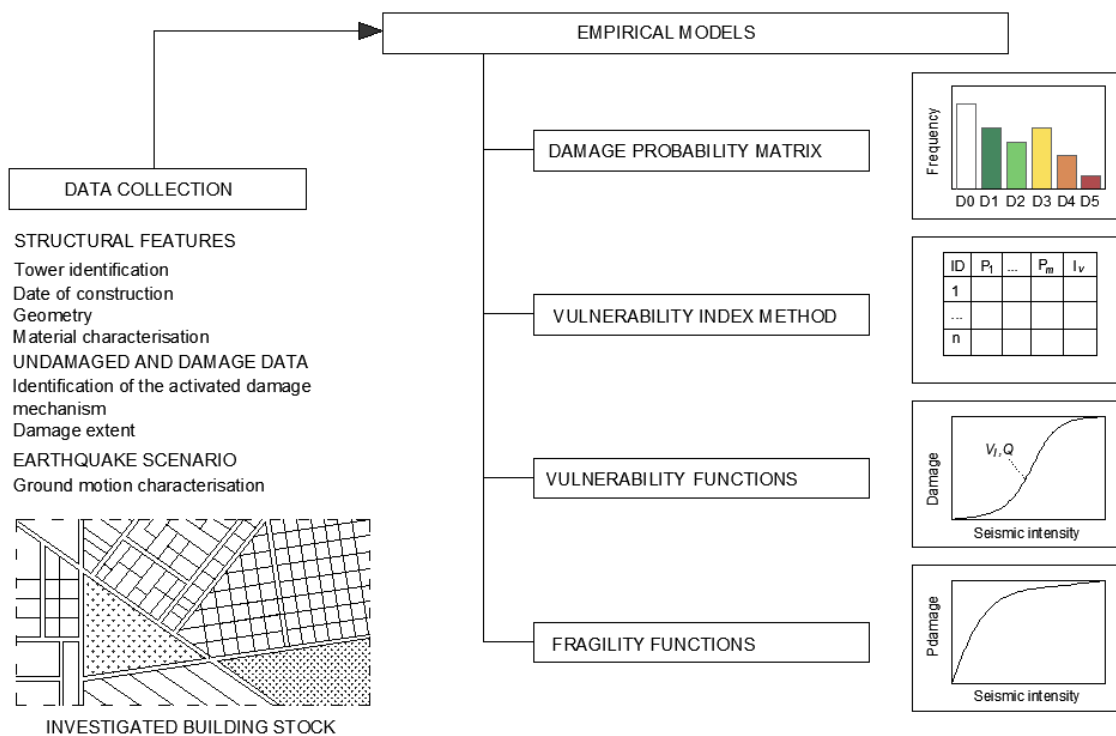


Figure 2.2: Empirical models.

These models differ from each other depending on how the data are used and which information is needed to extract. DPMs, originally proposed by (Braga et al., 1982; Whitman et al., 1973), are tabular representations, extensively adopted in empirical studies, that show the frequency of occurrence (probability) of different damage states according to a given scale for a building or structure typology, given a specific level of seismic intensity. The vulnerability functions are mathematical or graphical relationships that describe the expected damage level or loss (e.g., cost, downtime) as a function of a seismic intensity measure (Lagomarsino & Giovinazzi, 2006). The vulnerability index methods are procedures that assign a single numerical vulnerability score to buildings based on the main parameters that remarkably affect their seismic behaviour (Benedetti & Petrini, 1984). The definition of these relevant parameters and their weighted contribution to the overall vulnerability often result from expert opinions and heuristic approaches. Finally, the fragility functions are probabilistic models that represent the likelihood of a building or structure reaching or exceeding a specific damage state as a function of seismic intensity measures (Rossetto & Elnashai, 2003; Rota et al., 2008). Irrespective of the tool, numerical data can be used as surrogate or to integrate observational ones, supporting model development for specific building typologies. Further details on empirical methods are provided in Section 2.3.

Examples of analytical approaches for historic masonry buildings at various level of detail and complexity have been discussed in the Italian document for the preservation of the cultural heritage, namely the “Guidelines for the assessment and mitigation of the seismic risk of the Cultural Heritage” (DPCM, 2011). This document establishes a framework for structural analysis tailored to the peculiar characteristics of the built heritage. In particular, three analytical levels of evaluation (LV1, LV2 and LV3) with an increasing complexity are suggested for specific structural masonry building typologies, including masonry churches and masonry towers, among others. A summary of the three levels of evaluation proposed specifically for slender masonry structures is sketched in Figure 2.3.

Varying the level of analysis, with its corresponding complexity, affects the feasibility of the methods for the two distinct scales of assessment: the territorial and the individual building scale. More specifically, the first level of evaluation (LV1) is suggested for assessing the vulnerability of a large number of buildings over a defined territory, an urban centre or a region, with the main goal of identifying the most critical buildings, establishing the priorities for future interventions and supporting the seismic risk management (Bartoli, Betti, & Monchetti, 2017; Casapulla et al., 2018; Formisano & Marzo, 2017). The remaining two levels of evaluation (LV2 and LV3) are recommended for the vulnerability assessment of individual buildings when more refined analysis are required to better understand the

building's response and design future preservation strategies in a cost-effective manner (D'Ayala & Speranza, 2003; Lagomarsino & Cattari, 2015; Tanganelli et al., 2024; Torelli et al., 2020). In general, the first level of evaluation can be considered the preliminary step toward more accurate and detailed methods of analysis (DPCM, 2011). Further details on analytical methods are provided in Section 2.4.

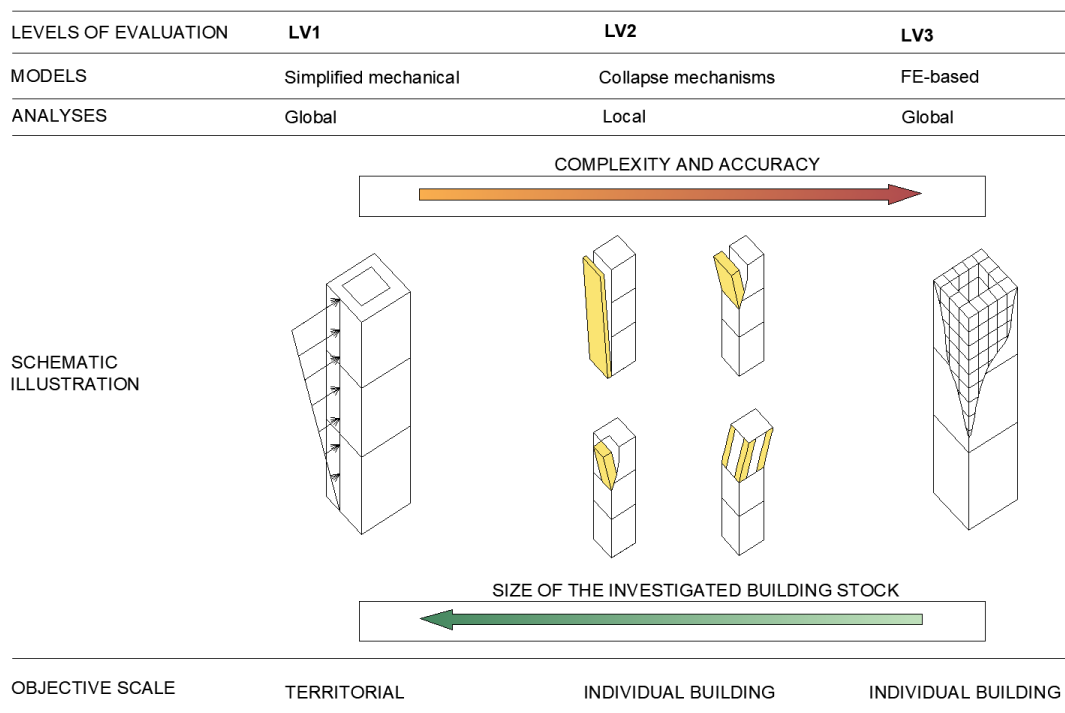


Figure 2.3: Summary of LV1, LV2 and LV3 levels of evaluation proposed by the Italian Guidelines for slender structures.

2.3 EMPIRICAL METHODOLOGIES

The development of empirical models requires damage data, in specific seismic events, for structures associated with specific building typologies, spanning at various territorial scales (i.e., urban, regional, national). The development of these models also requires information on the corresponding earthquake characteristics. Damage and earthquake data are typically collected and assessed following the seismic events, often as part of technical activities aimed at enabling rapid safety and usability verifications and supporting emergency management efforts. A typical timetable of these activities in case of ground shaking is provided in Table 2.1. Generally, the technical activities are coordinated by the Department of Civil Protection or a similar national agency, together with other organisations, authorised private entities, research centres and universities.

Earthquake data generally includes the processing of soil and building accelerometric data, as well as the site investigations to define the macroseismic Mercalli-Cancani-Sieberg (MCS) intensity and other intensity measures. To this end, most seismically active countries maintain a strategically deployed

monitoring network. Without the knowledge of the earthquake data, it is not possible to assess the seismic hazard and consequently the seismic risk. The MCS intensity scale, as other similar intensity scales like the Modified Mercalli (MM), the Medveded-Sponheuer-Karnik (MSK), the European Macroseismic Scale (EMS-98), and the Japan Meteorological Agency (JMA), is a predate modern instrumental method to record and characterise seismic intensity (Dolce et al., 2019, 2021). Therefore, it has been extensively used in the past to characterise the location and the intensity of an earthquake.

Table 2.1: Typical timetable of technical activities adopted after seismic events.

No.	Time	Main activities	Description
1	from 2 min up to 5-30 min	Epicentre and Magnitude evaluation	Collecting and processing seismometric network data
2	from 10 min up to 60 min	Simulated damage scenarios and data processing of monitoring systems	Software simulation of the earthquake impact on constructions; Collecting and processing soil and building accelerometric data.
3	from 6 hours up to 7-14 days	Site surveys for macroseismic effects	Site evaluation of macroseismic intensity; Geological surveys for landslides, surface faulting, and soil liquefaction.
4	from 6 hours up to 6-12 months	Temporary monitoring of soil and structures	Installing temporary soil and accelerometric stations and structure monitoring systems.
5	from 24 hours up to 6-12 months	Post-earthquake damage and safety assessment	Building inspections for damage and usability assessment; Technical evaluation for temporary houses.

MCS intensity is based on observed effects of an earthquake and their qualitative description, namely observed damage and human reaction, at a particular location (Musson, 2009). Intensity data can be collected from field surveys, eyewitness reports, and secondary sources, making it useful for regional or national risk assessments, especially in areas with limited instrumentation or to investigate past events prior to the introduction of the monitoring networks. For this reason, many traditional vulnerability assessment models have been originally developed by relying on these intensity measures. Nonetheless, several limitations of the assessment according to the MCS scale are gradually leading to its replacement by other intensity measures. Among these limitations, it is worth mentioning the subjectivity of the evaluation that relies on human observations, its resolution, often limited to coarse spatial scales and the impossibility of calculating it in real-time, supporting emergency response. Additionally, as it reflects the effects of the earthquake, it is not solely related to hazard, as an intensity measure typically should be, but it is also affected by the vulnerability of the local building stock (Musson, 2009). With the recent growth of instrumental monitoring, the use of PGA as seismic intensity measure has become very popular (Dolce et al., 2019, 2021). The PGA is a measure of the maximum acceleration experienced by the ground during an earthquake, as registered by a seismic station. The

local intensity of the earthquake at a specific site is thus characterised by the PGA along two orthogonal horizontal directions and the vertical direction. The characterisation of the seismic input through the PGA is widely accepted in the literature because of the direct relationship between PGA and inertial forces. PGA can be closely related to the largest dynamic forces induced in a certain type of structures. It should be noted that, when PGA is not available at a specific building site, the PGA can be estimated via ground motion predictive equations. As instrumentally recorded PGA data are not available for all the investigated assets and areas, several empirical models have been developed over time and calibrated to specific geographical regions and/or events. These models vary depending on the information required for the estimation, e.g., the granularity and type of geological, geotechnical, and other spatial data. Simplest correlations that can be adopted provide an estimation of the PGA as a function of the MCS intensity, as the well-established formula of (Faenza & Michelini, 2010). This formula has been employed in Chapter 3. For alternative similar laws the interested reader can refer to (Faccioli & Cauzzi, 2006; Gomez-Capera et al., 2020; Masi et al., 2020). More robust models, also called Ground Motion Models (GMMs) or attenuation laws, allow the estimation of seismic intensity measures, typically PGA, based on relevant features of the earthquake, the distance from the seismic source and a classification of the soil at the investigated site (Akkar & Bommer, 2010; Ambraseys et al., 2005; Joyner & Boore, 1981; Sabetta & Pugliese, 1987, 1996). An online compendium has been recently provided by (Douglas, 2018). Generally, the GMMs, developed by learning from the available data, are suggested to help the prediction of the PGA in specific regions, countries and continents. As an example, the GMM proposed by (Joyner & Boore, 1981) is suitable for North America, the GMMs from (Sabetta & Pugliese, 1987, 1996) are for Italy, and other GMMs developed by (Akkar & Bommer, 2010; Ambraseys et al., 2005) are for Europe and Middle East. Focusing on Italian scenarios, GMMs include, but are not limited to, SP96 (Sabetta & Pugliese, 1996), ITA08 (Bindi et al., 2010), and ITA10 (Bindi et al., 2011). These models, which have been widely used in literature, e.g., (Aloisio et al., 2024; Miano et al., 2020), have been considered as reference models in Chapter 5. While the SP96 has served as an initial reference in Italy, together with the SP87 (Sabetta & Pugliese, 1987), the ITA08 and ITA10 updated and enhanced accuracy by leveraging the Italian Accelerometric Archive (ITACA) database. Indeed, the SP96 has been trained using a limited number of earthquakes, i.e., 17, with magnitude ranging between 4.6 and 6.8, and distances up to 100 km, whereas the ITA08 and ITA10 have been proposed based on upon the original and updated version of the ITACA database. This initially included a large number of earthquakes, from 1972 to 2007, with magnitude ranging between 4.0 and 6.9, and similar distances. It was later updated by collecting a similar amount of earthquake data in a wider area, extending

through 2009. This underlines a continuous trend in recalibrating the GMM to improve robustness and reliability for seismic hazard assessment.

In the analysis of past events, ShakeMap is a common tool. This is a visual representation that depicts the spatial distribution of ground shaking intensity following an earthquake (Figure 2.4a). It is generated using real-time earthquake data from seismic monitoring networks combined with geological, geotechnical, and other spatial data. Similarly, to provide the spatial distribution of the effects of a ground shaking in a region, the MCS intensities observed at different locations are plotted on maps (Figure 2.4b), with contours of equal intensity (Iseismal map).

However, the PGA is a ground motion intensity measure that often does not fully explain the seismic damage to structures. This is particularly true for earthquakes characterised by peak accelerations occurring at high frequencies and with short duration. It is widely acknowledged that additional intensity measures are needed to accurately characterise strong ground motions, especially in terms of features that may be more closely related to damage onset and evolution (Gautam et al., 2018; Zucconi et al., 2020). To this aim, the SA, which is representative of the seismic forces on the structural system over a wide range of vibration periods, started to receive growing scientific interest to develop vulnerability models for large scale assessment. It should be noted that the SA, similarly to the PGA, can be also estimated through GMMs (Douglas, 2018). Other alternative intensity measures such as the Arias intensity, cumulative absolute velocity, Housner spectrum intensity, among other, are less common, as discussed in (Nguyen et al., 2020).

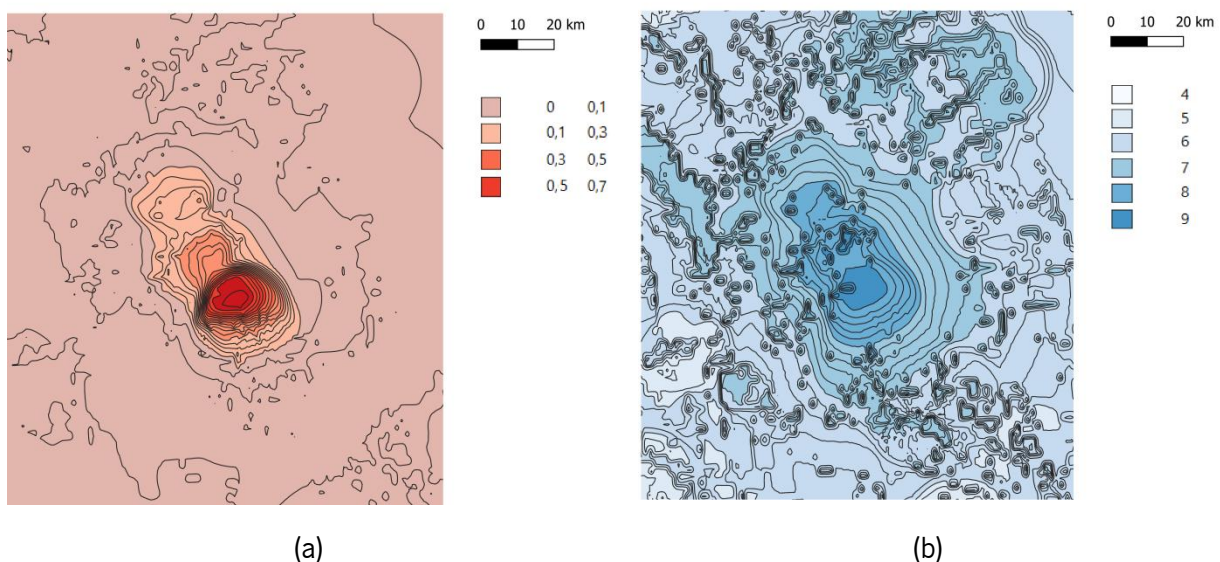


Figure 2.4: Shake map (a) and Iseismal map (b).

Besides the characterisation of the hazards, risk analysis based on empirical methods requires a collection of comprehensive information on the losses and damage caused to buildings. Post-

earthquake reconnaissance and damage survey activities have demonstrated to be valuable sources of information to assess the nature, type and extent of damage. In the context of emergency management, traditional methods rely on field surveys conducted by expert engineering teams to analyse, collect and document the structural conditions of inspected buildings in order to determine their safety and usability. This process is critical for guiding resource allocation and ensuring efficient response and recovery efforts. Technologies such as drones have become well-established tools to support in-field inspections. On the other hand, emerging strategies, such as remote satellite imaging, present a promising alternative to overcome the limitations of traditional methods and accelerate post emergency engineering activities (Macchiarulo et al., 2024).

Concerning a particularly relevant but vulnerable typology, namely masonry churches, the observational data collected in the aftermath of significant Italian earthquakes, starting from the 1976 Friuli Earthquake (Doglioni et al., 1994), allowed the definition and continuous improvement of robust procedures for rapid damage survey at territorial scale and paved the way for the development of sound statistical models for their risk assessment. The resulting models have been applied and improved after the 1997 Umbria-Marche Earthquake (Lagomarsino & Podestà, 2004b, 2004c), the 2002 Molise Earthquake (Lagomarsino & Podestà, 2004a), the 2003 Valle Scrivia Earthquake (Ruggieri et al., 2022), the 2009 L'Aquila Earthquake (Da Porto et al., 2012; De Matteis et al., 2016; De Matteis, Brando, & Corlito, 2019; De Matteis, Brando, Corlito, et al., 2019; Lagomarsino, 2012), the 2012 Emilia Earthquake (Sorrentino et al., 2014), the 2016-2017 Central Italy seismic sequence (Canuti et al., 2021; Ceroni et al., 2022; Cescatti et al., 2020; De Matteis & Zizi, 2019; Hofer et al., 2018; Penna et al., 2019; Sisti et al., 2023) and the 2017 Ischia Earthquake (Salzano et al., 2022). These experiences demonstrated that seismic damage to masonry churches is significantly influenced by the local response of their components, affected by their peculiar features. These factors contribute to their vulnerability and the complexity of their behaviour, which strongly deviates from the so-called box (integral or global) behaviour (Giuffrè, 1991). The survey form for masonry churches provided by the Italian Civil Protection supports the damage assessment by investigating this local response of the main structural components, namely dividing the structure into several distinct macroelements, i.e., façade, nave, lateral naves, transept, dome, apse, roof, lateral chapels and bell tower (Figure 2.5), that tend to exhibit independent behaviours when subjected to earthquake loading (MiBACT, 2015). For each macroelement the corresponding damage level is recorded. This is used to estimate the global damage level through a weighted average (DPCM, 2011). It is worth noting that the number of damage mechanisms expected for churches, thus considered in the survey form, has increased over time,

highlighting the lessons learned from past seismic experiences. Initially, 18 damage mechanisms were considered for the first version of the Civil Protection inspection form, while they increased to 28 in the current version.

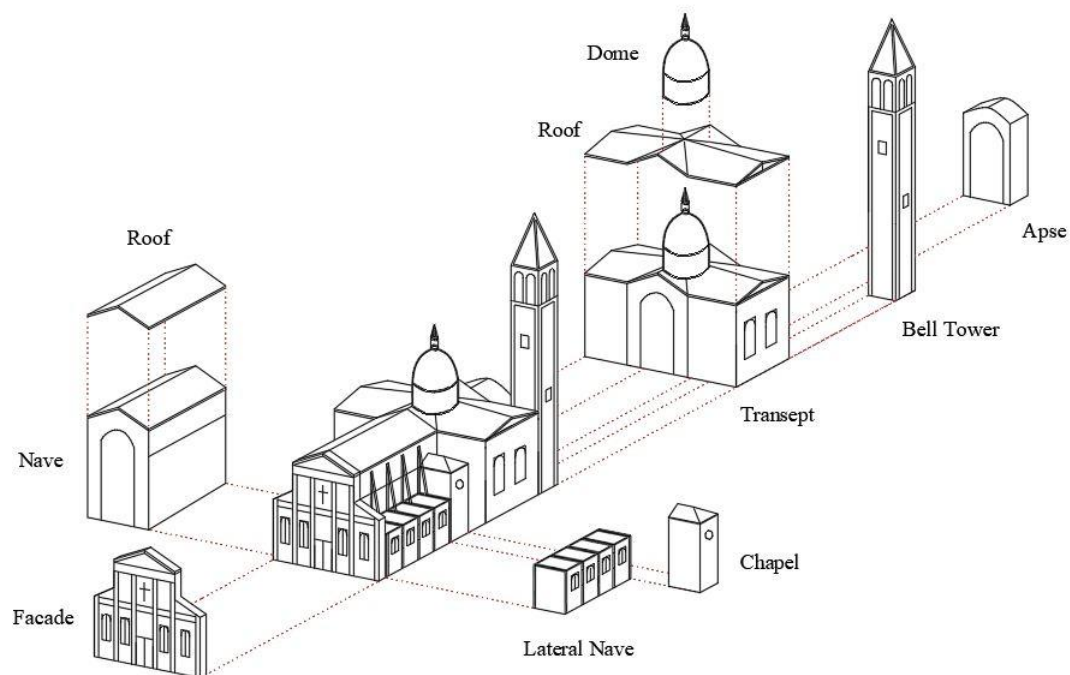


Figure 2.5: Recurrent macroelements in masonry churches.

Among the macroelements, historic towers demonstrated a rather high susceptibility to seismic damage. This can be positioned in different configurations within the overall structure of masonry churches, therefore it can be categorised into distinct typologies based on its interaction with the surrounding elements, as shown in Figure 2.6. Common typologies are isolated and bounded towers, where the latter can be distinguished into confined and integrated towers. A distinct configuration is constituted by the gable bell towers. An isolated tower is a free-standing structure, separated from the church. It is not physically connected and therefore stands independently near a church or within a complex. Differently, confined and integrated towers are structurally connected to the church. While a confined tower shares one or more walls with the adjacent building, an integrated tower is fully embedded into it. As a result, a confined tower depends partially on the adjacent buildings for support, whereas an integrated tower relies entirely on them. Finally, a gable bell tower, also called bell gable, is a planar projection of the church. Typically, it rises above the roofline, often at the front or back façade. This type of tower may either be built as part of the original design or added later, blending with the gable's structure.

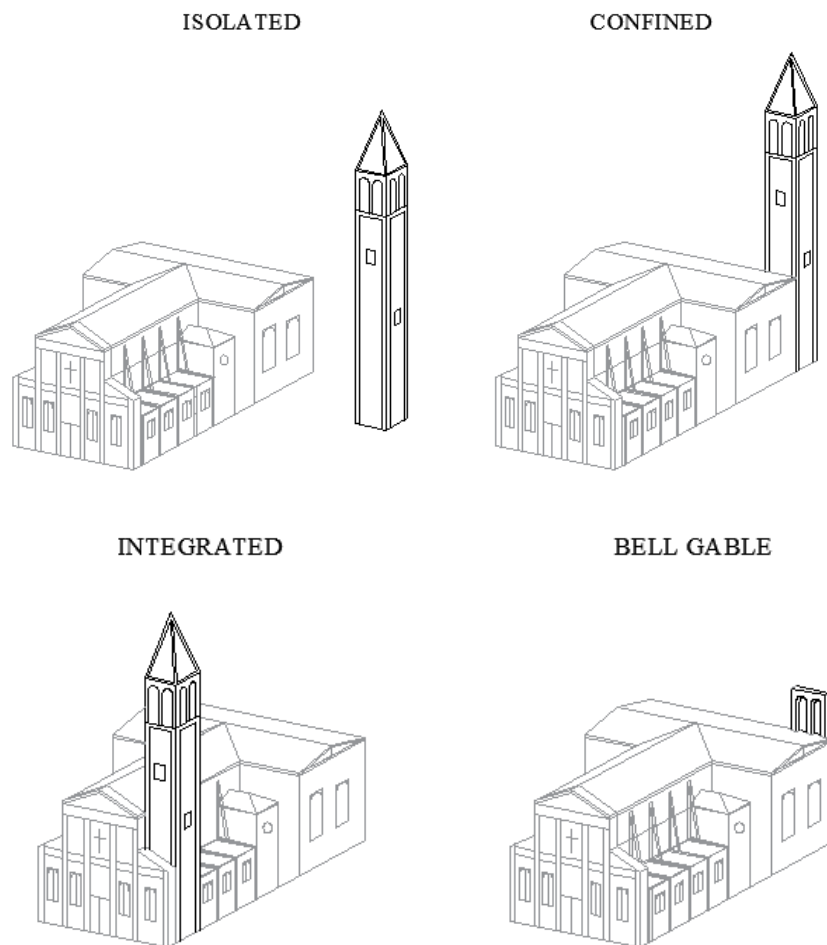


Figure 2.6: Tower typologies based on the interaction with surrounding elements.

Following the macroelement approach, as outlined in the post-earthquake inspection form of the Italian Civil Protection as well as the Italian Guidelines for the vulnerability assessment of Cultural Heritage (DPCM, 2011), the global observed damage to historic masonry towers can be described through three possible damage mechanisms, as shown in Figure 2.7.

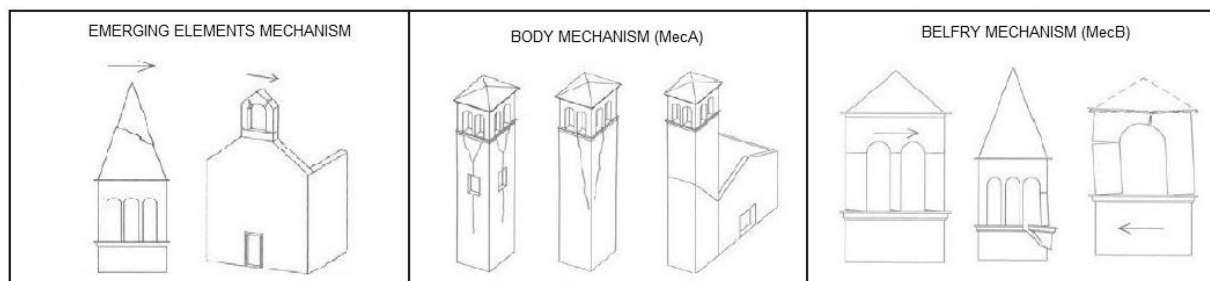


Figure 2.7: Damage mechanisms for the bell tower macroelement.

These are emerging elements mechanism, body (or shaft) mechanism, and belfry mechanism. In particular, the shaft represents the main structural system of the tower, whose damage could

compromise its entire stability. The belfries, sometimes missing, are structural parts at the top of the towers, having different characteristics and functions, whose damage is favoured by the abrupt variation in stiffness and capacity along the height. Finally, the emerging elements, such as spires, pinnacles and crenulations, are non-structural parts which could endanger human lives and properties when falling downwards. In the present thesis, the emerging elements mechanism is not included in the damage assessment of the towers, as the post-earthquake information collected through the inspection forms does not clearly distinguish when the surveyors identified this damage in the tower versus when it affected other emerging elements of the church. Therefore, following other similar studies (Canuti et al., 2021; Curti et al., 2008), only two damage mechanisms (shaft and belfry) are considered. These are listed as Mechanism 27 and Mechanism 28 in the last version of the A-DC survey form for the seismic damage to churches, provided by the Italian Civil Protection. For the sake of simplicity, they are renamed in this work as Mechanism A and Mechanism B, respectively.

Collecting empirical data, each mechanism is investigated to identify its activation, the type of damage and its extent. Different scales for classifying the damage extents observed in structural systems have been proposed over the years, including MCS, MM, MSK, and EMS-98. Any scale qualitatively grades the condition state of a generic building after seismic excitation. A comparison among them can be found in (Musson et al., 2010). Nowadays, the most commonly used damage classification scale is the EMS-98 (Grünthal, 1998). This scale establishes 6 sequential discrete damage grades (or levels/states) increasing progressively from D0 (no damage) to D5 (collapse), as shown in Figure 2.8.

NEGLIBLE-TO-SLIGHT DAMAGE	MODERATE DAMAGE	HEAVY DAMAGE	VERY HEAVY DAMAGE	DESTRUCTION (PARTIAL-TOTAL)
D1	D2	D3	D4	D5

Figure 2.8: EMS-98 Damage Classification (adapted for masonry towers).

After the identification of each occurring mechanism and the evaluation of its damage level according to the EMS-98 grading system (d_i), a global damage index for the whole tower is computed (i_d) with a maximum equal to the unit value, based on the following equation (weighted sum criteria):

$$i_d = \frac{1}{5} \frac{\sum_{i=1}^n \rho_i d_i}{\sum_{i=1}^n \rho_i} \quad (2.2)$$

where ρ_i denotes the weight to be assigned to each of the n possible damage mechanisms, based on its importance for the structural behaviour of the whole building.

Focusing in this work on the tower macroelement only, an importance weight equal to one is considered for both damage mechanisms (see Chapters 3, 4 and 5), as commonly done for the global assessment of churches, for instance in (De Matteis & Zizi, 2019). Therefore, the expression becomes:

$$i_d = \frac{1}{10} \sum_{i=1}^2 d_i \quad (2.3)$$

An alternative would be to give more relevance to the shaft mechanism, Mechanism A, as the consequences of failure are likely to be greater. However, this approach was not pursued in this work. The global damage index (i_d) is, thus, the average of the damage levels observed in the activated damage mechanisms. This is a continuous index within the 0-1 range. To obtain a discrete final global damage level (D_k) for each tower aligned with EMS-98 scale, the correlations provided in (Lagomarsino & Podestà, 2004c) and reported in Table 2.2 are used.

Table 2.2: Correlation between the global damage index and the global damage level.

Global damage index (i_d)	Global damage level (D_k)	Description
$i_d \leq 0.05$	0	No damage
$0.05 < i_d \leq 0.25$	1	Negligible to moderate damage
$0.25 < i_d \leq 0.40$	2	Moderate damage
$0.40 < i_d \leq 0.60$	3	Substantial to heavy damage
$0.60 < i_d \leq 0.80$	4	Very heavy damage
$i_d > 0.80$	5	Collapse

With the advent of digital technologies, significant efforts have been made to digitise this essential information about past events. Some of the most well-known databases available include the GEM

Database, the Cambridge Earthquake Damage and Casualty Database, the CATDAT Database, and the EM-DAT International Disaster Database (Rossetto & Ioannou, 2018).

In Italy, the Da.D.O. (Database of Observed Damage) web platform has been conceived by the Italian Civil Protection Department and realised by the European Centre for Training and Research in Earthquake Engineering to allow a long lasting and easy storage and retrieval of information (Di Meo et al., 2023; Dolce et al., 2019). The platform includes the digitisation of the survey forms from the physical inspection carried out during or following eight seismic events of national importance, namely, the 1997 Umbria-Marche, the 2002 Molise, the 2003 Piedmont, the 2004 Salò, the 2009 L'Aquila, The 2012 Emilia, the 2016-2017 Central Italy, and the 2017 Ischia earthquakes. In the cases of churches affected by these events, the damage data, catalogued with reference to the pre-defined damage mechanisms, can be displayed together with the characteristics of the events in terms of MCS intensity and in terms of PGA, elaborated by the National Institute of Geophysics and Volcanology (INGV).

The collected data on hazards and losses serve as foundational inputs for statistical analyses, which are used to derive damage distributions, develop vulnerability and risk models, and evaluate their predictive accuracy. For ordinary masonry buildings, ancient churches, and historic towers, commonly adopted methodologies include DPMs, vulnerability functions, vulnerability index methods, and fragility functions, each of which is discussed in detail in the following Sections.

2.3.1 Damage Probability Matrices (DPMs)

Among the earliest methods developed for risk analysis at the territorial scale, DPMs play a pivotal role in interpreting the vulnerability of asset typologies based on observed damage data. These matrices form the foundation of probabilistic risk assessment frameworks, offering a structured approach to quantify the occurrence of specific damage levels for various asset classes, given a range of intensity measure. To construct DPMs, observational data is typically aggregated into ranges of intensity measure, with the results visualised as tables or histograms for easier interpretation and application.

Emerging in the 1970s, after the 1971 San Fernando earthquake (Whitman et al., 1973), DPMs gained prominence following their application to analyse the damage from major seismic events. In the case of Italian earthquakes, DPMs were extensively adopted since the systematic study of data from the 1980 Irpinia earthquake (Braga et al., 1982). In the case of masonry towers, DPMs are used to describe the discrete probability distribution of the exceedance of specific damage levels, considering the individual collapse mechanisms (Mechanisms A and B) and the entire tower macroelement. In particular, when the DPMs are developed for the individual damage mechanisms, the mean damage μ_d is given by:

$$\mu_d = \frac{\sum_{i=1}^n d_{k,i}}{n} \quad k = [0, \dots, 5] \quad (2.4)$$

where $d_{k,i}$ is the damage level attributed to either the shaft or the belfry of the i -th tower, depending on the damage mechanism under study, and n denotes the number of towers investigated. When the DPMs are developed for the entire macroelement, the mean damage μ_D is given by:

$$\mu_D = \frac{\sum_{i=1}^n D_{k,i}}{n} \quad k = [0, \dots, 5] \quad (2.5)$$

where $D_{k,i}$ is the global damage level obtained for the i -th tower, and n denotes the number of towers analysed.

As DPMs represent a statistical relationship between the intensity measure of the hazard and the observed damage levels for a specific asset typology, their reliability depends on the consistency of these input data. If the assets within a typology share similar characteristics (e.g., construction materials, age, design standards) and the hazard's effects are uniform across the area of observation, the damage distributions are expected to exhibit relatively low scatter within each intensity range.

When applied to different building typologies, DPMs serve as an effective vulnerability screening tool, allowing for the preliminary identification of the most critical assets. However, DPMs provide a single output (distribution) per asset class, without explicitly accounting for unique features of the investigated building stock in their model definition, except as criteria for selecting homogeneous classes. Consequently, DPMs can be used to assess the seismic vulnerability of buildings not included in the original dataset and to predict their expected damage levels for events with specific intensity measures, only assuming that the buildings under assessment belong to the same typology, share similar characteristics, and will experience analogous hazard conditions.

Numerous studies have reported DPMs for different types of masonry structures, including churches subjected to various seismic events, since the 1976 Friuli and the 1997 Umbria-Marche earthquakes (Doglioni et al., 1994; Lagomarsino & Podestà, 2004c). Recent studies have also focused on specific typologies of churches, as for instance, one-nave (Ceroni et al., 2022; De Matteis & Zizi, 2019; Ruggieri et al., 2022) or three-nave (De Matteis et al., 2016; De Matteis, Brando, & Corlito, 2019). However, most of the studies have treated churches as a whole structural system, rarely reporting and discussing the individual behaviour of each single macroelement. This prevents critical insights into the localised vulnerabilities that significantly influence the overall seismic performance of these structures.

DPMs for distinct macroelements, including masonry towers, are provided by (De Matteis et al., 2016; De Matteis & Zizi, 2019), while (Canuti et al., 2021; Hofer et al., 2018; Ruggieri et al., 2022) also investigated distinct damage mechanisms (i.e., shaft and belfry). Therefore, the contribution of the distinct components and their damage mechanisms on the overall tower behaviour is not fully assessed in the literature. Additionally, in the case of masonry towers, the sources rarely provide a complete set of DPMs for various ranges of intensity measures, with just a few exceptions, e.g., (Canuti et al., 2021). The scarce sources available are extensively discussed in Chapter 3.

Finally, most of the existing studies developed DPMs using the MCS scale, which was traditionally one of the first measures adopted for seismic intensity. Nonetheless, recent advancements in territorial scale seismic risk assessment have shifted towards the use of alternative intensity measures that provide more quantifiable and objective metrics. In particular, PGA has been increasingly adopted to develop DPMs for masonry churches in general and the tower macroelements in particular (Ceroni et al., 2022; Sisti et al., 2023).

2.3.2 Vulnerability functions

While DPMs provide discrete probability distributions that quantify the frequency of specific damage levels for a given intensity measure, vulnerability functions aggregate this information into a single predictive curve, offering a continuous estimation of the expected damage, through a representative metric expressed as a function of the intensity measure. In this context, DPMs and vulnerability functions are closely related tools, with DPMs serving as a foundation for constructing the curves. Additionally, they are often complementary in a study as the vulnerability functions focus on a single metric of the damage for the class, while the DPMs allow an insight into the variability within the asset class. Commonly, the expected damage metric adopted is the mean damage, μ_d and the intensity measure follows the macroseismic scale, I_{MCS} .

Due to their ease-of-use and interpretation, vulnerability functions are likely the most common methods used in the literature for assessing the seismic vulnerability of ordinary and monumental masonry buildings, enabling a rapid screening of the structural condition across the considered study area. For instance, these functions have been extensively used within the RISK UE Project (Mouroux & Le Brun, 2008), “An advanced approach to earthquake risk scenarios”, to provide the seismic risk scenarios in some European cities, including Barcelona, Catania, Nice, and Thessaloniki. A similar approach has been adopted in Italy to estimate the expected damage to masonry churches and other monuments (DPCM, 2011).

To build a vulnerability model for a monumental masonry building typology, the function originally proposed by (Lagomarsino & Giovinazzi, 2006; Lagomarsino & Podestà, 2004c) is usually assumed, as follows:

$$\mu = 2.5 \left[1 + \tanh \left(\frac{I_{MCS} + \alpha V_I - \beta}{Q} \right) \right] \quad (2.6)$$

where μ is the expected mean damage $0 \leq \mu \leq 5$ correlated to the macroseismic damage levels defined by the EMS-98 scale, I_{MCS} is the MCS intensity varying from 5 up to 12, Q is the ductility index, V_I is the vulnerability index and α and β are coefficients that assume constant values of 6.25 and 13.1, respectively, irrespective of the building typology.

Therefore, the expected mean damage μ is correlated with increasing levels of macroseismic intensity I_{MCS} through the definition of two intrinsic parameters of the monumental building typology, namely, the ductility index, Q , and the vulnerability index, V_I . Q accounts for the structural response in nonlinear regime. It controls the rate of increase of the damage with the intensity and is typically defined for the building typology. The vulnerability index, V_I , may be assumed as equal to an average value, V_0 , for the entire class of assets, adjusted, if needed, to take into account relevant features of each specific building that influence their structural response. Average values of the vulnerability and the ductility indexes have been provided in literature for different building typologies according to expert judgement or by calibration to the real observed data collected from past seismic events. Different pairs of Q and V_0 values for monumental masonry structures suggested by (Despotaki et al., 2018; Lagomarsino, 2006; Lagomarsino et al., 2004) are shown in Table 2.3.

Table 2.3: Average values of V_0 and Q to be used in Eq. (2.6) for distinct building typologies.

Monumental building typology	Vulnerability index V_0	Ductility index Q
Arch bridges	0.296	2.30
Castles	0.456	2.30
Churches	0.890	3.00
Columns	0.456	1.95
Monasteries	0.736	2.30
Mosques	0.730	2.65
Obelisks	0.456	1.95
Palaces	0.616	2.30
Temples	0.500	1.95
Towers	0.776	2.30
Trilithes	0.456	1.95
Triumphal arches	0.456	2.30

The vulnerability functions obtained for the monumental masonry building classes with the coefficients in Table 2.3 are illustrated in Figure 2.9. Just by looking at the vulnerability functions, the expected mean damage computed for masonry churches is the greatest over the entire range of the intensity. Historic towers are one of the most vulnerable building typologies, together with mosques, palaces and masonry arch bridges. It is worth noting that bell towers are not included in the masonry tower class discussed by the authors.

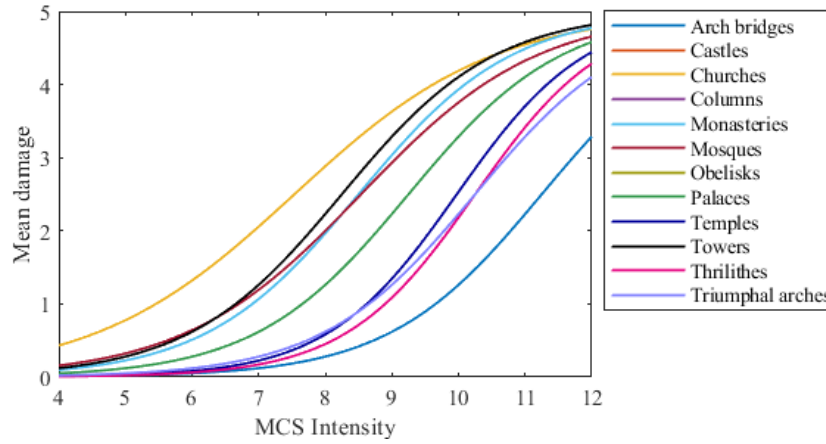


Figure 2.9: Simplified vulnerability functions for distinct monumental masonry building typologies.

In this framework, a list of corrective factors V_j were originally suggested for masonry churches by (Lagomarsino et al., 2004) and subsequently adopted for other historic structural typologies, including towers, by (Despotaki et al., 2018). These factors contribute to the estimation of the vulnerability index as:

$$V_I = V_0 + \sum V_j \quad (2.7)$$

Table 2.4 presents the parameters and categories considered along with their corresponding modifiers. With respect to the original list of parameters, the plan regularity is neglected for historic towers due to its minimal influence, as suggested by (Sepe et al., 2008).

The corrective factors V_j enable the adjustment of vulnerability functions for an entire asset class by incorporating specific building characteristics. This approach requires minimal knowledge, making it well-suited for large building stock and territorial-scale assessments. However, even the limited information necessary to accurately evaluate these corrective factors is often unavailable in large-scale assessments based on existing data sources. Standardised tools for data collection, such as inspection forms, in fact, frequently lack fields to capture all the required information. In this case, the vulnerability

index can be assumed equal to the average value for the class, as shown in Table 2.3, producing a less refined vulnerability assessment.

Table 2.4: Vulnerability modifier parameters.

Parameters	Classification			Vulnerability score		
State of preservation	Poor	Medium	Good	+0.04	0	-0.04
Damage level	Severe	Light	None	+0.04	+0.02	0
Architectural transformation	Yes	No	–	+0.02	0	–
Recent interventions	Yes	No	–	-0.02	+0.02	–
Site morphology	Ridge	Slope	Flat ground	+0.04	+0.02	0
Additional parameters for churches						
Position	Included	Additions	Isolated	-0.02	+0.02	0
Lateral wall height	Low (<6m)	Medium (>6 m and < 12m)	High (>12m)	-0.02	0	+0.04
Plan regularity (nave typology)	Central	One	Three	-0.02	0	+0.02
Elevation regularity (e.g., emerging elements)	Yes	No	–	+0.04	0	–

Besides the development of vulnerability functions for masonry towers as independent structural typology and following the seminal contribution provided by (Lagomarsino et al., 2004; Lagomarsino & Podestà, 2004c), few authors focused on the analysis of the vulnerability of the bell tower macroelements, leveraging a much larger datasets based on churches inspections carried out in the aftermath of the main Italian earthquakes and using the previously discussed inspection form of the civil protection (Canuti et al., 2021; Ceroni et al., 2022; Curti et al., 2008; Sepe et al., 2008).

A first noteworthy contribution is the work of (Curti et al., 2008), who exploited the collection of a wide dataset of historic masonry towers affected by different events, namely the 1976 Friuli, the 1997 Umbria-Marche, the 2002 Molise and the 2004 Lombardia earthquakes. This was used to calibrate ad-hoc vulnerability and ductility indexes for the two distinct set of mechanisms (shaft and belfry) by fitting the vulnerability curve, Eq. (2.6), proposed by (Lagomarsino & Giovinazzi, 2006; Lagomarsino & Podestà, 2004c), to the observed data. The two pairs of vulnerability and ductility indexes for the shaft and belfry mechanisms proposed by (Curti et al. 2008) are summarised in Table 2.5.

Table 2.5: Corrective coefficients proposed for the shaft and belfry mechanisms.

Monumental Building Typology	V_0	Q	Source
Tower macroelement damage	0.776	2.30	(Lagomarsino et al., 2004)
Damage Mechanisms			
Mechanism A (shaft)	0.890	2.00	(Curti et al., 2008)
Mechanism B (belfry)	0.940	1.49	(Curti et al., 2008)

The formulations suggested by (Curti et al., 2008) are illustrated in Figure 2.10 in comparison with the curve proposed for the tower building typology. From the comparison, the bell tower shafts (Mechanism A) and belfries (Mechanism B) are expected to undergo greater expected mean damage than the tower typology. Among the two components, belfry presents higher susceptibility than shaft.

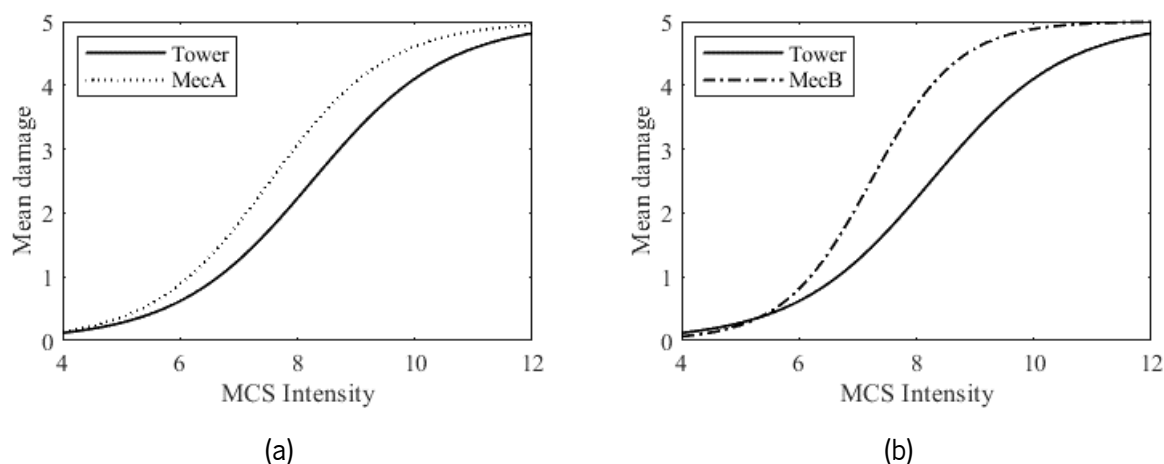


Figure 2.10: Vulnerability functions for the shaft (a) and the belfry (b) mechanisms, compared with the vulnerability function for the tower typology of monuments.

A multi-level methodology for assessing the seismic vulnerability of historic masonry towers was proposed in (Sepe et al., 2008), analysing a set of 107 historic towers. In the preliminary stage, the structural vulnerability was roughly estimated using limited information to identify the most critical towers within the investigated stock. Initial insights were gained by correlating the level of conservation of the towers with the MCS intensity, and by comparing the existing vulnerability function proposed by (Curti et al., 2008) with the observational data. Afterwards, the towers that deviated significantly from the expected trend were flagged for more detailed structural analysis.

(Canuti et al., 2021) exploited the surveys of 541 masonry churches in Marche region inspected in the aftermath of the 2016-2017 Central Italy seismic sequence to provide an overview on the occurred damage and structural vulnerability. Among the most recurrent mechanisms, bell towers received particular attention. The Authors reported the DPMs for distinct ranges of macroseismic intensity, separating failure mechanisms to the shaft and the belfry. Then, the distributions of the observed damage were compared with those estimated by the existing vulnerability models proposed by (Curti et al., 2008), revealing great consistency between observations and model predictions.

Although the use of macroseismic intensity in predictive vulnerability models is widespread, as already discussed for DPMs, several methodologies are now exploring the use of PGA as a ground motion intensity measure. PGA-based approaches are gaining increasing popularity (Ceroni et al., 2022; De Matteis & Zizi, 2019). For masonry towers, (Ceroni et al., 2022) proposed multiple vulnerability curves

as a function of the MCS intensity and PGA by fitting the Eq. (2.8)-(2.9) to the observed damage for 633 single nave churches struck by the 2016-2017 Central Italy sequence. The proposed calibration coefficients are shown in Table 2.6. Unlike the original formulation of the vulnerability functions, the Authors expressed the damage data in terms of mean damage index rather than mean damage level, thus having a continuous variable in the 0-1 range. Furthermore, the vulnerability and ductility indexes were not included in the novel predictive formulations.

$$\mu_{i_d} = 0.5 [1 + \tanh(a I_{MCS} - b)] \quad \text{MCS} \quad (2.8)$$

$$\mu_{i_d} = 0.5 [1 + \tanh(a' \log(PGA) - b')] \quad \text{PGA} \quad (2.9)$$

Table 2.6: Coefficients of Eq. (2.8)-(2.9) proposed for the shaft and belfry mechanisms.

Damage Mechanisms	MCS		PGA		Source
	a	b	a'	b'	
Mechanism A (shaft)	0.25	2.08	0.89	2.54	(Ceroni et al., 2022)
Mechanism B (belfry)	0.25	1.87	0.89	2.22	(Ceroni et al., 2022)

Besides these few mentioned cases, validations of existing vulnerability functions for historic masonry towers on post-earthquake damage data are quite rare in the literature. Therefore, in the present thesis, these existing models are extensively tested on empirical observations, evaluating their predictive performance and suggesting recalibrations for specific datasets at hand.

2.3.3 Vulnerability index methods

While DPMs and vulnerability functions have been extensively used to describe the probability distribution of damage levels based on empirical observations for classes of assets with expected homogeneous behaviour, a widespread family of methods, the so-called vulnerability index methods, have been developed by addressing the risk assessment problem from a different perspective. Indeed, vulnerability index methods provide a way to estimate the susceptibility of a building based on a set of qualitative or semi-quantitative criteria stressing the features that contribute the most to it. Each criterion is defined by a specific parameter, which is assigned a vulnerability score and an associated weight. These methods calculate the overall numerical vulnerability index, I_v , to an asset aggregating the scores of each parameter, weighted according to its relative importance. Therefore, these methods do not provide a direct estimation of the expected damage, rather highlight assets that are more susceptible to damage. Typical applications are aimed at identifying the critical structures for further detailed evaluation. These critical structures are generally detected as the ones with highest

vulnerability score obtained from the comparison among all the structures included in the investigated building stock (Sepe et al., 2008).

At the same time, the assessment of structural parameters related to the seismic behaviour allows a more refined analysis, capable of capturing more differences within a class of assets. The number of parameters and the type of assessment enable rapid application to large building stocks, with most parameters requiring only a visual inspection supported by basic tools. Nonetheless, when the building stock is extremely large the vulnerability index methods can be computationally and economically expensive. Furthermore, the judgment of the expert teams involved in the evaluation plays a dominant role in the results.

Several vulnerability index methods exist in literature. As previously mentioned, their basic functioning consists in the definition of a score I_v as the weighted sum of a set of relevant features:

$$I_v = \sum_{i=1}^N q_i w_i \quad (2.10)$$

where q_i is the score assigned to the i -th qualitative parameters (e.g., elevation layout, type and quality of the masonry, state of conservation) which might remarkably affect the seismic behaviour of the building typology under investigation, w_i is the weighted coefficients, and N the number of qualitative parameters. The global vulnerability score is then usually normalised between 0 and 100, for comparative purposes. The data may be collected by means of specific survey forms tailored to each method. The number of the qualitative parameters depends on the methodology adopted. Most of the existing methodologies for seismic vulnerability assessment descends from the procedure proposed by (Benedetti & Petrini, 1984) for unreinforced masonry buildings. Several researchers and scholars have adapted this method to other contexts, such as urban centres (Vicente et al., 2011), masonry aggregates (Formisano et al., 2015) and specific macroelements such as the façade (Ferreira et al., 2017). The method was also officially adopted in Italy by the National Group for Earthquake Defence (GNDT-SSN, 1994) and it was the reference for the procedure proposed by (Sepe et al., 2008) specifically for historic masonry towers. The parameters associated to the original methodology and the approach tailored to towers in (Sepe et al., 2008) are listed in Table 2.7, including the scores for each class and corresponding weights. Eleven parameters are needed for compiling the vulnerability survey form adopted by the GNDT II Level (Met1), whereas the vulnerability form addressing historic masonry towers (Met2) is the same as the GNDT, with the exclusion of the plan layout and maximum span

between walls, thus including a total of nine parameters. These features are assumed not significant for the seismic behaviour of this peculiar building typology.

Table 2.7: Existing list of parameters proposed for masonry buildings (Met1) and for historic masonry towers (Met2), together with scores and importance weights.

PARAMETERS		Method	Vulnerability classes and scores				Weight
			A	B	C	D	w
P1	Type and organisation of structural system	Met1	0	5	20	45	1.00
		Met2	0	5	20	45	1.00
P2	Capability of structural system	Met1	0	5	25	45	0.25
		Met2	0	5	25	45	0.50
P3	Conventional resistance	Met1	0	5	25	45	1.50
		Met2	0	5	25	45	0.80
P4	Soil morphology	Met1	0	5	25	45	0.75
		Met2	0	5	25	45	0.75
P5	Horizontal structures	Met1	0	5	15	45	0.5-1
		Met2	0	-	25	-	0.80
P6	Plan layout	Met1	0	5	25	45	0.50
		Met2	-	-	-	-	-
P7	Elevation layout	Met1	0	5	25	45	0.5-1
		Met2	0	5	25	45	1.50
P8	Max span between walls	Met1	0	5	25	45	0.25
		Met2	-	-	-	-	-
P9	Roof structure	Met1	0	15	25	45	0.5-1
		Met2	0	15	25	45	0.80
P10	Non-structural elements	Met1	0	0	25	45	0.25
		Met2	0	0	25	45	0.40
P11	Present state	Met1	0	5	25	45	1.00
		Met2	0	5	25	45	1.00

In broad terms, each parameter is labelled by an increasing scale of vulnerability into four classes (A, B, C, and D). Thus, each class has a corresponding increasing score. The two methods present slight variations in the decision criteria for classifying the vulnerability level of each parameter. The scores assigned to the parameters are then weighted according to the importance of each of them to the global seismic behaviour. The weights are also slightly different, being calibrated in Met2 based on the existing documentation of historic masonry towers. Consequently, the global vulnerability score that can range between 0 and 382.5 is updated for the case of historic towers to vary in a range between 0 and 323.75. Each of the 11 parameters is shortly described for a better comprehension, giving particular attention to the main differences between the two investigated methodologies.

The first parameter (P1), "Type and organisation of the structural system", describes synthetically the spatial organisation of the masonry load-bearing walls, considering factors such as the alignment to

current seismic code requirements, the presence of steel tie-rods, and the quality of connections among the vertical walls. For historic buildings, structural strengthening as reinforced concrete frame walls are common factors that can contribute to the vulnerability, therefore they are included in Met2. The second parameter (P2), “Capability of structural system”, refers to the quality and type of the masonry used, taking into account three specific aspects: type of material, shape of the units and type of pattern. When multi-leaves masonry walls are identified, type of infill material and level of connection along the thickness are added as potential sources of vulnerability. The general very low weight associated with this parameter is because the capability of the masonry is implicitly included in the third parameter as well. However, the value is increased for historic towers. The “Conventional resistance” is the third parameter (P3) and accounts for the shear strength capacity of the structure. The vulnerability class is determined through the computation of a coefficient α , related to the class of vulnerability. For the sake of brevity, the original formulation of the coefficient detailed in (Sepe et al., 2008) and expressed as a function of the wall thickness, floors number, and mechanical characteristics of the material, namely, material density, shear strength, among other, is not here reported. The importance weight associated with parameter P3 is reduced to 0.8 for towers due to the low shear effect in slender structures. The fourth parameter (P4) “Soil morphology”, evaluates the presence of the foundation, type and slope of the ground where the tower is located. The fifth parameter (P5), “Horizontal structures”, regards mainly the in-plane stiffness and level of connections among the vertical masonry walls, to assess their contribution to the structural resistance. For this parameter, only two classes (i.e., A and C) are considered for historic towers. The sixth parameter (P6), “Plan layout”, focuses on the plan arrangement of the masonry vertical walls, but, for the historic towers method, it is excluded in the analysis given its negligible effect on slender structures. The “Elevation layout” is the seventh feature (P7) through which the presence of discontinuities that might produce stiffness variations along the height of the tower (e.g., large openings, belfry, turrets) is assessed. Here, the slenderness (with thresholds of 2.65 and 4) is included as an additional condition for the vulnerability class assignment of historic towers. To underline the importance of this parameter in slender structures the weight is also increased from 1 to 1.5. On the contrary, the “Maximum span between walls” (P8) is not considered for historic masonry towers due to its poor influence. The ninth parameter (P9), “Roof structure”, analyses the characteristics of this component, specifically its typology (i.e., flat, no thrust, thrust) and weight, including the presence of steel tie-rods or ring beams. The “Non-structural elements” (P10) evaluates the presence of components that by falling could cause harm, namely bells, turrets, decorations, crenulations, and their state of connection to the structure. The weight is also increased

from 0.25 to 0.4 for historic towers given the potential danger of these elements when falling downwards from a considerable height. Finally, the last parameter (P11) is the “Present state” which regards the conservation state and the presence and nature of damage. For the original method, the vulnerability class is determined based on the crack pattern identified during the visual inspections (in terms of crack width). When adopting the methodology proposed for historic towers, the vulnerability class to associate to the parameter is instead given by the usability state or by the global damage level observed and estimated according to the EMS-98 damage severity scale.

Recently, an alternative vulnerability index method for assessing the seismic vulnerability of slender masonry structures (temples, towers, minarets, chimneys) has been proposed by (Shakya et al., 2018) who defined 12 parameters to estimate the vulnerability score. The definition and calibration of the parameters were carried out through experts’ opinion, post-earthquake damage observations, and parametric nonlinear static (pushover) analysis. The proposed methodology has been validated using information collected from extensive literature review to initially investigate the influence of each parameter on the global vulnerability score and then to develop simplified vulnerability curves of the investigated building typologies.

Several attempts have been made to correlate the vulnerability index with the vulnerability value used in vulnerability functions, as both aim to quantify a building's susceptibility to damage under seismic action. These efforts are focused on bridging the gap between the two methods, allowing a prediction of expected mean damage based on the estimated vulnerability index, thus enabling a building-specific assessment. Among the existing formulations, the correlation, Eq. (2.11), originally proposed by (Lagomarsino & Podestà, 2004c), also reported by (Curti et al., 2008) for tower’s collapse mechanisms, and the recent correlation, Eq. (2.12), derived by (Shakya et al., 2018), are given in the following:

Correlations	Vulnerability functions
$V_I = 0.67 + 0.55 I_v$	$\mu = 2.5 \left[1 + \tanh \left(\frac{I_{MCS} + 6.25V_I - 13.1}{2.3} \right) \right] \quad (2.11)$

$V_I = 0.46 + 0.0056 I_v$	$\mu = 2.5 \left[1 + \tanh \left(\frac{I_{MCS} + 3.4375V_I - 8.9125}{2.3} \right) \right] \quad (2.12)$
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It should be noted that the two correlations above are used in vulnerability functions different from each other, thus not directly comparable.

2.3.4 Fragility functions

Fragility functions are another well-established tool for risk analysis largely used in the present thesis. They estimate the probability that a structure or system will attain or exceed a certain level of damage when exposed to a specific seismic hazard intensity. These functions link hazard intensity measures (e.g., MCS intensity, Magnitude, PGA, PGV, PGD SA, SV, SD) to damage states. Derived from empirical data, analytical models, or expert judgment, fragility functions play a crucial role in disaster risk assessment, performance-based design, and resilience planning. In empirical studies, damage states are typically indicated according to EMS-98 scale and the PGA is the intensity measure most commonly used, although for this tool as well alternative intensity measures are gaining momentum.

Distinct statistical methods and optimisation techniques are today available for producing fragility functions based on the observational data. The most common methods can be grouped in two main categories: simplified strategies and data fitting procedures.

First category exploits the correlation of the cumulative damage probabilities in the form of Binomial Density Probability Functions (BDPFs) with the PGA to develop simplified fragility functions. The BDPF allows to predict the probability of reaching a given damage level, p_k , based on the knowledge of the mean damage μ (or the binomial coefficient $\mu/5$, for the six-level EMS-98 scale) observed in a given building typology, as follows:

$$p_k = \frac{5!}{k! (5-k)!} \cdot \left(\frac{\mu}{5}\right)^k \cdot \left(1 - \frac{\mu}{5}\right)^{5-k} \quad k = [0, \dots, 5] \quad (2.13)$$

This function is typically used in the literature to compare the predicted probabilities of damage for different discrete levels with the observed probabilities in the form of DPMs (De Matteis & Zizi, 2019), as in the example of Figure 2.11. The use of such distribution can be explained by the simplicity of the method whose input parameter is the observed mean damage only.

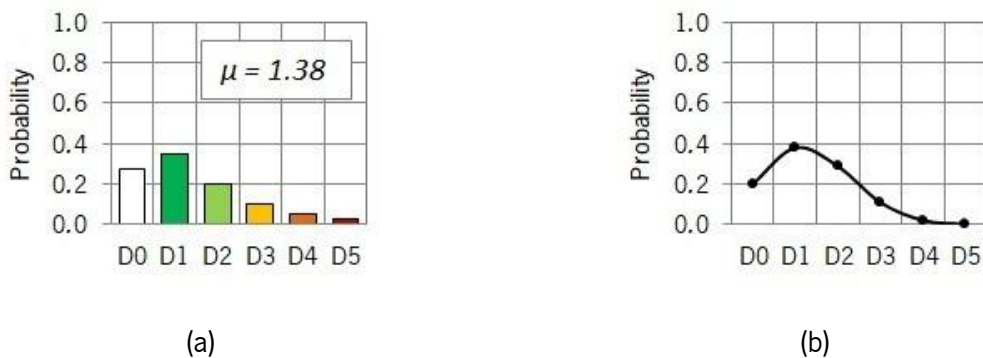


Figure 2.11: DPMs: (a) observed, and (b) predicted using the BDPF.

For the development of fragility functions, the observed mean damage is computed by separating the samples in different bins representing increasing values of PGA (Figure 2.12a). Afterwards, the BDPF is applied to estimate the probability of reaching a given damage level for different bins of PGA and exploited in the form of cumulative probability function (Figure 2.12b).

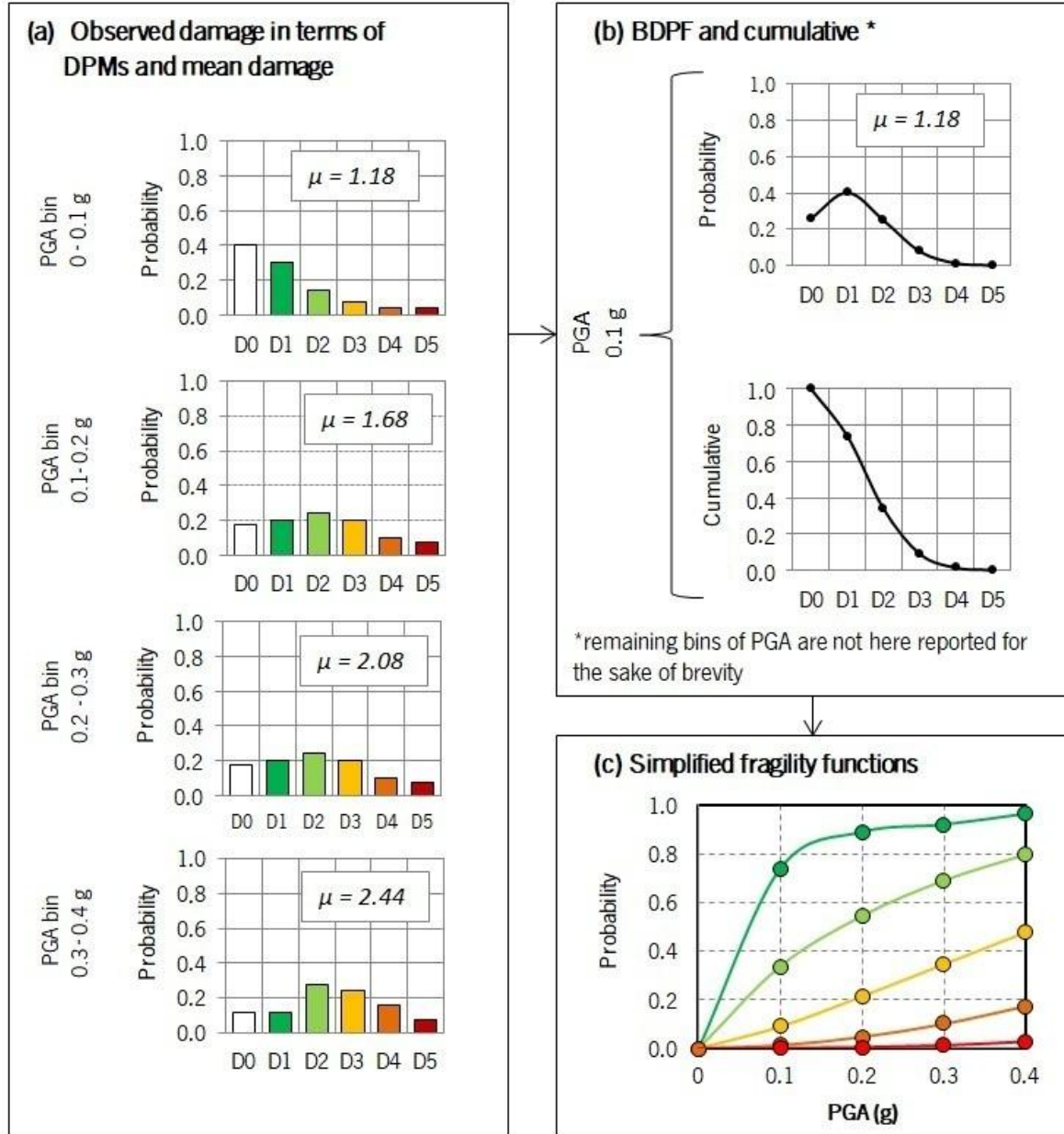


Figure 2.12: Fragility curves based on a simplified approach.

The classic expression of the cumulative probability function is reported as follows:

$$P_k = \sum_{i=k}^5 p_i \quad (2.14)$$

where P_k is the cumulative probability for a given damage level k , and p_i is the probability computed according to the Eq. (2.13) for i that varies between the considered damage level and the maximum attained damage level. Finally, the obtained cumulative probabilities are correlated with the corresponding bins of PGA to develop the fragility functions (Figure 2.12c), one for each damage level (from D1 to D5).

This approach requires only the knowledge of the mean damage observed across the entire range of predefined bins of PGA. The bins are often defined in order to ensure a consistent number of samples in each one, forming a homogenous distribution. Nonetheless, the expected trend, increasing mean damage for increasing values of PGA, may not be captured due to data incompleteness (e.g., lack of data, data scatter). To address such issues, a viable approach, well consolidated in the literature, consists in using the expected mean damage, rather than the observed mean damage, computed via simplified formulations mentioned in Section 2.3.2. This approach has been pursued in Chapter 3.

Alternatively, data fitting procedures can be used to directly extrapolate the fragility functions from the observed probabilities of exceeding a certain level of damage through specific analytical or statistical functions, such as the exponential function or the lognormal cumulative probability distribution function. Two common optimisation procedures exist, namely the Least Square Estimation (LSE) methods and the Maximum Likelihood Estimation (MLE) methods (Porter, 2021).

Irrespective of the fitting function and optimisation method, the observed probabilities are first computed. To this end, samples are organised accounting for the distinct levels of damage and for increasing values of PGA (Figure 2.13a and b). Indeed, the observed probability of damage are computed as the ratio between the number of towers with damage equal or larger than a specific grade and total number of towers for each level (or bin) j of PGA. This calculation is conducted for each state k of damage (from D1 to D5) as follows (Figure 2.13c):

$$P(D_k|PGA_j)_{obs} = \frac{\text{no. damaged towers} \geq D_k}{\text{no. total towers}} \quad (2.15)$$

In such applications, a typical function used for fitting the observed probabilities is the lognormal cumulative distribution function (Figure 2.13d), expressed as in the following:

$$P(D_k|PGA_j) = \phi \left(\frac{\ln(PGA/\theta)}{\beta} \right) \quad (2.16)$$

where $P(D_k|PGA_j)$ denotes the fragility function, namely the probability of reaching a damage state D_k (from D1 to D5) for a given PGA value j (or bin), Φ represents the standard normal Cumulative Distribution Function, θ denotes the PGA median value, and β constitutes standard deviation of the logarithms of the PGA.

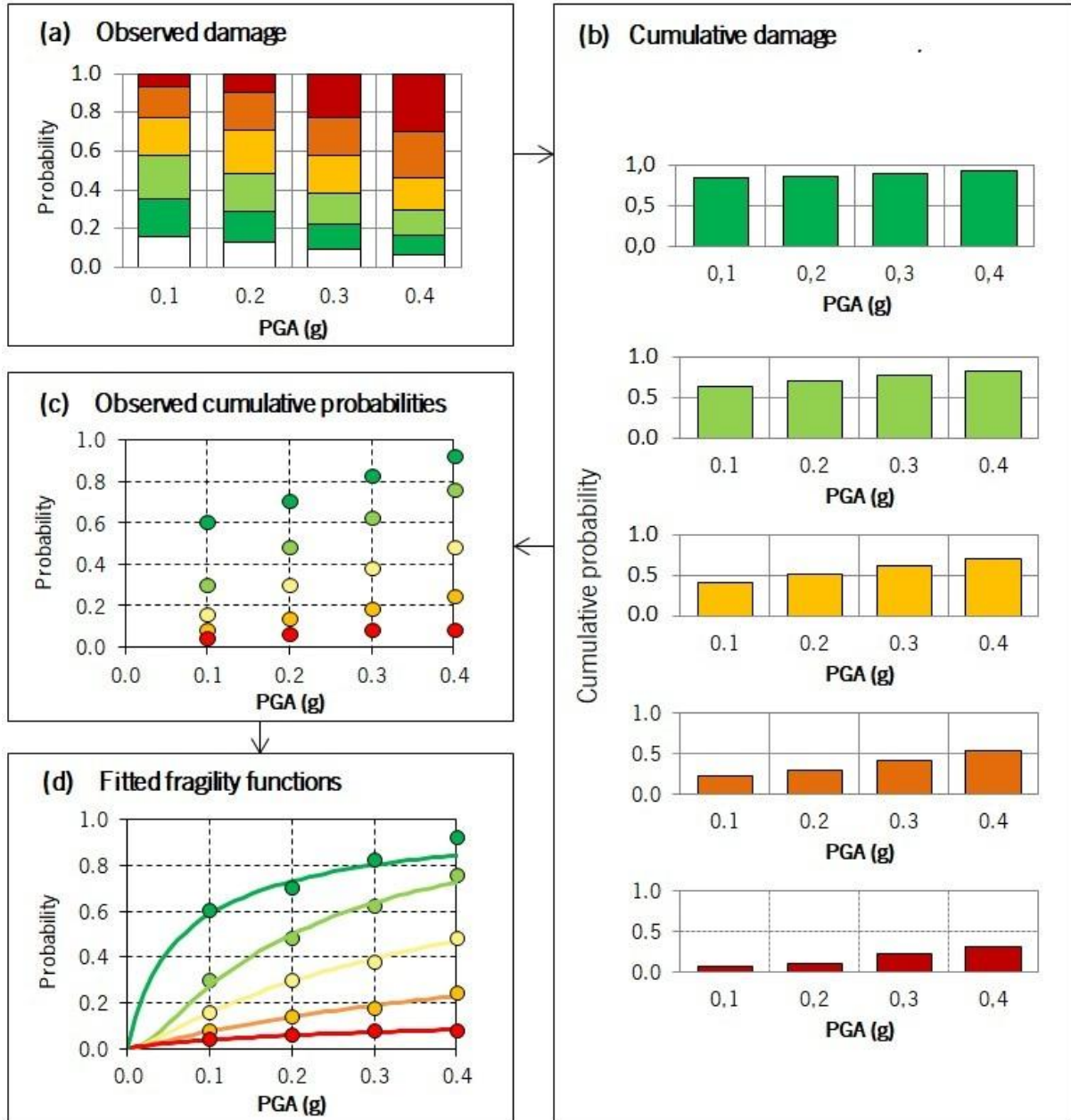


Figure 2.13: Fragility curves based on data fitting.

The lognormal cumulative probability distribution function is widely considered a reliable and established method in the scientific community for several reasons. It is straightforward to apply, extensively used in earthquake engineering, and often provides a good fit to observed data. Specifically,

the lognormal distribution's characteristic right-skewed shape can better represent observed probabilities, which tend to cluster at lower levels of ground shaking.

The procedure, described and adopted in this study in Chapters 4 and 5, fits the lognormal cumulative probability distribution to the empirical data by means of the MLE method, as originally proposed by (Baker, 2015), and widely adopted in the literature in similar works. The MLE method allows for the estimation of the median and standard deviation of the lognormal cumulative distribution function, which are the two unknowns in Eq. (2.16). These parameters define the fragility function that most likely generated the observed probability data. It is important to note that, in common applications, the MLE optimisation process is iterated to derive multiple fragility functions, each corresponding to different sequential damage levels. Other optimisation procedures commonly found in the literature for data fitting are not recommended for this type of data. For example, the LSE method does not account for the variance of the observed probabilities. For further details on this topic, the reader is encouraged to refer to (Baker, 2015).

Mathematically, the probability of attaining a certain damage level is given by the binomial distribution, expressed as follows:

$$P(D_k|PGA_j) = \binom{n_j}{z_j} p_j^{z_j} (1 - p_j)^{n_j - z_j} \quad (2.17)$$

where n_j denotes the number of total samples, z_j the number of damaged samples, and p_j the true probability of reaching a damage level for a given PGA value j (or bin), calculated according to Eq. (2.16).

To account for the different PGA values (or bins), the likelihood function is given by the product of the probabilities for each PGA value as:

$$Likelihood = \prod_{j=1}^m \binom{n_j}{z_j} \phi \left(\frac{\ln \left(\frac{PGA_j}{\theta} \right)}{\beta} \right)^{z_j} \left(1 - \phi \left(\frac{\ln \left(\frac{PGA_j}{\theta} \right)}{\beta} \right) \right)^{n_j - z_j} \quad (2.18)$$

To ensure the parameter estimation with the highest probability of reproducing the observed data, the maximum value of the maximum likelihood function is then computed as:

$$\{\theta, \beta\} = \underset{\theta, \beta}{argmax} \prod_{j=1}^m \binom{n_j}{z_j} \phi \left(\frac{\ln \left(\frac{PGA_j}{\theta} \right)}{\beta} \right)^{z_j} \left(1 - \phi \left(\frac{\ln \left(\frac{PGA_j}{\theta} \right)}{\beta} \right) \right)^{n_j - z_j} \quad (2.19)$$

The applicability of such procedure is very straightforward requiring a very few input parameters which are: (i) the number of total samples (n_j); (ii) the number of damaged samples (z_j); and (iii) the range (or bins) of PGA.

It is important to note that fragility functions, when developed for distinct damage grades, may exhibit a cross-behaviour. This occurs when the estimated parameters for different curves cause their intersection. This behaviour may be due to the dataset being poorly distributed across the different PGA bins. However, when the damage grades are sequential, as in the case of the EMS-98 scale, ranging from D0 to D5, the intersection of the curves implies that the probability of exceeding a higher damage grade is greater than the probability of exceeding a lower damage grade, for certain values of PGA. Therefore, avoiding crossing of the fragility curves for sequential damage states is critical for ensuring the reliability of the resulting applications.

To address the intersection of the fragility curves, two distinct approaches (Approach 1 and 2) have been suggested by (Porter, 2021). Both are tested and discussed in Chapters 4 and 5. The simplest approach (Approach 1) requires a revision of the parameters posterior to the separated fitting for each damage state. The procedure involves the computation of a new logarithmic standard deviation, equal for all damage states, and new adjusted median values, one for each damage state. In particular, the following equations are adopted:

$$\beta_{new} = \frac{1}{k} \sum_{D=1}^k \beta_k \quad k = [1, \dots, 5] \quad (2.20)$$

$$\theta_{new} = \theta_k \cdot \exp[0.842 \cdot (\beta_{new} - \beta_k)] \quad (2.21)$$

Approach 2 is considered more advanced and it involves developing the fragility curves simultaneously using the MLE, resulting in a constant logarithmic standard deviation across all damage states, while the median varies, with a distinct value for each damage state. To address the issue of fragility curve intersections, recent contributions worth mentioning include the work by (Sisti et al., 2023) for masonry churches and the work of (Tatangelo et al., 2024) for ordinary constructions. Nevertheless, the significance of this issue remains somewhat underappreciated within the scientific community.

In the literature, applications based on simplified approaches to develop typological fragility curves are numerous (Chieffo et al., 2022; Lagomarsino et al., 2021). Although fragility functions have been mostly resorted to PGA as seismic intensity measure, alternative applications using the macroseismic intensity can also be found, as an example in (De Matteis, Brando, & Corlito, 2019; Lagomarsino &

Podestà, 2004c; Vicente et al., 2011). With the recent progresses in the field of monitoring and territorial scale assessment, especially in light of the recent seismic experiences, earthquake and damage data have become increasingly accessible. As a result, fitting procedures have become standard practice for deriving fragility functions for various types of buildings and structures. While a substantial body of work exists reporting empirical fragility curves for historic masonry churches (Ceroni et al., 2022; Cescatti et al., 2020; Hofer et al., 2018; Lagomarsino et al., 2021; Marotta et al., 2021; Sisti et al., 2023), limited attention has been given to historic masonry towers, with just a few exceptions (Marotta et al., 2021; Sisti et al., 2023). While alternative seismic intensity measures, such as the SA, are gaining increasing scientific interest for the development of fragility models for reinforced concrete (Rossetto & Elnashai, 2003) and unreinforced masonry (Gautam et al., 2018; Zucconi et al., 2020) buildings, their application to masonry churches and historic masonry towers remains relatively limited.

2.4 ANALYTICAL METHODOLOGIES

Empirical methods rely on observations of actual damage from past earthquakes to develop vulnerability relationships or fragility curves. While the use of actual damage data makes the results realistic and directly linked to observed outcomes, this approach requires extensive, high-quality damage data, which may not be available in many regions. Moreover, this data, even when available, reflects local construction traditions, materials, and seismic characteristic, thus, could be hardly applicable to regions with different building practices or seismicity and/or could overlook evolving construction standards or retrofitting efforts. To overcome these limitations, adjustment to the methods or novel strategies based on expert judgment and qualitative assessments may be explored and applied across diverse regions with modifications to suit local conditions and tackle data-scarcity. However, these methods may be less precise and the variability in expert judgment can lead to inconsistent outcomes across studies. A valid alternative is offered by analytical methods. These involve detailed simulations or calculations of building responses to seismic actions assessing the vulnerability across a wide range of geographical areas, scenarios and structural types at various level of detail, accuracy and complexity. While these methods address all the limitations of the previous strategies, they are less simple to implement, requiring significant computational resources, advanced expertise in structural engineering and computational modelling. Moreover, their scalability is affected by the level of accuracy demanded to the models, reflected into the level of knowledge available or pursued of the building. The knowledge path affects both the modelling stage and the seismic demand evaluation. In general, for the development of a mechanical model, the representative features of the towers such as geometry (in

plan and elevation), material characteristics, boundary conditions, and state of conservation are necessary. Typically, those can be retrieved either from existing documentations and codes, or from observations and measurements acquired by in situ inspections. Regarding the seismic input characterisation, features such as location and soil type, topography and morphology need to be identified to extract the code spectra to be used for the estimation of the seismic demand. Therefore, highly reliable models are suitable for individual structures only but impractical for larger scale in a reasonable timeframe, due to resource constraints to collect the information required and carry out the analyses. To ensure their viability for territorial scale assessment, simplifications must be introduced at the risk of overlooking important aspects of the seismic response of the building.

To this end, the Italian Guidelines (DPCM, 2011) for the assessment and mitigation of the seismic risk of the cultural heritage introduces a classification of the modelling strategies according to three increasing levels of complexity. Particular attention is given to the first level of evaluation (LV1), further described hereafter with specific focus on masonry towers, for its suitability to territorial scale analysis. However, a significant body of existing research in the literature employs more accurate evaluations (LV2 and LV3) for historic towers. At these levels, commonly each study investigates one or few case studies in detail. For the second level of evaluation (LV2), potential collapse mechanisms and local failures occurring in historic masonry towers have been analysed through limit analysis by (Bartoli, Betti, & Monchetti, 2017; Chisari et al., 2022; Curti et al., 2012; Faccio et al., 2011b; Sarhosis et al., 2018; Tanganelli et al., 2024; Torelli et al., 2020) and, more recently, using finite element models by (Degli Abbati et al., 2024; Mehrotra et al., 2023; Milani, 2019). Typical mechanisms in historic masonry towers included in these studies are presented and discussed in Chapter 3. For the third level of evaluation (LV3), illustrative applications resorting to finite element models are (Bartoli, Betti, & Monchetti, 2017; D'Ambrisi et al., 2012; Degli Abbati et al., 2024; Micelli & Cascardi, 2020; Milani & Clementi, 2021; Tanganelli et al., 2024; Torelli et al., 2020; Valente & Milani, 2016). Advanced 3D models are developed to accurately estimate the structural response, typically by means of nonlinear static analysis. Methodologies, advantages and limitations of these two levels of evaluations (LV2 and LV3) are not here addressed due to their relevance at the individual scale analysis. Although the first level of evaluation (LV1) reduces the number of input parameters when compared with the higher levels of evaluation, its practical use for seismic vulnerability assessment at the territorial scale remains still a field of research not fully explored, with just a few examples (Bartoli, Betti, & Monchetti, 2017; Chisari et al., 2022; Curti et al., 2008; Sarhosis et al., 2018). This is likely due to the level of knowledge required by this seismic safety evaluation, which is often constrained by the challenges of addressing

the unique characteristics of each tower, particularly those arising from geometry complexities and material variations within each asset (Degli Abbati et al., 2024; Faccio et al., 2011b; Lagomarsino et al., 2014; Tanganelli et al., 2024; Torelli et al., 2020).

The first level of evaluation (LV1) proposed by the Italian Guidelines (DPCM, 2011), for masonry towers, analyses the seismic response based on a very simplified mechanical model, namely a cantilever masonry beam, subjected to both vertical loads simulating the dead loads and static horizontal forces simulating the earthquake effects. Each masonry tower is then subdivided in portions with uniform geometrical and constructive characteristics at different heights, taking into account peculiar structural features, including interaction with surrounding buildings, tapered sections and presence of significant openings.

In this approach, the resultant of the lateral forces F_h , representative of the seismic demand, is expressed as follows:

$$F_h = 0.85 \frac{S_e(T_1)W}{qg} \quad (2.22)$$

where $S_e(T_1)$ is the ordinate of the elastic response spectrum to be computed as a function of the fundamental vibration period (or natural frequency) of the tower, W is the total weight of the tower, q is the behaviour factor and g denotes the gravitational acceleration.

This force, corresponding to the base shear, is then applied statically in the form of horizontal concentrated forces F_i , whose distribution grows linearly along the height of the tower, and therefore:

$$F_i = \frac{w_i z_i}{\sum_{k=1}^n w_k z_k} F_h \quad (2.23)$$

where w_i and w_k are the weights of the portion i and k respectively, z_i and z_k are the heights of the centre of mass of portions i and k with respect to the foundation.

Subsequently, the resultant of the seismic forces F_{hi} acting on the i -th sector are determined as follows:

$$F_{hi} = \frac{\sum_{k=1}^n w_k z_k}{\sum_{k=1}^n w_k z_k} F_h \quad (2.24)$$

The height z_{Fi} to which F_{hi} is applied can be computed as:

$$z_{Fi} = \frac{\sum_{k=i}^n w_k z_k^2}{\sum_{k=i}^n w_k z_k} - z_{Fi*} \quad (2.25)$$

where z_{hi*} denotes the height of the verification portion with respect to the tower base.

The behaviour of the masonry material is assumed to have no tensile resistance with a non-linear distribution of the compressive stresses. Under this assumption, structural failure is expected in a generic cross-section of the tower due to crushing of a compressed masonry region. The ultimate resisting bending moment M_{Rd} at the base of each portion is:

$$M_{Rd,i} = \frac{\sigma_i A_i}{2} \left(b_i - \frac{\sigma_i A_i}{0.85 a_i f_d} \right) \quad (2.26)$$

where f_d is the compressive strength, a_i and b_i are the transversal and longitudinal dimensions of the i -th portion with respect to the direction of the horizontal forces, A_i is the area of the cross-section of the i -th portion, $\sigma_i = W_i/A_i$ is the average compressive stresses of the i -th portion due to the gravity loads, and W_i is the weight of the portion of the tower above the analysed cross-section. To account for all the uncertainties included in the modelling stage, the ultimate resisting bending moments are corrected through a confidence factor F_c that varies according to the level of knowledge of the structure.

A first preliminary evaluation to determine the safety level of the towers via simplified mechanical models can be performed by comparing the acting bending moment (demand) with the ultimate resisting bending moment (capacity), both computed along the tower height. The safety is ensured when the structural capacity is greater than the demand. The demand at each portion base is:

$$M_{Ed,i} = F_{hi} z_{Fi} \quad (2.27)$$

However, for comparative assessment purposes, the safety level of the towers is more commonly expressed in terms of safety index, $I_{S,LS}$ and acceleration factor, $f_{a,LS}$.

The safety index, $I_{S,LS}$, is the ratio between the return period T_{ULS} of the seismic action that brings the structure to the considered Limit State, commonly the Life Safety Ultimate Limit State (ULS), and the expected return period of the earthquake of the site, corresponding to the same Life Safety ULS, $T_{R,ULS}$, as follows:

$$I_{S,ULS} = \frac{T_{ULS}}{T_{R,ULS}} \quad T_{R,ULS} = 475 \text{ year} \quad (2.28)$$

The return period T_{ULS} of the seismic action that brings the structure to the ULS can be computed as the return period that determines a spectral ordinate such that the weakest cross-section of the structure would meet the ULS threshold. By setting the equivalence between the acting bending moment and the ultimate resisting bending moment, the spectral ordinate for the i -th portion can be computed as follows:

$$S_{d,ULS,i}(T_1) = \frac{qgM_{Rdi} \sum_{k=1}^n z_k W_k}{0.85W (\sum_{k=1}^n z_k^2 W_k - z_i^* \sum_{k=i}^n z_k W_k) F_C} \quad (2.29)$$

Once the minimum SA is identified among the portions considered for the tower, the return period is computed according to an iterative procedure resorting to a linear interpolation. This interpolation relies on data reported in the Appendix of the Italian building code, where values needed for the calculation of the spectrum are reported for each point of the topographic network and for increasing return period, from 30 to 2475 years.

On the other hand, the acceleration factor, $f_{a,ULS}$, is defined as the ratio between the horizontal maximum ground acceleration that brings the structure to the ULS, a_{ULS} , and the reference ground acceleration associated with the ULS, $a_{g,ULS}$, as follows::

$$f_{a,ULS} = \frac{a_{ULS}}{a_{g,ULS}} \quad (2.30)$$

While the estimation of $a_{g,ULS}$ is straightforward, a_{ULS} being function of the response spectrum ordinate, can be obtained by inverting the elastic response spectrum formulas, as follows:

$$a_{ULS} = \begin{cases} \frac{S_{d,ULS}}{SF_0} & T_B \leq T_1 \leq T_C \\ \frac{S_{d,ULS}}{SF_0} \frac{T_1}{T_C} & T_C \leq T_1 \leq T_D \end{cases} \quad S_{d,ULS} = \frac{S_{e,ULS}}{q} \quad (2.31)$$

A general framework for the development of simplified mechanical models is illustrated in Figure 2.14. Several applications of the LV1 approach to the seismic vulnerability assessment of masonry towers have been discussed in the literature, often to compare its effectiveness with methods presenting higher level of accuracy and to test and validate the strategies admitted by the Italian standard (Bartoli, Betti, &

Monchetti, 2017; Degli Abbati et al., 2024; Faccio et al., 2011a; Tanganelli et al., 2024; Torelli et al., 2020). A first illustrative application was given by (Faccio et al., 2011b) who assessed the seismic vulnerability of the bell tower of the St. Antonin church in Venice through the safety index and acceleration factor associated with ULS. Different assumptions in the boundary conditions to investigate the effect of adjacent buildings have been made when computing the safety value.

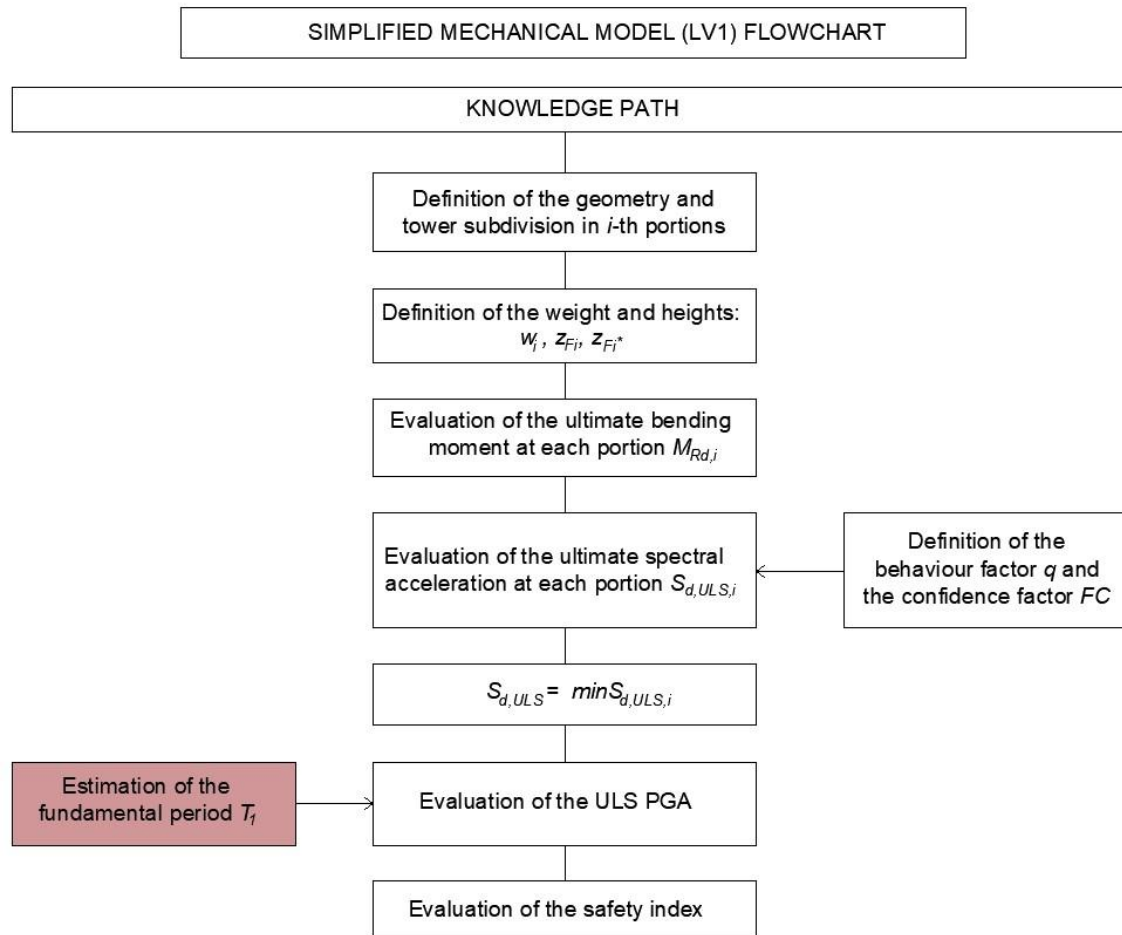


Figure 2.14: LV1 approach suggested by the Italian Guidelines for slender masonry structures.

Another noteworthy application was conducted within the scope of the research project RISEM, Seismic Risk of Monumental Buildings, (Bartoli, Betti, & Monchetti, 2017), that aimed at evaluating the seismic safety indexes and acceleration factors of four monumental masonry towers in San Gimignano. The evaluation included the influence of the adjacent buildings by considering different heights of the towers and by estimating the fundamental period of the towers using distinct simplified equation. In addressing the seismic vulnerability of the Cugnanesi tower, also located in San Gimignano, (Torelli et al., 2020) evaluated the safety index by examining the effect of the uncertainties in the visual identification of the masonry typology. Following the approach outlined in the Italian guidelines, this led to variations in the

assumed masonry material properties, resulting in significant variability in the results. A more recent work, carried out by (Tanganelli et al., 2024) evaluated the safety indexes of the Giotto Bell tower in Florence using different simplified equations for the estimation of the first natural frequency and assessing their influence on the definition of the seismic demand. Finally, (Degli Abbati et al., 2024) adopted the LV1 approach for the global assessment of the bell tower of Saint Lawrence Cathedral relying on the experimentally identified natural frequencies through ambient vibration testing, and analysing the impact of different boundary conditions. Although the method has been extensively tested on Italian case studies, where it is recommended by the national standard, some applications to heritage assets in other countries can also be found in the literature, as in (Magalhães et al., 2012).

Even though the LV1 approach in the literature mainly addresses the assessment of single towers, a few studies have discussed the applications of this simplified modelling strategy at the territorial scale (Chisari et al., 2022; Curti et al., 2008).

31 bell towers struck by the 1976 Friuli Earthquake were investigated through this method by (Curti et al., 2008). In particular, the acceleration factor was evaluated for each tower and the results were compared with the damage levels observed in the post-earthquake surveys. The comparison showed a clear discrepancy as towers with greater observed damage levels were characterised, by contrast, by higher values of safety indexes, highlighting the limitation of the simplified mechanical approach. Very recently, (Chisari et al., 2022) proposed a novel simplified vulnerability model suitable for territorial scale assessment of historic masonry towers. In particular, the development of this method required the construction of a parametric mechanical model, based on an enhanced version of the LV1 approach, to investigate the tower collapse for a wide number and range of parameters affecting it, in order to assess their influence and provide a reduced set of the most significant ones. The definition of the parameters and their range followed a detailed survey of 56 historic masonry towers in the area of the city of Naples, and the final method was tested against 31 towers found in the literature.

The analysis of the existing studies shows that the seismic safety evaluation using LV1 benefits from employing multiple analytical models and sensitivity analysis, providing a more detailed understanding of the effects of uncertainties and varying assumptions on the estimated structural performance. This is often focused on the boundary conditions (Bartoli, Betti, & Monchetti, 2017; Faccio et al., 2011a), the first natural frequency (Faccio et al., 2011a; Tanganelli et al., 2024) or the masonry mechanical properties (Torelli et al., 2020), especially when their direct evaluation based on experimental tests is not available.

At the same time, higher levels of assessment (LV2 and LV3) are generally exploited for comparative purposes, validating the obtained results. These comparisons have often shown that the LV1 method, due to its simplified assumptions, may overlook the effect of specific failure mechanisms, that are less related to the bending of the structure, potentially neglecting the occurrence of collapse caused by them.

To address this, enhanced versions of the LV1 method have been proposed, aiming at a balance between the computational efficiency of the approach and improved accuracy of the results. In particular, as previously mentioned, (Chisari et al., 2022) modified the LV1 approach by not only considering the hypothesis of a cantilever beam with bending and compression failure, but also accounting for the shear failure of the tower. The formula used for the shear strength of the portion was based on the one originally provided by (Turnšek & Čačovič, 1971):

$$V_{Rd,i} = 1.5\tau_{0d} \sqrt{1 + \frac{\sigma_{0,i}}{1.5\tau_{0d}} \sum_k \frac{l_{ik}t_{ik}}{b_{ik}}} \quad (2.32)$$

where $\sigma_{0,i} = \frac{N_i}{A_i}$ with N_i vertical load and A_i the area of the cross-section at each portion i , τ_{0d} is the shear strength of the masonry, l_{ik} and t_{ik} are the length of the masonry panel k at each portion i and its thickness, respectively, and finally $b_{ik} = \frac{h_{ik}}{t_{ik}}$ ranges between 1 and 1.5. The ultimate shear was then used to compute the corresponding ultimate bending moment for conducting the safety evaluation.

CHAPTER 3

VALIDATION AND IMPROVEMENT OF EXISTING VULNERABILITY MODELS THROUGH AN OPEN DATASET

ABSTRACT

This Chapter focuses on existing vulnerability functions for masonry towers, especially bell towers, empirically developed based on post-disaster surveys. In particular, the validation and improvement of the existing models is addressed by relying on a database of historic masonry bell towers struck by past earthquakes, which is the result of an extensive literature review on post-earthquake reconnaissance reports and scientific publications. The observed damage, locally classified into the damage mechanisms of the two main components of the tower (i.e., shaft and belfry) and globally computed for the tower macroelement as a function of a damage index, was analysed to give insight into the seismic behaviour and vulnerability of such peculiar structural typology. This enabled the development of damage probability matrices, providing the frequency of damage occurrence. Existing vulnerability functions were tested against the dataset and a recalibration was suggested when a poor performance emerged. Finally, fragility functions were derived to help the prediction of possible future damage scenarios. Due to the sources of uncertainties that affect vulnerability and fragility functions, a strategy based on random sampling was implemented to ensure their robustness.

3.1 INTRODUCTION

The present Chapter addresses the first activities carried out within the thesis work in the field of territorial scale seismic assessment of masonry towers and slender structures (see Figure 3.1).

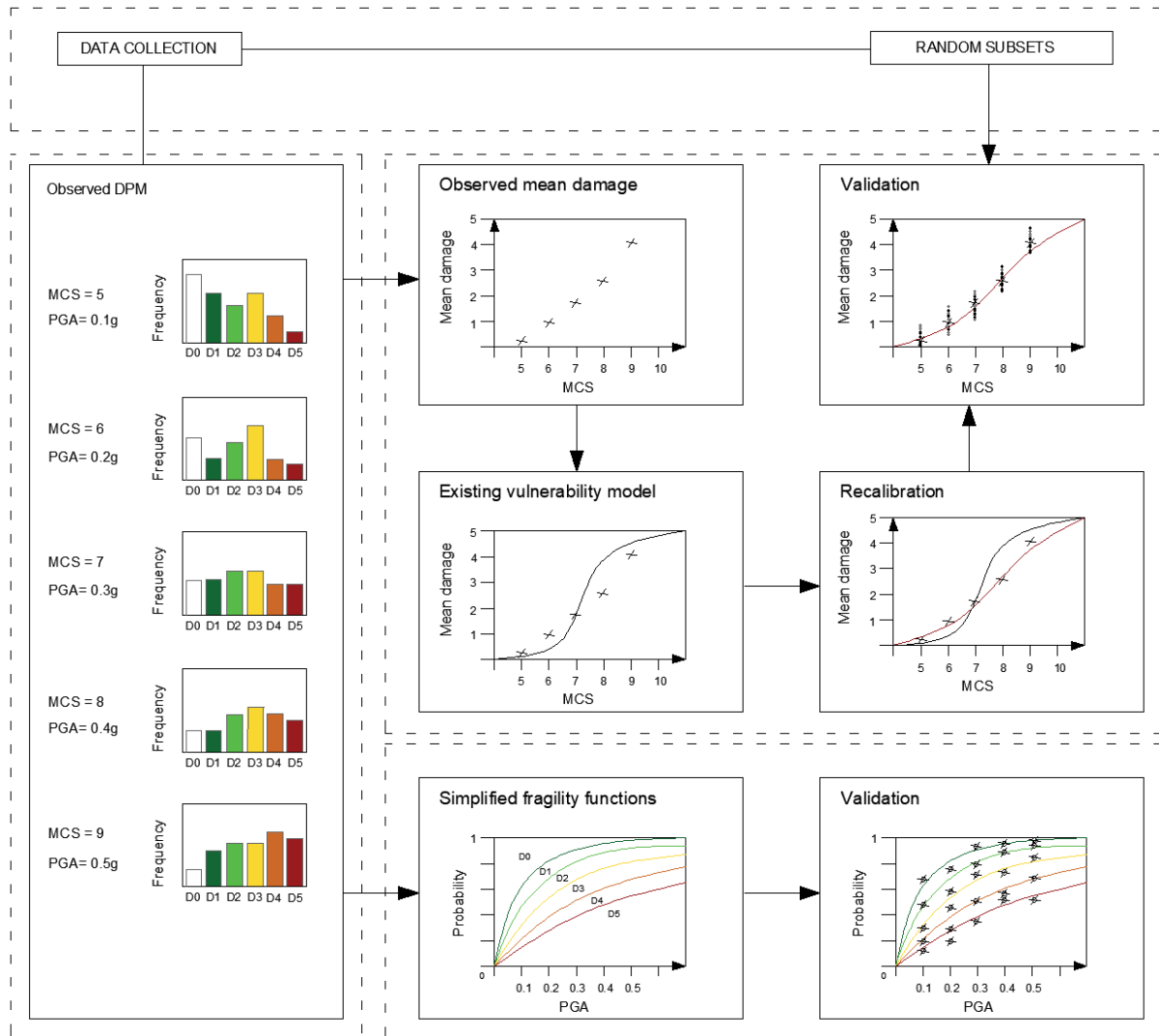


Figure 3.1: Flowchart methodology adopted in this Chapter.

In the first stage, the lack of reliable and open access large collections of data on real masonry towers struck by past earthquakes has been addressed by conducting an extensive literature review of post-earthquake surveys of historic masonry bell towers in Italy. The main aspects of this documentary search and the collected database with its statistical analysis is described in Section 3.2, and is exploited in Section 3.3, to provide the global vulnerability of the investigated towers through DPMs for different damage mechanisms, levels of intensity measure, seismic events and homogenous clusters of towers. A comparison with results of similar studies is also included. Subsequently, vulnerability curves are obtained, in Section 3.4, first testing and validating the existing formulations suggested in the literature for historic towers (Curti et al., 2008; Lagomarsino et al., 2004), and then recalibrating their

parameters in case of unacceptable performance against the specific dataset. To this end, a regression analysis is performed by fitting the samples through a function with the same form of the existing equation, and new coefficients are proposed for the specific dataset in hand. In Section 3.5, fragility curves are developed by correlating the cumulative probability function of damage exceedance with the PGA; while the former is obtained by applying the Binomial Probability Density Function (BPDF), the latter is retrieved using an empirical correlation between PGA and macroseismic intensity according to the MCS scale ($PGA-I_{MCS}$) proposed by (Faenza & Michelini, 2010).

3.2 OPEN DATABASE OF MASONRY TOWERS STRUCK BY EARTHQUAKES

The database encompasses 129 historic masonry bell towers, hit by six moderate to strong earthquakes for which post events reconnaissance reports and scientific publications have been found. The extensive literature review of technical and scientific publications ensures a reliable identification of the occurred damage mechanisms and extents. In particular, the classification of the collapse mechanisms follows the macroelement approach presented by the Italian Guidelines for Cultural Heritage (DPCM, 2011), described in Chapter 2. The search focused on Italian events due to the combination of a large exposed heritage and the significant seismic hazard. This has allowed the identification of a relevant number of towers analysed and reported. The database, openly available as an Appendix to the present thesis (see Appendix A), gathers and expands similar existing collections that have mainly focused on a single seismic event, allowing to consider different earthquake scenarios at the same time, thus, to cover a wider range of intensity measures. This open database is intended to provide diverse and numerous samples for future studies but also ensures the comparison between different characteristics of the seismic input and the affected towers.

Table 3.1 summarises the characteristics of the earthquakes that affected the investigated towers, namely regions that suffered the most severe consequences, date and time, epicentral coordinates, hypocentral depth and moment magnitude, obtained from the Italian Earthquake Catalogue (Rovida et al., 2020). Figure 3.2 shows the map of the epicentres. The selected earthquakes comprise the most severe events since 1997 Umbria-Marche earthquake, namely, the 2002 Molise, the 2009 L'Aquila, the 2012 Emilia and the 2016 Central Italy earthquakes, plus the previous 1976 Friuli earthquake. The latter was a key event for the development of the macroelement approach in Italy, as well as the territorial scale investigation of churches vulnerability. For these earthquakes, data about structural damage observed in historic masonry towers is largely available, allowing a collection of a sufficient number of cases for the open dataset. The MCS intensity scale (I_{MCS}) is here adopted to characterise the seismic events at the specific location of each analysed tower.

Table 3.1: Earthquake events considered in the present study.

No.	Earthquake Event	Date	Origin Time (UTC)	Epicentre		Hypocentre	M_w
				Lat.	Long.	Depth (km)	
1	Friuli	6 May 1976	20:00	46.241	13.119	5.7	6.5
2	Umbria-Marche	26 Sep 1997	09:40	43.014	12.853	9.8	6.0
3	Molise	31 Oct 2002	10:32	41.716	14.893	25.1	5.7
4	L'Aquila	6 Apr 2009	01:32	42.309	13.510	8.3	6.3
5	Emilia	29 May 2012	07:00	44.841	11.065	8.1	5.9
6	Central Italy	24 Aug 2016	01:36	42.698	13.233	8.1	6.2



Figure 3.2: Distribution of the analysed earthquakes events.

Table 3.2 provides a summary of the maximum and the range of intensity registered for each earthquake together with the corresponding source according to the Italian Macroseismic Database (DBMI15) (Locati et al., 2022). The Molise and Emilia earthquakes provides an insight into the behaviour of the bell towers for a moderate macroseismic intensity and magnitude, while the Friuli, the Aquila and the Central Italy earthquakes are essential to investigate the bell towers behaviour for stronger intensity and magnitudes. It is worth noting that despite the maximum magnitude of the Central Italy earthquake being lower than the same in other events reported in the database, the seismic sequence generated in 2016 featured various strong shocks localised in the same area,

contributing to an intensification and accumulation of the damage. Due to the emphasis on this event in the second part of the study, a more detailed characterisation of the hazard is provided in Chapter 4.

Table 3.2: Maximum MCS intensity associated to each earthquake event and range of intensity at the location of the investigated towers.

No.	Earthquake Event	I_{MCS}^{max}	Source
1	Friuli (1976)	9-10	(Archivio Macrosismico, 1995)
2	Umbria-Marche (1997)	9	(Boschi et al., 2000)
3	Molise (2002)	8-9	(Bosi et al., 2002)
4	L'Aquila (2009)	9-10	(Galli & Camassi, 2009)
5	Emilia (2012)	7-8	(Galli et al., 2012)
6	Central Italy (2016)	10-11	(Galli et al., 2016)

A total of 129 historic masonry towers, affected by the aforementioned events, was collected, grouping the towers by the corresponding earthquake. The groups of the analysed towers are distributed over the Italian territory in Figure 3.3.

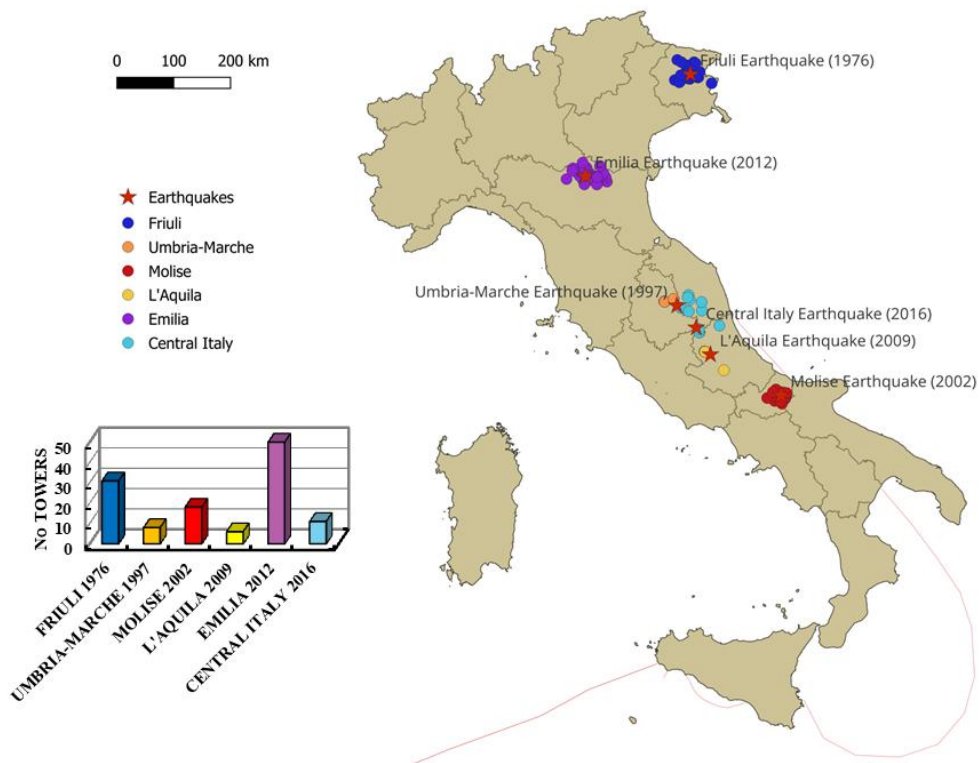


Figure 3.3: Geographical locations of towers across the Italian territory, grouped by earthquake event.

In particular, 31 towers were struck by the 1976 Friuli seismic event, 8 towers by the 1997 Umbria-Marche, 18 towers by the 2002 Molise, 6 towers by the 2009 L'Aquila, 55 towers by the 2012 Emilia, and the remaining 11 towers by the 2016 Central Italy sequence. The distribution of the analysed towers considering the different MCS intensities recorded in the location is shown in Figure 3.4 upon a

classification in five distinct groups for increasing levels of MCS intensity: Group I includes 40 towers with $5 \leq I_{MCS} < 6$, Group II comprises 29 towers with $6 \leq I_{MCS} < 7$, Group III is composed of 25 towers characterised by $7 \leq I_{MCS} < 8$, Group IV includes 22 towers with $8 \leq I_{MCS} < 9$, and finally Group V consists of 13 towers with I_{MCS} equal to or greater than 9. It should be noted that whenever the MCS intensity associated with the tower location was not available in the sources listed in Table 3.2, the intensity associated with the Municipality where the tower is located was assigned.

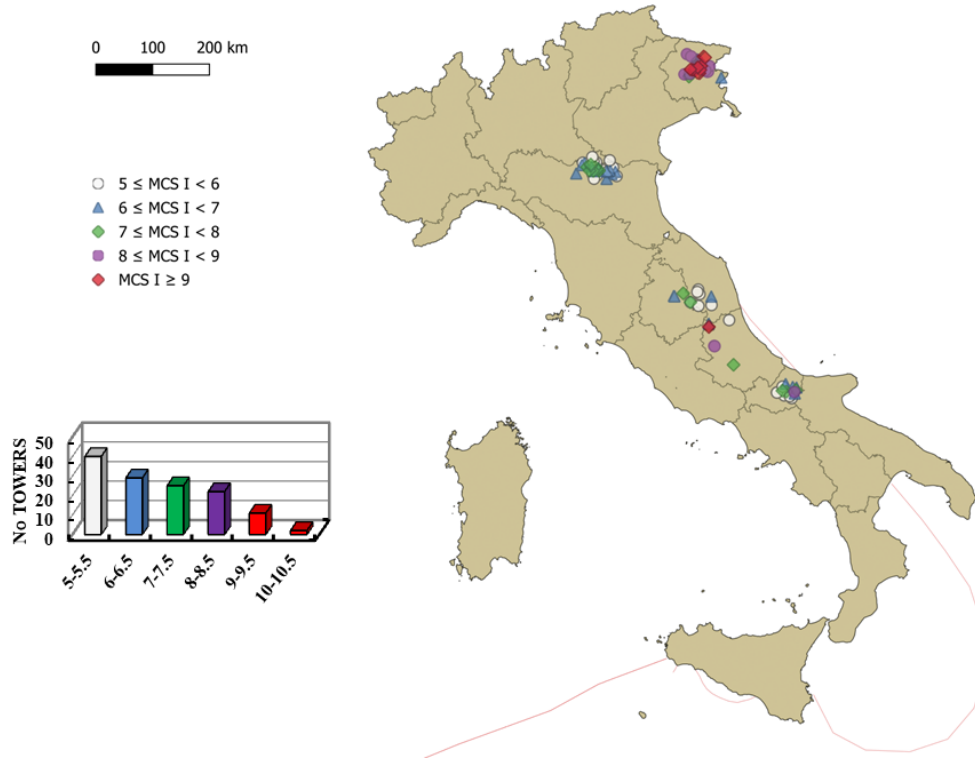


Figure 3.4: Map highlighting the tower distribution grouped by MCS intensity.

To give more insight into the seismic behaviour of the historic masonry towers included in the database, reducing the scatter in their response and damage level, the investigated structures were classified according to the tower typology. In this regard, as mentioned in Chapter 2, the towers can be generally grouped into three categories according to their plan configuration (or plan layout): isolated towers, bounded towers, and gable bell towers. The bounded towers can be additionally classified into subgroups according to their location in the aggregate (i.e., confined and integrated) as in (Ceroni et al., 2022; Sepe et al., 2008). Still, due to the limited number of towers and information available for their characterisation, an excessive fragmentation of the dataset is avoided to ensure that all subsets are statistically representative. Therefore, the towers were grouped in two main classes only: isolated and bounded. This typological classification was conducted based on the gathered documentation and using the (Google Earth Pro, 2022) software in case of missing information in the investigated studies. It

should be noted that the bell gable typology was not considered, given the focus of the thesis on historic towers and slender structures.

The damage mechanisms and extents for each historic masonry tower included in the present database were collected from the results of post-earthquake survey activities conducted by field engineers and researchers focusing on masonry churches using the macroelement approach, according to the Italian Guidelines for Cultural Heritage (DPCM, 2011). Macroelements can be activated during the ground shaking leading to different failure mechanisms. Therefore, different classifications of macroelements have been proposed and included into survey forms over time. Concerning the tower as a single macroelement, a division into mechanisms affecting its main components, namely shaft (or main body) and belfry, as shown in Figure 3.5, was introduced and systematically adopted, simplifying the collection of information from the sources of the database.

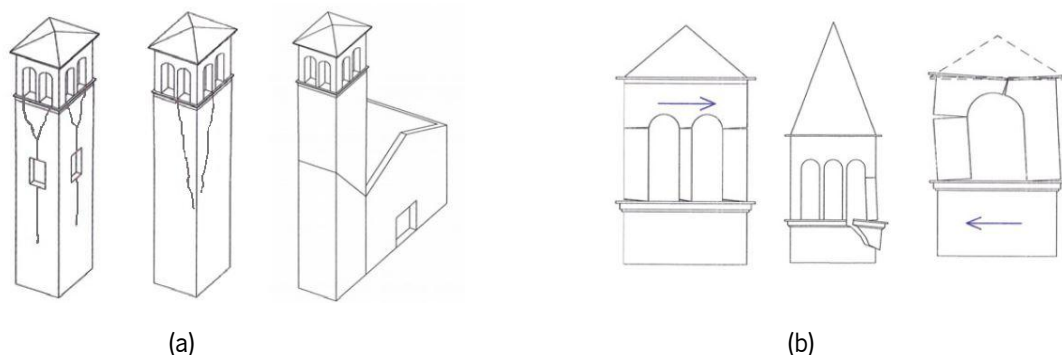


Figure 3.5: Failure mechanisms associated with the tower macroelement: (a) tower mechanism (Mechanism A) and (b) belfry mechanism (Mechanism B).

The two distinct failure mechanisms in Figure 3.5, highlighted as Mec27 and Mec28 in the last version of the A-DC survey form for the seismic damage to churches, provided by the Italian Civil Protection (DPCM, 2011), are in this work renamed Mechanism A and Mechanism B, respectively. A mechanism of non-structural emerging elements (listed as Mec26 of the same form) records damage to the spires, pinnacles, crenulations and other decorations of the tower or its belfry. This mechanism is not included in the data collection as the information available does not allow to clearly distinguish when the surveyors identified this damage in the tower and when it affected other emerging elements of the church. This is a well-known intrinsic limitation of the A-DC survey form that could not be addressed by the present work. Additionally, it is worth noting that, although historic towers are apparently regular structures typically with a heavy weight that may be beneficial for structural stability, experience shows that, when subjected to ground shaking, they exhibit damage patterns that can vary widely due to specific structural features (e.g., the slenderness, the presence of openings, the configuration of the belfries, the interaction with adjacent buildings and soil).

The aforementioned Mechanisms A and B, indeed, collect a series of typical failure mechanisms that occurred in historic masonry towers (Figure 3.6). as extensively discussed in (Doglioni et al., 1994).

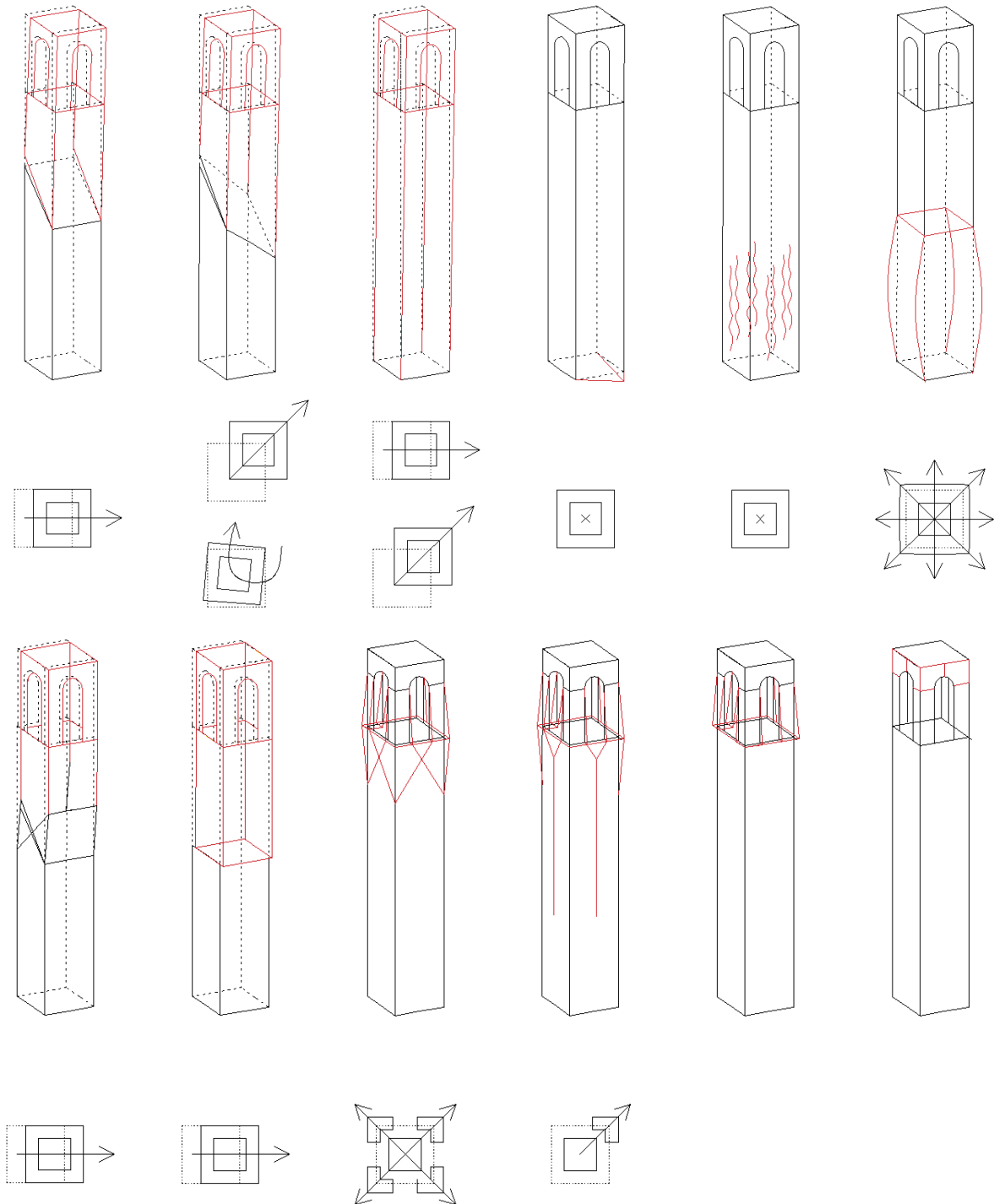


Figure 3.6: Typical damage mechanisms observed in historic masonry towers.

In particular, several combinations of damage patterns can affect the main body of the tower. In-plane shear cracking may lead to a rocking mechanism, thus affecting the entire superstructure. The crack patterns and the subsequent formation of a hinge depend on the orientation of the shear planes, as

they can form a horizontal axis of rotation along one side, converge towards a corner or fragment the panels upon the formation of double-diagonal (X-type) cracks. Very rare is the case of failure due to sliding. Nonetheless, some examples of towers that suffered this type of damage exist, showing remarkable crack patterns but without causing total failure. Finally, the shaft may fail due to crushing. Cracking in the belfry is likely the most common damage observed in bell towers. In most cases, the damage is due to rocking of the upper part because of the formation of horizontal cracks in the masonry pillars of the belfry, often involving some detachment of masonry corners in the level below. In some cases, the tower can undergo a shear failure right below the belfry combining the two mechanisms. Other possible combinations of mechanisms depend on the distribution of the openings. Indeed, vertical cracks may occur in the presence of aligned openings and lead to splitting of the tower. The identification of the two mechanisms (i.e., Mechanism A and Mechanism B) for masonry bell towers dates back to the seminal studies on the macroelement behaviour of masonry churches, in the aftermath of the 1976 Friuli Earthquake (Doglioni et al., 1994). The masonry towers affected by such a severe event and included in the database have been investigated by (Curti et al., 2008), according to this distinction in the tower and the belfry mechanisms. For the 1997 Umbria-Marche Earthquake and the 2002 Molise Earthquake, information about the damage to the towers derived from technical reports, (Spence et al., 1998) and (Cifani et al., 2005), respectively. For the following events included in the database, the damage records are extracted from scientific publications. In particular, for the 2009 L'Aquila Earthquake, the main sources were (Augenti & Parisi, 2010; Brandonisio et al., 2013; Criber et al., 2015). For the towers affected by the 2012 Emilia Earthquake, relevant information is provided by (Ferrari, 2020; Paupério et al., 2012; Sorrentino et al., 2014; Valente et al., 2017). Finally, the damage observed in the towers after the 2016 Central Italy Earthquake was obtained from the works of (Acito et al., 2021; Clementi et al., 2020; De Matteis & Zizi, 2019; Giordano et al., 2019; Jain et al., 2020).

Unlike the classification of the mechanisms recurring among different sources, regarding the extent of the damage, documents referring to different earthquakes typically adopt different discrete scales, until the diffusion and adoption of the EMS-98 six-level scale (Grünthal, 1998), with the damage ranging from 0 up to 5. Therefore, an essential task conducted during the formation of the present dataset consisted in the reclassification of the damage level, based on the grade assigned in the original sources and/or other available information, such as descriptions in the texts, photos or drawings.

For around 65% of the identified case studies, the aforementioned sources provided the damage levels, e.g., (Criber et al., 2015; Curti et al., 2008; Ferrari, 2020; Sorrentino et al., 2014), while in the remaining 35% the damage level was assigned based on the visual inspection of the gathered

photographic documentation, e.g., (Augenti & Parisi, 2010; Brandonisio et al., 2013; Cifani et al., 2005), according to the EMS-98 (Grünthal, 1998). These cases are highlighted in the database with the letter *c* (Appendix A).

Finally, it should be noted that for a few towers included in the database no damage level was reported associated with one or both failure mechanism(s) (Appendix A). These cases for which no clear information was obtained from the sources are properly highlighted in the database table, since it was not possible to confirm the presence of the specific portion of macroelement or the activation of the mechanism and, in case, its severity. These towers belong to the 1976 Friuli and 2012 Emilia earthquakes affecting the entire range of MCS intensities, specifically between 5 and 9. The final number of towers is reported in Table 3.3, including the subgroups associated with each earthquake, each MCS intensity group and each failure mechanism (Mechanism A, Mechanism B).

Table 3.3: Number of towers included in the database to which a damage score has been assigned, grouped by earthquake event, damage mechanism and macroseismic intensity.

Earthquake	Damage Mechanism	Macro-seismic Intensity MCS						Total
		5.0/5.5	6.0/6.5	7.0/7.5	8.0/8.5	9.0/9.5	10.0/10.5	
Friuli (1976)	Mechanism A	0	0	2	12	8	0	22
	Mechanism B	0	0	2	13	8	0	23
Umbria-Marche (1997)	Mechanism A	0	4	4	0	0	0	8
	Mechanism B	0	4	4	0	0	0	8
Molise (2002)	Mechanism A	7	5	5	1	0	0	18
	Mechanism B	7	5	5	1	0	0	18
L'Aquila (2009)	Mechanism A	0	0	1	5	0	0	6
	Mechanism B	0	0	1	5	0	0	6
Emilia (2012)	Mechanism A	23	13	9	0	0	0	45
	Mechanism B	23	15	8	0	0	0	46
Central Italy (2016)	Mechanism A	7	2	0	0	0	2	11
	Mechanism B	7	2	0	0	0	2	11
Total	Mechanism A	37	24	21	18	8	2	110
	Mechanism B	37	26	20	19	8	2	112

The distribution of the final number of towers obtained by considering their corresponding seismic events and MCS intensity is given in Figure 3.7, for the case of damage mechanism to the body of the tower (Mechanism A). The distribution of the towers affected by the damage mechanism to the belfry (Mechanism B) is very similar, thus, it is not reported for the sake of brevity. Many towers included in the database are associated with the Friuli, the Molise and the Emilia events, while a few of them are affected by the Umbria-Marche, the L'Aquila, and the Central Italy earthquakes. Additionally, many towers were affected by a MCS intensity range between 5 and 7.5. These towers refer mainly to the Umbria-Marche, the Molise, the Emilia, and the Central Italy events. In the remaining range (MCS

intensity 8-10.5), the number of towers is mainly given by the Friuli, the L'Aquila and the Central Italy earthquakes.

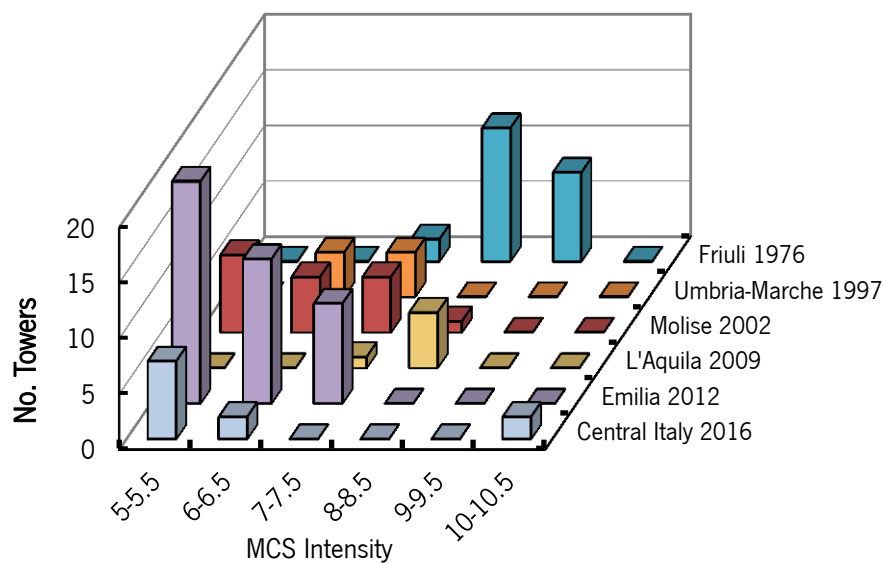


Figure 3.7: Distributions of towers affected by Mechanism A according to earthquake event and MCS intensity.

The distribution of the analysed towers distinguishing them in isolated and bounded is given in Figure 3.8, revealing that most, among the investigated building stock, are bounded towers and that the isolated towers data mainly belong to the Friuli and Emilia earthquakes; the distribution in Figure 3.8a refers to towers affected by the shaft mechanism (Mechanism A), whereas the distribution in Figure 3.8b refers to towers affected by the belfry mechanism (Mechanism B).

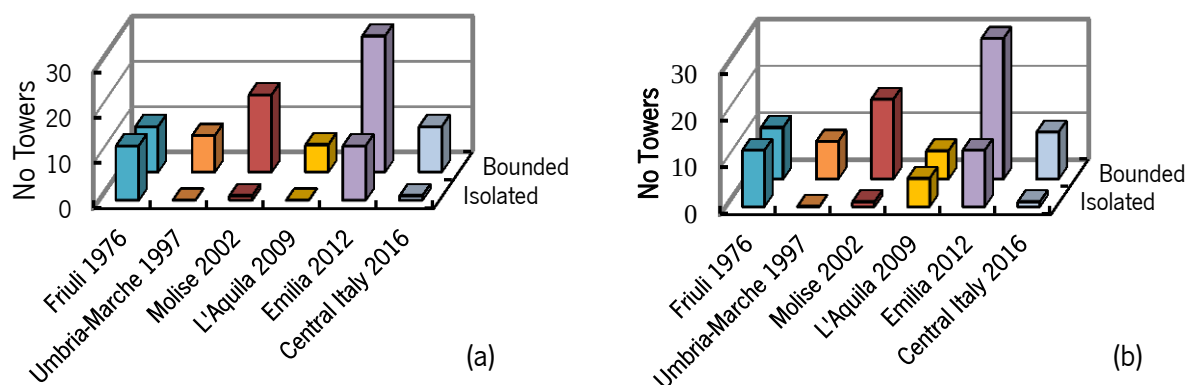


Figure 3.8: Distributions of the towers according to earthquake event and structural typology: towers affected by Mechanism A (a), and towers affected by Mechanism B (b).

The isolated and bounded towers were then sorted according to the MCS intensity. Table 3.4 outlines the total number of the isolated and bounded towers within each intensity group, together with the distinct damage mechanisms (Mechanisms A and B). As can be observed, similar distributions are

obtained for the towers affected by distinct damage mechanisms. Once again, it is worth highlighting that, due to missing information, the number of towers for which the actual damage level is known for Mechanisms A and B does not always correspond and both are less than the entire size of the dataset, as shown in the following table.

Table 3.4: Number of towers included in the database.

Plan Layout		Macro-seismic MCS Intensity						Total Known	Total in Database
		5.0/5.5	6.0/6.5	7.0/7.5	8.0/8.5	9.0/9.5	10.0/10.5		
Isolated	Mechanism A	8	4	2	4	7	1	26	32
	Mechanism B	7	5	2	4	7	1	26	32
Bounded	Mechanism A	29	20	19	14	1	1	84	97
	Mechanism B	30	21	18	15	1	1	86	97
Total	Mechanism A	37	24	21	18	8	2	110	129
	Mechanism B	37	26	20	19	8	2	112	129

After the identification of the damage levels observed in the distinct mechanisms, the global assessment of the tower as a single macroelement was carried out using the method adopted in similar studies which follows the weighted sum criterion, as mentioned in Section 2.3 and in (Testa et al., 2024). The global damage level of the macroelement, hereafter called Macroelement damage, was computed only for the towers whose damage levels (ranging from 0 to 5) are available for both damage mechanisms (i.e., 106 towers out of the total 129). When the damage level related to one of the two distinct failure mechanisms was not known (i.e., empty cell in the database), the global damage level was not computed. It is stressed that an empty cell does not indicate absence of damage, instead indicates lack of information.

3.3 DAMAGE PROBABILITY MATRICES (DPMs)

The DPMs were produced for each combination and grouping the towers discussed in the previous Section. The DPMs were also compared with the distribution of the probability of damage estimated using the Binomial Density Probability Function (BDPF) at a given level, p_k , based on the knowledge of the observed mean damage μ , as follows:

$$p_k = \frac{5!}{k! (5-k)!} \cdot \left(\frac{\mu}{5}\right)^k \cdot \left(1 - \frac{\mu}{5}\right)^{5-k} \quad k = [0, \dots, 5] \quad (3.1)$$

3.3.1 Entire dataset and individual events

The observed DPMs and the related BDPFs are reported in Figure 3.9, considering the entire set (i.e., $5 \leq I_{MCS} < 11$). Similar values emerge for the mean damage associated to the two damage

mechanisms and to the whole tower macroelement: 2.69 and 2.53 for Mechanism A and Mechanism B, respectively, and 2.77 for Macroelement damage.

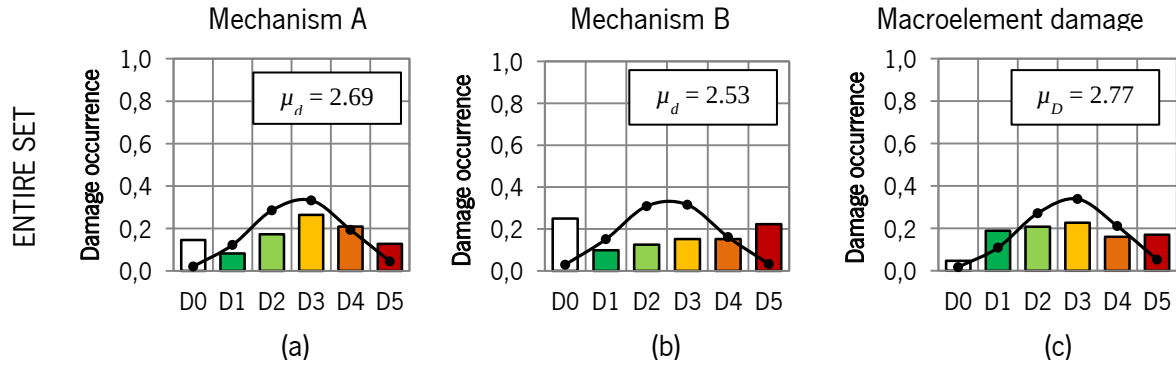


Figure 3.9: DPMs for distinct damage mechanisms (a and b), and for the entire macroelement (c).

Although similar values of the observed mean damage are obtained, the distributions of the collected levels of damage are quite different. In particular, for Mechanism A, a unimodal (i.e., a single peak) almost symmetrical distribution of the damage with a peak at level 3 emerges, while Mechanism B exhibits a growing trend in the distribution with a large number of towers presenting a damage score equal to 5. This trend suggests a higher susceptibility to severe damage of the belfry when compared with the shaft. The last case, Macroelement damage, reveals an almost uniform distribution of damage with slightly larger occurrence of D3 and slightly more cases with low damage than severe damage. The displayed damage histograms for Mechanism A and Macroelement damage have an acceptable fit by the binomial distribution, whereas the distribution of Mechanism B does not seem to match the BDPFs. The value obtained for the global mean damage is larger than the individual mechanisms mean damage. To better investigate this unexpected trend, the analysis was repeated by taking into account the same number of towers (i.e., 106) instead of the 110 and 112 towers originally considered for Mechanism A and Mechanism B, respectively. Thus, all the towers for which no information was available for one of the two mechanisms were excluded from the calculation.

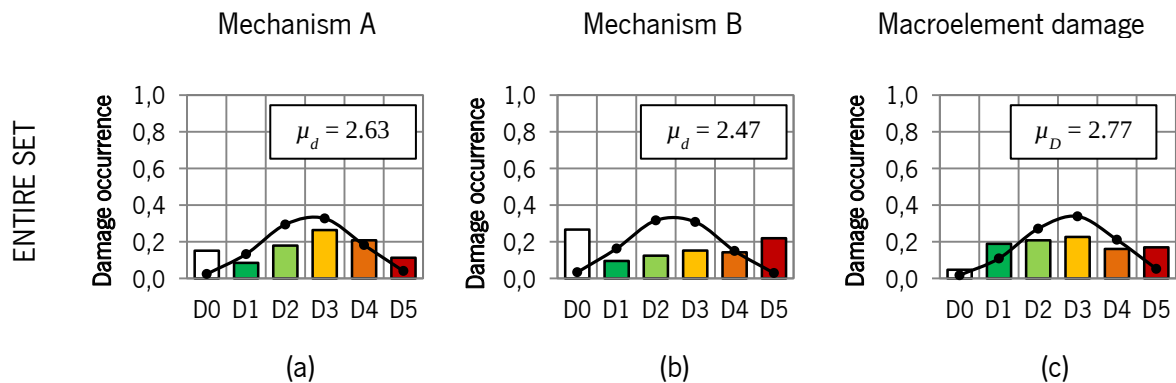


Figure 3.10: DPMs for distinct damage mechanisms (a and b), and for the entire macroelement (c) considering 106 towers only.

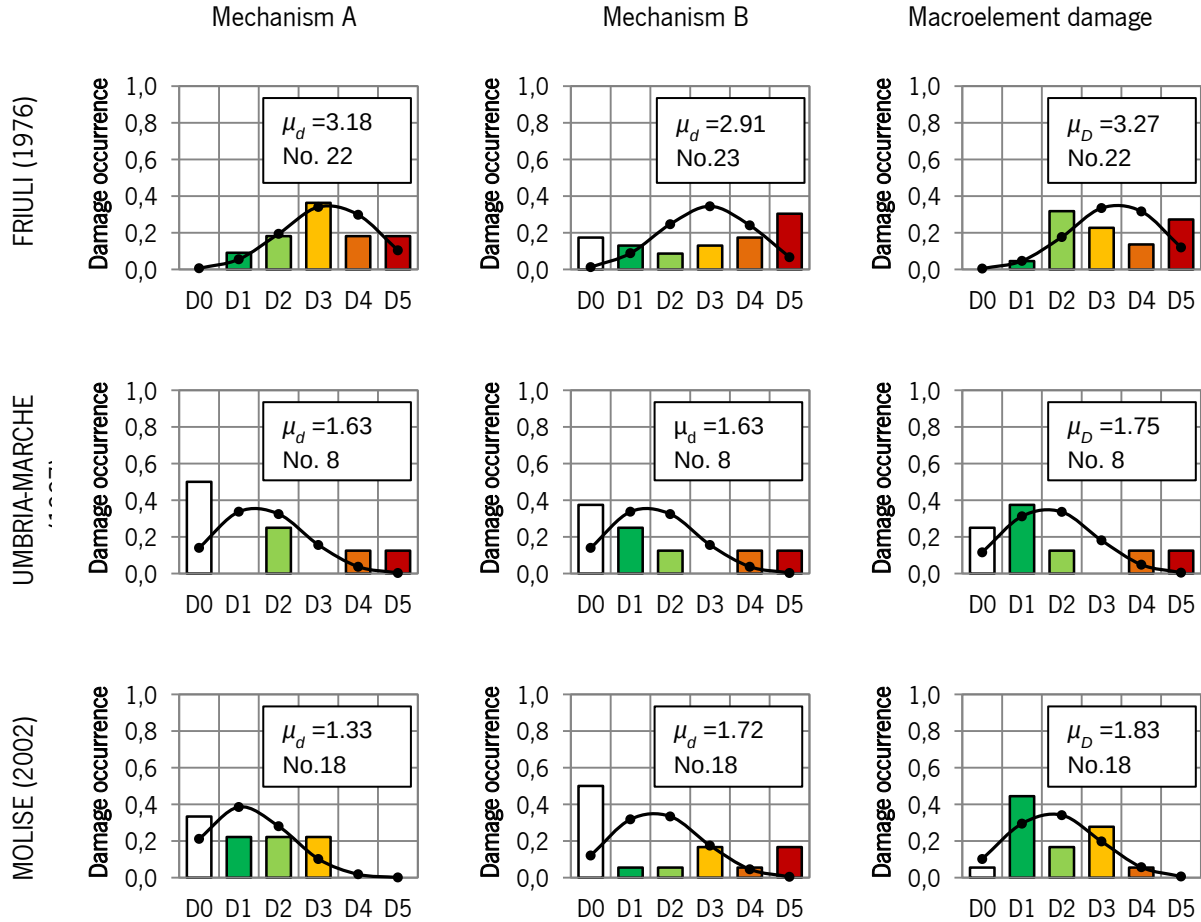


Figure 3.11: DPMs of the investigated towers developed for Friuli, Umbria-Marche and Molise events, and by considering both the damage levels observed in each damage mechanism and macroelement.

The new DPMs and the BDPFs are illustrated in Figure 3.10, which provide the same unexpected result. No significant variation in the mean damage values and corresponding damage distributions emerges and the mean global damage continues to be larger than the mean damage obtained for the single individual mechanisms. This is likely ascribed to the equation used to combine the damage levels into the global damage index (Eq. 2.3) and to the related correlation with the global damage level (Table 2.2). The overestimation of the global damage with respect to the average of the individual mechanisms' values provided by this approach, when adopted for the case of towers, is likely influenced by the weight assigned to each mechanism and by the correlation used to transform the continuous global damage index into the discrete global damage level. Alternative approaches to the weighted sum criterion have been proposed in literature, especially for reinforced concrete structures. A study that focuses on the evaluation of how the global damage index using the existing approaches affect the damage level estimates is given in (Zucconi et al., 2022). However, in the present study, both weights and correlation are maintained as in the original approach by (Lagomarsino & Podestà, 2004b,

2004c) and in similar works as in (De Matteis & Zizi, 2019); variations to this approach are left as future scopes of the research. Considering possible differences in the towers behaviour due to the peculiar characteristics of each earthquake and local constructions, additional DPMs were developed by taking into account the earthquake events individually are shown in Figure 3.11 (Friuli, Umbria-Marche and Molise) and Figure 3.12 (L'Aquila, Emilia and Central Italy).

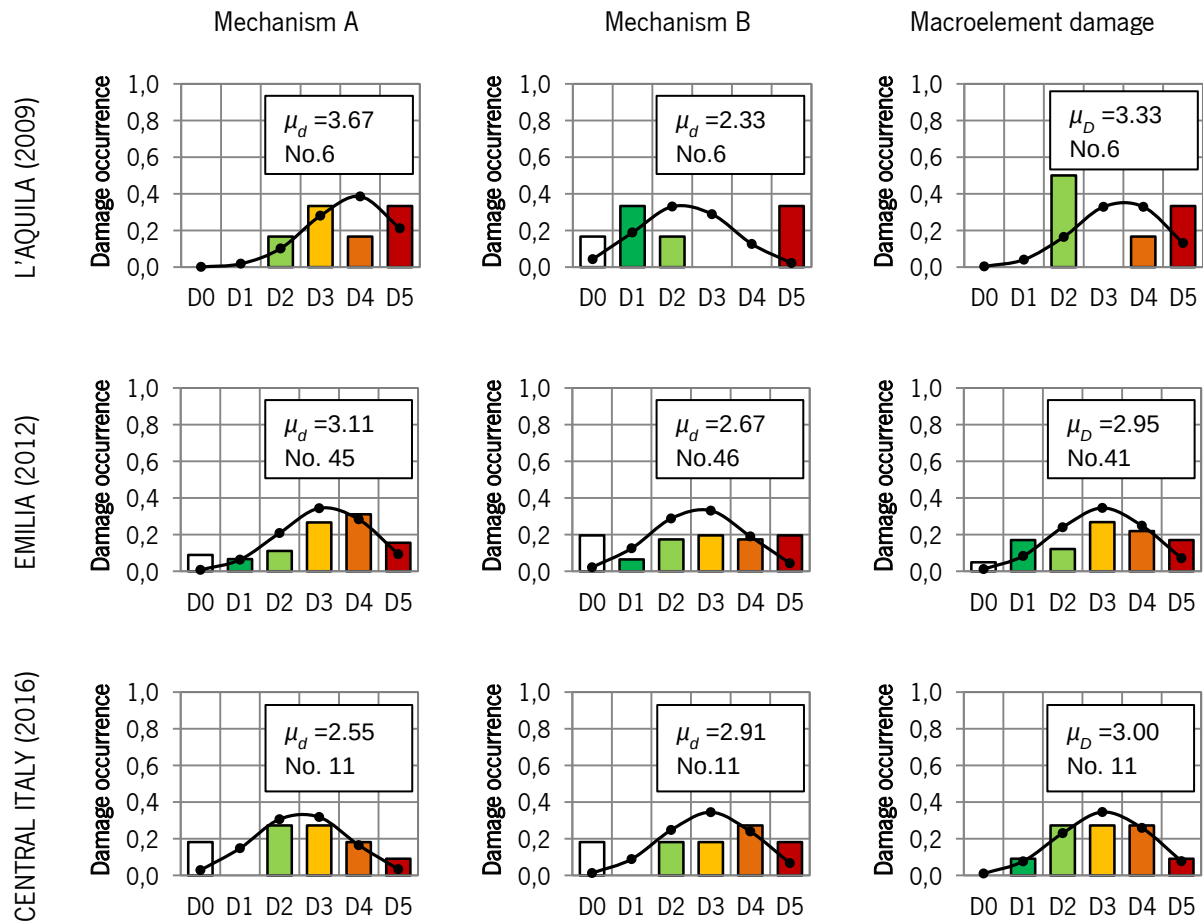


Figure 3.12: DPMs of the investigated towers developed for L'Aquila, Emilia and Central Italy events and by considering both the damage levels observed in each damage mechanism and the global damage levels.

The DPMs developed by classifying the investigated towers based on the set of earthquakes aim to perform an in-depth examination of the towers' behaviour and condition against the corresponding seismic event they experienced. The observed damage, for the cases of the Umbria-Marche and the Molise earthquakes, is mostly characterised by occurrences of no damage or small damage extents. For the former event, the limited number of known towers in the database may affect the evaluation, although the histogram shows a clear trend. For the Central Italy event, the observed damage is likely normally distributed, especially for Mechanism A and Macroelement damage. For the remaining cases (i.e., the Emilia, the Friuli and L'Aquila earthquakes), the observed damage is left-skewed distributed,

showing an increasing occurrence of larger damage extents. This trend is less pronounced for the towers struck by the Emilia earthquake and more evident in the Friuli and L'Aquila earthquakes. These results are similar when comparing the two damage mechanisms, except for the L'Aquila earthquake, which are, however, likely biased by the reduced size of the sample. For this case, the damage distribution associated with the belfry mechanism is shifted leftward although the presence of towers with a high damage extent is highlighted. The final trends associated with the global damage occurrence do not differ from the separated trends revealing just a few fluctuations. Nonetheless, dissimilar values of the mean damage computed for the distinct damage mechanisms and the global mean damage are obtained for the considered cases even when a constant number of samples for Mechanism A and Mechanism B is available (e.g., the Umbria-Marche earthquake). Furthermore, the observed damage distribution is in agreement with the damage predicted using the binomial distribution when the number of investigated towers becomes significant (e.g., the Friuli and the Emilia earthquake). For the remaining sets (e.g., L'Aquila earthquake), as expected, the binomial probability performed poorly, likely due to the limited number of towers available that makes these sets less representative of the tower population for the affected regions. The differences in the behaviour of the towers in each seismic event may depend on the peculiar characteristics of the seismic actions, the soil conditions of the affected region, the intrinsic structural and architectural features of the towers due to local traditions and material available (e.g., tower height, masonry patterns and quality, natural or artificial masonry units, characteristics of the mortar, tower interactions with surrounding buildings, etc.) and the state of maintenance. However, the representativeness of the samples and other sources of uncertainties in the data collection may play a key role in some biasing of the results.

In order to investigate the uncertainties due to the data collection process, the representativeness of the sources and the reliability of the results obtained for the distinct earthquakes, a comparison with the DPMs and the global mean damage in historic masonry tower macroelements obtained from similar studies in the literature was conducted and the results are given in Table 3.5 and Table 3.6. Four values of the global mean damage were found in (Cianchino et al., 2021; De Matteis et al., 2016; De Matteis & Zizi, 2019; Sorrentino et al., 2014), each addressing one-single seismic event, namely the 2009 L'Aquila earthquake, the 2012 Emilia earthquake or the 2016 Central Italy earthquake. A large difference is obtained for the L'Aquila earthquake and the Central Italy earthquake, highlighting the need to expand these datasets including a larger number of towers. Indeed, the higher mean damage obtained in the present database may be caused by a systematic underestimation of the number of slightly or non-damaged towers. This bias is commonly present in empirical works where slightly

damaged buildings are often uninspected and unrecorded (Perelli et al., 2019; Rossetto & Ioannou, 2018) and is likely more detrimental in the present work that has access to a limited group of sources that already carried out a selection of the case studies. Uncertainties related to this bias are further analysed hereafter. Unlike the other cases, the mean damage for the Emilia earthquake is in good agreement with the ones reported in the literature by (Sorrentino et al. 2014), confirming the reliability of this dataset. This agreement could be explained by the fact that the towers reported in that study have been included in the present data, constituting a significant portion of it.

Table 3.5: Comparison of the mean damage found with literature values for masonry towers.

Seismic Event	Present Thesis	Other Sources	
L'Aquila (2009)	3.33	1.109	(De Matteis et al., 2016)
Emilia (2012)	2.95	3 - 4	(Sorrentino et al., 2014)
Central Italy (2016)	3.00	1.74	(De Matteis & Zizi, 2019)
Central Italy (2016)	3.00	0.632	(Cianchino et al., 2021)

DPMs for the same events were found in the literature (Cianchino et al., 2021; De Matteis et al., 2016; De Matteis & Zizi, 2019) and are shown in Table 3.6. Recent studies (Ceroni et al., 2022; Sisti et al., 2023) have reported the DPMs specifically for bell towers hit by the Central Italy seismic event as function of the PGA instead of the MCS intensity, thus, they cannot be directly compared with the other datasets and have not been presented hereafter.

Table 3.6: DPMs for towers as entire macroelements (Macroelement damage).

Source	Seismic Event	MCS	Observed Damage Level (D_k)					
			D0	D1	D2	D3	D4	D5
Authors' database	(see Section 3.2)	V-XI	5%	19%	21%	23%	16%	17%
(De Matteis et al., 2016)	L'Aquila (2009)	IV-IX	40%	24%	20%	10%	4%	2%
(De Matteis & Zizi, 2019)	Central Italy (2016)	V-VII	30%	17%	28%	8%	8%	9%
(Cianchino et al., 2021)	Central Italy (2016)	V-VII	74%	7%	8%	7%	3%	1%

Among the investigated studies, (De Matteis et al., 2016) analysed 64 three-nave masonry churches affected by the 2009 L'Aquila earthquake to provide the DPMs for the entire set and for each structural macroelement including bell towers. Their results obtained for the towers as entire macroelements indicate that 60% of the towers suffered damage. Similarly, (De Matteis & Zizi, 2019) developed DPMs based on the damage data of 68 one-nave masonry churches collected during the inspection activities after the 2016-2017 Central Italy seismic sequence. According to their results obtained for the macroelement, structural damage was observed in 70% of the towers.

The DPMs of the present database can be also compared with the results of (Cifani et al., 2005; Hofer et al., 2018; Ruggieri et al., 2022), who specifically developed DPMs for the distinct damage mechanisms. In particular, (Cifani et al., 2005) presented the results of 437 churches inspected after the 2002 Molise earthquake. (Hofer et al., 2018) reported the damage distribution for the belfry mechanism by analysing a database of 196 masonry churches after the 2016 Central Italy earthquake, whereas (Ruggieri et al., 2022) provided the damage distribution for both damage mechanisms included in a database of 90 one-nave masonry churches inspected after the 2003 Valle Scrivia earthquake. This moderate event occurred on April 11, 2003, in the Piemonte region, with epicentral coordinates of 44.68 (Lat.) and 8.82 (Long.), hypocentral depth of 26 km and magnitude of 4.9 and caused significant damage to buildings and churches. It is worth noting that such towers and in general this event were not included in the database, because the available information was insufficient.

The results are summarised in Table 3.7 and Table 3.8, including a comparison with the present database, taking into account the damage mechanisms separately.

Table 3.7: DPMs for tower damage mechanism (Mechanism A).

Source	Seismic Event	MCS	Observed Damage Level (d_k)					
			D0	D1	D2	D3	D4	D5
Authors' database	(see Section 3.2)	V-XI	15%	8%	17%	26%	21%	13%
(Ruggieri et al., 2022)	Valle Scrivia (2003)	IV-VII	82%	10%	8%	0%	0%	0%
(Cifani et al., 2005)*	Molise (2002)	V-VII	73%	10%	6%	5%	5%	1%

*Percentages provided by (Cifani et al., 2005) for the entire dataset of churches, here recalculated only for churches with towers

From the comparison, it emerges that the present database includes a rather large number of towers although specific events may be underrepresented. In this regard, the existing studies analysed masonry churches, and the exact number of towers for which the DPMs have been developed is not always explicitly specified, since not all the churches present a tower. A few exceptions are worth to be mentioned: in the work of (Hofer et al., 2018) the number of activated belfry mechanisms is reported (i.e., in 17 out of 196 churches). In (Cifani et al., 2005), the two portions of the microelement, shaft and belfry, are present in 49.3% and 49% of the cases, respectively. Finally, in the work of (Ruggieri et al., 2022) a total percentage of towers within the dataset (i.e., about 70%) is provided without the distinction of the activated damage mechanisms.

Table 3.8: DPMs for belfry damage mechanism (Mechanism B).

Source	Seismic Event	MCS	Observed Damage Level (d_k)					
			D0	D1	D2	D3	D4	D5
Authors' database	(see Section 3.2)	V-XI	25%	10%	13%	15%	15%	22%
(Hofer et al., 2018)*	Central Italy (2016)	-	-	6%	35%	29%	18%	12%
(Ruggieri et al., 2022)	Valle Scrivia (2003)	IV-VII	74%	10%	11%	4%	1%	0%
(Cifani et al., 2005)**	Molise (2002)	V-VII	72%	13%	6%	4%	1%	3%

*MCS and D0 data not found in the work of (Hofer et al. 2018).

**Percentages provided by (Cifani et al., 2005) for the entire dataset of churches, here recalculated only for churches with towers.

Moreover, from the comparison, it is also noteworthy that the present database presents a significant percentage of cases with severe damage (from D3 to D5), and at the same time, a rather low number of towers that are undamaged (D0). This, on one hand, highlights the importance of providing a database that encompasses several events, since a single event may present a narrower range of macroseismic intensity and fewer cases with higher damage extent. On the other hand, the results further stress the aforementioned risk of availability bias. In this light, data collection and analysis must address not only the towers characterised by the occurrence of damage but also those with no damage, since the latter are paramount for an unbiased assessment. As an example, (Cianchino et al., 2021), analysing the damage data of 87 churches hit by the 2016 Central Italy earthquake, reported a low percentage of mechanism activation, with about 74% of the towers presenting no damage. This result is quite different from similar DPMs provided for the same seismic event. The comparison of distinct works on the effect of this same earthquake demonstrates the need of an adequate definition of representative samples for the analyses.

It is also worth noting that the existing studies for the same seismic event, especially when the same is characterised by a long sequence with several moderate to strong shocks, may refer to inspections carried out at different times during the earthquake sequence, thus leading to dissimilar results. In this regard, a further source of discrepancy can be linked to the typologies of towers included in the analysis. Indeed, some of the works (Ceroni et al., 2022; Cianchino et al., 2021; De Matteis et al., 2016; De Matteis & Zizi, 2019) seem to develop DPMs including various types of towers (e.g., isolated bounded, and bell gable), differently from the present database which encompasses isolated and bounded only.

3.3.2 Typology of tower and individual intensity groups

To further investigate the damage scenarios observed in the towers included in the database, DPMs were also developed by taking into account the tower typology, as shown in Figure 3.13, over the entire set of earthquakes, and then compared with the results obtained from the application of the binomial

probability function. This analysis allowed to investigate the effects of the adjacent buildings on the vulnerability, thus, on the local and global damage of the investigated towers.

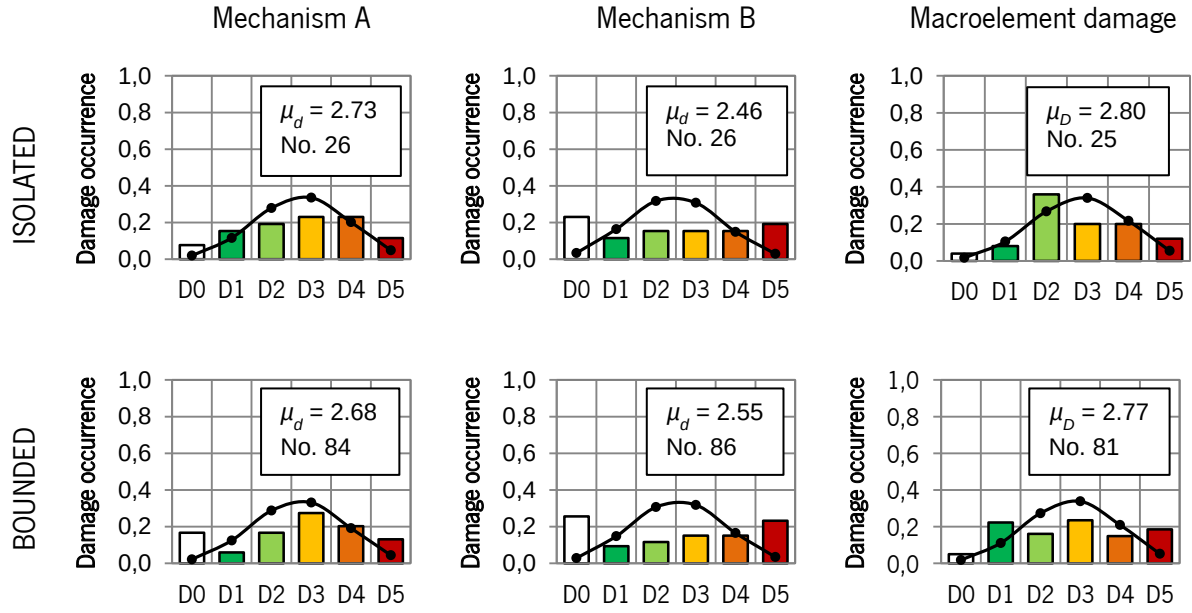


Figure 3.13: DPMs of the investigated towers developed according to their typology, and by considering both the damage levels observed in each damage mechanism and the global damage levels.

The results revealed the potential damage that can occur in historic masonry towers when distinguishing them in isolated and bounded. Although the number of towers included in the two subsets is largely dissimilar, comparable trends can be observed in the distributions of damage occurrence for each damage mechanism. In particular, the distributions of damage occurrence are unimodal and almost normally distributed for Mechanism A and are bimodal with peaks in D0 and D5 for Mechanism B. However, when comparing the two structural configurations, Mechanism A presents a peak with higher level of damage (D4) in case of isolated towers and Mechanism B highlights higher percentage of collapse occurrence (D5) in the case of bounded towers. Consequently, the mean damage value of Mechanism A is slightly higher for isolated towers, whereas, in the case of Mechanism B, is slightly higher for the bounded towers. This could imply that the isolated towers are more prone to suffer structural damage to the shaft, likely presenting a closer behaviour to a cantilever beam when compared with bounded towers, whereas in the bounded towers the interactions with adjacent buildings are likely affecting the shaft positively, thus increasing the likelihood of damage in the belfry. However, the differences are minimal, and more research is needed to support such a distinction in the seismic behaviour. Focusing on the global damage (Macroelement damage), the distributions of damage occurrence obtained are rather different. In particular, the isolated towers show higher damage

occurrence in D2 and D4, whereas the bounded towers are distributed with an almost constant damage occurrence, with an increasing percentage in D1, D3 and D5, when compared with the isolated towers. Finally, it can be noted that the binomial distribution seems to fit well the observed data for Mechanism A, but presents a large error for the macroelement and even larger for Mechanism B.

Considering the broad range of intensity affecting the investigated towers, the reliability of the statistical analysis was further increased by developing separated DPMs for the five aforementioned MCS intensity groups, as shown in Figure 3.14(a-q). The results show an unexpected increase in the mean damage for the lowest MCS intensity (Group I, $5 \leq I_{MCS} < 6$), which is higher than the mean damage obtained for Group II ($6 \leq I_{MCS} < 7$). This result may be due to two main non-mutually exclusive reasons, on one hand, it is possible that some towers, due to their characteristics and conditions, have shown significant damage in an area that has been then characterised by low macroseismic intensity, on the other hand, the way the database has been collected, by relying on published surveys of damaged towers, may have led to a bias in availability of results, as already mentioned. In fact, many cases of undamaged towers, which would have reduced the mean damage for a given intensity, may have been overlooked. However, an increase in the observed mean damage with increasing intensity appeared from Group II, through Group III ($7 \leq I_{MCS} < 8$), to Group IV ($8 \leq I_{MCS} < 9$), which is expected and consistent with similar results obtained from the literature. Group V ($I_{MCS} \geq 9$) presents a lower value of mean damage when compared with the mean damage of Group IV. This is likely due to the small, non-representative, number of towers included in Group V.

The DPMs for the two damage mechanisms (Mechanism A and Mechanism B) are compared with those reported in the work of (Canuti et al., 2021), who developed the DPMs accounting for distinct groups of macroseismic intensity (see Table 3.9 and Table 3.10). This comparison is more informative than the previous ones, since it is carried out according to the distinct macroseismic intensity groups for a better characterisation of the samples. However, this information is not readily available in most of the reviewed papers. It is worth noting that, in (Canuti et al., 2021; Hofer et al., 2018; Ruggieri et al., 2022), the percentages are approximated values, being extrapolated from the graphs. The comparative evaluation confirmed the discussed data availability bias especially for the lowest macroseismic intensity.

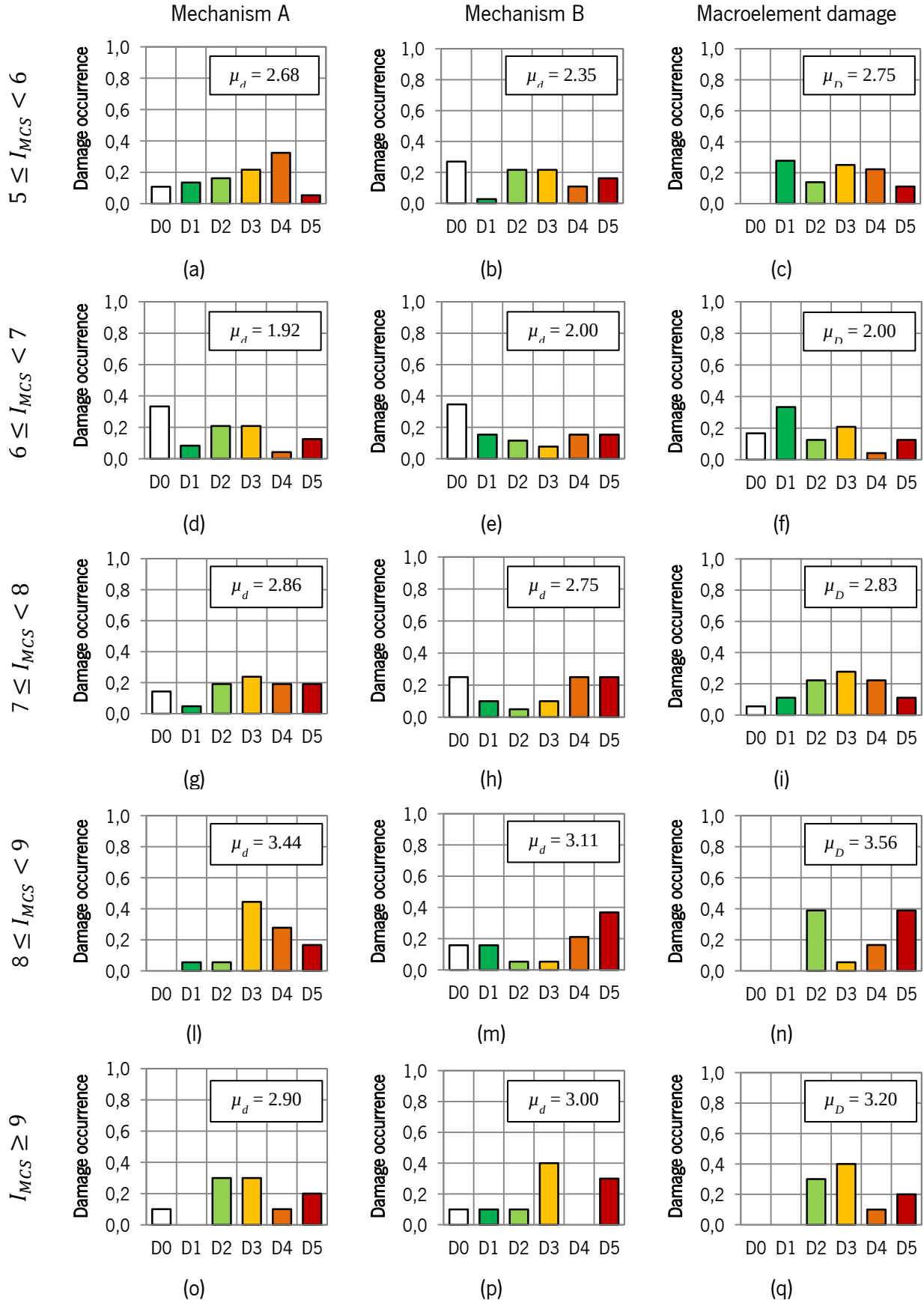


Figure 3.14: DPMs for different MCS intensities.

Table 3.9: DPMs for the tower mechanism (Mechanism A), grouped by macroseismic MCS intensity.

Source	Seismic Event	Observed Damage Level (d_k)					
		D0	D1	D2	D3	D4	D5
$I_{MCS} = 5$							
Authors' database	(see Section 3.2)	11%	14%	16%	22%	32%	5%
(Canuti et al., 2021)	Central Italy (2016)	60%	16%	24%	0%	0%	0%
$I_{MCS} = 6$							
Authors' database	(see Section 3.2)	33%	8%	21%	21%	4%	13%
(Canuti et al., 2021)	Central Italy (2016)	58%	18%	12%	8%	4%	0%
$I_{MCS} = 7$							
Authors' database	(see Section 3.2)	14%	5%	19%	24%	19%	19%
(Canuti et al., 2021)	Central Italy (2016)	50%	22%	10%	5%	2%	11%
$I_{MCS} = 8$							
Authors' database	(see Section 3.2)	0%	6%	6%	44%	27%	17%
(Canuti et al., 2021)	Central Italy (2016)	9%	9%	0%	39%	22%	22%

Table 3.10: DPMs for the belfry mechanism (Mechanism B), grouped by macroseismic MCS intensity.

Source	Seismic Event	Observed Damage Level (d_k)					
		D0	D1	D2	D3	D4	D5
$I_{MCS} = 5$							
Authors' database	(see Section 3.2)	27%	3%	22%	22%	11%	15%
(Canuti et al., 2021)	Central Italy (2016)	65%	21%	10%	4%	0%	0%
$I_{MCS} = 6$							
Authors' database	(see Section 3.2)	35%	15%	12%	8%	15%	15%
(Canuti et al., 2021)	Central Italy (2016)	48%	21%	15%	12%	2%	2%
$I_{MCS} = 7$							
Authors' database	(see Section 3.2)	25%	10%	5%	10%	25%	25%
(Canuti et al., 2021)	Central Italy (2016)	61%	10%	12%	8%	7%	2%
$I_{MCS} = 8$							
Authors' database	(see Section 3.2)	16%	16%	5%	5%	21%	37%
(Canuti et al., 2021)	Central Italy (2016)	9%	19%	27%	9%	27%	9%

3.4 VULNERABILITY FUNCTIONS

As an alternative to the DPMs, vulnerability curves are a powerful tool to rapidly estimate the expected mean damage, μ , for a given level of macroseismic intensity, I_{MCS} . To build a vulnerability model, the function originally proposed by (Lagomarsino & Giovinazzi, 2006; Lagomarsino & Podestà, 2004c) was here adopted, as follows:

$$\mu = 2.5 \left[1 + \tanh \left(\frac{I_{MCS} + 6.25V_I - 13.1}{Q} \right) \right] \quad (3.2)$$

where μ is correlated with increasing levels of I_{MCS} through two physical parameters intrinsic of a specific investigated asset typology, component or subclass, namely the ductility index, Q , and the vulnerability index, V_I .

In particular, values for vulnerability and ductility indexes for historic masonry bell towers were recommended by (Curti et al., 2008; Lagomarsino et al., 2004) and have been here used as a reference, see Table 3.11.

Table 3.11: Vulnerability and ductility indexes for historic masonry towers.

	V_I	Q	Source
Damage Mechanism			
Mechanism A	0.890	2.00	(Curti et al., 2008) (1)
Mechanism B	0.940	1.49	(Curti et al., 2008) (2)
Entire Macroelement			
Macroelement damage	0.776	2.30	(Lagomarsino et al., 2004)

As already mentioned in the state of the art, (Lagomarsino et al., 2004) proposed a vulnerability function, also recently reported by (Despotaki et al., 2018) for masonry towers and slender buildings, not specifically for bell towers, here used as reference for the entire tower macroelement only, while (Curti et al., 2008) developed vulnerability functions specifically addressing all the damage mechanisms of towers. It should be noted that the modifier parameters of the vulnerability index, discussed in Chapter 2, have not been taken into account due to the lack of data.

Figure 3.15 shows the vulnerability curves obtained by applying these existing equations against the present dataset. The curves are compared with the values of mean damage observed in the investigated towers for different groups of MCS intensity, further grouped by seismic events, through a colour key.

It should be noted that the towers characterised by maximum MCS intensity have been distinguished into two subgroups: $9 \leq I_{MCS} < 10$ and $10 \leq I_{MCS} < 11$. The plots also report the values of the mean damage considering the full set of earthquakes for each group of MCS intensity as the Global Mean. This value was preferred for validation and calibration purposes to overcome a possible bias given by the different number of towers observed in distinct earthquakes. Nonetheless, this approach could not completely solve the unexpected high mean damage in the range of $5 \leq I_{MCS} < 6$ and the extremely low mean damage in the range of $10 \leq I_{MCS} < 11$. Samples in these ranges may be not representative of the expected real population of damaged towers; therefore, for the calibration of new vulnerability curves, they were discarded.

The existing predictive functions for the individual damage mechanisms fit well the observed mean damage, especially in the range $6 \leq I_{MCS} < 10$. A certain scatter between the predicted and the observed damage levels emerges but the same is reduced when the Global Mean values are

considered. The predictive function for the entire macroelement, instead, appears to be non-conservative, producing an underestimation of the mean damage.

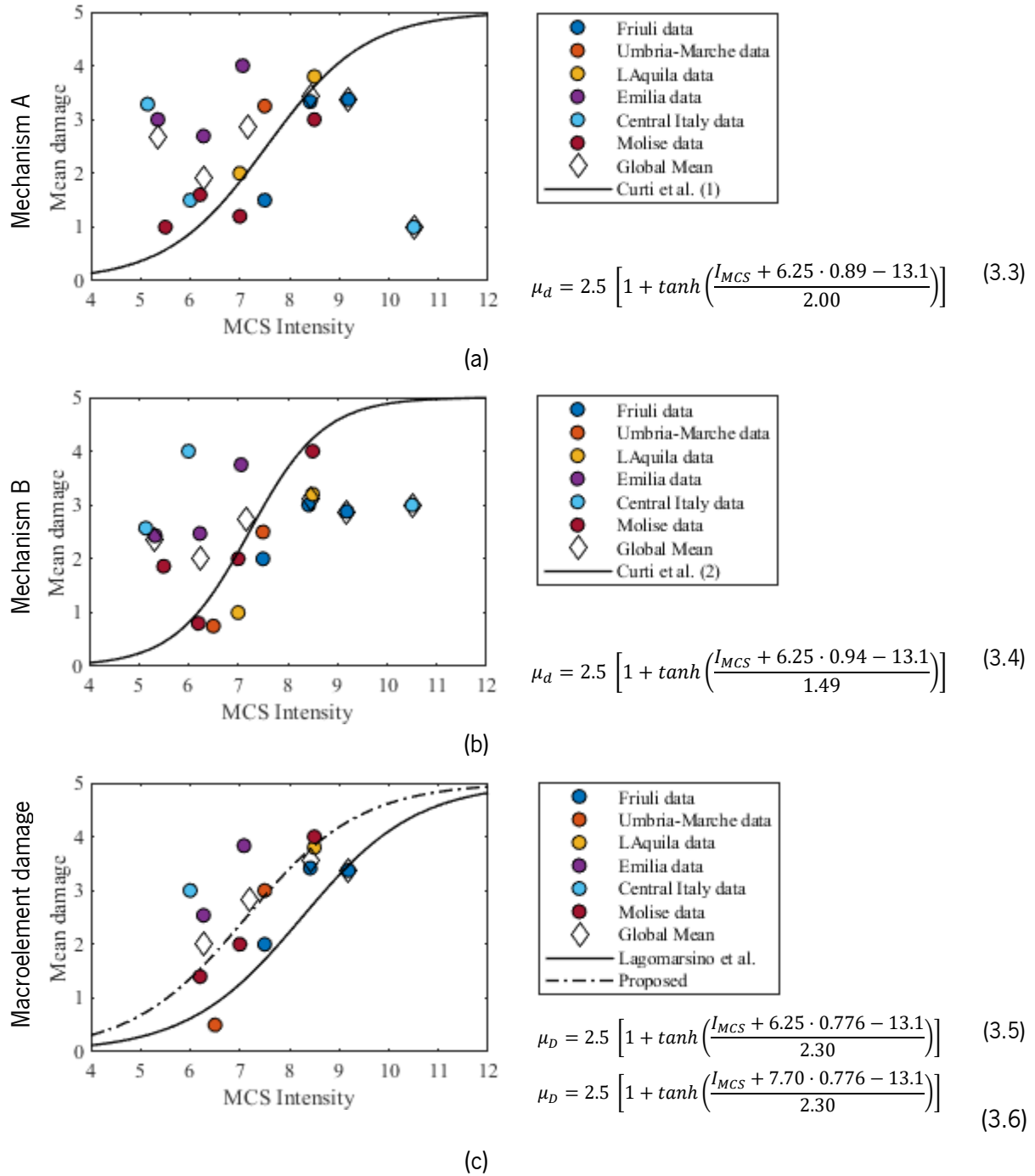


Figure 3.15: Comparison between the existing vulnerability functions and the observed mean damage for distinct mechanisms (a and b) and for the entire macroelement (c).

In light of these considerations, further analyses were conducted to fit the observed mean damage using the well-known formulation of the vulnerability curve but recalibrating its parameters. Initially, the calibration focused on the parameters α , β , Q , as shown in Eq. (3.7), one by one, while keeping the vulnerability index from literature. These parametric analyses performed on the vulnerability equations

provided a graphic tool useful to identify the best curve according to specific data in hand. This procedure is similar to (De Matteis, Brando, & Corlito, 2019).

$$\mu = 2.5 \left[1 + \tanh \left(\frac{I_{MCS} + \alpha V_I - \beta}{Q} \right) \right] \quad (3.7)$$

The results obtained for the vulnerability curves varying the parameters over a wide range are illustrated in Figure 3.16. The lower and upper bound for the parameters (α , β , Q) of the three existing formulations and the intervals in their variation, considered for the definition of the curves are reported in Table 3.12 (here Δ is the step adopted for the different curves shown). It can be observed that while the variations of the parameters α and β produce a shift of the vulnerability curves, where the effect on the curve of increasing α corresponds to the effect of reducing β and vice-versa, the variation of the ductility index Q generates a change in its slope. Among the calibration coefficients, α and β play a major role in fitting the curves to the present dataset.

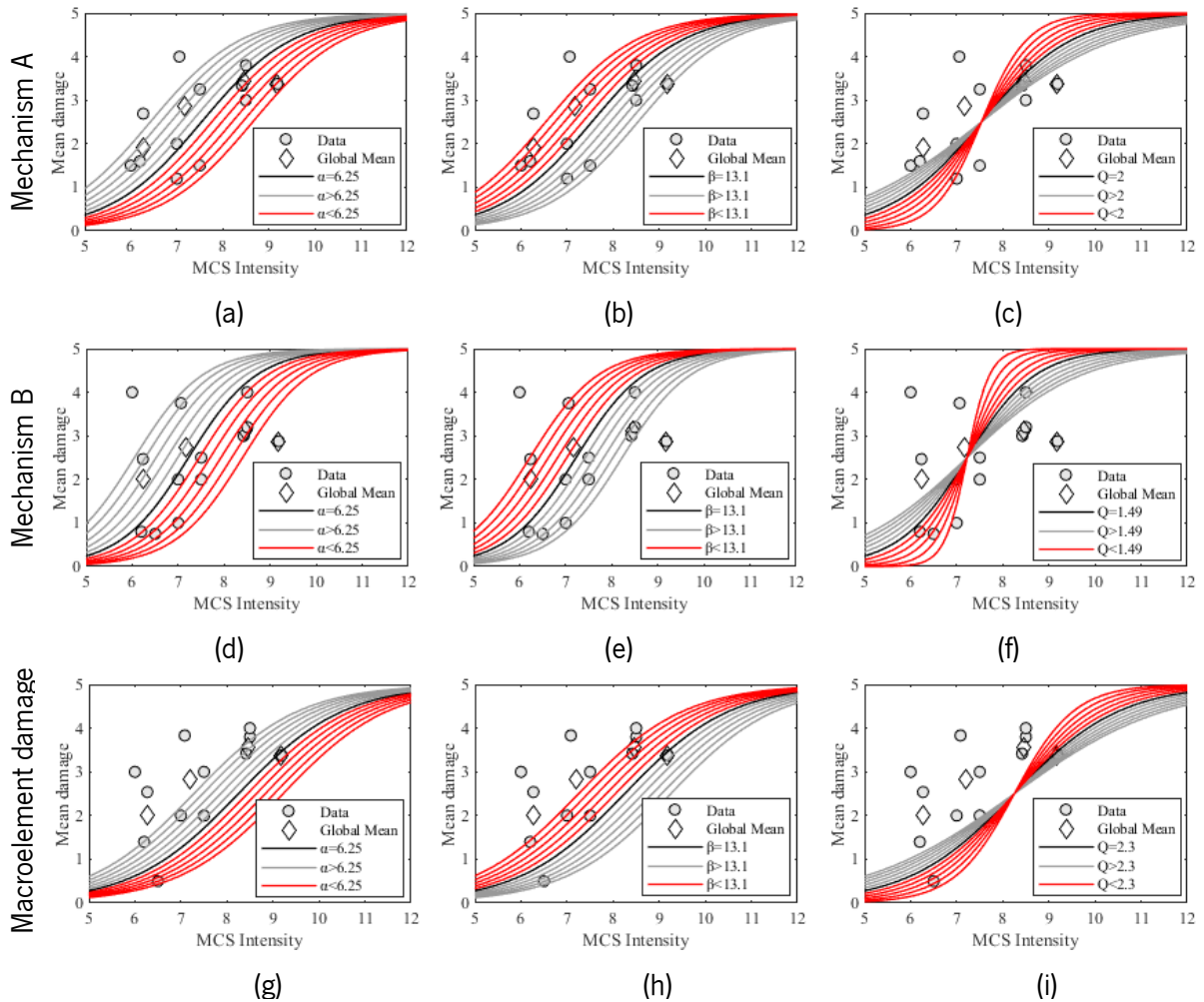


Figure 3.16: Parametric analyses of the existing vulnerability functions for distinct damage mechanisms, (a) to (f), and for the whole macroelement, (g) to (i).

Table 3.12: Constraints adopted in the analysis.

	Initial values	Min	Max	Δ
α	6.25	5.00	7.50	0.25
β	13.10	12.10	14.10	0.20
Q (Mechanism A)	2.00	1.00	3.00	0.20
Q (Mechanism B)	1.49	0.49	2.49	0.20
Q (Macroelement damage)	2.30	1.30	3.30	0.20

Upon this preliminary sensitivity analysis, a regression analysis was conducted to develop new functions fitting the observed Global Mean damage, by choosing α as a variable to calibrate. The results related to the individual damage mechanisms (Mechanism A and Mechanism B) show values similar to the ones proposed by (Curti et al., 2008), Eq. (3.3) and (3.4), confirming their capability for damage prediction. When fitting the dataset for the entire tower macroelement (Macroelement damage), instead, the obtained Eq. (3.6) produces a significant improvement with respect to the original formulation proposed by (Lagomarsino et al., 2004), given in Eq. (3.5), for the specific dataset in hand.

Figure 3.17 shows the performance plots, namely predicted versus observed mean damage values, obtained by applying both the existing and proposed formulations, together with the Residual Sum of the Square (RSS) metric, expressed as follows:

$$RSS = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (3.8)$$

where n is the number of observations, y_i denotes the actual observed damage extent, and $\hat{y}_i$ denotes the predicted value estimated via the vulnerability function. This metric shows a significant improvement of the predictive capability upon recalibration of the vulnerability function coefficient α , considering both the Global Mean values (plotted as diamonds), very well predicted, and the mean values for the samples grouped by earthquake (plotted as circles) that, despite a larger dispersion, are better predicted as well.

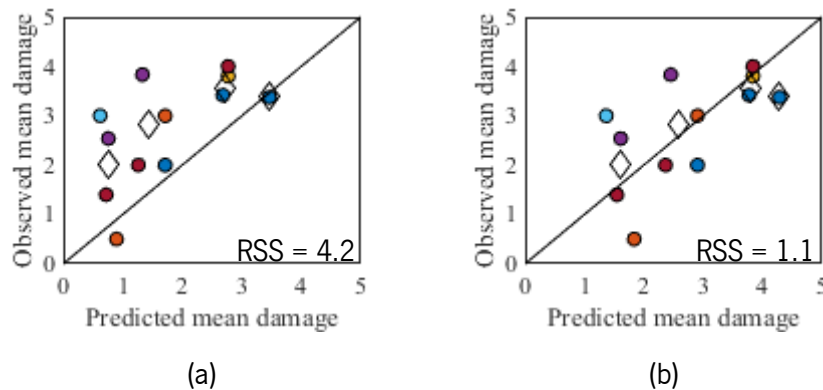


Figure 3.17: Performance plots before (a) and after (b) recalibration. RSS estimated for the Global Mean value.

Finally, to highlight the statistical relevance of the present database and to further validate the existing and proposed vulnerability functions, investigating the effect of the size and characteristics of the tower samples on their performance, multiple statistical analyses were performed by randomly generating several subsets of historic masonry towers. More than 200'000 subsets for each analysed instance (Mechanism A, Mechanism B and Macroelement damage) were generated following distinct sampling strategies to randomly exclude towers from the whole database, defining several groups of towers of different sizes.

Exclusions based on the specific events or the specific sources of information, despite being potentially valid alternative approaches for the generation of the subsets, were discarded as they could introduce biases in the dataset at hand. Indeed, sources like Curti et al. (2008), Acito et al. (2021) or Jain et al. (2020) provide only information for high macroseismic intensity while others, like De Matteis and Zizi (2019) or Valente (2017), inform about low macroseismic intensity. Moreover, as demonstrated by the DPMs categorised by earthquake and by intensity, the information is likely not well distributed over the damage levels. Therefore, combining all sources together was deemed essential for a correct inference of the towers behaviour against real earthquake events.

First, 10'000 distinct subsets of 100, 102, and 96 historic masonry towers were generated for Mechanism A, Mechanism B and Macroelement damage, respectively, by randomly removing, in each subset, 10 towers from the initial whole dataset. All the possible combinations of leave-10-out subsets were too numerous to be managed (i.e., in the order of tera). The Global Mean damage values were computed for all the subsets and then clustered accounting for each macroseismic intensity as shown in Figure 3.18(a, c and e). The resulting distribution of Global Mean damage values appear to be very narrow considering both the interquartile range and the spread of the whiskers in the boxplots, which account for a 99% coverage assuming a normal distribution of the data. Nonetheless, a few significant outliers emerge. It should be also noted that the distribution of the Global Mean damage for high macroseismic intensity features the largest spread. To statistically measure the performance of the existing and novel formulations, the RSS metric was evaluated over all the random subsets, and the obtained distributions, which were also fitted by a Gaussian probability density function, are illustrated in Figure 3.18(b, d and f).

The RSS distributions present lower mean values for Mechanism A than for Mechanism B and Macroelement damage. In Figure 3.18e and f), beside the original formulation of the vulnerability function for Macroelement damage, the suggested recalibration and its performance in terms of RSS are shown. Here, the RSS comparative results confirm a significant improvement upon recalibration of

the curve for Macroelement damage, as their mean decreases and their distribution becomes narrower. A recalibration of the curve coefficients for Mechanism B data has been attempted as well but has led to a negligible improvement of the performance. Indeed Figure 3.18(c) suggests that a recalibration of the ductility factor Q would be more effective than the attempted calibration of the coefficient α . However, a tentative calibration of the factor Q led to a value too far from its acceptable physical range, likely due to the uncertainties in the dataset, and was discarded.

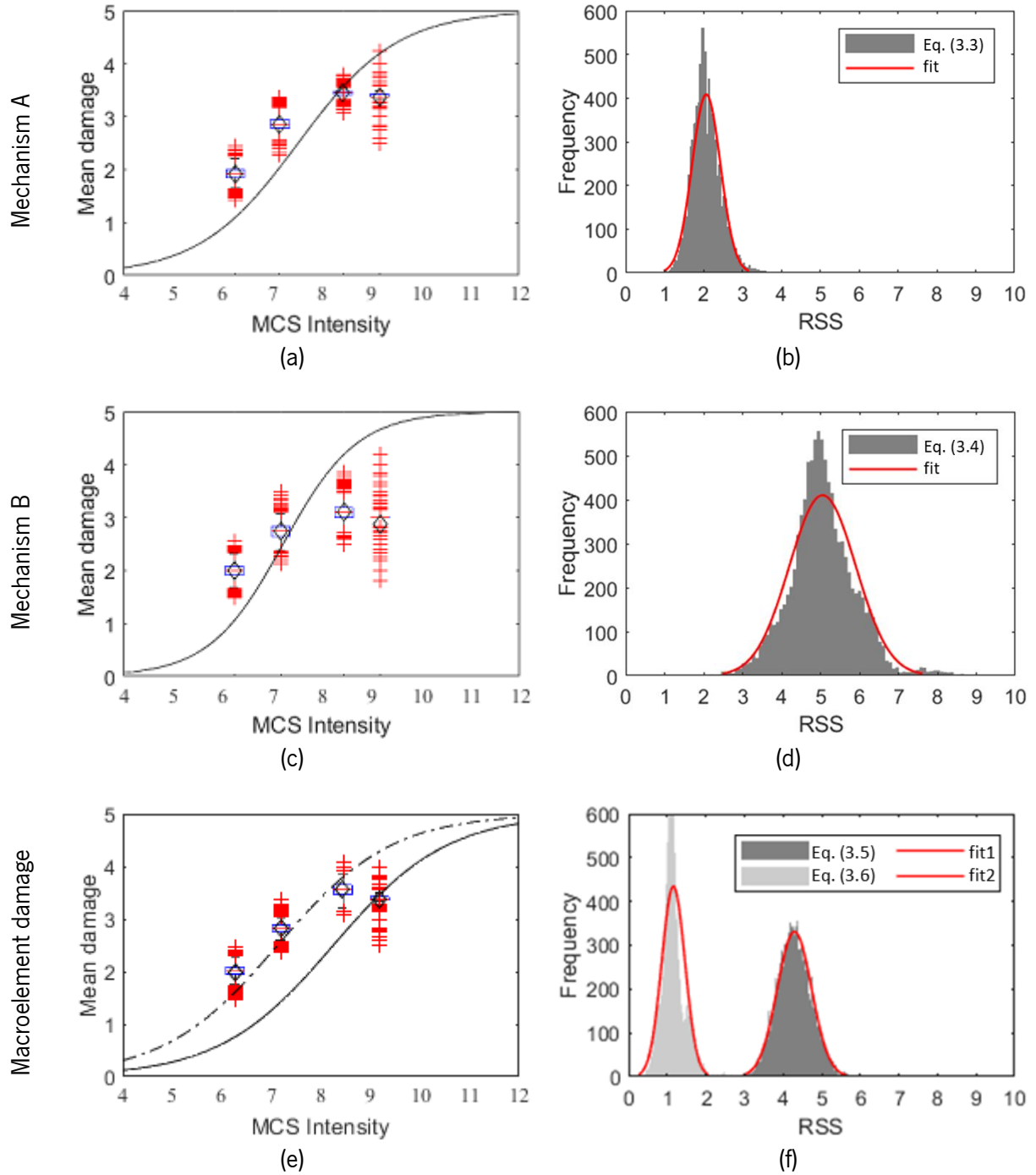


Figure 3.18: Validation of the existing (a-d) and the proposed (e-f) vulnerability functions against 10'000 subsets generated by randomly removing 10 towers each time.

In a second stage, randomly generated samples of different sizes were investigated. In particular, 3 and 20 towers at a time were randomly removed from the initial whole dataset.

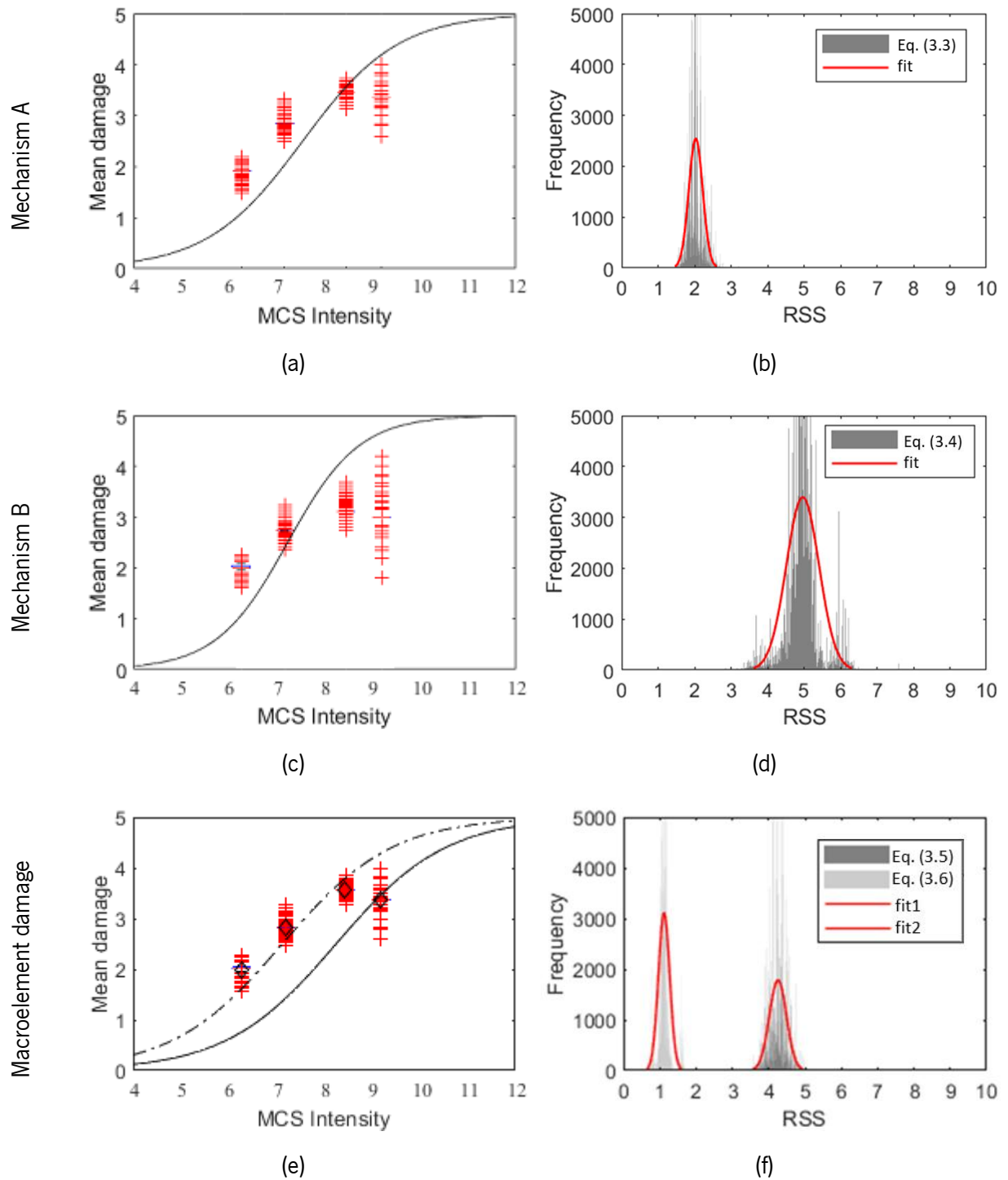


Figure 3.19: Validation of the existing (a-d) and the proposed (e-f) vulnerability functions against more than 200'000 subsets generated by randomly excluding 3 towers each time.

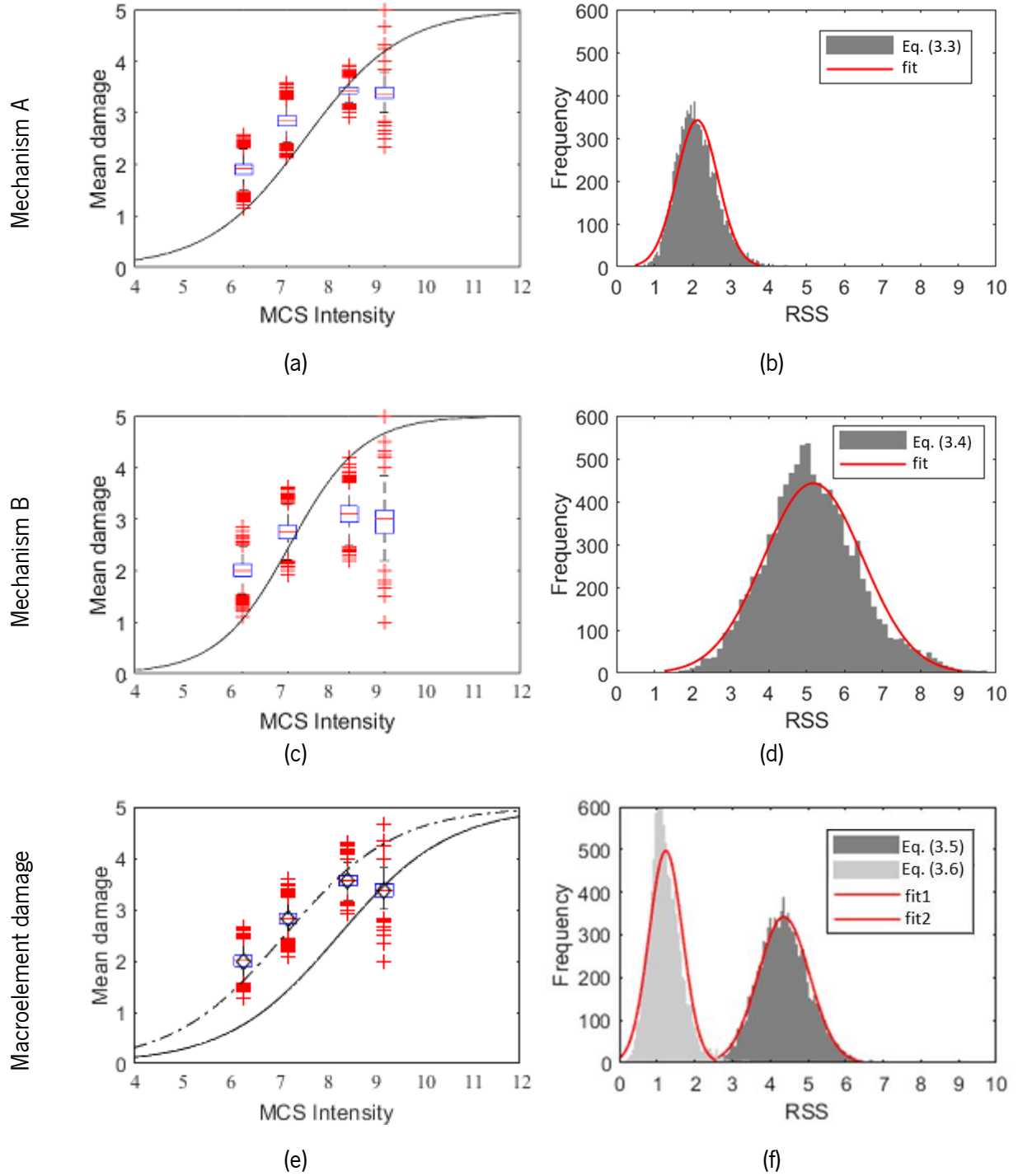


Figure 3.20: Validation of the existing (a-d) and the proposed (e-f) vulnerability functions against 10'000 subsets generated by randomly removing 20 towers each time.

Exclusion of 3 towers produced smaller subsets of 107, 109, and 103 towers for Mechanism A, Mechanism B and Macroelement damage, respectively. All the possible combinations of 3 samples out of the original database were considered, summing up to 215'820, 227'920, and 192'920 distinct random subsets for Mechanism A, Mechanism B and Macroelement damage, respectively (Figure 3.19). Exclusion of 20 towers produced smaller subsets of 90, 92 and 86 towers for Mechanism A,

Mechanism and Macroelement damage, respectively. All the possible combinations were too numerous to be managed (i.e., in the order of zetta). Therefore, as in the previous case, 10'000 distinct random combinations were generated (Figure 3.20).

The results confirm the rather narrow distribution of the Global Mean damage that tends to spread with more outliers as the number of removed samples increases and, in general, for higher intensity, likely due to the cardinality of the subsets for each macroseismic intensity level that significantly reduces when more samples are removed.

These analyses suggest that the database used for the training ensures a sufficiently robust vulnerability curve, in the case of the recalibrated one, and allows to generally conclude that the three investigated functions, for Mechanism A, Mechanism B and Macroelement damage, present an acceptable performance. At the same time, the presence of outliers in the distribution of the Global Mean damage values and the RSS values, whose extent increases when more samples are removed, stresses the need for future extensions of the database, fostered by distributions available in open access, and for a recalibration of the curves if necessary.

3.5 FRAGILITY FUNCTIONS

The open database was finally employed to define fragility functions. However, instead of defining the actual probability of exceeding a certain damage level for increasing values of the intensity measure based on the actual samples, the simplified method, adopted in similar studies, e.g., (Lagomarsino & Podestà, 2004c), and described in Section 2.3.4 was followed. The two existing vulnerability functions for the mechanisms Mechanism A and Mechanism B, Eq. (3.3) and (3.4), and the novel proposed formulation for the global behaviour of the macrolelement, Eq. (3.6), were exploited for the estimation of the expected mean damage for each intensity ($5 \leq I_{MCS} \leq 12$). These expected mean damage values were then introduced in the BPDFs, Eq. (3.9), upon the derivation of the PGA, used as ground motion parameter for the fragility functions, through the existing PGA-I empirical formulation proposed by (Faenza & Michelini, 2010), expressed as:

$$I_{MCS} = 2.58 \cdot \log PGA + 1.68 \quad (3.10)$$

$$\log PGA = -0.65 + 0.39 \cdot I_{MCS} \quad (3.11)$$

The adopted approach is preferred at this stage of the work, given the limited availability of data and their potential limitations, already stressed in the present Chapter. Nonetheless, more accurate estimation of the PGA and more robust strategies for the development of the fragility functions by fitting

statistical functions to the empirical data (Baker, 2015) are exploited in the subsequent works, as discussed in Chapter 4.

Figure 3.21(a and b) show the fragility curves developed for the case of damage level associated with the two distinct damage mechanisms, and Figure 3.21(c) illustrates the fragility curves built for the case of global damage level associated with the tower as an entire structural macroelement.

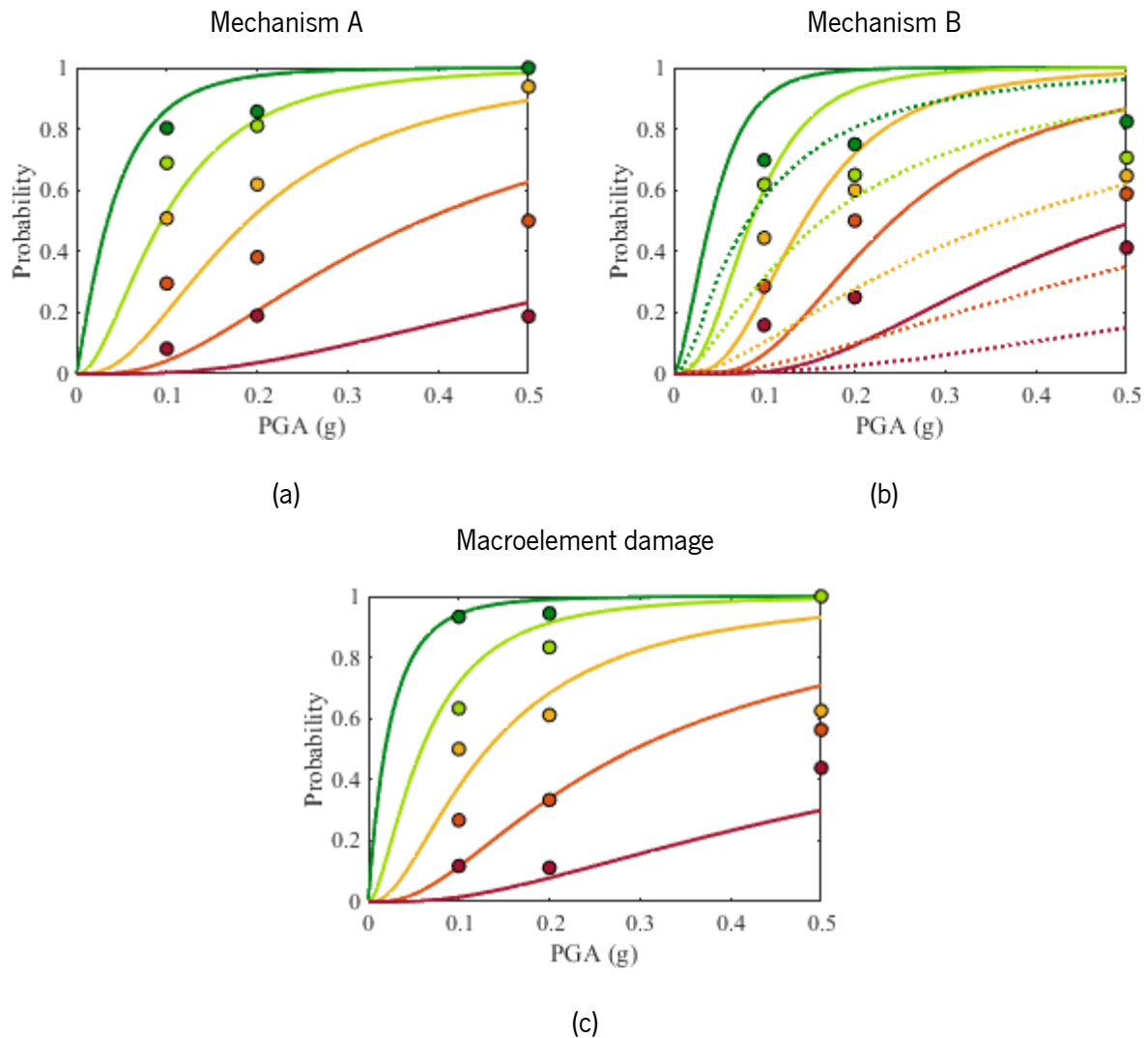


Figure 3.21: Fragility curves for distinct damage mechanisms (a and b) and for the entire macroelement (c). Here, solid lines are the curves from the present database and the dashed lines are from (Marotta et al., 2021). The observed damage points are included for validation.

It can be observed that the probability of exceeding a certain level of damage for Mechanism A is generally lower than the probability obtained for Mechanism B, namely the probability of damage occurrence is higher for the belfry mechanism, as a consequence of the larger vulnerability of such element. The fragility curves produced for the entire macroelement exhibit an intermediate behaviour.

In Figure 3.21, the observed damage points calculated based on the samples in the database are also reported, upon estimating the PGA at their site using the same empirical equation. This comparison allowed to evaluate the validity of the adopted approach and possible biases included in the collected data. It is worth noting that the estimation of PGA using the empirical equation presents a gap in the range of 0.3-0.4g. This gap is not caused by the characteristics of the database at hand, but represents a limitation of the empirical equation itself due to the link between the discrete MCS intensity values and the continuous PGA values.

The comparison indicates that the fragility functions for the distinct damage mechanisms obtained from the BPDFs overestimate the probability of the mean damage for D1 and underestimate the probability of the mean damage for D4-D5 in the range of PGA between 0.1-0.2g. In the remaining bin of PGA (0.5g), the comparison for the shaft mechanism, Mechanism A, presents acceptable prediction, while the fragility functions for the belfry mechanism, Mechanism B, overestimate the probability of the mean damage along all the possible damage scenario extents (D1-D5). Regarding the belfry mechanism, fragility functions have been already developed by (Marotta et al., 2021) using a fitting procedure based on the lognormal cumulative distribution method over their damage data. These proposed fragility functions, presented in Figure 3.21(b), underestimate the probability of the mean damage of the present database, especially for D3 to D5, when compared with the fragility functions developed in this study. The trends emerged by comparing observational damage points and fragility functions are likely due to the aforementioned underrepresentation of slightly and non-damaged towers, which should be more numerous in the lower PGA range, leading to higher probability of the points. The even poorer accuracy against Mechanism B observed damage may be explained by the use of BPDFs to generate the curves, considering the emerged difference between the BPDFs and the DPMs for the case of Mechanism B (see Section 3.3).

On the other hand, the rather larger percentage of towers affected by severe damage (e.g., from D3), when compared with other datasets in literature, could explain the higher observed probabilities with the respect to the curves. Finally, comparing the fragility functions with the observed mean damage for the entire macroelement, the results, in Figure 3.21(c), show good agreement, expected consequence of the use of a recalibrated vulnerability function to derive the fragility models.

3.6 CONCLUSIONS

The present Chapter discussed the results of the first activities carried out within the thesis work in the field of territorial scale seismic assessment of masonry towers and slender structures. These activities, encompassed: (i) the collection of a database of 129 historic masonry bell towers struck by six past

seismic events that occurred in Italy; (ii) the development of new Damage Probability Matrices (DPMs) and comparison with similar ones in literature; (iii) validation and enhancement, through recalibration, of existing vulnerability functions for masonry towers; (iv) formulation of fragility functions through a simplified approach; (v) extensive analysis of the sources of uncertainties affecting the database and, consequently, the developed vulnerability and fragility models.

The database was gathered to tackle the lack of reliable and open access large collections of data on real masonry towers struck by past earthquakes. To this end, an extensive literature review of technical and scientific literature allow to identify various towers, with relevant information such as their geographic location, the MCS intensity characterising the seismic input, their typology (isolated or bounded), and the observed damage using a macroelement approach, thus organised into mechanism emerging in the shaft, in the belfry and overall damage level to the macroelement.

DPMs have been developed for the entire dataset, and for subsets of towers with likely homogenous behaviour, namely grouped for earthquake events, for tower typologies and for different intensity levels, always considering the two individual mechanisms and the macroelement global damage. The fragmentation of the overall dataset into smaller homogeneous classes, despite showing interesting insight in the susceptibility of the tower typologies to specific damage mechanisms or the different likelihood of damage occurrence for different mechanisms at higher intensity, stressed potential limitations of the dataset and the data collection, in particular: (i) an overestimation of the damage level for low intensity, due to missing information regarding slightly or non-damaged towers; (ii) an underestimation of the damage level for very strong intensity, due to statistically non-representative sample size; (iii) a non-representativeness of the samples for specific earthquakes.

Nonetheless, the database has been used as a testbed for simplified vulnerability models existing in the literature, paying particular attention to evaluate and mitigate the effect of the uncertainties and the biases induced by the characteristics of the dataset. The results revealed that, while the formulations of vulnerability functions proposed for individual damage mechanisms performed well against the collected data, the formulation proposed for the entire macroelement had a comparatively lower predictive capability. A wide range of curves has been also reported to support future users in fitting specific data in hand. Still, a new calibration of the vulnerability function coefficients has been carried out to propose an improved formulation for the entire macroelement. The validation of the existing and newly proposed vulnerability curves has been extensively carried out against a large number of reduced sets generated from the initial database by randomly removing 3, 10 and 20 samples. This allowed to

explore the statistical relevance of the dataset at hand, and at the same time to better interpret the performance of the vulnerability functions.

Finally, fragility functions have been derived by correlating the cumulative probability function, in the form of the Binomial Density Probability Function based on the expected mean damage, with the PGA, retrieved using an existing empirical correlation between PGA and macroseismic intensity (PGA-I). The proposed fragility curves indicated the higher susceptibility to damage of the belfry when considering the distinction between the two failure mechanisms.

The emerged limitations call for a careful analysis of the representativeness of the datasets used to develop essential rapid predictive models such as the vulnerability and the fragility curves. To this end, an extension of the database, especially to include undamaged towers and to increase the number of cases for lower and higher macroseismic intensities, is essential to ensure that the samples are fully representative of the entire data population. Additionally, the development of more effective predictive models may benefit from the implementation of more robust strategies for the estimation or instrumental measurement of the PGA, combined with the availability of a larger database which will allow the development of fragility functions based on observed data fitting. These paramount aspects are further addressed in the next Chapter.

CHAPTER 4

VULNERABILITY MODELS FOR CENTRAL ITALY: CUMULATIVE DAMAGE AND KEY FEATURES

ABSTRACT

The present Chapter expands and deepens the study of seismic vulnerability of masonry towers at territorial scale, overcoming most of the shortcomings identified in the previous Chapter regarding uncertainties and biases induced by the characteristics of the database. To this end, a single seismic sequence was investigated and a wider sample was analysed. In particular, a set of 794 bell towers surveyed during the Central Italy seismic sequence was collected, filtering and cleaning a database of more than 3000 inspected churches. This seismic sequence, occurred between 2016 and 2017, comprises several moderate to severe shocks. The characteristics of the dataset allowed to investigate subsets of towers affected by distinct seismic scenarios, enabling the distinction between towers inspected after a single main shock and towers likely subjected to cumulative damage. Additionally, subsets of towers were extrapolated considering key features such as location, geometry, interactions, maintenance, to explore their impact on the vulnerability. DPMs, vulnerability and fragility functions were derived to this purpose.

4.1 INTRODUCTION

The present Chapter addresses the most relevant shortcomings that emerged during the analyses discussed in Chapter 3 improving the reliability of territorial scale seismic vulnerability models for masonry towers and slender structures (see Figure 4.1).

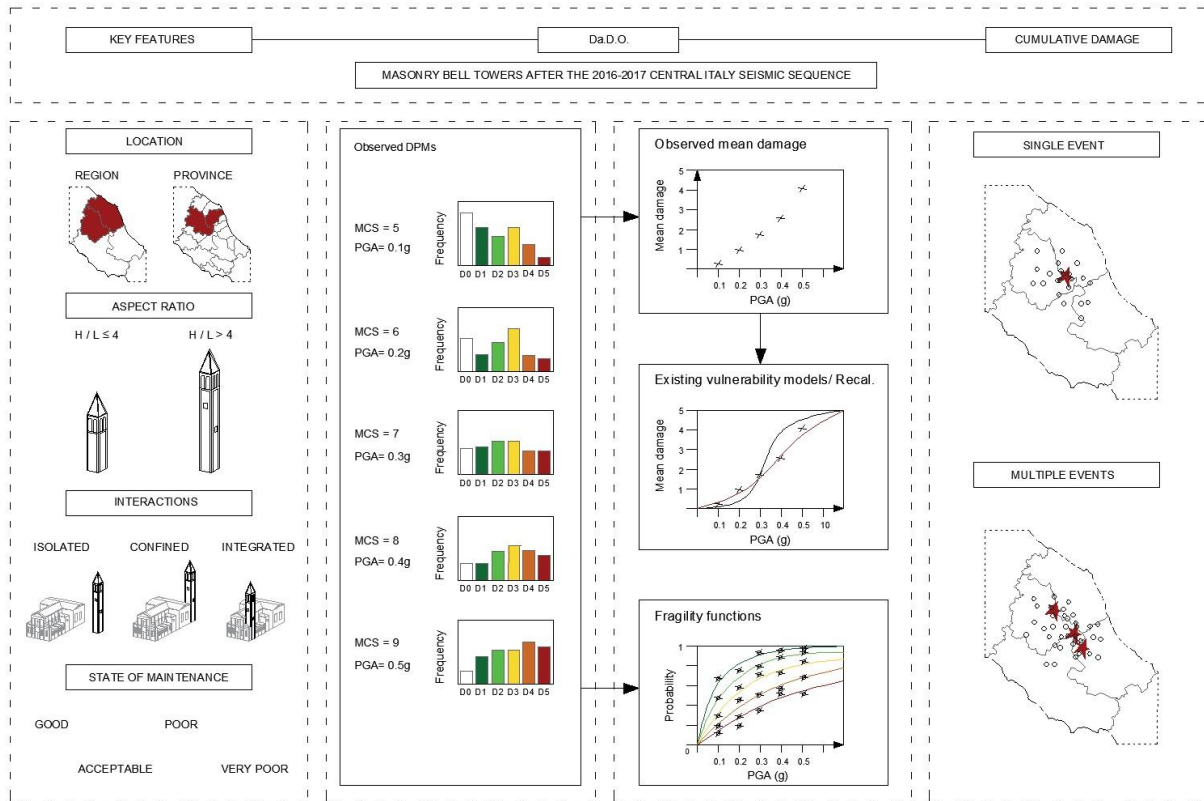


Figure 4.1: Flowchart methodology adopted in this Chapter.

In the first stage, historic masonry bell towers affected by the 2016-2017 Central Italy seismic sequence have been collected, filtering the entire set of churches resulting from the Database of Observed Damage (Da.D.O.) from (Dolce et al., 2019). The final database with its statistical analysis is described in Section 4.2. Successively, vulnerability models (DPMS, vulnerability and fragility functions) have been produced in Section 4.3. The entire dataset was considered and divided in multiple subsets accounting for distinct seismic scenarios, namely single seismic shock and multiple seismic shocks, to provide an insight into the seismic risk of masonry bell tower under cumulative damage. Finally, additional models have been developed, in Section 4.4, taking into account key features such as location, geometry, interactions, and maintenance, seeking to identify towers with more likely homogenous behaviour.

4.2 DESCRIPTION OF CENTRAL ITALY SEISMIC EVENTS

4.2.1 Seismic sequence

The Central Italy seismic sequence struck a wide area at the boundary among Lazio, Abruzzo, Marche and Umbria regions with a series of moderate to strong normal-fault earthquakes. The first main shock occurred on August 24, 2016 with epicentre in the area of Monti della Laga near Accumoli, Lazio region, and a magnitude M_w 6.2, followed by several aftershocks recorded during remaining 2016 and 2017. Two severe aftershocks hit on October 26, 2016, in the area of Valnerina with epicentre near Castelsantangelo sul Nera, Marche region, with M_w 5.5 and 6.1. The main shock of the sequence occurred four days later on October 30, 2016, with M_w 6.6 and epicentre in Norcia, Umbria region. Finally, on January 17, 2017, three events occurred with the strongest having magnitude M_w of 5.7, all with epicentre near Capitignano, Abruzzo region (Rovida et al., 2020; Sisti et al., 2023). The recovery plan and reconstruction works are still on-going almost 10 years after the events (PCM, 2025). Table 4.1 shows the general information of the main seismic events considered in this study, according to the Italian Earthquake Catalogue (Rovida et al., 2020). The epicentres of these events are illustrated over the Italian territory in Figure 4.2.

Table 4.1: General information about the seismic sequence.

No. Event	Epicentral Area	Date [dd-mm-yyyy]	Time (UTC)	Lat [°]	Long [°]	Depth [km]	M_w [-]
1 st	Monti della Laga	24/08/2016	01:36	42.698	13.233	08.1	6.2
2 nd	Valnerina	26/10/2016	17:10	42.874	13.124	08.1	5.5
3 rd	Valnerina	26/10/2016	19:18	42.904	13.090	09.6	6.1
4 th	Valnerina	30/10/2016	06:40	42.830	13.109	10.0	6.6
5 th	Aquilano	18/01/2017	10:14	42.531	13.283	09.6	5.7

Damage surveys were conducted in three main phases. Initially promoted after the first main shock on August 24, 2016, the activities were interrupted by the severe earthquakes in October to start again only in November. Finally, the third phase was triggered by the last strong shock in January 2017 (Carbonari et al., 2019). Second and third phases, in some cases, comprised also repetition of previous inspections.

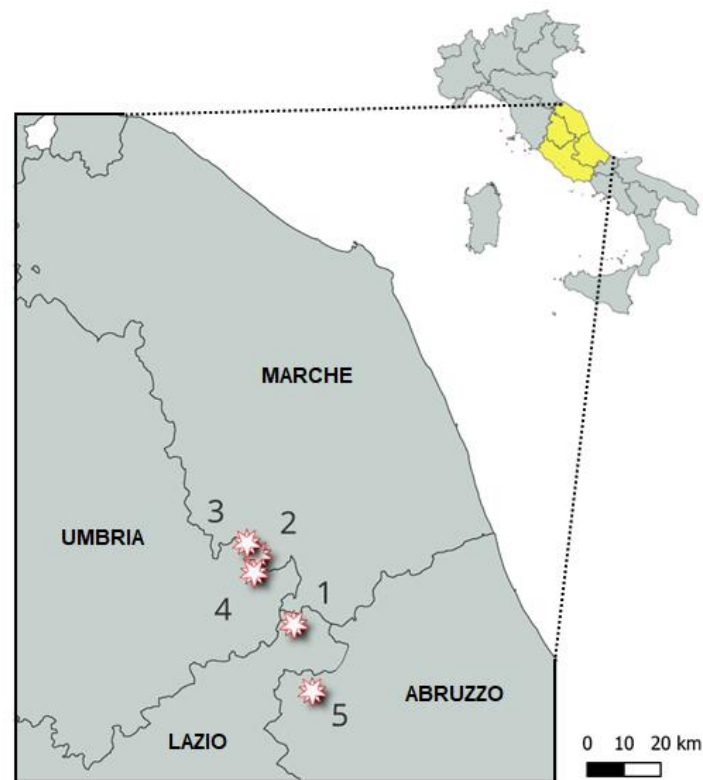


Figure 4.2: Map of the epicentres.

4.2.2 Dataset

Data about the investigated bell towers were extrapolated from the Da.D.O., Database of Observed Damage (Dolce et al., 2019), within a collaboration with a research group from the University D'Annunzio of Chieti-Pescara.

Da.D.O. is the Italian WEB-GIS platform that stores and catalogues the digitised inspection forms of unreinforced masonry and monumental buildings, namely masonry churches, among other assets struck by past seismic events. The extensive sample gathering is paramount and allows not only to assess the seismic risk at different scales based on past events, but also to develop models to forecast future possible damage scenarios, thus supporting the civil protection activities.

The collected data were registered during field inspections by means of specific survey forms for each building typology (e.g., regular buildings, churches, palaces, etc.). The forms have been conceived by the Italian Civil Protection Department for post-earthquake damage assessment within the framework of the emergency technical activities and comprise information about: location (site characteristics, urban context, and address), architectural and structural features (construction date, plan layout vertical system, material, existing cracks, recent interventions, and state of maintenance), damage (observed damage levels, in the most recent versions according to the EMS-98 scale), seismic input

characterisation (macroseismic intensity and PGA) and relevant exposure-related features (use, time usage and peak time), among other details, such as the date and the level of accuracy of the survey. The platform allows filtering for distinct seismic events and building typologies. Information about the historic masonry towers are retrieved from Da.D.O. as part of the broader surveys of the churches affected by the seismic event. In the church survey form, specific features strictly referring to the bell towers or, in any case, relevant for their assessment, can be extrapolated, namely: location, geometry, tower typology, damage grade, and intensity measures characterising the seismic input. Beside the specific coordinates, the form directly reports the region and province/district where the church is located. The available geometrical data refer to the length of the two base sides (rectangular or squared cross-section) and height, measured or estimated. The tower typology indicates the type of interaction between the towers and the surrounding buildings, by identifying recurring plan configurations, namely isolated, confined, integrated, and bell gable, as introduced in Chapter 2. As for the previous applications of the present thesis, target mechanisms comprise damage to the shaft and belfry, neglecting the non-structural elements (e.g., pinnacles, spires and decorations), with their corresponding damage levels assigned in the form through expert judgment during site inspections. Finally, the intensity measures included in Da.D.O. for each asset are MCS intensity and PGA. These values, provided for the churches, are assumed to be the same for the towers macroelements.

At the time of the consultation of the database, a total of 3356 churches were recorded for the Central Italy seismic sequence. However, different filters were applied to obtain a more reliable dataset focused on masonry bell towers only. In particular, to ensure that the same number of recorded damage levels was available for the shaft and belfry mechanisms, all the cases characterised by absence of one of the two elements or missing damage data were excluded. Moreover, the bell gable typology was not considered, given the focus of the thesis on historic towers and slender structures, as in Chapter 3. For the sake of clarity the remaining three classes of interaction are exemplified in Figure 4.3. Finally, to avoid issues related to completeness of the survey, towers located in areas with macroseismic intensity (I_{MCS}) lower than 5 or no information available were discarded. Indeed, as the area of low and very low intensity expand drastically, far from the epicentre, the likelihood of missing survey data for many undamaged assets increases dramatically, with the consequence that most of the recorded surveys in this wide region may have addressed churches because of their peculiarities or of an anomalous damage level at such intensities.

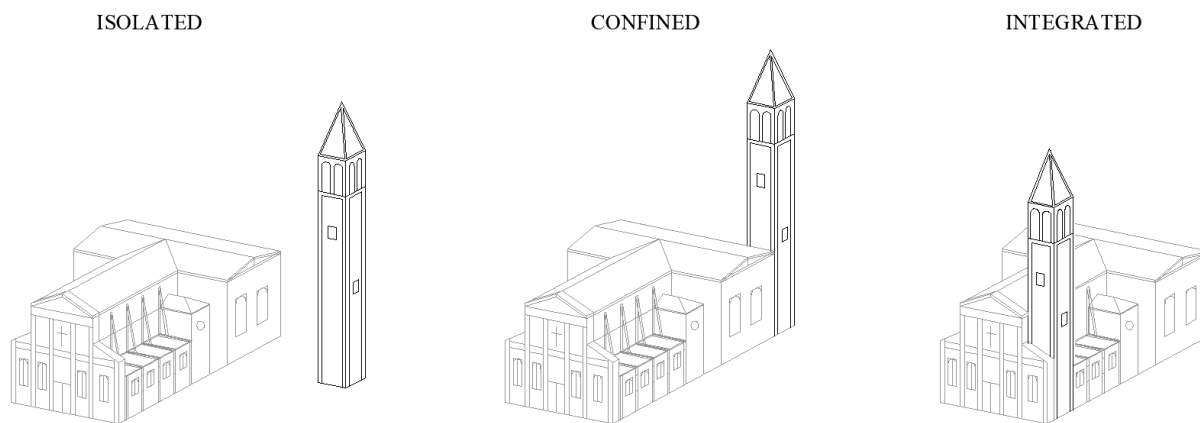


Figure 4.3: Tower typologies considered in the present Chapter.

This issue, well-known in literature (Perelli et al., 2019; Rossetto & Ioannou, 2018; Tatangelo et al., 2024; Zucconi et al., 2022), may be tackled for residential buildings relying on census data. However, effective strategies for monumental buildings have not been developed or extensively applied yet and are outside the scope of the present thesis. The filtering process allowed to gather 794 historic masonry towers with potential activation of both damage mechanisms. After cleansing the data, the global damage index for the tower macroelement was computed as in Chapter 3 and in (Testa et al., 2024). These 794 towers constitute the entire dataset analysed hereafter, also indicated for simplicity GT0.

Focusing on the date of the inspection, compared to the date of the main shocks of the sequence, it was possible to identify four groups of towers. Most of them were inspected between the first and second shocks, or after all main shocks of the seismic sequence. Very few towers were inspected either between the third and the fourth shocks, or between the fourth and the fifth shocks. Since two seismic shocks have occurred on October 26, 2016 and the available documentation does not allow to identify the hour of the inspection, it is assumed that the towers were inspected after both seismic events occurring in the same day. Figure 4.4(a) illustrates the spatial distribution of the epicentres of the main events during the seismic sequence and the towers, identified according to the last main event prior to the survey date. The recorded towers are mainly concentrated to the north and north-east of the epicentres.

The long duration of the seismic sequence and the relationship between the date of inspection and the main shocks of the sequence require a careful definition of the intensity measure value adopted to characterise the input to the tower. In particular, the Da.D.O. database provides a single value of the macroseismic intensity for each tower location but five values of PGA, one per each main event. Following a well-established approach (Carbonari et al., 2019; Rossetto & Ioannou, 2018; Sisti et al., 2023), the PGA at the tower site that was considered for the analyses corresponds to the maximum

acceleration among the events previous to the inspection date. Figure 4.4(b) shows the spatial distribution of the towers, distinguished by the specific event, prior to inspection, in which the maximum PGA at the site was registered. In general, the maximum PGA at the site of the towers was induced by the seismic events with greater magnitudes, namely, the shocks occurred on August 24 and October 30, 2016. A non-negligible number of towers suffered the maximum PGA during the third shock, namely on October 26, also characterised by a rather strong intensity, due to their proximity to its epicentre.

Based on the experienced PGA and the date of inspection, the investigated towers of the whole dataset (GT0) were grouped into four partially overlapping subset: (i) 219 towers inspected between first and second shocks (GT1); (ii) 487 towers inspected after last main shock (GT2); (iii) 349 towers inspected after fourth main shock for which the second highest PGA in the events prior to inspection is lower than 0.1g (GT3); (iv) 212 towers inspected after fourth main shock for which the second highest PGA in the events prior to inspection is higher than 0.1g (GT4). The characteristics of these datasets and their sampling are further described in the following analyses.

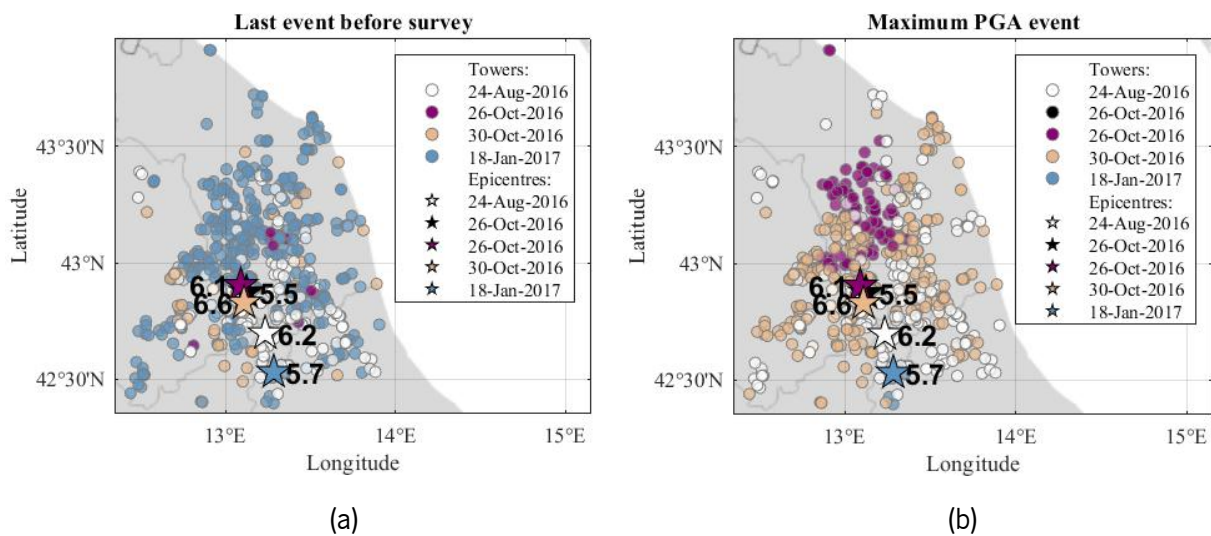


Figure 4.4: Localisation of the epicentres with their magnitude and geographical distribution of the towers distinguished by: (a) last seismic event before the survey date; (b) seismic events causing the maximum PGA at site before survey date.

Besides analysing the potential accumulation of seismic effects during the sequence, in the present Chapter an attempt is made to identify homogeneous classes of towers based on their attributes described in the inspection form. In Figure 4.5, a first classification based on the geographical location is carried out, illustrating the distribution of all inspected towers across the four regions and the ten provinces affected by the Central Italy seismic sequence.

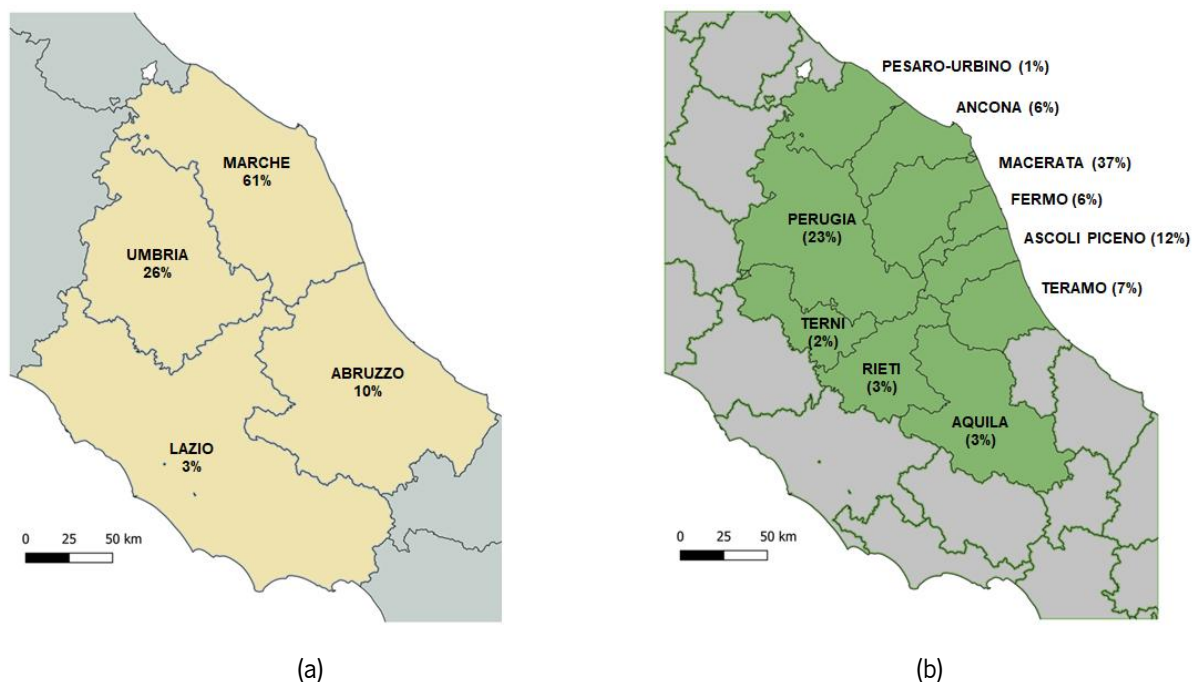


Figure 4.5: Towers geographical distribution: (la) regional-based; (b) province-based.

Most of the towers are located in the Marche and Umbria regions, representing 61% and 26% of the dataset, respectively. A similar trend is observed at the provincial level, where the majority of towers are situated in the Macerata province of the Marche region and the Perugia province of the Umbria region, accounting for 37% and 23% of the dataset, respectively. The remaining regions and provinces contain fewer towers, with a maximum of 12% of the dataset found in any other province. Subsets based on geographical location were created, taking into account possible peculiarities in local construction traditions and historical events. For this purpose, the dataset was divided into four subsets, which include towers from the Marche and Umbria regions, and towers from the Macerata and Perugia provinces, respectively. Other regions and provinces were not considered due to the limited sample sizes, which is not statistically significant.

A second classification focused on the geometry. Some records pertain to inspections conducted at different times for the same churches (e.g., Abbazia SS. Rufino & Vitale, Cathedral of San Catervo or Church of Madonna delle Grazie). A comparison of the geometric and typological data registered in the survey forms from different inspections revealed discrepancies, which were corrected by checking the original sources and remotely inspecting the towers. When the data could not be reliably corrected, the affected towers were excluded from the following subsets based on geometry. A similar process was applied to instances where available geometrical data appeared unrealistic. This reflects the well-known issue of potential errors in data collection, transmission and digitisation during field post-earthquake investigations, likely due to the limited number and experience of technical personnel, affecting the

accuracy and reliability of the information (Baggio et al., 2007; Rossetto & Ioannou, 2018). It is possible that other cases within the dataset may require attention due to errors. However, a detailed evaluation of these cases is beyond the scope of this study.

Four continuous independent variables were identified: three directly extracted from the Da.D.O., namely, the minimum base side (L_{min}), the maximum base side (L_{max}), and the total height (H_{tot}), and one indirectly computed as the ratio of the total height to the minimum side length at the base, referred to as aspect ratio (H_{tot}/L_{min}). A summary of their statistics is reported in Table 4.2. Additionally, the number of samples with available data is reported for each variable. This number, 396 out of the 794 towers in the entire dataset, highlights a certain lack of completeness in the forms. This is a common issue when many assets must be quickly inspected by a few operators within a very short timeframe, a situation often seen in post-disaster surveys (Rossetto et al., 2015; Rossetto & Ioannou, 2018). The variable distributions are illustrated in Figure 4.6. The distributions are right-skewed and unimodal. L_{min} and L_{max} present very similar statistics, except for the maximum values that for L_{max} reach 14 m, likely due to outliers in the distribution, since most of the towers have square cross-section and both side lengths are often shorter than 5 m, mostly in the range 2-4 m. The height ranges from a minimum of 8 m to a maximum of 53 m, with a concentration of towers below 20 m high. Finally, the aspect ratio varies between 2 and 11, with most of the samples presenting a ratio smaller than 9 and a high concentration in the range 3-6. Besides the maximum range of variation of the distribution, Table 4.2 reports the 5 and 95 percentiles of the geometrical data. This range provides a better insight into the parameter variations, reducing the effect of few extreme values especially in the lower bound, characterised by particularly small-sized and stocky configurations. At the same time, due to the right-skewness, the 95 percentile limits significantly the upper bound. The statistics here reported are in good agreement with those computed for other datasets of existing masonry towers analysed within the present thesis (see Chapter 5, Section 5.3.2).

Table 4.2: Main statistics of the geometrical data.

Variable	No. Samples	μ	m	Min	Max	σ	5%	95%
L_{min}	396	3.67	3.50	1.50	9.80	1.32	2.00	6.00
L_{max}	396	3.82	3.50	1.50	14.00	1.46	2.00	6.00
H_{tot}	396	18.11	16.00	8.00	53.00	7.37	10.00	34.00
H_{tot}/L_{min}	396	5.16	5.00	1.82	11.00	1.74	3.00	8.52

μ = mean; m = median; σ = standard deviation; 5% = fifth percentile; 95% = ninety-fifth percentile.

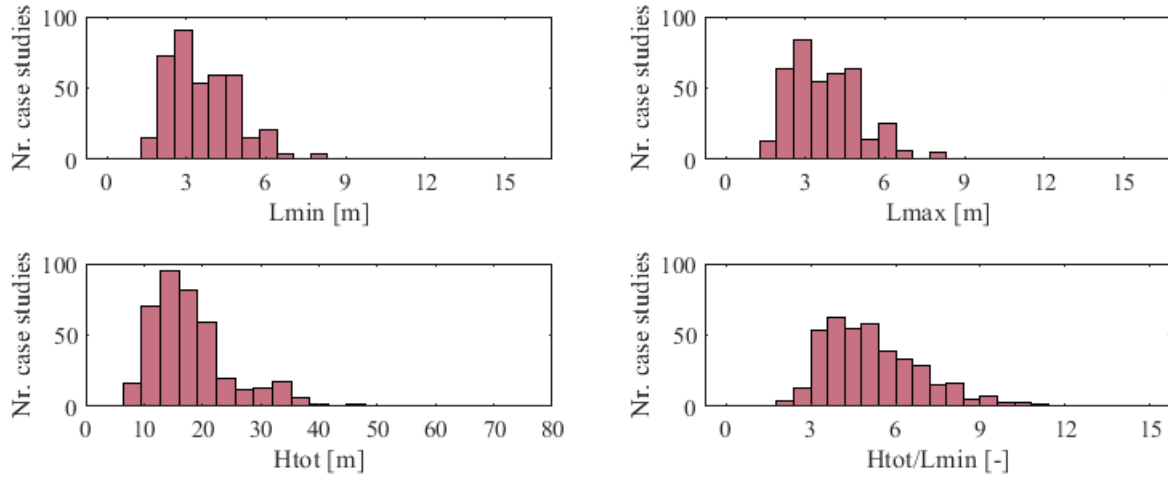


Figure 4.6: Distributions of the geometrical characteristics of the towers.

Among the geometric data, the aspect ratio is an indirect parameter that significantly affects the seismic behaviour of towers. A higher aspect ratio tends to result in a cantilever-like response, characterised by larger lateral deformations at the top. Conversely, lower aspect ratio values lead to an overall shear behaviour, primarily involving the resisting masonry walls. Although the aspect ratio is crucial for describing the response of this building typology, the literature lacks studies defining thresholds that explain the differing structural behaviours. Notably, (Sepe et al., 2008) classifies the towers by proposing an aspect ratio threshold of 4. This threshold was adopted in the present study to define two subsets, which include 120 towers (15% of the dataset) with aspect ratio equal or smaller than 4, and 276 towers (35% of the dataset) with ratio larger than 4, respectively. It is worth noting that for about 50% of the samples included in the dataset the available information does not allow an estimation of the aspect ratio.

A third classification focused on the plan configuration due to its significant impact on the seismic response of historic masonry towers. While towers often exhibit a regular arrangement in plan due to their vertical development, lateral interactions can influence their seismic behaviour, particularly due to adjacent buildings, causing sudden variations of stiffness along the height. Although it is impossible to account for all the possible real-world scenarios in plan configuration, the inspection form groups the towers based on the type of interactions in three main categories: isolated (no interaction), confined (partial interaction), and integrated (full interaction). Therefore, two subsets were created, which include 312 confined towers (39% of the dataset), and 233 integrated towers (29% of the dataset), respectively. Isolated towers are very rare (3% of the dataset, 20 towers), thus they cannot be considered for a statistical analysis. Likewise for the geometry, a significant lack of completeness of the form was

observed, with information about the plan configuration unavailable for 29% of the dataset, namely 229 towers.

Finally, the towers were classified according to their state of maintenance. The inspection form provides a qualitative assessment of the general condition of the churches in the Da.D.O., using the following categories: Good, Acceptable, Poor, Very Poor, and Work in Progress. Similar to the previous features, the state of maintenance is an indirect parameter expected to influence the seismic response of historic masonry towers. Although the assessment in the form does not directly refer to this macroelement but to the whole church, hereafter it is assumed that the same evaluation is applicable to the tower. Therefore, three subsets were created, which include 419 towers in a good state of maintenance (53% of the dataset), 234 towers in acceptable state of maintenance (29% of the dataset), and 85 towers in poor condition (11% of the dataset), respectively. Towers in very poor conditions were not enough to form an independent subset for statistical analysis, comprising only 22 samples (3% of the dataset). Similarly, a very small percentage of towers presented work in progress (1% of the dataset, 7 towers). Thus, none of them were grouped into any subset. Information about the state of maintenance of the remaining towers was not available (4% of the dataset, 29 towers).

4.2.3 Critical aspects

Several assets in the area under study were previously damaged by the destructive 1997 Umbria-Marche earthquake, being then subjected to repairs and strengthening (Carbonari et al., 2019). In many churches that suffered extensive damage, effective mitigation measures were implemented before the new events, including light ring beams, tie rods and other metal restraints, limiting the formation of various mechanisms. During the Central Italy sequence, these low-impact interventions led to the migration of damage to the discontinuities caused by the repairs or to other areas of the structures. Thus, damage emerged according to mechanisms for which no intended strengthening was implemented and likely for a higher level of seismic action (Parisi et al., 2018; Sferrazza Papa & Silva, 2018). This peculiar characteristic of the churches in the area and their macroelements, including the bell towers, should be properly considered in the evaluation of the vulnerability towards a generalisation of the results.

Additionally, it is worth mentioning that studies conducted in the area suggested a relevant contribution of site effects to the damage level, with both stratigraphic (e.g., localised alluvial and softer deposits in areas otherwise featuring outcropping rock and shallow bedrock) and topographical characteristics (e.g., ridges and sharp hills) likely causing amplifications in some of the municipalities closer to the epicentres (e.g., Accumuli, Amatrice, Visso or Camerino) (Sextos et al., 2018).

A further source of variability is related to the characteristics of the seismic input in the various shocks. The spectra of the records in the August 24 and October 30, 2016, events showed a maximum amplification in the short period range, with a clear peak within 0.1 s and 0.5 s. However, in the case of October 30, the spectra presented an amplification extended to the medium period range until about 1.0s (Acito et al., 2023). As the first bending modes of many bell towers fall in the 0.3-1.0 s range (Testa et al., 2023b), the amplification in the October 30 shock may justify a dramatic increase of damage with respect to the first event of the sequence. Indeed, the October 30 shock may have affected slender towers, whose natural periods already fell within the amplification range of this ground motion, and towers whose original lower periods lengthened due to prior damage, bringing them into the same range. The interaction between the effects of structural strengthening and amplification due to site effects or earthquake-to-earthquake variability requires further investigation. This is evidenced by the performance of the Civic Tower and the bell tower of S. Emidio Church in Amatrice, which were the only structures to survive, albeit with significant damage, in an area otherwise almost completely destroyed (Acito et al., 2023; Jain et al., 2020). However, in the current study, the available information and the scale of the analysis do not permit a comprehensive exploration of these factors. Their thorough investigation remains an important focus for future research.

4.3 VULNERABILITY ASSESSMENT: CUMULATIVE DAMAGE

In many seismically active countries, most recorded major earthquakes have been characterised by at least two or more strong shocks of comparable intensity during a period of time, ranging from the same day or a few days to several months, and in a limited geographical area (Bessason et al., 2022; Bessason & Bjarnason, 2016; Casolo, 2017; Kazama & Noda, 2012; Ruiz-García, 2012; Senaldi et al., 2015). As a result, settlements located within a few tens of kilometres from the epicentres have a significant probability of being affected by multiple strong events. Often, these events occur so close together in time that there is insufficient opportunity for inspections or, more importantly, for implementing temporary stabilisation measures. Therefore, worsening of structural conditions due to the cumulative effect of repeated seismic events is likely. The evolution of damage in masonry structures has demonstrated to often progress non-linearly, with a tendency for abrupt increases in damage extent. This is driven by the brittle behaviour of the structures and depends on the effectiveness of the mitigation measures implemented (Sextos et al., 2018). The present Section investigates the effect of cumulative damage providing a systematic attempt to analyse how the towers response evolved during the Central Italy seismic sequence.

4.3.1 Damage levels and Damage Probability Matrices (DPMs)

Figure 4.7 shows the spatial distribution of the entire towers dataset (GTO) grouped by damage grades assigned to the individual damage mechanisms as well as the overall macroelement. To enhance visualisation, damage points are represented using a well-established colour scale, moreover, larger points represent more extensive damage.

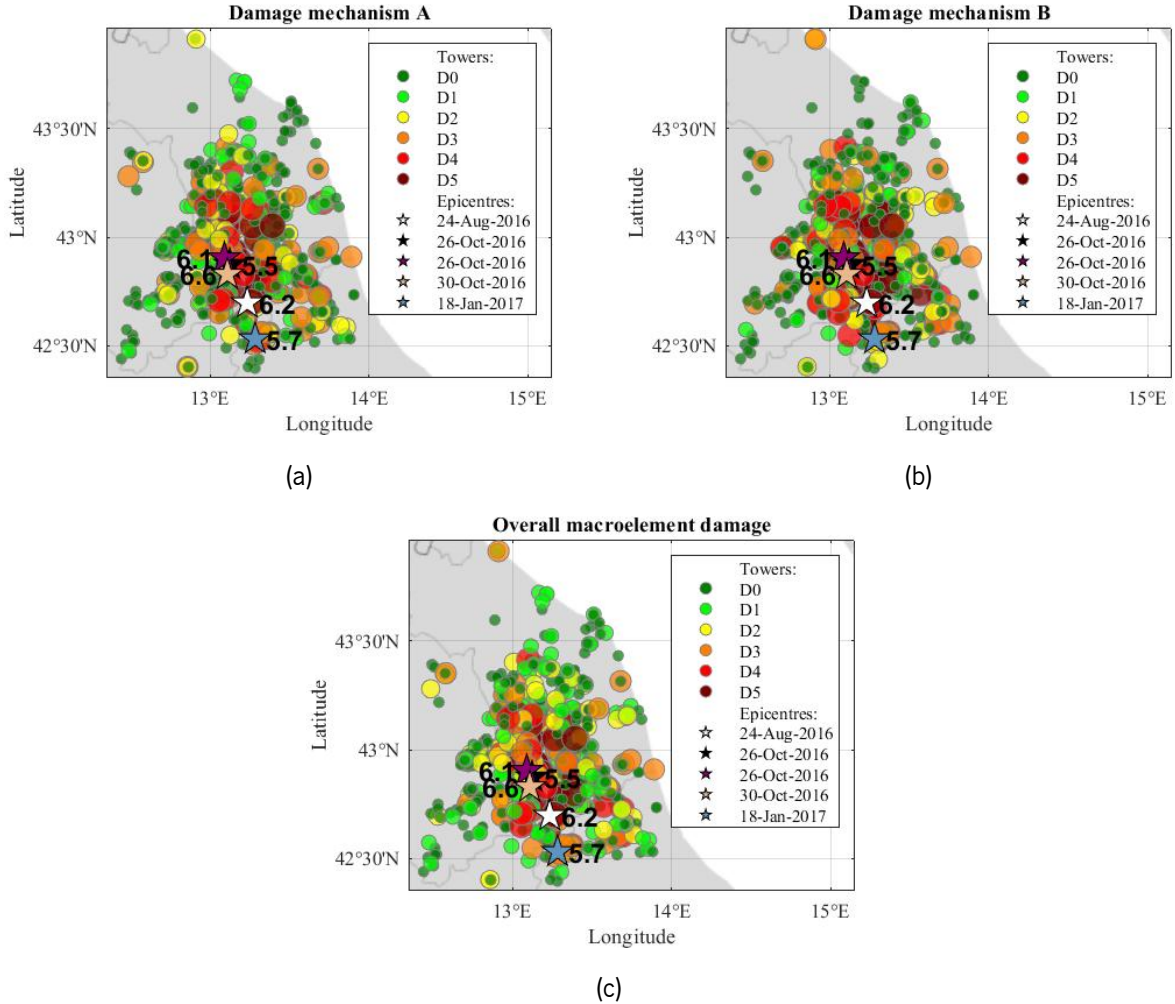


Figure 4.7: Geographical distribution of the towers grouped by the damage grades assigned to: (a) mechanism A; (b) mechanism B; (c) overall macroelement.

The towers in GTO dataset cover the entire range of damage grades. Higher levels of damage are concentrated near the epicentres of the first and fourth seismic events, mainly to the north and north-east. As expected, there is a visible correlation between damage severity and distance from the epicentres of the main shocks, although some cases of significant damage at greater distances are also observed. To further investigate this trend, the damage grades of each tower (for both individual mechanisms and overall macroelement) were correlated with the distance to the epicentre of the event that caused the maximum PGA at its site prior to inspection. These results are grouped into bins of 10

km based on the epicentral distance. The average damage grades are then computed and plotted against the average distance of the towers in each bin (represented by red diamonds in the graphs). The results are illustrated in Figure 4.8. Additionally, a plot of the maximum PGA at the tower sites against the epicentral distance is provided together with the average value for each 10 km bin. The average values for very high distances (> 80 km) are considered statistically non-representative due to the limited samples available, thus ignored subsequently.

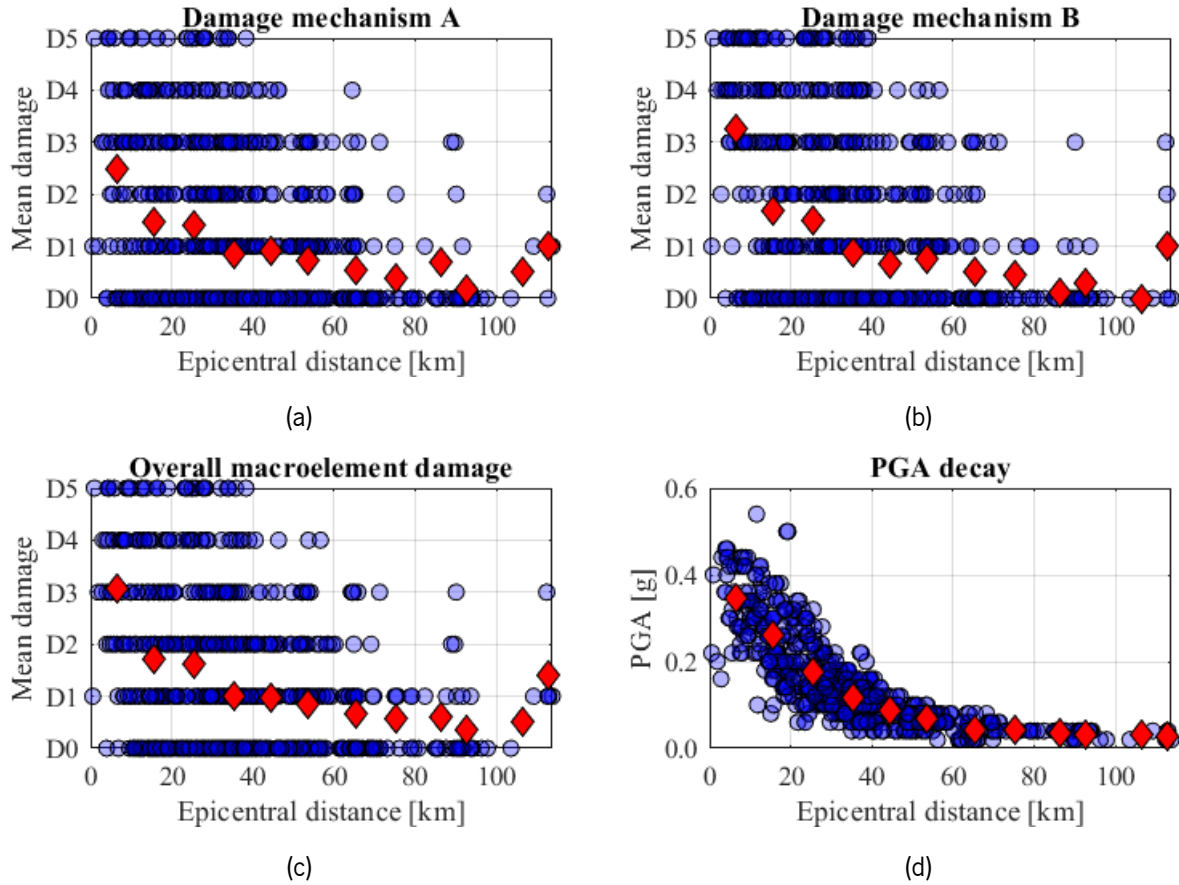


Figure 4.8: Decay of seismic effects with the epicentral distance: (a) damage grades for mechanism A; (b) damage grades for mechanism B; (c) damage grades for overall macroelement; (d) maximum PGA registered at the tower site. Scatter plot (blue points) and mean values for 10 km bins (red diamonds).

A large scatter can be observed in the damage grades of different towers at the same range of distance from the event causing the maximum PGA at their site, especially within few tens of kilometres. However, a clear overall decay emerges in all the average values, that drops quickly within the first 20 km and goes below D1 between 30 and 40 km (Figure 4.8a, b, and c). Interestingly, at the same distance, the maximum PGA, which presents less scatter and a clear and constant reduction moving away from the epicentre, goes below 0.1g (Figure 4.8d). Only within the first 20-30 km, the average damage for mechanism B appears higher than the one for mechanisms A, suggesting a likely larger

susceptibility to damage of the belfry with respect to the shaft. However, this difference in the average damage becomes quickly negligible for larger distances.

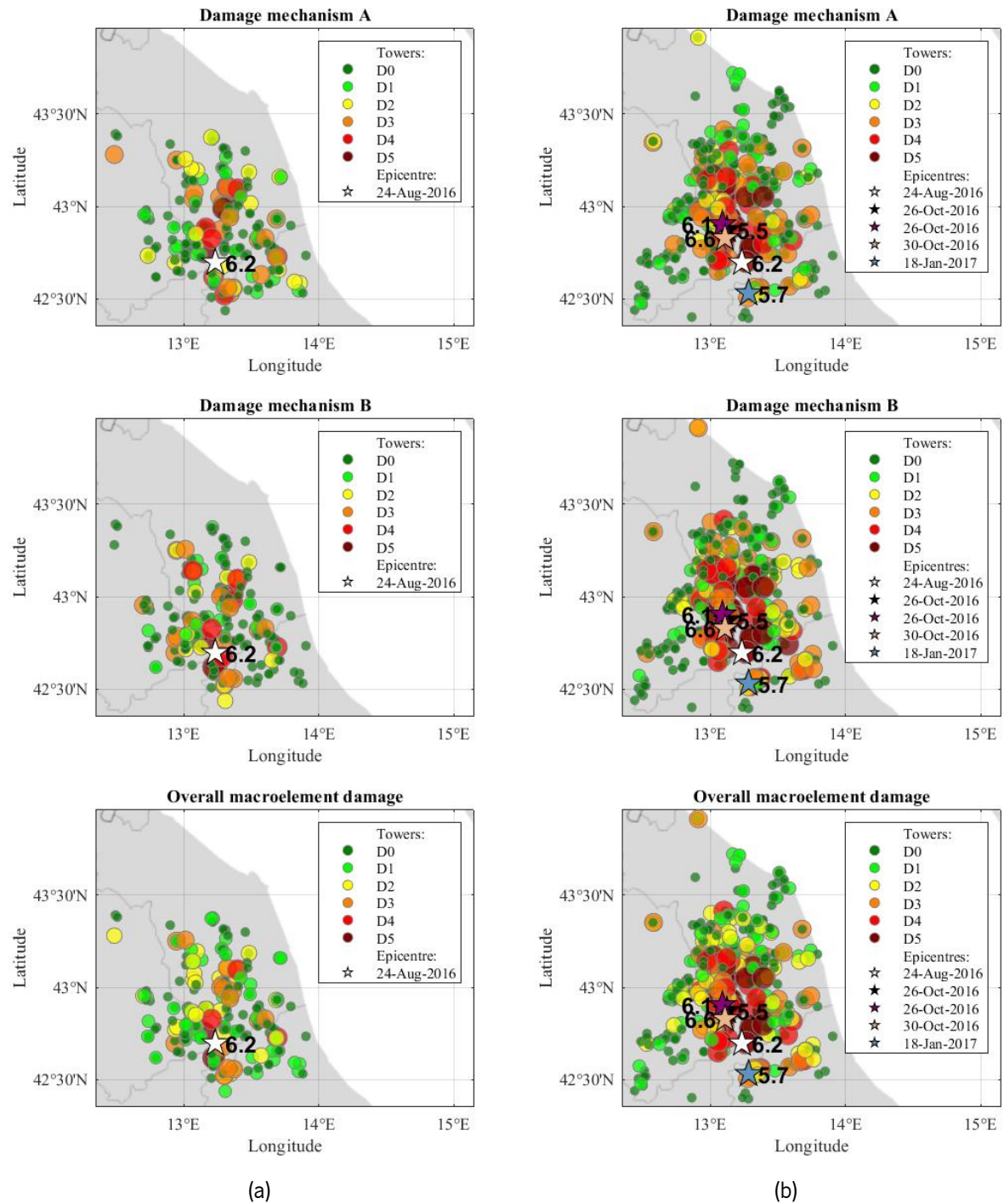


Figure 4.9: Geographical distribution of the towers grouped by the damage grades assigned to individual mechanisms and overall macroelement: (a) GT1, towers inspected after the first shock and before the second shock; (b) GT2, towers inspected after the five main shocks of the seismic sequence.

Figure 4.9 shows the spatial distribution of the 219 towers solely affected by the first main shock occurred on August 24, 2016 (GT1) and the 487 towers likely subjected to cumulative damage caused by multiple seismic events, being inspected after the fifth main shock occurred on January 18, 2017

(GT2). These two datasets were selected because they are the most numerous. They also contain towers with damage grades ranging from D0 to D5. However, in GT1 (Figure 4.9a), towers experiencing higher damage grades are rare and located closer to the epicentre, whereas in GT2 (Figure 4.9b) higher levels of damage affected a greater number, even at larger distance from the epicentres.

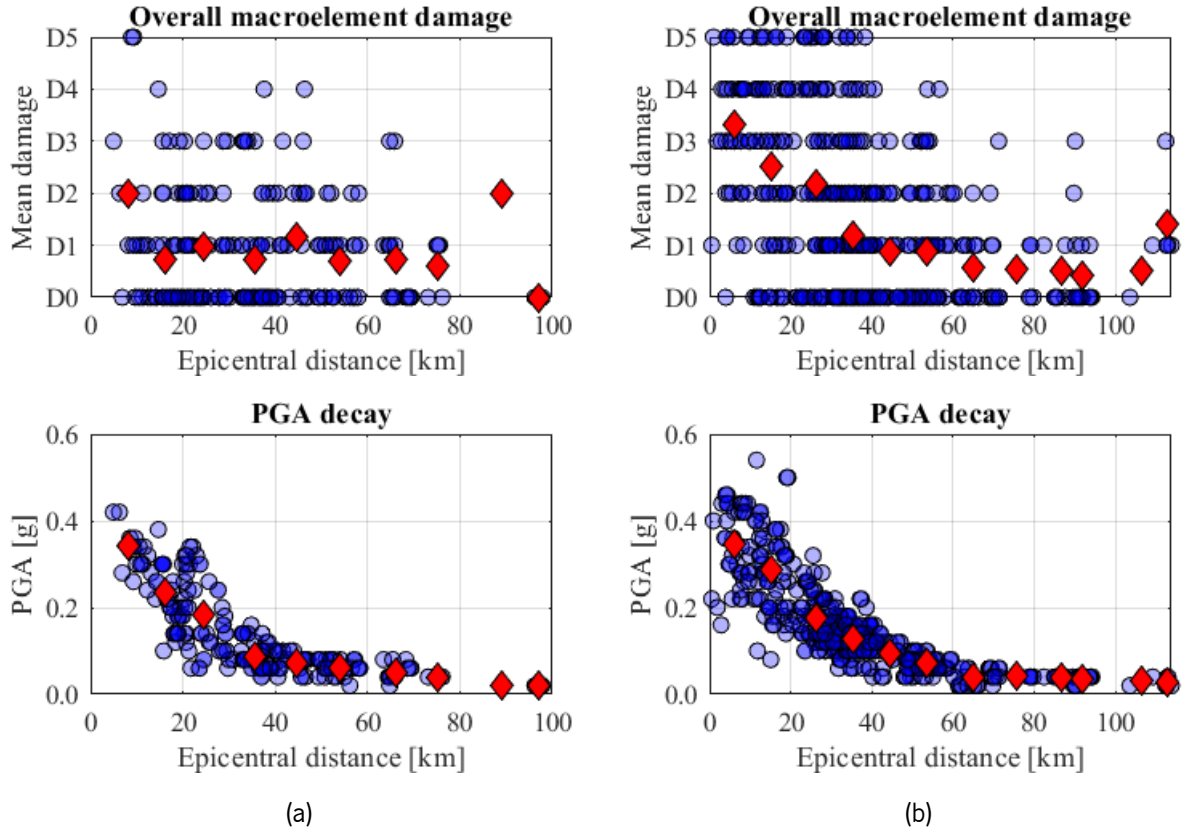


Figure 4.10: Decay of seismic effects with the epicentral distance in terms of overall macroelement damage and maximum PGA registered at the tower site: (a) GT1; (b) GT2. Scatter plot (blue points) and mean values over 10 km steps (red diamonds).

Similarly to GT0, the damage grades of each tower were correlated with the epicentral distance, calculating the average damage for increasing distance bins of 10 km. The results are presented in Figure 4.10, focusing on the overall macroelement for the two distinct subsets. The correlation between the maximum PGA at the sites of the towers and the epicentral distance is also included. It should be noted that the trends obtained for the overall macroelement are representative of those obtained for individual damage mechanisms, and thus, for brevity, the latter are not reported. Again, values after 80 km are ignored due to the limited number of surveyed towers in this range, especially for GT1.

Although there is some variability in the response of individual towers, with a few severely damaged cases within approximately 40 km of the epicentre, most towers of GT1 (Figure 4.10a) sustained moderate to slight damage, despite the magnitude of the first event ($M_w = 6$). The highest average damage, approximately D2, is observed within 10 km from the epicentre. After an initial drop, the

average damage stabilises at approximately D1 for greater distances. In GT2 (Figure 4.10b), instead, the scatter in the damage grade is visibly larger, presenting various towers with severe damage. The average damage within the first 10 km is significantly higher than the one obtained for GT1 and reduces smoothly to less than D1 beyond 40 km of epicentral distance after which it remains almost constant. The noticeable increase in average damage and actual sample damage levels, compared to GT1 can only be partially attributed to the intensity of seismic input at the tower locations. While the maximum PGA is higher in GT2, with several instances exceeding 0.4g, the average PGA experienced by the towers in both cases is similar, showing a comparable decreasing trend that drops below 0.1g at 30-40 km from the epicentre. Comparing Figure 4.10(a and b) with Figure 4.8(c), it can be observed that, within 10 km from the epicentre, the average damage in the entire dataset (GT0) is primarily influenced by GT2. Beyond this distance, instead, GT0 exhibits an intermediate behaviour between GT1 and GT2. DPMs were firstly developed for the entire dataset of towers (GT0), grouped by the individual mechanisms and the overall macroelement damage (see Figure 4.11). In Figure 4.12 and Figure 4.13, the DPMs are developed upon grouping the towers by macroseismic MCS intensity. While the MCS intensity groups follows the discrete values of the MCS scale and PGA ranges with values reported in the database every 0.5, the continuous PGA ranges are defined to ensure a statistically significant number of cases within each bin. Adopting a well-establish mathematical notation, the ranges of PGA are indicated as $[\cdot)$ to stress that the lower bound is included in the range while the upper bound is excluded.

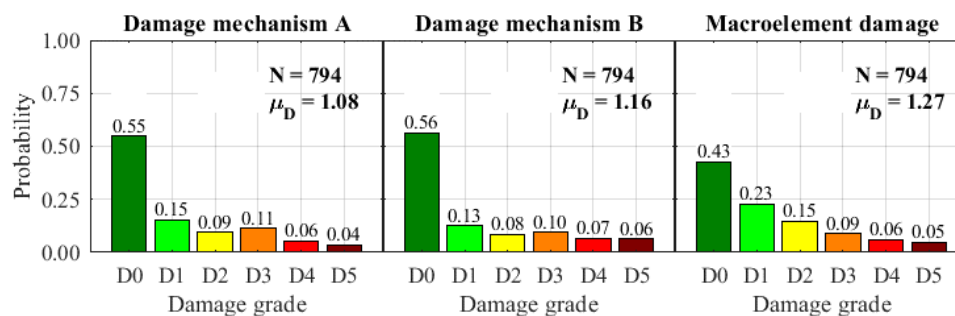


Figure 4.11: DPMs for the entire dataset GT0.

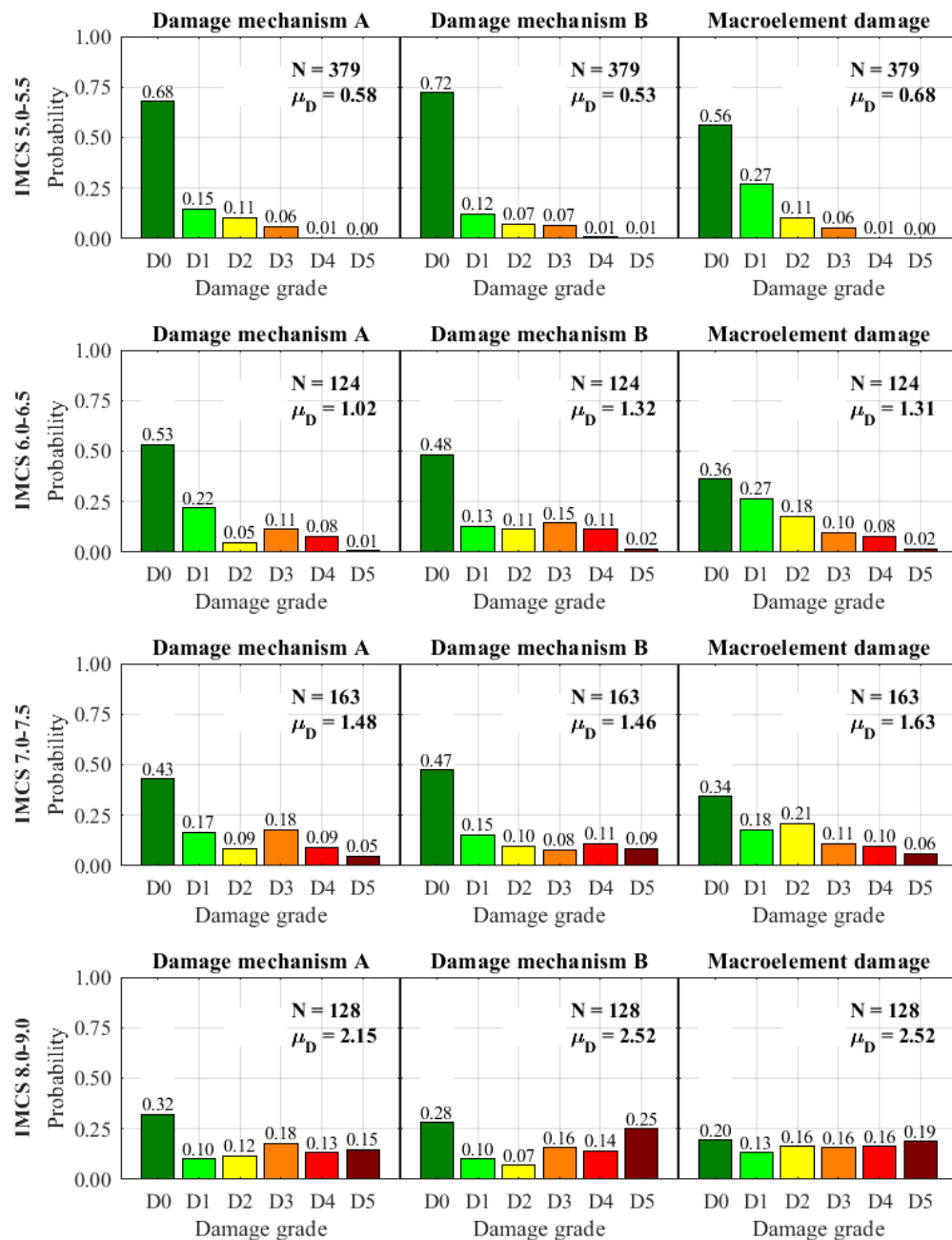


Figure 4.12: DPMs as functions of MCS intensity for GT0.

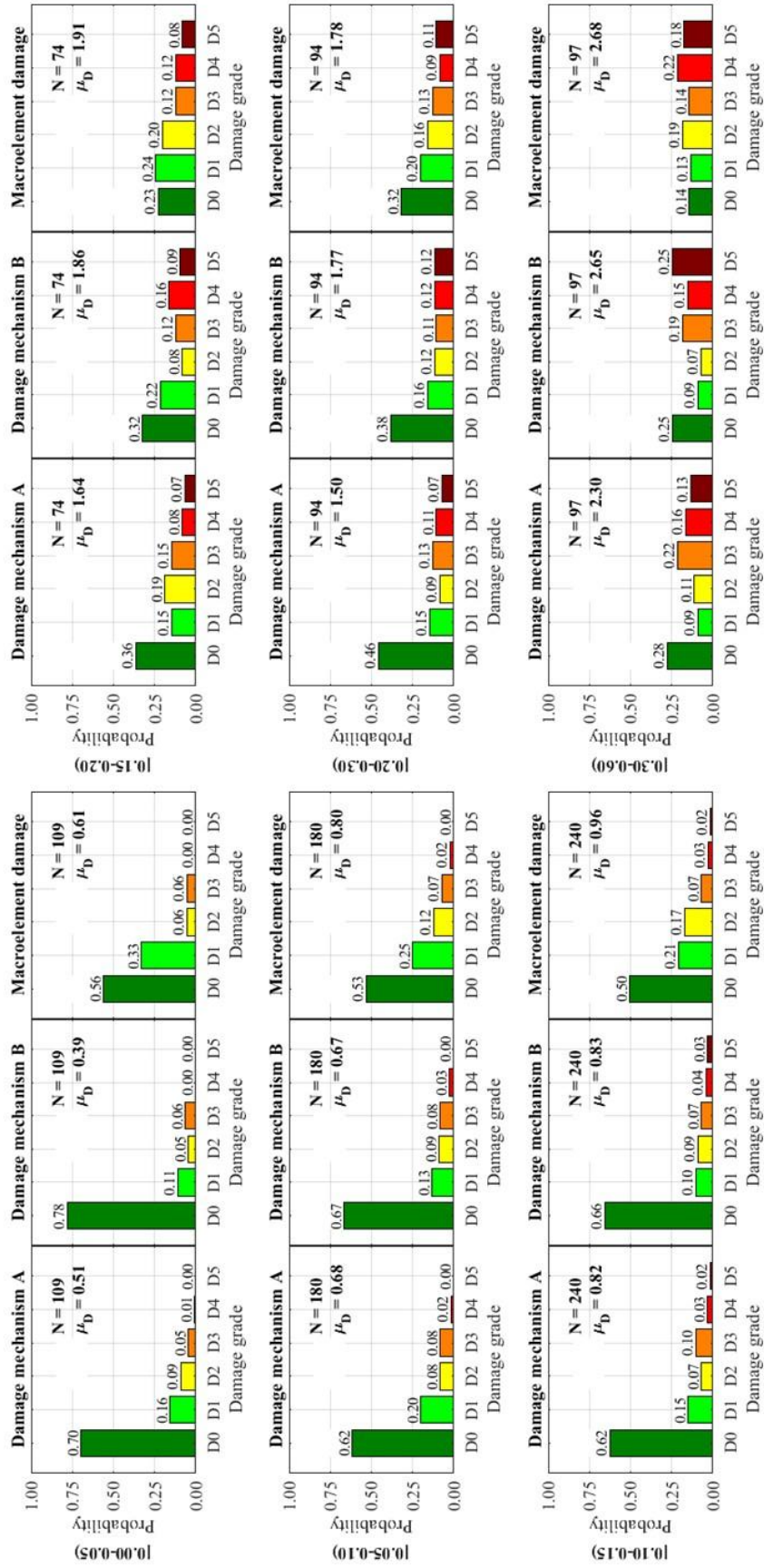


Figure 4.13: DPMs as functions of PGA for GT0.

In both cases, the DPMs are well distributed, showing an agreement with the expected behaviour, as the size and characteristics of the dataset allows to overcome several issues emerged with the open database analysed in Chapter 3. Indeed, while for lower seismic intensity the damage distribution is expected to be right-skewed generally unimodal and showing peaks for lower damage extents, for higher seismic intensity the damage distribution is expected to be left-skewed and showing peaks for greater damage extents. These trends are respected with a few exceptions. In particular, a high number of undamaged towers can be observed for lower seismic intensities, whereas the number of damaged towers grows for greater seismic intensities. This is also reflected in the values of mean damage which become significant for increasing intensity values. However, for the maximum values of the intensity measures, the observed damage is distributed quite uniformly or normally across the different damage grades. Only in the case of mechanism B, a clear left-skewed distribution appears. This reflects the tendency, more evident when considering the PGA rather than the MCS intensity, for the damage distributions and global mean damage values to increase more significantly for mechanism B than for mechanism A, starting from certain seismic intensity thresholds (e.g., 0.1/0.15g in terms of PGA). These results suggest a higher vulnerability of the belfry compared to the shaft of the towers. Additionally, this trend highlights the better correlation between PGA and observed damage.

A condensed form of the DPMs as functions of the PGA, for the individual mechanisms and the overall macroelement damage, is reported in Figure 4.14(a, b, c) repeating the results for GT0 and comparing them with GT1 and GT2.

GT2 samples confirm the findings discussed for GT0 and, in particular, the clear shift from a right skewed distribution of the damage levels for small intensity to a left skewed for high intensity in the case of mechanism B. Mechanism A, instead, is characterised by a lower percentage of collapses, even at the largest intensity, and an unimodal distribution with peaks in D3, for GT0, or in D4, for GT1. The DPMs for GT1, on the other hand, present a different behaviour, similar for most bins of PGA, except for the highest range. It is worth noting that the distribution of damage level for GT1 in the range 0.2-0.3g is influenced by a reduced number of samples.

These obtained results suggest that the greater severity of damage in GT2 was primarily due to the cumulative effects of multiple seismic events.

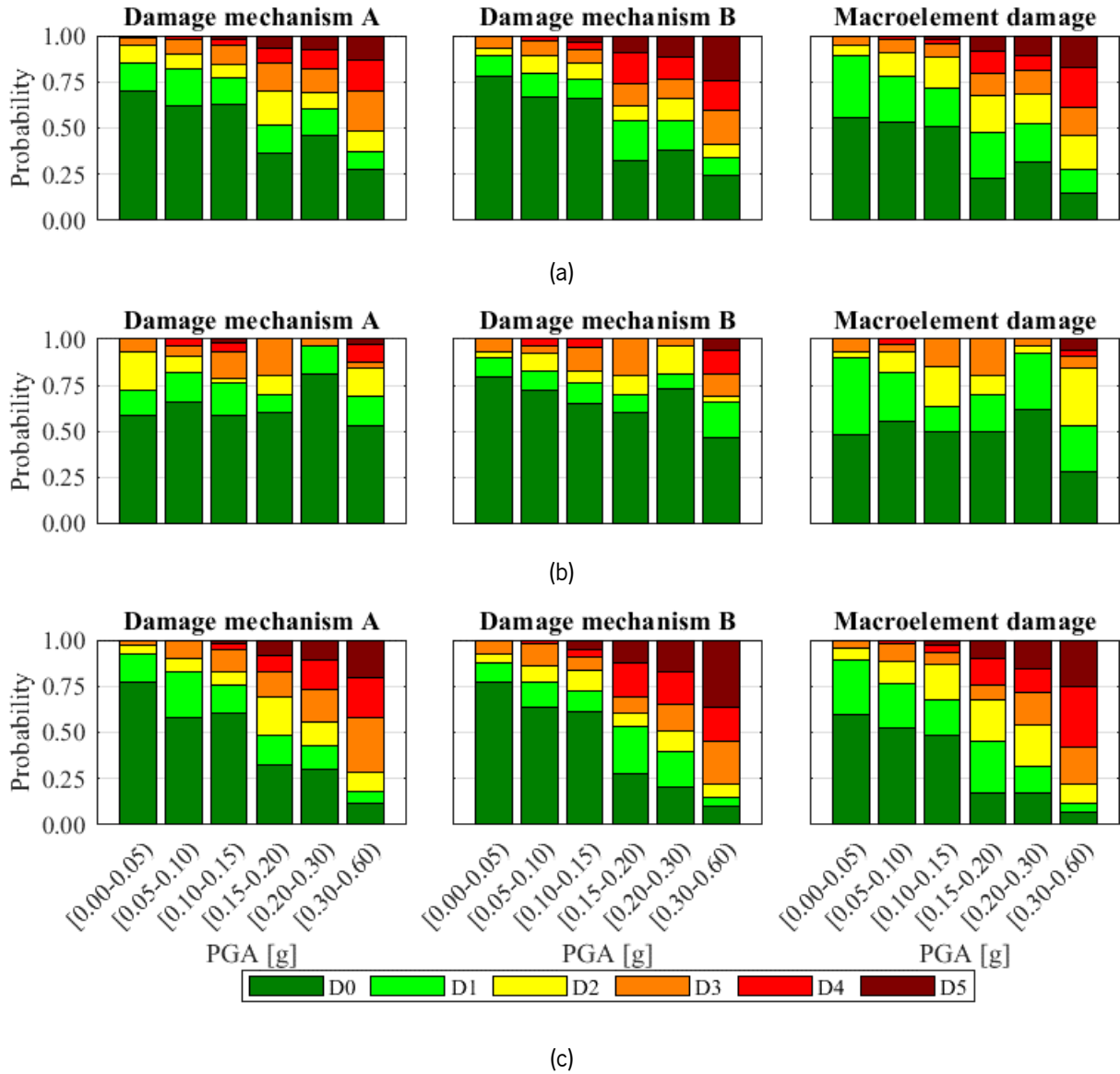


Figure 4.14: DPMs as functions of the PGA for different subsets: (a) GT0; (b) GT1; (c) GT2.

To explore this hypothesis, two new subsets, namely GT3 and GT4 were defined. Both were drawn from towers surveyed after either the 4th or 5th main shock. GT3 consists of towers likely affected by a single event. To this end, it contains 349 samples where the second-highest PGA recorded at each site is less than or equal to 0.1g. As discussed earlier, this threshold is used to expect a mean damage level above D1, suggesting that towers in GT3 likely experienced at most one shock capable of causing at least moderate damage. On the other hand, GT4 includes 212 towers where the second-highest PGA exceeds 0.1g, thus, impacted by at least two seismic events potentially damaging. It is important to note that while GT0 is large, the number of samples becomes too small for a meaningful statistical analysis when attempting to identify towers that experienced first a larger event and then a weaker one

(mainshock-aftershock sequence). Therefore, towers in GT4 are collected regardless of the chronological order of the seismic inputs.

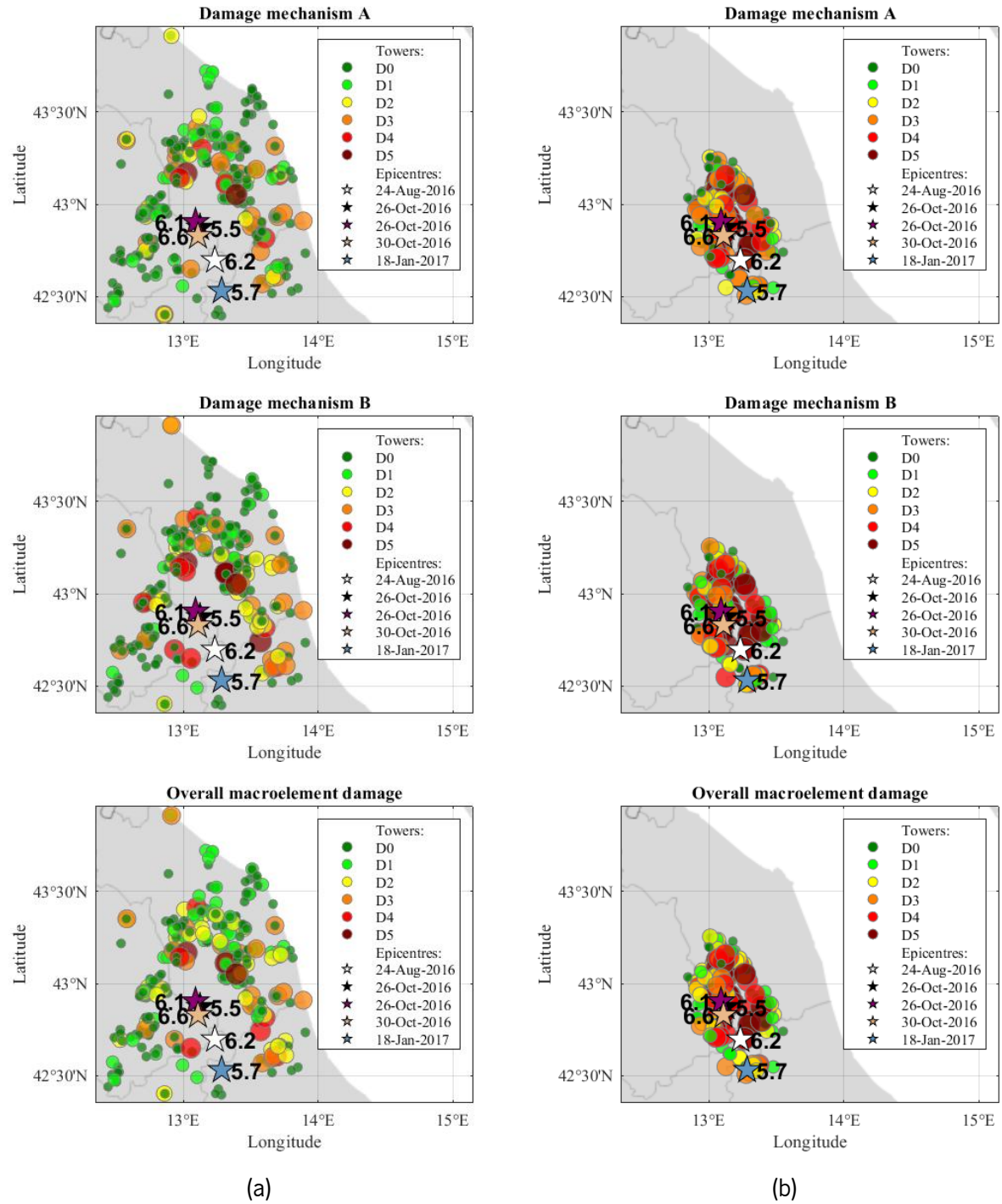


Figure 4.15: Geographical distribution of the towers categorised by the damage grades assigned to individual mechanisms and overall macroelement: (a) GT3, towers affected by a single seismic event (second highest PGA $\leq 0.1g$) ; (b) GT4, towers affected by multiple seismic events (second highest PGA $> 0.1g$).

Figure 4.15 shows the spatial distribution of these two subsets, while Figure 4.16 illustrates the correlation of both damage grade and maximum PGA with the epicentral distance, including the mean

values (red diamonds in the graphs) for increasing distance bins of 10 km. As for GT1 and GT2, the correlation is presented for the overall macroelement damage only.

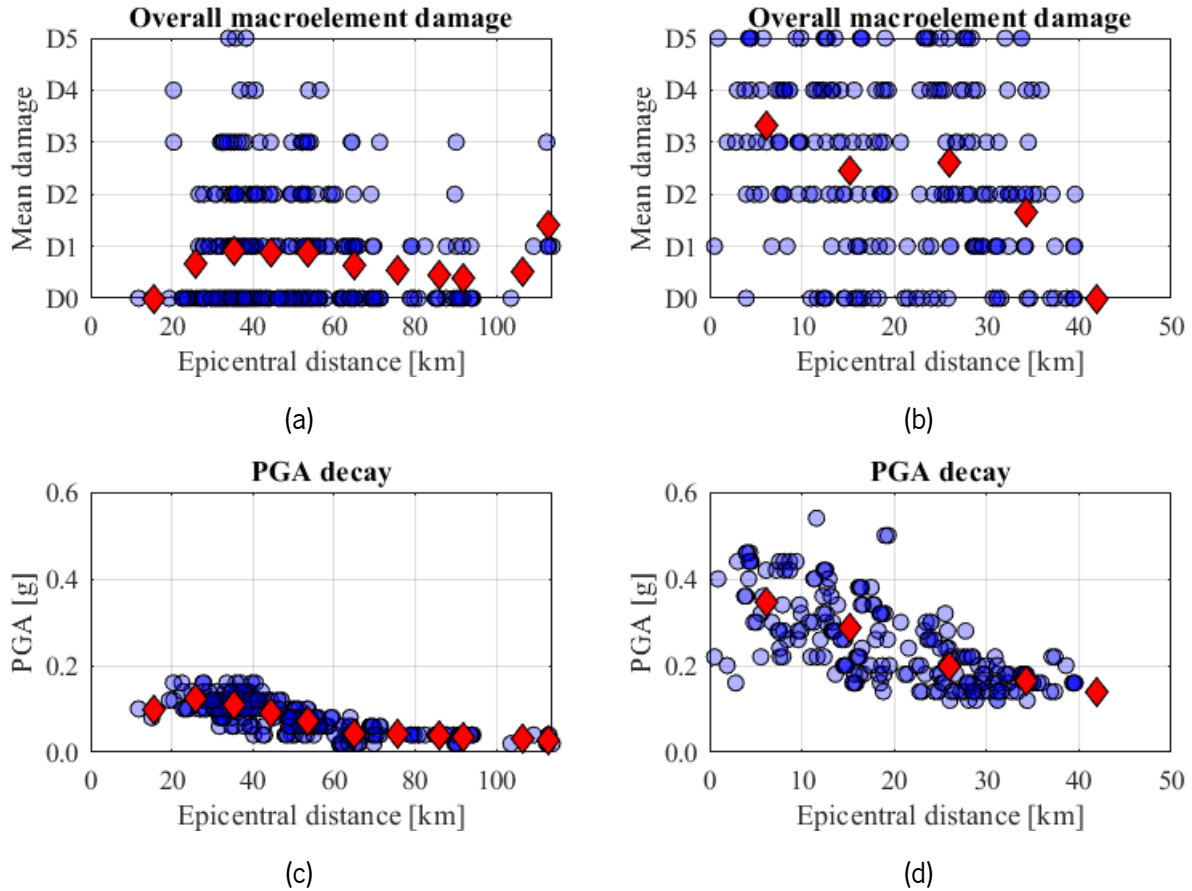


Figure 4.16: Decay of seismic effects with the epicentral distance in terms of damage mechanisms (a, b) and maximum PGA registered at the tower site (c, d) for GT3 (a, c) and GT4 (b, d).

The spatial distribution of the two subsets relative to the epicentres differs significantly. In particular, the towers in GT3 are located at more than 10 km from the epicentre of the event that caused the highest PGA with only a few closer than 25 km. Instead, they are spread over a very wide area, ranging from about 10 km to 100 km. Moreover, with few exceptions, the towers present smaller damage grades. The average damage consistently remains below D1, showing an almost constant trend up to 60 km, followed by even lower values. In contrast, the towers in GT4 are concentrated in a narrow area, mostly within 35-40 km of the epicentre of the event that caused the highest PGA. Only a few cases are located between 40 and 45 km, with none beyond this distance. This distribution reflects the sampling strategy and the trend of PGA values with increasing epicentral distance. While the average PGA for GT3 is around 0.1g or less, GT4 exhibits a wider range of PGA values. The average PGA per 10 km bin constantly decreases with distance, from a peak of approximately 0.4g within 10 km to about 0.2g at 40 km. Beyond this point, no towers are affected by more than one major event. These moderate to

high PGA values induce significant damage, with an average damage grade above D3 within 10 km from the epicentre and above D2 up to 30 km. A noticeable reduction in average damage occurs only beyond 30 km.

Finally, the two highest values of the PGA at each tower location are correlated in Figure 4.17, and the samples are grouped based on the damage grades, through a colour key, considering only the towers featuring a second-highest PGA above 0.1g (GT4).

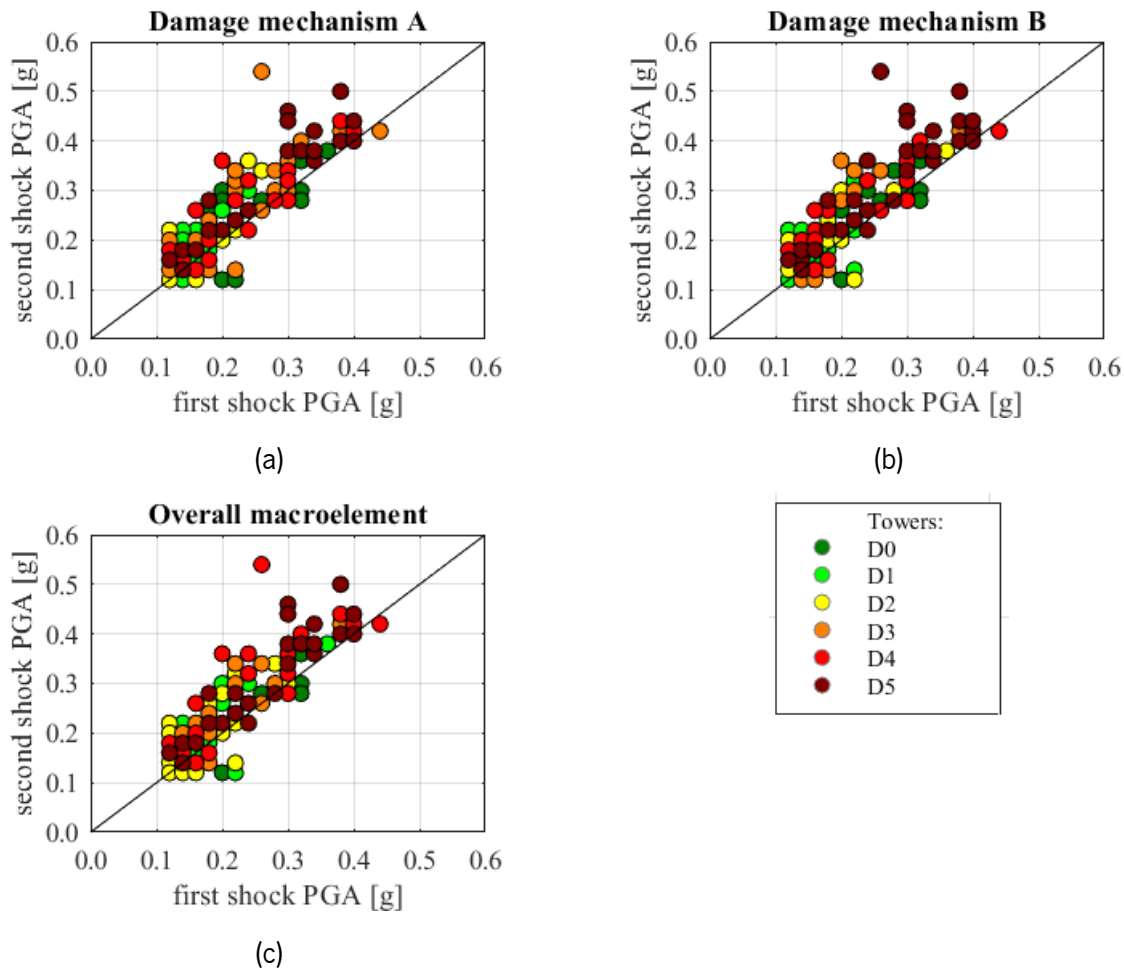


Figure 4.17: First and second-highest PGA correlations for both individual damage mechanisms (a, b) and overall macroelement (c) for GT4, together with the damage grades.

This correlation shows the sequential order between the events that caused the first and second highest PGA, since, in the plot, x-axis refers to the first chronological event and y-axis to the second. The results show that most of the towers included in this subset are characterised by highest PGA in the second event, as, for the Central Italy event, shocks with higher intensity occurred in the middle of the seismic sequence. The classification of the chronological order does not provide any evidence of trends or clustering of the tower damage grades.

4.3.2 Vulnerability functions

The values of the global mean damage obtained from the analysis of the DPMs for increasing seismic intensities (in terms of MCS and PGA) are plotted as indicators of the seismic vulnerability and compared against a set of existing vulnerability models, including the formulation derived in Chapter 3, developed specifically for historic masonry bell towers (Figure 4.18).

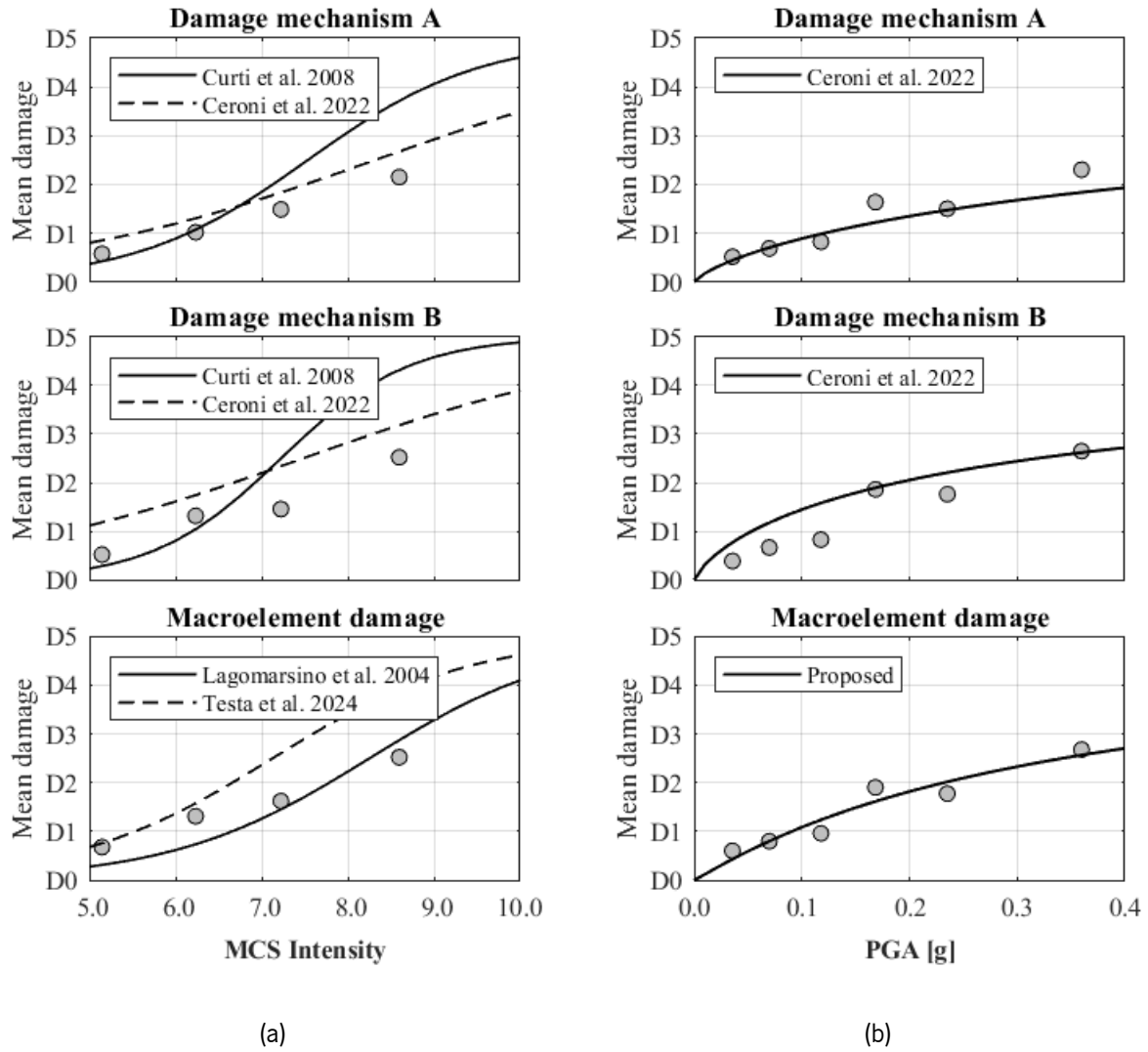


Figure 4.18: Observed mean damage, plotted with circles, compared with existing and novel calibrated (proposed) vulnerability functions for the entire dataset (GT0) as functions of: (a) MCS macroseismic intensity; (b) PGA.

The vulnerability functions are identified through the publication where they have been proposed (Ceroni et al., 2022; Curti et al., 2008; Lagomarsino et al., 2004), and the model developed in Chapter 3 is indicated as in (Testa et al., 2024). For the MCS intensity measure, the observed damage is comparable to other events at lower seismic intensities. However, at higher seismic intensities, the bell towers exhibited a better response than expected, as shown by the comparison with the vulnerability

curves proposed by (Curti et al., 2008; Testa et al., 2024), which were calibrated against various seismic events. The model proposed by (Lagomarsino et al., 2004) for masonry towers and slender buildings, not specifically bell towers, here applied to the entire microelement, presents a better predictive performance in the higher intensity range. Nevertheless, a slight underestimation can be observed in the lower range, where, in any case, the mean damage level remains very low. For the PGA intensity measure, the models identified in literature present a rather high predictive performance when compared with the observed mean damage values. This result is however expected since these models have been calibrated over a similar dataset of towers affected by the same sequence, except for a different filtering and sampling strategy leading to the final investigated set. In the absence of a vulnerability curve for the entire macroelement as a function of the PGA, a recalibration of the parameters a' and b' of the formulation suggested by (Ceroni et al., 2022), Eq. (4.1), is proposed, by fitting the global mean damage-PGA pairs, as given in Table 4.3.

$$\mu_d = 2.5 [1 + \tanh(a' \log(PGA) + b')] \quad \text{Eq. (4.1)}$$

Table 4.3: Calibration of vulnerability function coefficients.

Mechanism	a'	b'	Reference
Mechanism A	0.89	-2.54	(Ceroni et al., 2022)
Mechanism B	0.89	-2.22	(Ceroni et al., 2022)
Macroelement damage	1.20	0.56	Proposed recalibration

At this stage, the available information does not allow for a deeper investigation into the causes of the better performance of the towers compared to other events. It is plausible that this result is influenced by the characteristics of the assets in the area, many of which were retrofitted and strengthened following the Umbria and Marche earthquake. Additionally, the characteristics of the seismic input, at least for some of the shocks, with peaks in a period range that likely did not affect the towers' first modes may have played a role. However, more detailed research is needed to draw definitive conclusions.

In order to provide a better insight into the distinct behaviour of the towers affected by a single main shock and multiple shocks, Figure 4.19 shows the mean damage of GT1 and GT2 plotted against increasing seismic intensity values, in terms of MCS and PGA.

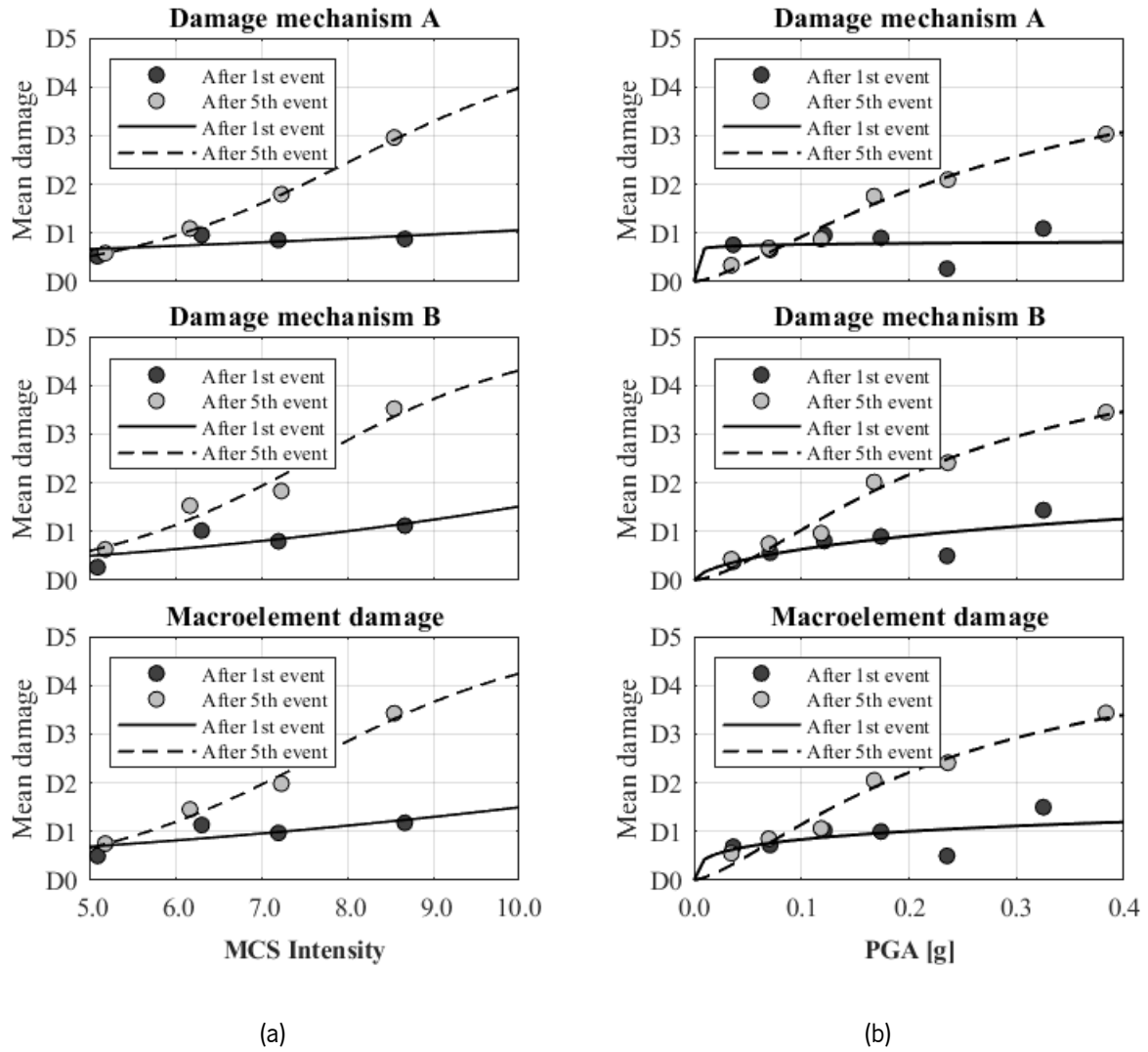


Figure 4.19: Observed mean damage compared with newly calibrated vulnerability models as functions of the MCS macroseismic intensity (a) and PGA (b) for GT1 and GT2.

A recalibration was conducted to propose new formulations for historic masonry towers to predict the expected damage caused by two distinct scenarios: single shock and multiple shocks. The fit of these new formulations with the observational data for both earthquake scenarios is shown in Figure 4.19, with the calibration coefficients provided in Table 4.4 using the same form of the previous equation, Eq. (4.1), in terms of PGA, and, Eq. (2.8), in terms of MCS. It should be noted that the formulation coefficients are calibrated by fitting the global mean damage (according to the EMS98 scale)-MCS pairs, differently from the Eq. (2.8) which instead links the MCS to the global damage index (in 0-1 scale). The intersection of the vulnerability functions in the lower range of the intensity measures likely results from the larger uncertainties in the available data for these input values. This outcome contradicts the physical interpretation of the two curves, as it suggests that the average damage from cumulative events could be lower than from a single event. However, as demonstrated by the decay curves in

Figure 4.10, the threshold of 0.1g average PGA corresponds to a decrease in average damage to D1, observed at around 40 km from the epicentre. It is reasonable to assume that structural damage typically requires, on average, a seismic input greater than 0.1g, and if the maximum input in a sequence is 0.1g, no cumulative damage effect is expected. Therefore, the vulnerability functions for GT2 are considered valid only for intensity values above this threshold.

Table 4.4: Recalibration coefficients for the two distinct subsets and for the seismic intensity measures.

		MCS		PGA	
		a	b	a'	b'
Mechanism A	GT1	0.055	-1.207	0.059	-0.796
	GT2	0.351	-2.832	1.620	0.878
Mechanism B	GT1	0.136	-1.777	0.699	-0.265
	GT2	0.382	-2.907	1.790	1.117
Macroelement damage	GT1	0.098	-1.405	0.371	-0.431
	GT2	0.360	-2.740	1.627	1.021

Finally, GT3 and GT4 were used for validation, by comparing their mean damage correlation with seismic intensity to the trends observed for GT1 and GT2, as well as their vulnerability functions (Figure 4.20). For the sake of brevity, this analysis focuses on PGA only, but similar results have been obtained for MCS macroseismic intensity data. The towers in GT3 are concentrated in the lower range of PGA, below 0.2g. The distributions of mean damage across the entire range of intensity is consistent with the behaviour observed in GT1 up to a PGA of 0.15g. However, it is worth noting that, within this range, GT2 shows a very similar trend, as the PGA values are on average too low to cause significant damage. The number of towers in the highest PGA bin for GT3 (0.15-0.2g) may be insufficient to determine whether the visible increase in damage with respect to the GT1 response, especially for mechanism B and for the overall macroelement behaviour, is statistically significant. Interestingly, this increase could link the mean damage estimated in GT3 with the one in GT4. Investigating the PGA at the location of these towers during the most severe shocks of the sequence and their overall seismic response could help refine the thresholds that distinguish towers affected by single or multiple events. This detailed analysis is a critical area for future research. On the other hand, towers in GT4 show a distribution of the mean damage across the entire intensity range that aligns closely with the trend observed in GT2. This result is likely due to significant overlap between the towers in GT2 and GT4. The vulnerability functions proposed by fitting GT1 and GT2 samples, namely Eq. (4.1) adjusted with values in Table 4.4, are validated over the observational data in GT3 and GT4 in the same Figure 4.20. These vulnerability functions are, thus, suggested as promising tools to model and predict the mean damage in case of towers affected by a single main shock or by a combination of more shock inducing cumulative effects.

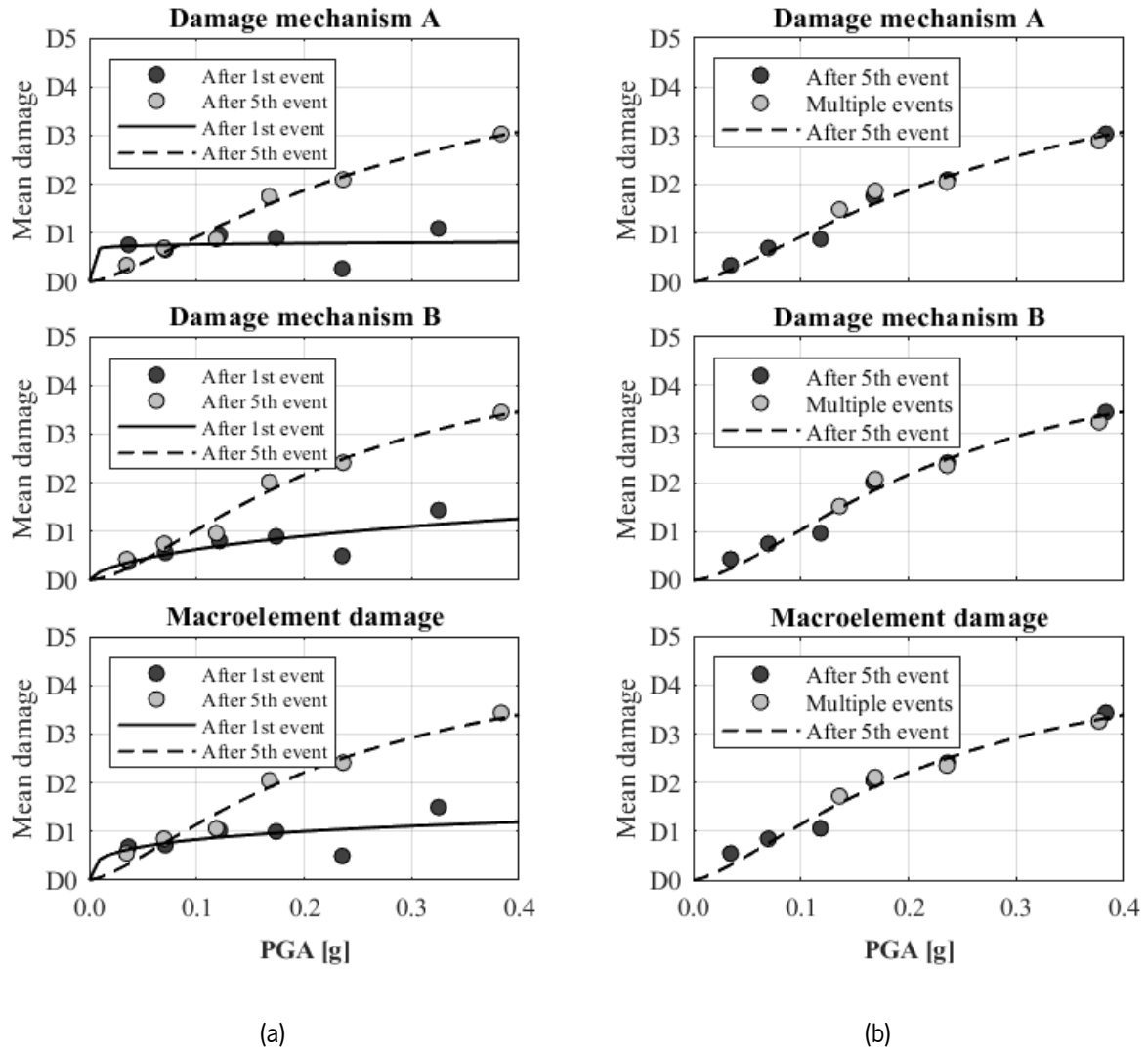


Figure 4.20: Validation of the newly calibrated vulnerability curves as functions of the PGA for: (a) single event, GT3; (b) multiple events, GT4.

4.3.3 Fragility functions

Overcoming another limitation of the study presented in Chapter 3, the fragility functions, presented hereafter, were derived assuming a lognormal cumulative distribution function. The parameters of this distribution were estimated by fitting the function to the observed probability of exceedance of the damage grade at multiple levels of intensity using as optimisation procedure the Maximum Likelihood Estimation (MLE) method (Baker, 2015). The fundamentals of the adopted procedure enabling the development of fragility functions are those described in Section 2.3.4. The fragility functions obtained using the entire dataset of bell towers (GT0) are illustrated in Figure 4.21.

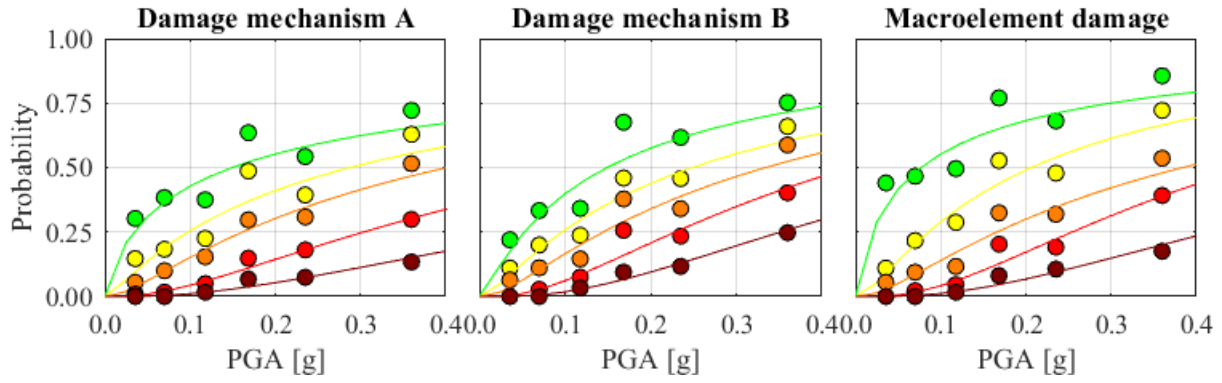


Figure 4.21: Fragility functions for GT0.

Table 4.5 summarises the lognormal cumulative distribution parameters, median and standard deviation, estimated by maximising the likelihood function, representative of the best fit to the observed probability of damage exceedance. Values are provided for the different damage states (from D1 to D5), according to the considered damage mechanisms.

Table 4.5: Median and standard deviation for the distinct damage grades.

	Parameters	D1	D2	D3	D4	D5
Mechanism A	θ_{D_k}	0.15	0.29	0.40	0.63	1.03
	β	2.21	1.61	1.35	1.07	1.01
Mechanism B	θ_{D_k}	0.15	0.25	0.33	0.43	0.64
	β	1.55	1.40	1.24	0.95	0.89
Macroelement damage	θ_{D_k}	0.08	0.21	0.38	0.46	0.77
	β	2.02	1.31	1.24	0.88	0.90

When comparing the results for the two damage mechanisms, differences emerge in the probabilities of exceeding the different damage states as PGA increases. In particular, except for D1, the belfry consistently shows a lower median, indicating a tendency to suffer damage at lower PGA values. While the probabilities of exceeding damage states from D1 to D3 do not exhibit significant variations, they increase substantially for D4 and D5 in the case of mechanism B. When examining the tower as a macroelement, the fragility functions tend to display an intermediate behaviour. It is worth noting that the optimisation is carried out by considering each damage level independently. This approach, commonly adopted in literature, especially for the analysis of churches and their macroelements, can lead to crossing of fragility functions as seen in (Sisti et al., 2023). However, for a sequential damage scale, like EMS-98, crossed curves imply that higher damage levels, above a given value of the intensity measure, have a higher probability of occurring than lower damage levels. This result is not acceptable. This issue may arise due to a dataset that is unevenly distributed across the different PGA bins. In the present case, the fragility functions of Figure 4.21 cross each other at very high PGA values

approaching a probability of exceedance equal to 1. These values fall outside the range of observational data and are thus not plotted. The distance between the PGA values investigated and the crossing points suggests that the presented functions provide a reasonable approximation of fragility for the observed intensity levels. Nonetheless, two approaches to avoid the crossing of the fragility curves suggested by (Porter, 2021), and further presented in Section 2.3.4, are hereafter adopted, to analyse advantages and disadvantages. The first approach (Approach 1) involves a rotation of the fragility functions, as shown in Figure 4.22, to avoid crossing. The probability shifts resulting from this approach, especially for D1-D2, are noticeable, with the functions shifting upward to prevent crossing. The second approach (Approach 2), illustrated in Figure 4.23, is more advanced and optimises all damage levels simultaneously.

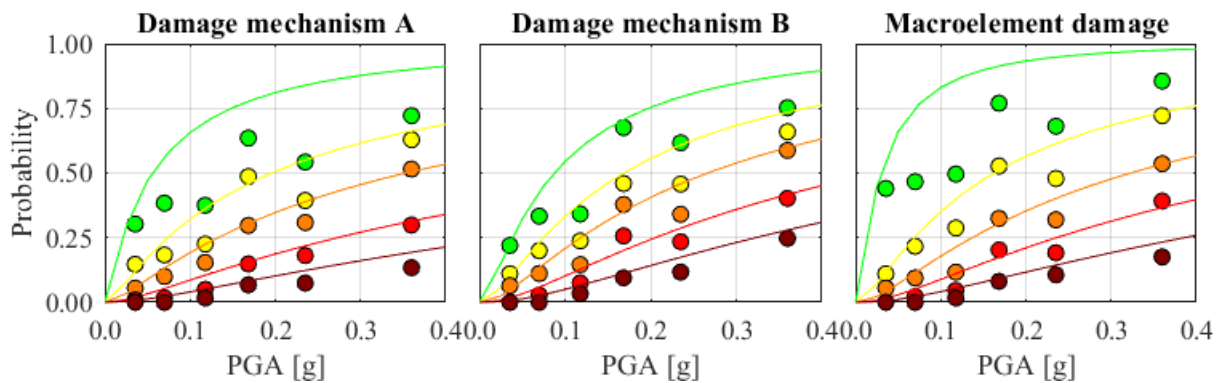


Figure 4.22: Fragility functions for GT0 derived using Approach 1.

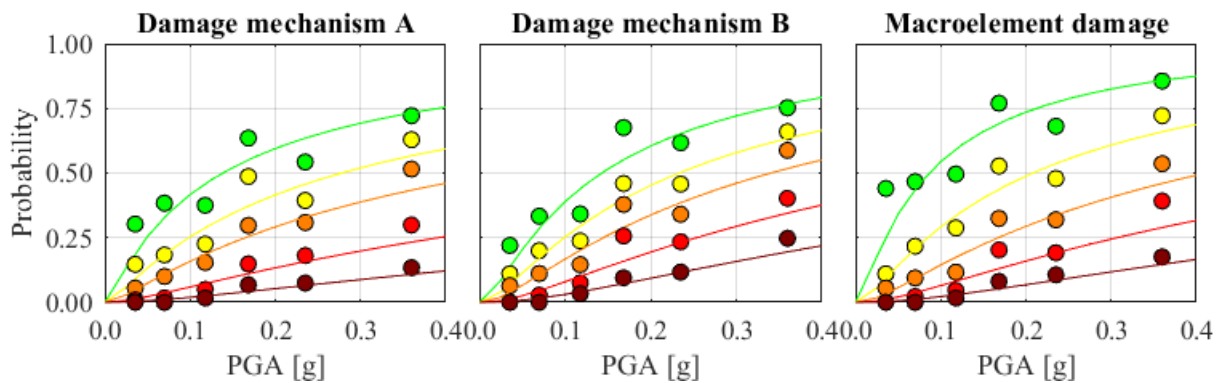


Figure 4.23: Fragility functions for GT0 derived using Approach 2.

When comparing the later fragility functions with those in Figure 4.21, the probability variations are minimal, confirming that the results obtained from optimising each function separately were acceptable for this specific case study. The results of the calibration for GT1 and GT2 with the approach 2 are presented in Figure 4.24, while the estimated parameters are reported in Table 4.6, together with the parameters for GT0.

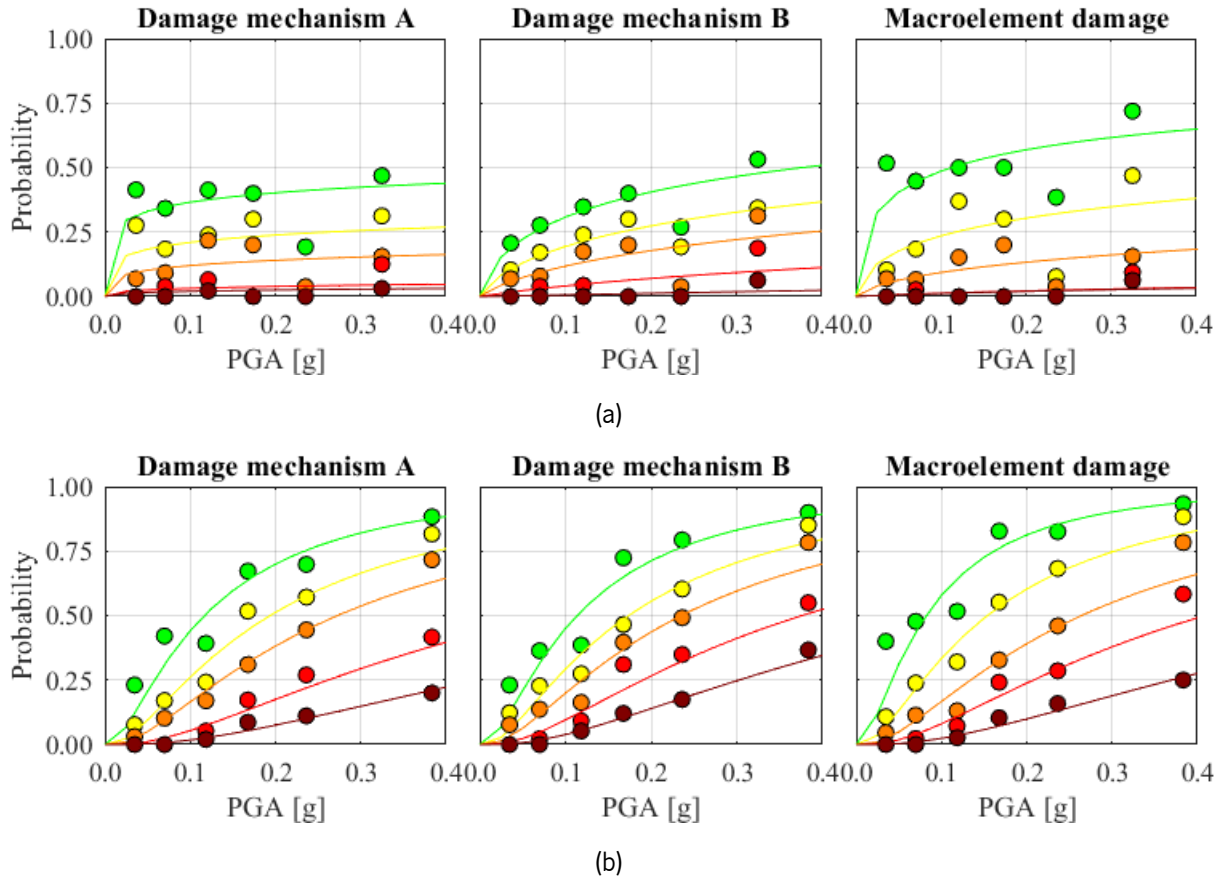


Figure 4.24: Fragility functions: (a) GT1; (b) GT2.

Table 4.6: Cumulative lognormal distribution parameters for the fragility functions.

Par	D1			D2			D3			D4			D5		
	GT0	GT1	GT2	GT0	GT1	GT2	GT0	GT1	GT2	GT0	GT1	GT2	GT0	GT1	GT2
Mechanism A															
θ_{D_k}	0.14	1.21	0.12	0.28	35.32	0.19	0.46	510.06	0.27	1.10	7.32e04	0.52	2.41	3.00e05	0.88
β	1.54	7.27	1.03	1.54	7.27	1.03	1.54	7.27	1.03	1.54	7.27	1.03	1.54	7.27	1.03
Mechanism B															
θ_{D_k}	0.14	0.38	0.11	0.23	0.98	0.17	0.34	2.29	0.24	0.60	9.86	0.38	1.07	73.94	0.60
β	1.27	2.64	1.01	1.27	2.64	1.01	1.27	2.64	1.01	1.27	2.64	1.01	1.27	2.64	1.01
Macroelement damage															
θ_{D_k}	0.09	0.11	0.08	0.21	1.10	0.15	0.41	8.00	0.26	0.75	161.16	0.41	1.46	183.56	0.73
β	1.33	3.32	1.00	1.33	3.32	1.00	1.33	3.32	1.00	1.33	3.32	1.00	1.33	3.32	1.00

To further stress the importance of a calibration strategy that accounts for all the damage grades of each mechanism simultaneously, its results are compared with the traditional calibration (Figure 4.25), where each curve is derived independently.

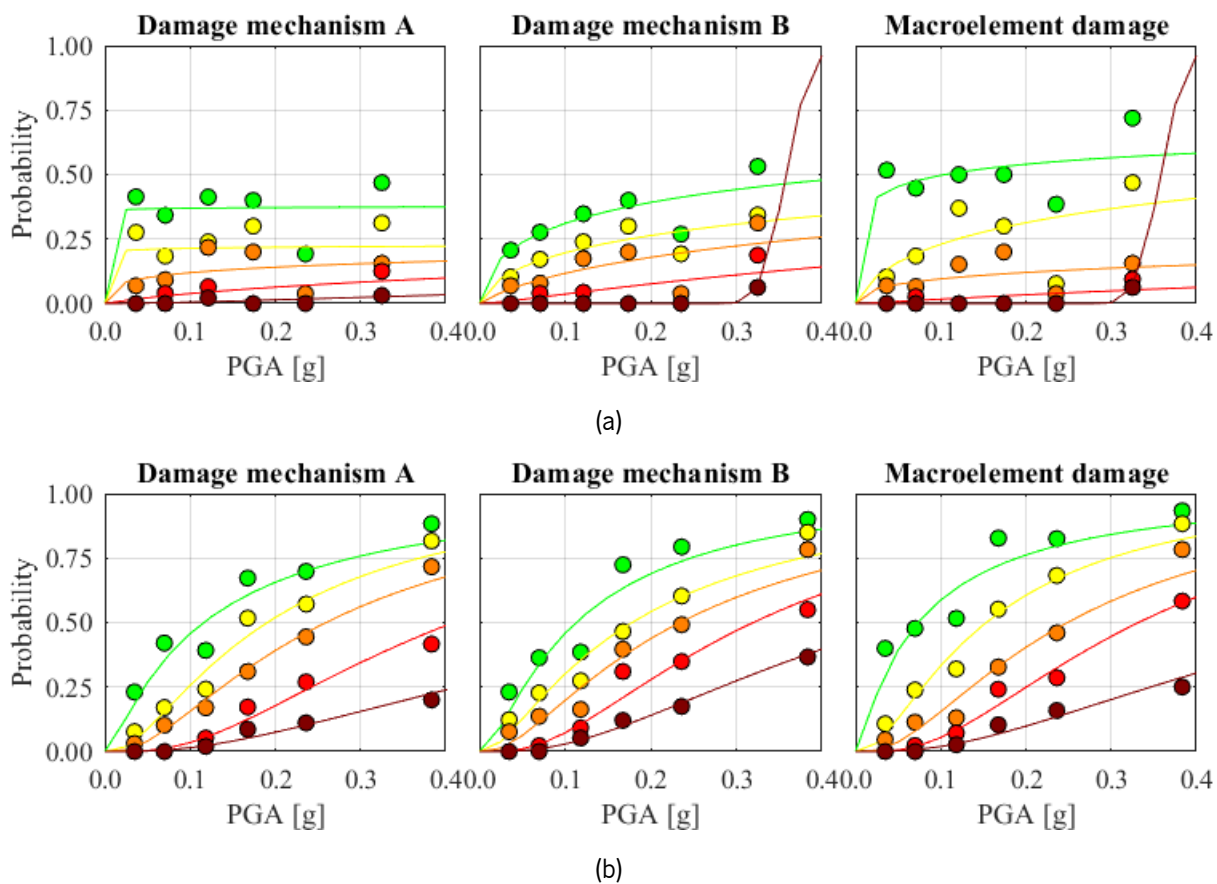


Figure 4.25: Fragility functions for GT1 (a) and GT2 (b), calibration of individual damage grade exceedance.

For GT0 and GT2, the adopted strategy does not significantly alter the trajectory and shape of the curve within the investigated range of PGA, since the crossing occurs at larger values of PGA, closer to a probability of exceedance of 1. In contrast, for a traditional calibration strategy applied to GT1, the crossing of the curves related to exceedance of D5 occurs within the range of PGA recorded for the event, considering both the belfry mechanism and the global macroelement damage. Additionally, the fragility functions for D1 and D2 present a behaviour dissimilar from what is typically expected. Both of these issues are likely due to the distribution of the observed probability of damage in GT1 across the PGA bins. Indeed, a very low percentage of samples exceeds higher damage grades, while the percentage of samples with minor damage remains almost constant across the considered PGA range. The characteristics of the dataset GT1 stresses the limitation of the simplified Approach 1 as well. Here, the rotation of the curves coupled with the shortcoming of the distributions at hand for very low and very high damage levels, leads to an unacceptable set of curves, when compared to the observational data (Figure 4.26).

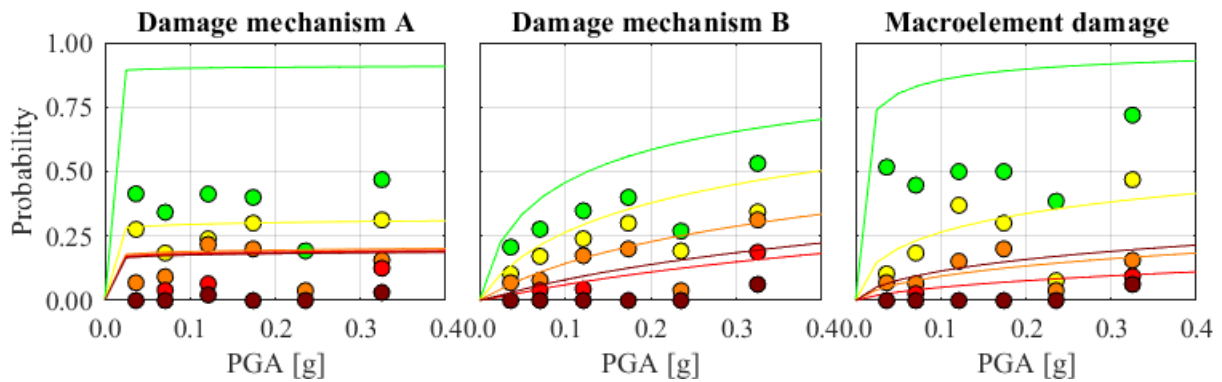


Figure 4.26: Fragility functions for GT1 derived using Approach 1.

4.4 VULNERABILITY ASSESSMENT: KEY FEATURES

The effectiveness of the risk analysis models (i.e., DPMs, vulnerability and fragility functions), namely their reliability and usefulness, is constrained by the ability to identify homogeneous asset classes. Within these classes, distinguishing the attributes and features that influence the overall susceptibility to damage and the peculiar behaviour against the hazard is critical (Lagomarsino et al., 2021; Rossetto & Ioannou, 2018). This is essential to reduce the large scatter in the response of assets within the same class under the same intensity measure. While ordinary buildings may exhibit greater homogeneity, monuments often cannot be adequately represented by broad asset classes, as these fail to capture the intrinsic variability within each typology. Key attributes of a typology may be reflected through vulnerability corrective factors, which adjust the vulnerability score to improve predictions of mean damage and better represent the susceptibility of each asset to seismic hazards. Typical vulnerability modifiers discussed in the literature include factors such as maintenance, material quality, structural regularity, size and aspect ratio, interaction with adjacent structures, and the presence of retrofitting interventions (Lagomarsino, 2006; Lagomarsino et al., 2004).

In order to better understand the seismic response of the towers, this Section investigates specific attributes and features described in the inspection forms, seeking to identify subsets with more homogeneous behaviour. However, the dataset's available information limits the number and type of features that can be analysed. Consequently, several subsets of the entire GTO were derived, grouping the towers based on the following attributes, among the ones described in Section 4.2.2, due to their relevance to towers as potential sources of vulnerability and/or hazard amplification: (i) maintenance; (ii) geometry; (iii) tower typology and interaction with surrounding buildings; and (iv) geographical location. It is important to note that the state of maintenance, in the inspection form, refers to the entire masonry church. However, here, it is assumed to apply to the associated towers and is therefore included in the analysis. Given the utility of PGA in developing predictive models and its strong

correlation with the tower damage grades, the following Sections focus exclusively on this intensity measure.

The effectiveness of vulnerability and fragility functions was explored for both damage mechanisms A and B and entire macroelement. In particular, the existing and proposed vulnerability functions were reported against the mean damage evolution observed for the distinct subsets, together with the mean damage evolution for the entire dataset (GTO). The fragility functions were compared, highlighting, for the sake of clarity, the damage levels D3 and D4 only. These two damage levels, in fact, ensure the availability of a sufficient number of samples across the PGA range, providing consistency and significance. At the same time, they represent a range of damage in which the collapse mechanism is clearly activated, preventing any possible misinterpretation. Hereafter, only the results of the calibration through the approach 2 by (Porter, 2021) are discussed. Table 4.7 summarises the median and standard deviation estimated by maximising the likelihood function for the development of the lognormal cumulative distribution functions representative of the best fit to the observed probabilities. However, for the sake of completeness, the DPMs and the fragility functions developed for all considered subsets, damage levels and calibration approaches, namely traditional approach and optimised one (approach 2), are individually presented in the Appendix B of the present thesis.

Table 4.7: Median and standard deviation for damage states D3 and D4 of the distinct subsets accounting for the different key features considered in this study.

Parameters	Location								Geometry				Structural Features											
	Region				Province				Aspect Ratio				Tower Typology				State of Maintenance							
	Marche		Umbria		Macerata		Perugia		H/L ≤ 4		H/L > 4		Confined		Integrated		Good		Acceptable		Poor			
	D3	D4	D3	D4	D3	D4	D3	D4	D3	D4	D3	D4	D3	D4	D3	D4	D3	D4	D3	D4	D3	D4		
Mechanism A																								
θ_{D_k}	0.37	0.80	0.67	1.22	0.26	0.50	0.68	1.28	0.62	2.29	0.66	1.95	0.69	2.88	0.51	1.07	0.70	1.69	0.64	1.71	0.21	0.62		
β	1.43	1.43	1.40	1.40	1.13	1.13	1.44	1.44	2.11	2.11	1.91	1.91	1.92	1.92	1.65	1.65	1.60	1.60	2.03	2.03	1.30	1.30		
Mechanism B																								
θ_{D_k}	0.33	0.56	0.34	0.55	0.24	0.37	0.33	0.49	0.33	0.77	0.39	0.73	0.41	0.89	0.30	0.44	0.43	0.84	0.37	0.63	0.21	0.36		
β	1.29	1.29	1.05	1.05	1.05	1.05	0.92	0.92	1.27	1.27	1.25	1.25	1.61	1.61	1.02	1.02	1.30	1.30	1.50	1.50	0.86	0.86		
Macroelement damage																								
θ_{D_k}	0.36	0.64	0.54	0.82	0.25	0.40	0.50	0.75	0.47	1.23	0.51	1.01	0.54	1.33	0.40	0.66	0.60	1.14	0.43	0.95	0.23	0.41		
β	1.32	1.32	1.18	1.18	1.01	1.01	1.10	1.10	1.56	1.56	1.42	1.42	1.70	1.70	1.21	1.21	1.37	1.37	1.64	1.64	1.00	1.00		

4.4.1 Influence of maintenance

The influence of the maintenance condition of the towers at the time they were affected by the earthquake on their susceptibility of damage was investigated referring to three states of maintenance (Figure 4.27): good , acceptable , and poor.

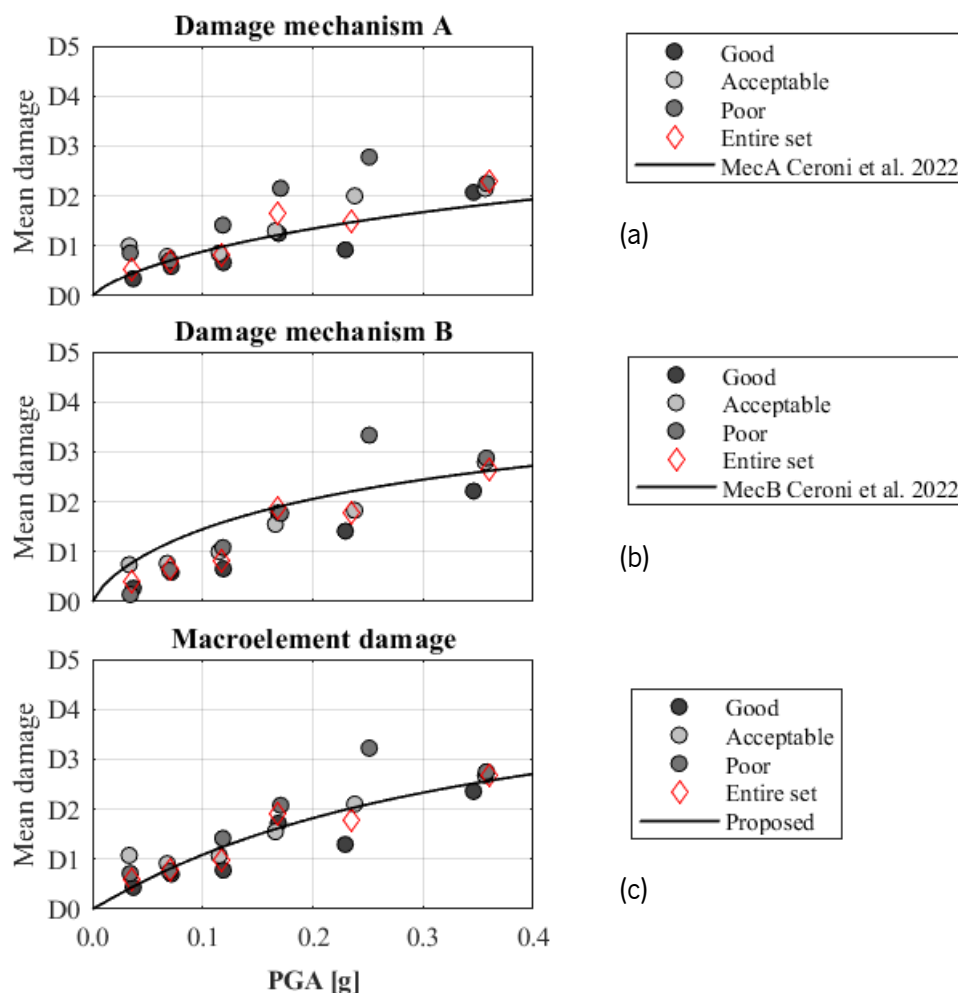


Figure 4.27: Observed mean damage compared with the vulnerability functions accounting for the maintenance states; (a) mechanism A; (b) mechanism B; (c) overall macroelement.

Comparing the overall distribution of the mean damage observed for good, acceptable and poor states,, despite their similarities, an emerging difference in behaviour can be detected. In particular, the mean damage for good state is distributed among lower damage levels (with a maximum level not exceeding D3). For poor state, the mean damage consistently reaches higher levels (exceeding D3 for a PGA of approximately 0.25g, but presenting a lower value for higher PGA). Meanwhile, the mean damage for acceptable state falls between good and poor ranges. The different damage distribution can be better highlighted by examining the DPMs (see Appendix B). The obtained results revealed that the state of

maintenance of the analysed towers can be considered an explanatory variable which can explain the variations characterising the towers with different behaviour under similar intensity of the seismic action. In this light, the mean damage predicted for increasing values of PGA through the existing and proposed vulnerability functions has a limited potential, as it cannot fully capture the actual damage distribution characterising the towers with different maintenance states. Therefore, the formulation coefficients were recalibrated, allowing to propose novel formulations to better explain the different behaviour accordingly. The recalibrated coefficients are summarised in Table 4.8.

Table 4.8: Recalibration coefficients for the three distinct subsets based on the state of maintenance.

		a'	b'
Good state	Mechanism A	1.26	0.31
	Mechanism B	1.32	0.49
	Macroelement damage	1.22	0.44
Acceptable state	Mechanism A	0.83	0.20
	Mechanism B	1.23	0.57
	Macroelement damage	0.94	0.41
Poor state	Mechanism A	1.00	0.52
	Mechanism B	1.89	1.20
	Macroelement damage	1.36	0.88

To further check the obtained results, an additional analysis involved the comparison of the predictive performance between the vulnerability functions proposed specifically for the towers that feature an acceptable state of maintenance with the existing and novel vulnerability functions for the entire dataset, allowing to check if the formulation for acceptable maintenance state is sufficiently close to the average behaviour of the entire set of towers. The results, illustrated in Figure 4.28(a), show a negligible difference in the predictive performance.

Based on the obtained results, the predictive performance of the formulations proposed specifically for subsets of towers characterised by poor and good states of maintenance can be considered as upper and lower bounds respectively, as shown in Figure 4.28(b).

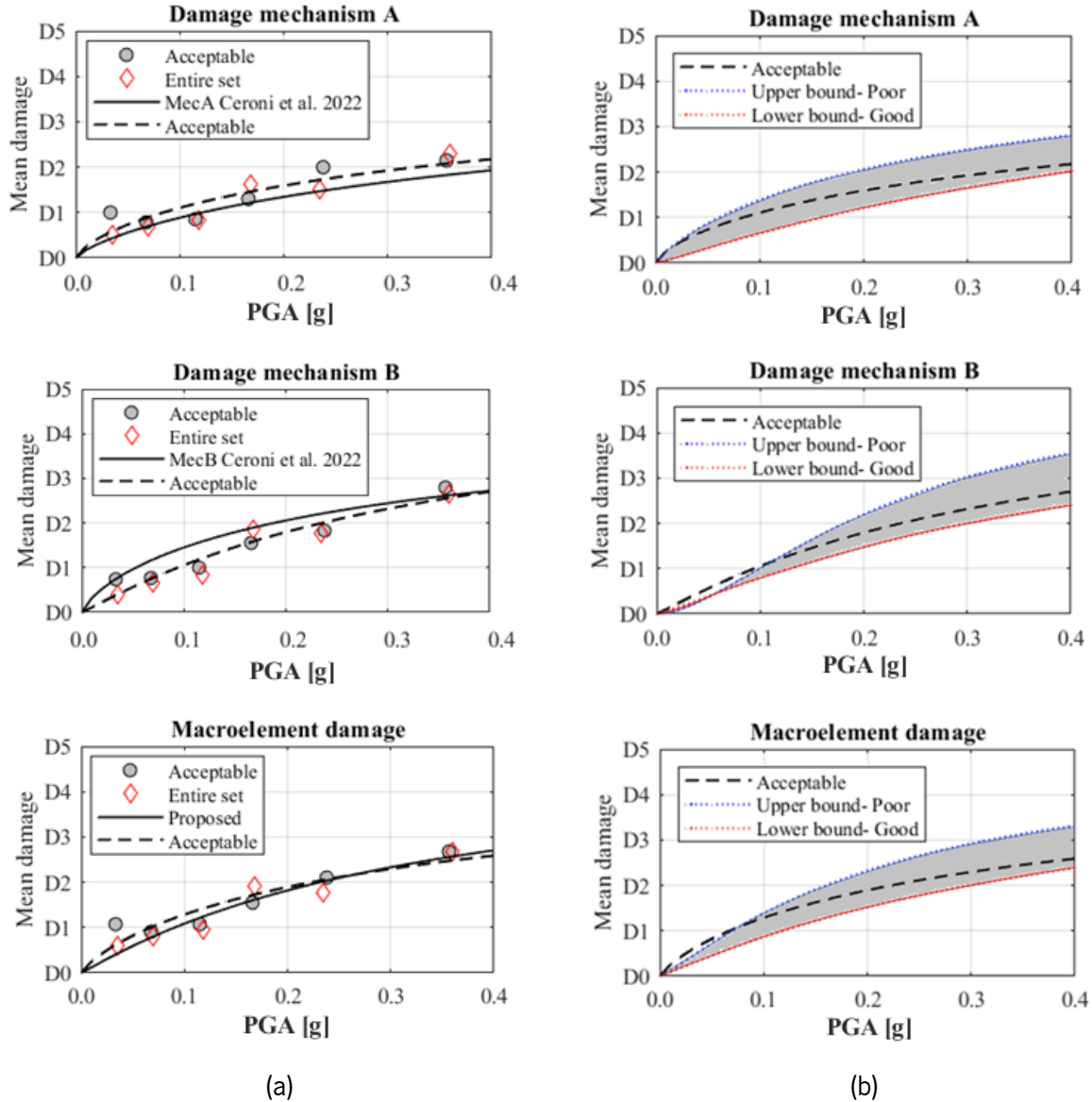


Figure 4.28: Observed mean damage compared with the vulnerability functions for the acceptable state of maintenance and for the entire set (a); Predictive performance of the formulations, together with upper and lower bounds provided by formulations proposed for the poor and good states of maintenance (b).

When comparing the fragility curves obtained for towers with different maintenance (Figure 4.29), the results of the vulnerability functions are confirmed, as significant differences can be observed in the probabilities of exceeding the considered damage states (D3 and D4) for increasing values of PGA. Analysing D3, the probabilities of exceeding the damage states for good (dashed curves) and acceptable states (solid lines) show similar increasing trends, peaking at around 50% for a maximum PGA of approximately 0.4g. In contrast, for poor state (dotted curves), the probabilities follow an increasing trend as well but reach a significantly higher peak of about 70% for the same maximum PGA.

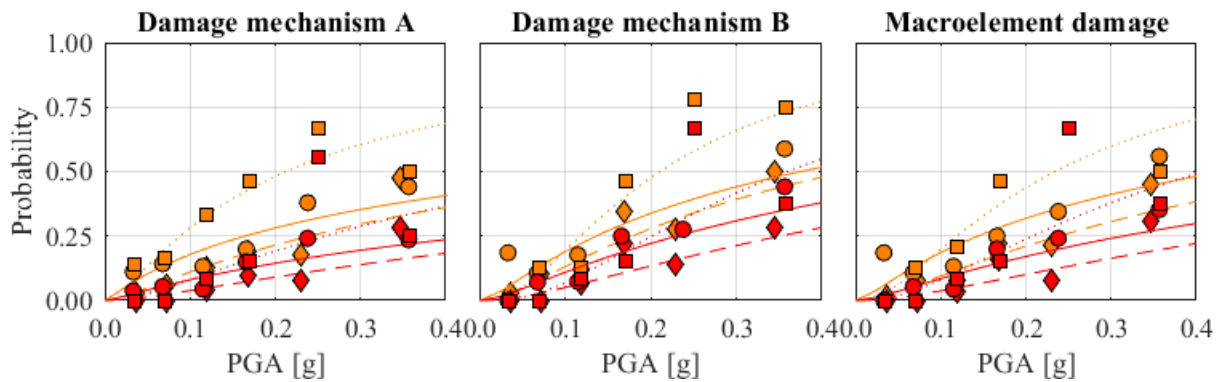


Figure 4.29: Fragility curves of D3 and D4 exceedance fitted to good (diamond points), acceptable (circle points), and poor (squared points) maintenance states.

To support the conclusions regarding the influence of the state of maintenance on the vulnerability, a possible bias in the results was checked to ensure that differences in damage levels across the three subsets were not influenced by the varying presence of towers affected by a single main shock or cumulative effects. The results are summarised in Table 4.9.

Towers in good, acceptable, and poor maintenance states represent different proportions of the entire dataset (53%, 29%, and 11%, respectively), reflecting a significant difference in the number of towers. However, when analysing the towers affected by single or multiple shocks, the subsets of towers with distinct maintenance states show comparable distributions. Specifically, the proportions of towers inspected after first main shock are 27%, 29%, and 31%, respectively, while, for multiple shocks, they are 55%, 61%, and 63%, respectively. This confirms that including the state of maintenance provides a reliable indicator for better understanding the vulnerability behaviour of this particular building typology.

Table 4.9: Statistics of the subsets considering the maintenance states based on type of damage.

Subsets	Key Feature		
	State of Maintenance		
	Good	Acceptable	Bad
	419 (53%)	234 (29%)	85 (11%)
First Event	115 (27%)	68 (29%)	26 (31%)
Last Event	230 (55%)	143 (61%)	54 (63%)

4.4.2 Influence of geometry and interactions

The towers were grouped according to two additional parameters accounting for their geometry and interaction with surrounding structures. Regarding the former, the ratio of the total height with the minimum side length at the base, referred to as aspect ratio (H_{tot}/L_{min}), was considered to distinguish two classes: the first class characterised by towers with ratios lower than or equal to 4 and the second class by towers with ratios greater than 4. The impact of the aspect ratio on the mean

damage evolution is presented in Figure 4.30. It can be generally observed that the mean damage values observed for the first class ($H_{tot}/L_{min} \leq 4$) are very similar to those observed for the second class ($H_{tot}/L_{min} > 4$).

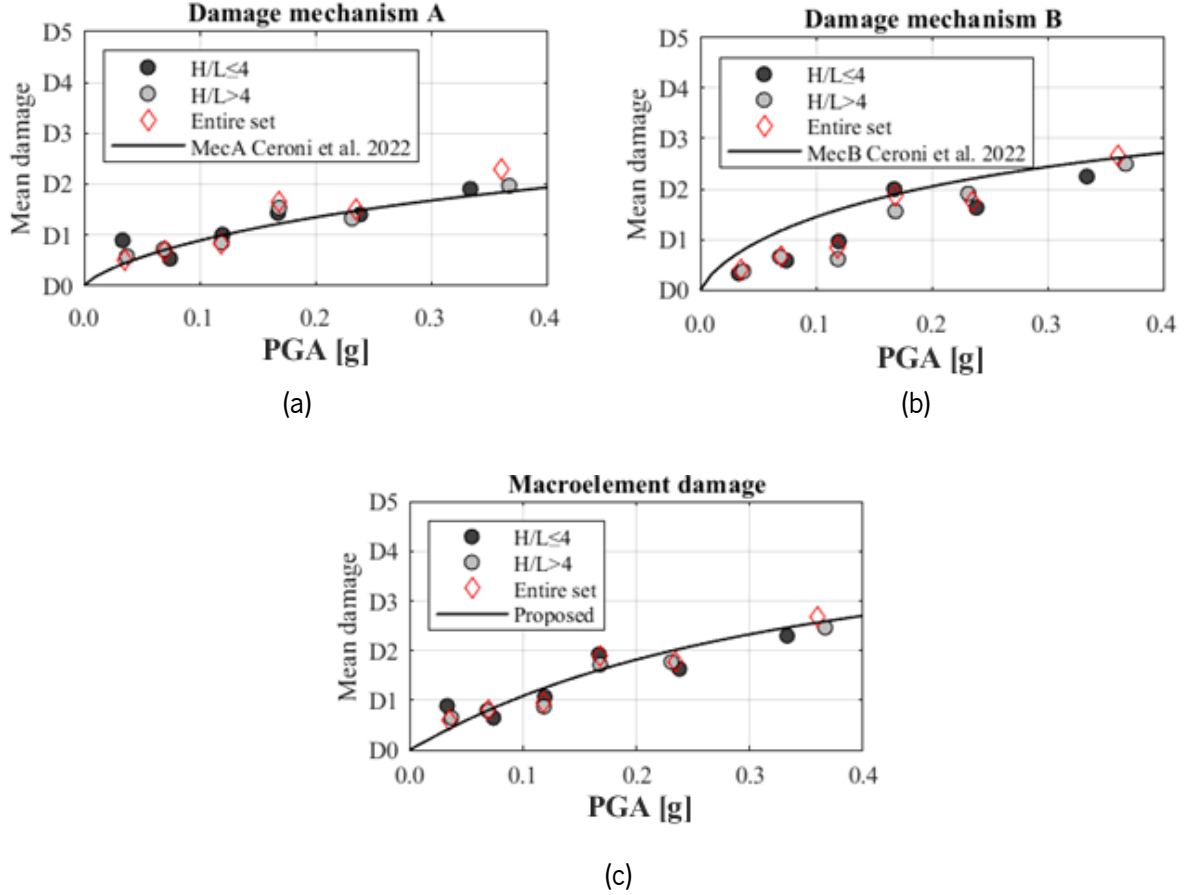


Figure 4.30: Observed mean damage compared with the vulnerability functions accounting for the aspect ratio threshold; (a) mechanism A; (b) mechanism B; (c) overall macroelement.

This is confirmed by the comparison of the fragility curves obtained for the different aspect ratios (Figure 4.31), where no significant differences in the probabilities of exceeding the considered damage states (D3 and D4) for increasing values of PGA can be observed. Based on the results, the aspect ratio of the analysed towers can be considered an explanatory variable which has relatively low influence on the damage data at hand.

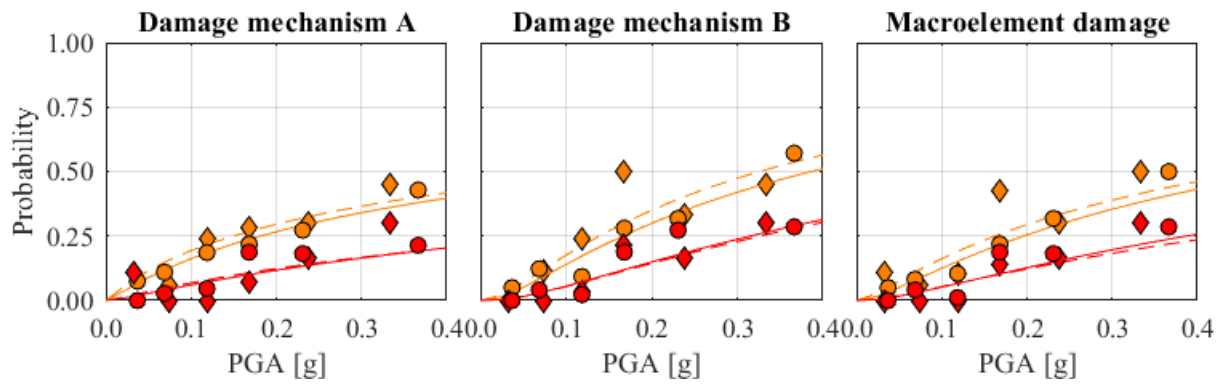


Figure 4.31: Fragility curves of D3 and D4 exceedance fitted to towers with $H_{tot}/L_{min} \leq 4$ (diamond points) and towers with $H_{tot}/L_{min} > 4$ (circle points).

The interaction between the towers and the surrounding buildings, instead, was investigated by analysing two typologies of bounded towers, namely confined and integrated. The influence of the tower typology on the mean damage evolution is given in Figure 4.32. As for the previous case, the mean damage observed for confined towers is very comparable with the mean damage observed for integrated towers.

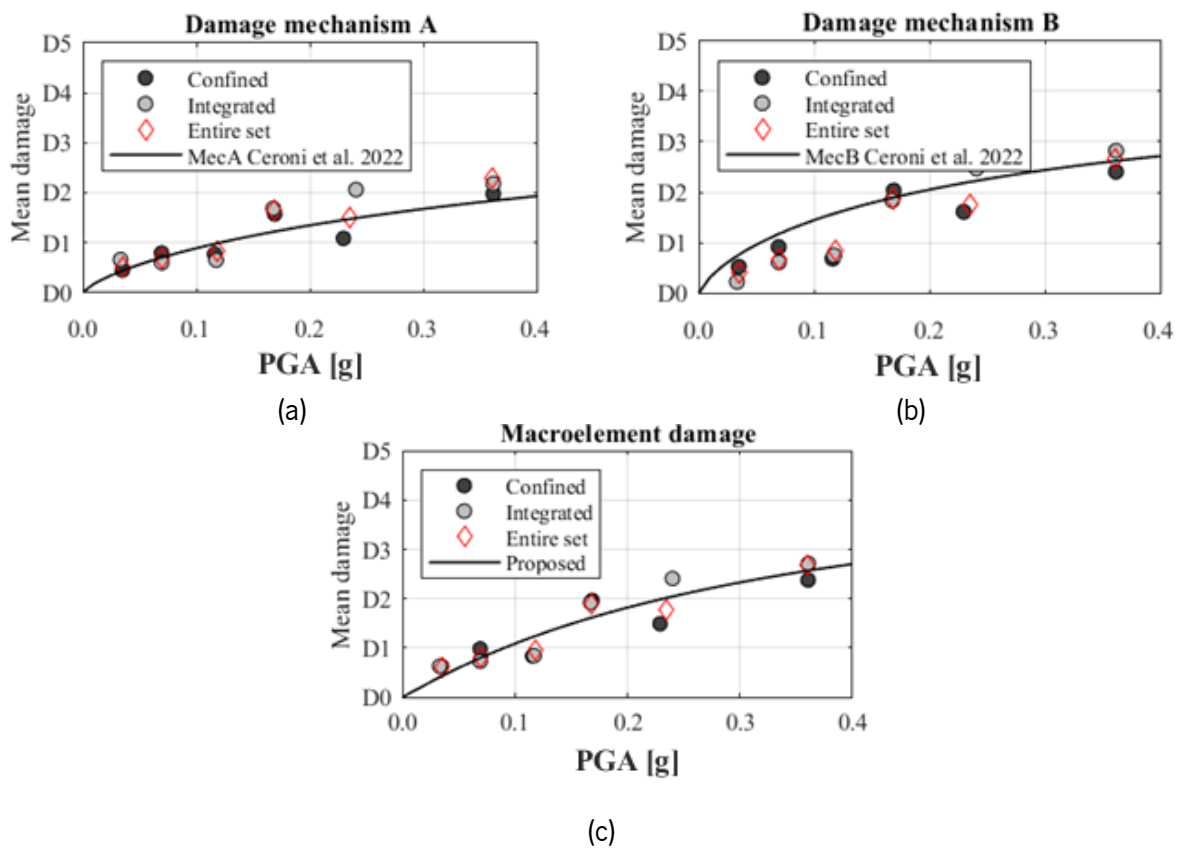


Figure 4.32: Observed mean damage compared with the vulnerability functions accounting for the tower typology (confined and integrated); (a) mechanism A; (b) mechanism B; (c) overall macroelement.

However, when comparing the fragility curves (Figure 4.33), slight differences in the probabilities of exceeding the considered damage states (D3 and D4) for increasing values of PGA can be observed. For confined (dashed curves) and for integrated (solid curves), the probabilities are very close to each other until reaching values of PGA above 0.15g for which the probabilities are larger for integrated than for confined. The functions tend to diverge more for damage mechanism B. Despite the slight difference in the fragility data, the tower typology is at this stage considered an irrelevant explanatory variable in the behavioural response of the dataset, but more research is needed to provide a better insight into the influence of the configuration on the seismic behaviour.

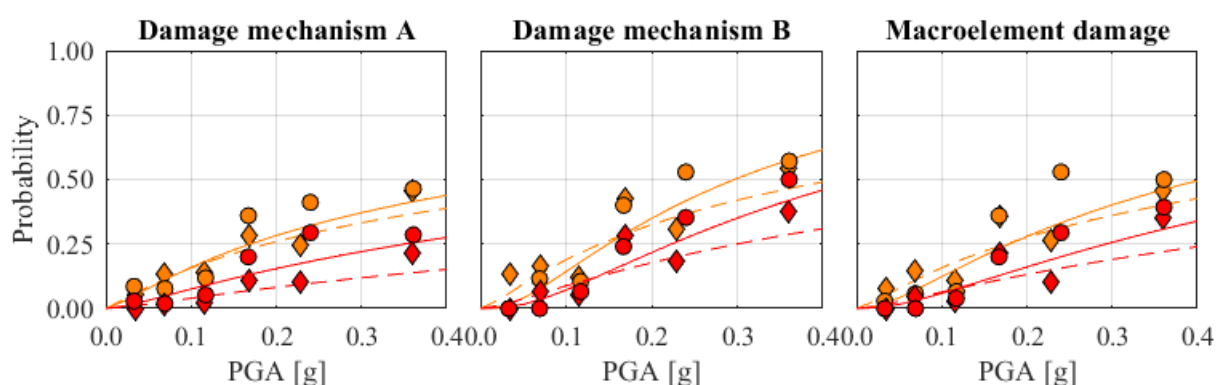


Figure 4.33: Fragility curves of D3 and D4 exceedance fitted to confined (diamond points) and integrated (circle points).

4.4.3 Influence of location

Finally, two explanatory variables, namely the region and the province, were identified to account for the location feature. The effect of the location on the mean damage evolution is reported in Figure 4.34. The mean damage observed for Marche region is higher than the mean damage observed in Umbria region (Figure 4.34a). Similarly, the mean damage observed for Macerata province is greater than the mean damage observed in Perugia province (Figure 4.34b). The comparable trends can be justified by the similar statistics between the considered region datasets and the provinces that constitute their main subsets. The results suggest higher vulnerability of towers belonging to the Macerata province and Marche region in general, irrespective of the individual damage mechanism and macroelement, over the entire range of PGA. This can be better highlighted by examining the different DPMs developed for the different regions and provinces (see Appendix B). While the towers in Macerata province and Marche region present mean damage values, for both mechanism A and B, only slightly larger than the entire dataset (GT0), the towers in Perugia province and Umbria region exhibit a significantly better behaviour until 0.25-0.30g.

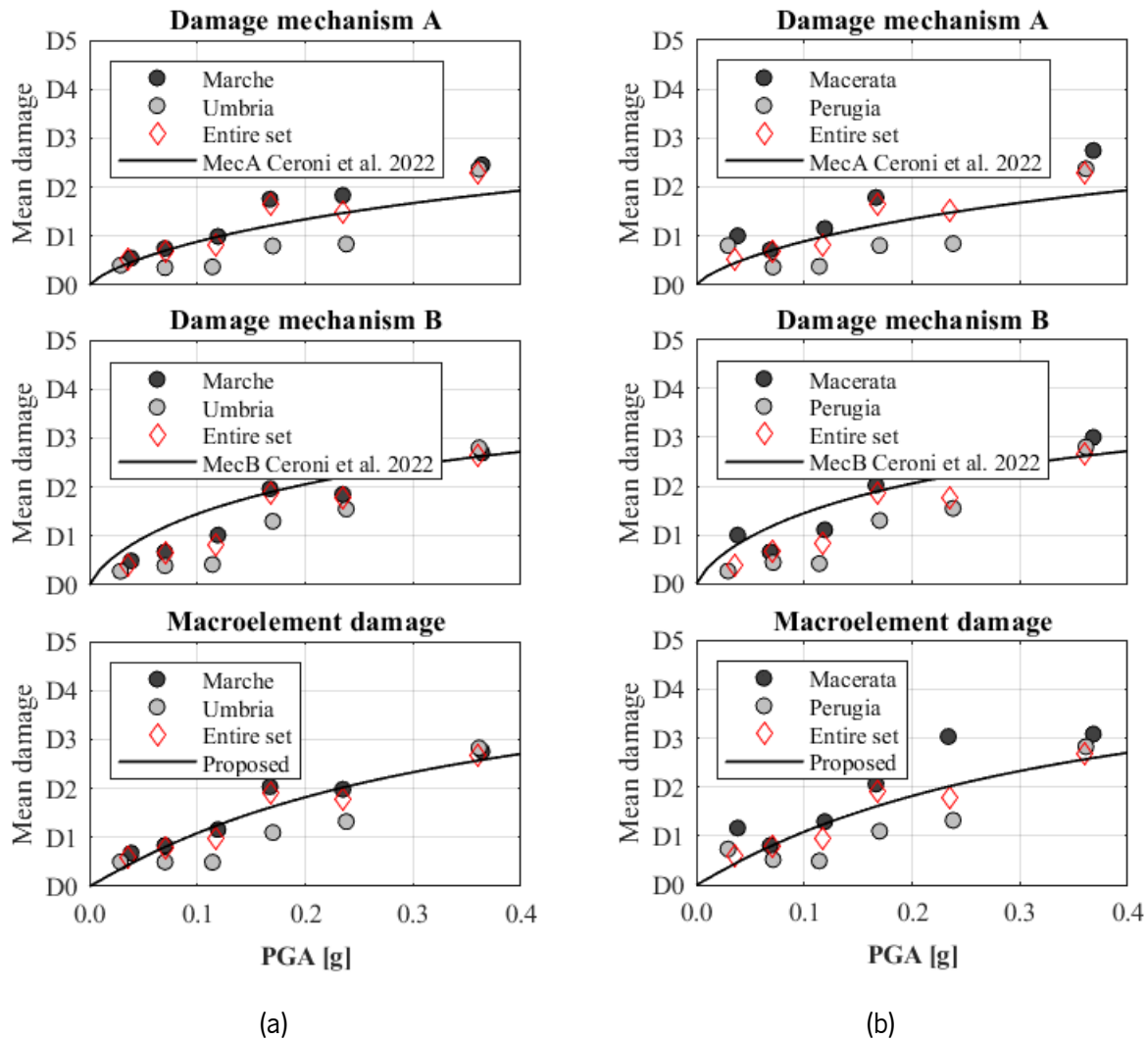


Figure 4.34: Observed mean damage compared with the vulnerability functions: (a) regional-based analysis and (b) provincial-based analysis.

When comparing the fragility curves obtained for the different regions (Figure 4.35), and provinces (Figure 4.36), significant differences can be observed in the probabilities of exceeding the considered damage states (D3 and D4) for increasing values of PGA. As expected, the models developed for the regions are reflected similarly in the models for the provinces. The results for Macerata province show the highest probability of exceeding the damage states, slightly larger than that for the entire Marche region. Although towers in Perugia province and entire Umbria region exhibit lower probability of damage exceedance, this trend reverses for some levels of damage and mechanisms at higher PGAs, closer to 0.4g.

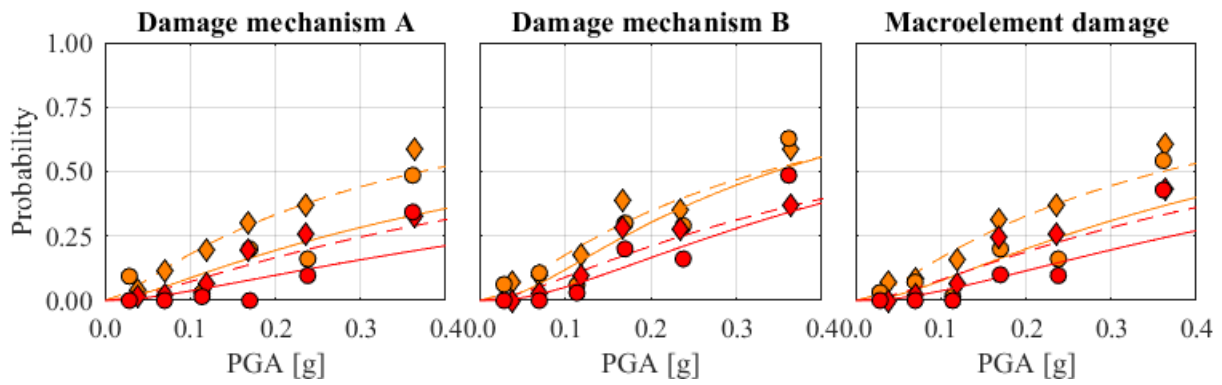


Figure 4.35: Fragility curves of D3 and D4 exceedance fitted to towers in Marche (diamond points) and Umbria (circle points).

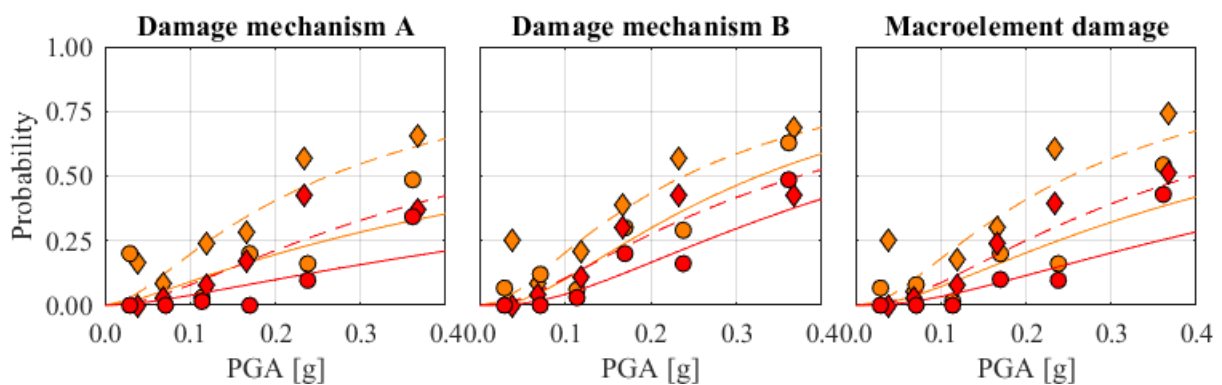


Figure 4.36: Fragility curves of D3 and D4 exceedance fitted to towers in Macerata (diamond points) and Perugia province (circle points).

Similarly to the analysis of the maintenance state, the results could be biased by the influence of other parameters strongly correlated with damage level at a given intensity measure, as the number of severe shocks experienced or the maintenance level prior to the seismic sequence. Thus, the distributions of these factors in the datasets from the four geographical areas were investigated. The results are summarised in Table 4.10.

As previously mentioned, the Marche and Umbria subsets correspond to 61% and 26% of the entire dataset, respectively. Although the two datasets differ in size, the increased vulnerability observed in towers located in the Marche region compared to those in Umbria may be attributed to the larger proportion of towers in the Marche subset affected by cumulative damage (70%, inspected after the last event) compared to the Umbria subset (53%), and the smaller number of towers inspected after first event (22% and 31%, respectively). Very similar results were obtained when analysing towers located in the provinces of Macerata and Perugia. Based on these findings, the investigation of the cumulative effect in the analysed towers, categorised by region and province, helps explain the variations in the vulnerability behaviours of towers within the considered subsets.

Table 4.10: Statistics of the subsets considering the location based on the type of damage.

	Key Feature			
	Location			
	Region		Province	
Subsets	Marche	Umbria	Macerata	Perugia
Entire	486 (61%)	203 (26%)	295 (37%)	183 (23%)
First Event	106 (22%)	63 (31%)	52 (18%)	61 (22%)
Last Event	338 (70%)	108 (53%)	224 (76%)	92 (50%)
Good	251 (52%)	114 (56%)	149 (51%)	105 (57%)
Acceptable	152 (31%)	55 (27%)	98 (33%)	49 (27%)
Bad	50 (10%)	19 (9%)	26 (9%)	16 (9%)

4.5 CONCLUSIONS

The present Chapter, relying on a wide database of historic masonry bell towers, primarily focused on the effect of cumulative damage, and successively on the influence of relevant features and attributes on the towers vulnerability, developing and analysing DPMs, vulnerability and fragility functions. The database exploited in this Chapter is composed of 794 historic masonry towers struck by the 2016-2017 Central Italy seismic sequence, obtained from a larger dataset of churches inspected and collected in Da.D.O. (Database of Observed Damage). Access to this source was obtained through a collaboration with the University of Chieti-Pescara.

To investigate the cumulative damage effect, the DPMs, vulnerability, and fragility functions were developed for towers under different seismic scenarios, including those affected by a single seismic shock and those subjected to multiple severe shocks. This approach aimed to identify ranges of intensity measures for which the cumulative effect of subsequent shocks may be considered negligible and establish a methodology applicable to similar seismic regions where repeated events are common. The comparisons highlighted and confirmed the importance of considering multiple seismic shocks rather than focusing solely on the main seismic event. This enabled a quantification of the increase in vulnerability and damage caused by cumulative effects.

DPMs, vulnerability, and fragility functions were also developed for distinct clusters of towers with likely homogeneous behaviour, depending on relevant features and attributes, recorded in the inspection forms, such as location, geometry, interactions, and state of maintenance. The results varied depending on the cluster analysed. Specifically, the state of maintenance proved to be a key factor, enabling the identification of clusters with effectively homogeneous behaviour. In contrast, the interaction with surrounding structures and the aspect ratio led to only minor variations among classes. The effect of location, although highlighting a significant variation in the damage levels for the same intensity measures, was probably affected by an uneven distribution of towers subjected to multiple severe

shocks in the subsets. Future studies should further address the effect of local response on the seismic behaviour and the damage observed, accounting for topographic conditions, soil stratigraphy and input signal variability. Additional characteristics which may influence the vulnerability and could not be properly investigated in this work, due to the limited available information, include the construction materials and the presence of recent repair and strengthening actions.

CHAPTER 5

VULNERABILITY MODELS FOR CENTRAL ITALY BASED ON SPECTRAL ACCELERATION

ABSTRACT

The present Chapter further explores the study of seismic vulnerability of masonry towers at territorial scale, using the Spectral Acceleration (SA) as seismic intensity measure, relying on the set of bell towers damaged after the Central Italy seismic sequence presented in Chapter 4. To this end, first, a Ground Motion Model (GMM) available in the literature and calibrated over Italian past earthquake data was employed to estimate the SA at the tower sites, upon validating its accuracy. Then, DPMs, vulnerability and fragility functions were derived accordingly. As the spectral acceleration depends, among other parameters, on the fundamental frequency of the investigated structures, novel empirical predictive equations were proposed for masonry towers, based on the largest dataset ever collected, which was gathered before. Finally, DPMs, vulnerability and fragility functions were compared, in terms of various performance metrics, with their counterparts derived using the PGA as intensity measure.

5.1 INTRODUCTION

The present Chapter, starting from the Central Italy dataset analysed in Chapter 4, focuses on the use of an alternative seismic intensity measure, namely the Spectral Acceleration (SA), for the development of territorial scale vulnerability models for masonry towers and slender structures (see Figure 5.1).

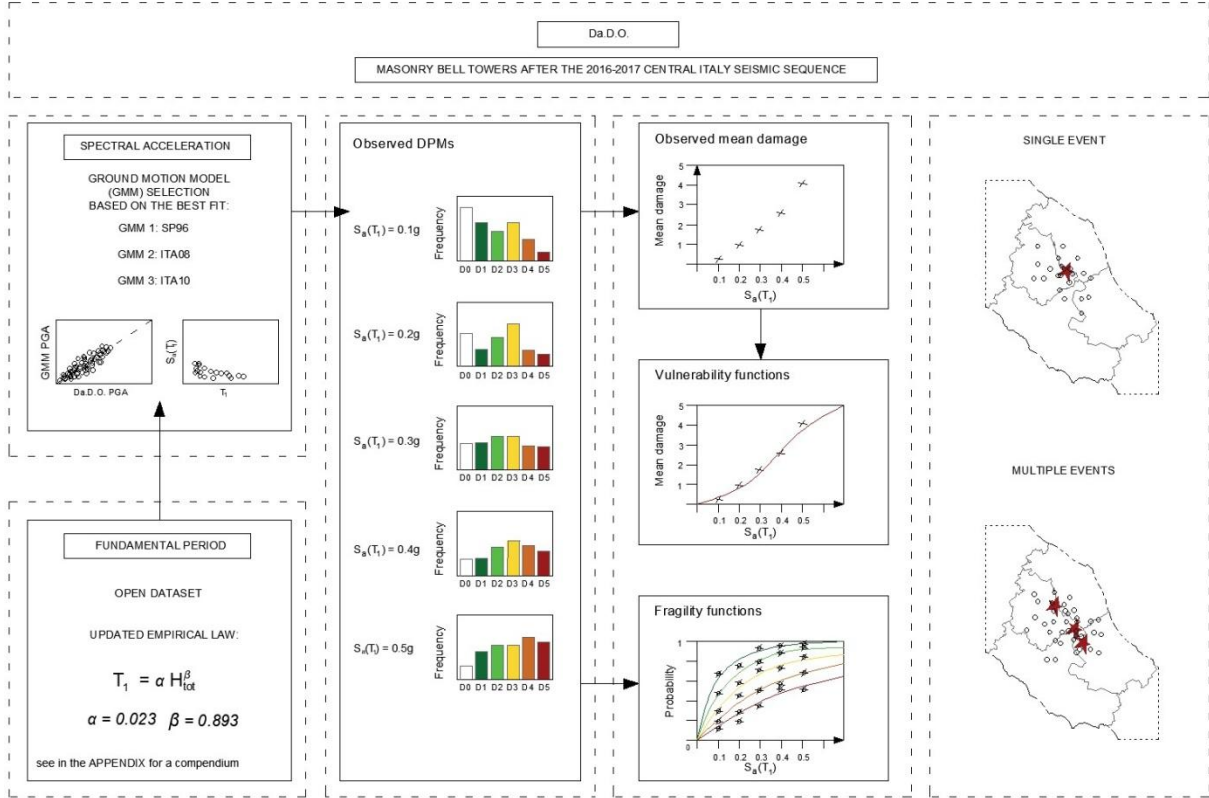


Figure 5.1: Flowchart of the methodology adopted in this Chapter.

First, existing Ground Motion Models (GMMs) recently proposed specifically for Italy were reviewed and a preliminary analysis was carried out to validate the PGAs predicted through these models with the values included in the Da.D.O. database at each tower location. Afterwards, the optimal GMM, selected based on the highest correspondence with the data, was used to estimate the SA as a function of the earthquake magnitude, epicentral distance, and fundamental frequency of the towers. The latter was calculated through an updated empirical formulation, developed within the scope of this thesis. A detailed discussion of the formulation, along with its validation against a large dataset of historic masonry towers, is presented in the Chapter. In the final stage of the work, territorial scale seismic vulnerability models (DPMs, vulnerability and fragility functions) were developed as a function of SA, and compared with similar ones, derived using the PGA, relying on various performance metrics, allowing to identify the best formulation for SA-based models and discuss possible impacts of the seismic intensity measure selection.

5.2 SEISMIC INTENSITY MEASURES

Ground motion models (GMMs), also termed as Ground Motion Predictive Equations or attenuation laws, are empirical models that allow the estimation of the probability density function of ground motion characteristics, namely seismic intensity measures, at a specific site, given a set of properties of the earthquake source, wave propagation and site response. The primary model parameters are the earthquake magnitude M , source-to-site distance R , soil class at site S , and the fault rupture type F . Typical seismic intensity measures that can be computed via GMM are the Peak Ground Acceleration (PGA), Peak Ground Velocity (PGV), Pseudo-Spectral Velocity (PSV) and Spectral Acceleration (SA), among others.

Several GMMs have been proposed in the literature for specific regions, countries and continents, and an online compendium of well-established models can be found in (Douglas, 2018). Among them, the GMMs available for Italy include, but are not limited to SP96 (Sabetta & Pugliese, 1996), ITA08 (Bindi et al., 2010), and ITA10 (Bindi et al., 2011). They were selected for this thesis due to their widespread use and the robustness demonstrated in the context of Central Italy earthquakes, as already mentioned in Chapter 2. While Section 5.2.1 provides a thorough description of these models, Section 5.2.2 presents a comparative analysis to determine the most suitable one for estimating the SA at the sites of towers affected by the Central Italy seismic sequence.

5.2.1 Ground Motion Models (GMMs)

SP96, ITA08 and ITA10 models are hereafter presented and described in detail, providing a definition of the inputs and the outputs, as well as the regression coefficients for their implementation. The SP96 model is expressed as follows:

$$\log_{10}(Y) = a + bM + c \cdot \log_{10}\sqrt{R^2 + h^2} + e_1S_1 + e_2S_2 \quad (5.1)$$

where Y is the seismic intensity measure to be computed, M refers to the surface-wave magnitude (M_s) when $M \geq 5.5$ or the local magnitude (M_L) when $M < 5.5$, and R is the site-to-source distance, in km. S_1 and S_2 are coefficients that account for the site classification based on three categories of site and take value of: $S_1 = 1$ and $S_2 = 0$ for shallow sites (i.e., depth $H \leq 20$ m, alluvium with shear wave velocity $400 \leq V_s \leq 800$ m/s), $S_1 = 0$ and $S_2 = 1$ for deep sites (i.e., depth $H > 20$ m, alluvium with $400 \leq V_s \leq 800$ m/s), and $S_1 = 0$ and $S_2 = 0$ for stiff sites (i.e., $V_s > 800$ m/s). Finally, a , b , c , h , e_1 and e_2 are regressive coefficients that vary depending on the seismic intensity measure considered and the type of selected distance. These values are reported in

(Sabetta & Pugliese, 1996) for PGA (in g), PGV (in cm/s) and Pseudo-Spectral Velocity (PSV) for a 5% damped response spectra (in cm/s), allowing to adopt either the epicentral distance (i.e., between the tower site and the earthquake epicentre) or the Joyner-Boore distance (i.e., the shortest distance between the tower site and the surface projection of the fault rupture), and to consider multiple directions, including the vertical and the larger of the two horizontal components of the motion. As an example, regressive coefficients suggested for the horizontal PGA and PSV when using the epicentral distance as site-to-source measurements are given in Table 5.1 and Table 5.2. The applicability of the SP96 model is suggested for earthquake magnitudes that range between 4.6 and 6.8, and for distance up to 100 km.

Table 5.1: SP96 attenuation law coefficients for the horizontal maximum PGA (g).

Coefficients values					
a	b	c	h	e_1	e_2
-1.845	0.363	-1	5.0	0.195	0

Table 5.2: SP96 attenuation law coefficients for the horizontal maximum 5% damped PSV (cm/sec) for different fundamental periods (natural frequencies) of the investigated structures.

T (s)	f (Hz)	Coefficients values for ranges of vibration period of the structure					
		a	b	c	h	e_1	e_2
4.00	0.25	-2.500	0.725	-1.00	2.60	0.000	0.100
3.03	0.33	-2.250	0.715	-1.00	3.00	0.000	0.108
2.00	0.50	-1.900	0.687	-1.00	3.60	0.000	0.150
1.49	0.67	-1.647	0.660	-1.00	4.00	0.010	0.175
1.00	1.00	-1.280	0.612	-1.00	4.40	0.050	0.208
0.75	1.33	-1.000	0.570	-1.00	4.70	0.120	0.190
0.50	2.00	-0.595	0.500	-1.00	5.00	0.230	0.124
0.40	2.50	-0.281	0.445	-1.00	5.20	0.222	0.078
0.30	3.33	0.100	0.377	-1.00	5.40	0.185	0.020
0.20	5.00	0.296	0.323	-1.00	5.70	0.161	0.000
0.15	6.67	0.222	0.310	-1.00	5.90	0.161	0.000
0.10	10.00	-0.019	0.304	-1.00	6.20	0.161	0.000
0.06	15.00	-0.312	0.304	-1.00	6.30	0.161	0.000
0.04	25.00	-0.817	0.330	-1.00	4.70	0.161	0.000

The ITA08 model is represented by the following equation:

$$\begin{aligned}
 \log_{10}(Y) = & a + b_1(M_w - M_{ref}) + b_2(M_w - M_{ref})^2 \\
 & + (c_1 + c_2(M_w - M_{ef})) \cdot \log_{10}\sqrt{R^2 + h^2} + e_0S_0 \\
 & + e_1S_1 + e_2S_2 + f_0F_0 + f_1F_1 + f_2F_2
 \end{aligned} \tag{5.2}$$

where Y is the seismic intensity measure to be estimated, M_w is the Moment Magnitude, M_{ref} is equal to 4.5 and R is the site-to-source distance. S_0 , S_1 and S_2 refer to the site classification using three possible categories, following the SP96 approach, and take value of: $S_0 = 0$, $S_1 = 1$ and $S_2 = 0$ for shallow sites, $S_0 = 0$, $S_1 = 0$ and $S_2 = 1$ for deep sites, $S_0 = 1$, $S_1 = 0$ and $S_2 = 0$ for stiff sites. F_0 , F_1 and F_2 refer to the fault mechanism classification, and take value of: $F_0 = 1$, $F_1 = 0$ and $F_2 = 0$ for Normal fault, $F_0 = 0$, $F_1 = 1$ and $F_2 = 0$ for Strike-Slip fault, $F_0 = 0$, $F_1 = 0$ and $F_2 = 1$ for Reverse fault. Finally, a , b_1 , b_2 , c_1 , c_2 , h , e_0 , e_1 , e_2 , f_0 , f_1 and f_2 are regressive coefficients that are defined depending on the seismic intensity measure considered and the type of distance.

In (Bindi et al., 2010), these values are provided for PGA (in cm/s^2), PGV (in cm/s) and SA for a 5% damped response spectra (in cm/s^2), using either the epicentral or the Joyner-Boore distance, for multiple directions, including both the vertical and the larger of the two horizontal components of the motion. As an example, regressive coefficients suggested for the estimation of the horizontal PGA and SA when using the epicentral distance as site-to-source distance are shown in Table 5.3 and Table 5.4.

Table 5.3: ITA08 attenuation law coefficients for the horizontal maximum PGA (cm/sec^2).

Coefficients values								
a	b_1	b_2	c_1	c_2	h	e_0	e_1	e_2
3.75	0.118	-0.1147	-1.9267	0.4285	10.0497	0	0.2297	0.1022

The ITA08 model is applicable to a wider range of earthquake magnitudes, namely between 4.0 and 6.9, and for the same range of distance of the SP96 model. It should be noted that, according to the authors of the model, the fault mechanism classification may be neglected in the regression due to its small influence, thus, coefficients f_0 , f_1 and f_2 have not been provided in the original study. Therefore, Eq. (5.2) becomes:

$$\begin{aligned}
 \log_{10}(Y) = & a + b_1(M_w - M_{ref}) + b_2(M_w - M_{ref})^2 + (c_1 \\
 & + c_2(M_w - M_{ref})) \cdot \log_{10}\sqrt{R^2 + h^2} + e_0S_0 + e_1S_1 \\
 & + e_2S_2
 \end{aligned} \tag{5.3}$$

Table 5.4: ITA08 attenuation law coefficients for the horizontal maximum 5% damped SA (cm/sec^2) for different fundamental periods (natural frequencies) of the investigated structures.

T (s)	f (Hz)	Coefficients values for ranges of vibration period of the structure								
		a	b_1	b_2	c_1	c_2	h	e_0	e_1	e_2
0.03	33.33	3.3836	-0.0723	-0.1043	1.9618	0.4346	10.5707	0	0.2211	0.0842
0.04	25.00	3.8461	0.1056	-0.0993	-1.9232	0.4040	10.0637	0	0.2260	0.0742
0.07	14.29	3.9944	0.1111	-0.0962	-1.9128	0.3835	10.2906	0	0.2122	0.0367
0.10	10.00	3.9926	0.1360	-0.0839	-1.8447	0.3492	10.3528	0	0.2609	0.0560
0.15	6.66	4.0596	0.1713	-0.1061	-1.8525	0.3646	10.6030	0	0.2667	0.0610
0.20	5.00	4.0725	0.1252	-0.1124	-1.8780	0.4187	10.6263	0	0.2158	0.1191
0.25	4.00	3.9793	0.1856	-0.1177	-1.8747	0.4025	10.3088	0	0.1654	0.1431
0.30	3.33	3.8899	0.1709	-0.1186	-1.8837	0.4298	9.7877	0	0.1448	0.1605
0.35	2.86	3.8082	0.1670	-0.1153	-1.8929	0.4480	9.4708	0	0.1329	0.1868
0.40	2.50	3.6486	0.2120	-0.1262	-1.8463	0.4439	9.2789	0	0.1185	0.2168
0.45	2.22	3.5930	0.2225	-0.1119	-1.8654	0.4352	9.3832	0	0.0983	0.2612
0.50	2.00	3.5320	0.2612	-0.1081	-1.8738	0.4193	9.3706	0	0.0802	0.2722
0.60	1.66	3.3660	0.3051	-0.0931	-1.8534	0.4061	9.2463	0	0.0581	0.2955
0.70	1.43	3.2342	0.3272	-0.0968	-1.8420	0.4096	9.1689	0	0.0489	0.3197
0.80	1.25	3.1072	0.3532	-0.0708	-1.8367	0.3911	8.9420	0	0.0481	0.3387
0.90	1.11	3.0662	0.4113	-0.0663	-1.8733	0.3612	9.4254	0	0.0445	0.3645
1.00	1.00	3.0468	0.4210	-0.0666	-1.9096	0.3627	9.8637	0	0.0389	0.3772
1.25	0.80	2.8175	0.4168	-0.0708	-1.8762	0.3838	9.6593	0	0.0285	0.4079
1.50	0.66	2.8253	0.2885	-0.0923	-1.9731	0.5055	9.9835	0	0.0035	0.4089
1.75	0.57	2.8399	0.2188	-0.0878	-2.0782	0.5756	10.7942	0	0.0041	0.4124
2.00	0.50	2.7171	0.2378	-0.0990	-2.0787	0.5868	10.3772	0	-0.0012	0.3830

Finally, the third model considered in this study, namely ITA10, is expressed in a general form as the sum of five terms:

$$\log_{10}(Y) = e_1 + F_D(R, M) + F_M(M) + F_S + F_{sof} \quad (5.4)$$

where Y is the seismic intensity measure, e_1 is a regressive coefficient, $F_D(R, M)$ is the distance function, $F_M(M)$ is the magnitude scaling, F_S is the term that accounts for the site amplification, and F_{sof} is the corrective coefficient that takes into account the type of fault. These terms are described in detail hereafter.

More specifically, the distance function is formulated as:

$$F_D(R, M) = \left(c_1 + c_2(M_w - M_{ref}) \right) \cdot \log_{10} \left(\sqrt{R^2 + h^2} / R_{ref} \right) - c_3 \left(\sqrt{R^2 + h^2} - R_{ref} \right) \quad (5.5)$$

where M_w is the moment magnitude and M_{ref} is equal to 5.0. R is the Joyner-Boore distance (in km), limiting the use of the epicentral distance when the fault geometry is unknown and, if possible, only for $M \leq 5.5$, and R_{ref} is equal to 1 km. Finally, c_1 , c_2 , h , and c_3 are regressive coefficients.

The magnitude scaling expression is the following:

$$F_M(M) = \begin{cases} b_1(M_w - M_h) + b_2(M_w - M_h)^2 & \text{for } M_w \leq M_h \\ b_3(M_w - M_h) & \text{otherwise} \end{cases} \quad (5.6)$$

where $M_h = 6.75$, b_1 and b_2 are regressive coefficients and b_3 is kept equal to 0 as suggested in the original study.

The site amplification term is given by:

$$F_S = s_j C_j \quad j = 1, \dots, 5 \quad (5.7)$$

where s_j are regressive coefficients and C_j are dummy variables used to denote five site classes, from A to E, according to the Eurocode 8 (EC8) (CEN, 2004), differently from the previous GMM models: $C_A = 1$ and other $s_j C_j = 0$ for soil A ($V_{s,30} > 800 \text{ m/s}$), $C_B = 1$ and other $s_j C_j = 0$ for B ($360 \leq V_{s,30} \leq 800 \text{ m/s}$), $C_C = 1$ and other $s_j C_j = 0$ for C ($180 \leq V_{s,30} \leq 360 \text{ m/s}$), $C_D = 1$ and other $s_j C_j = 0$ for D ($V_{s,30} \leq 180 \text{ m/s}$), $C_E = 1$ and other $s_j C_j = 0$ for E.

Similarly to the ITA08 model, the term related to the faulting type correction is given by:

$$F_{sof} = f_j E_j \quad j = 1, \dots, 4 \quad (5.8)$$

where f_j are regressive coefficients and E_j are dummy variables used to denote four types of fault: $E_1 = 1$ and other $f_j E_j = 0$ for Normal fault, $E_2 = 1$ and other $f_j E_j = 0$ for Reverse fault, $E_3 = 1$ and other $f_j E_j = 0$ for Strike-Slip fault, and $E_4 = 1$ and other $f_j E_j = 0$ for Unknown fault.

In (Bindi et al., 2011), the values of the regressive coefficients are reported for the estimation of PGA (in cm/s^2), PGV (in cm/s) and SA for a 5% damped response spectra (in cm/s^2) for multiple directions, including the vertical and the larger of the two horizontal components of the motion without accounting for the distinct site-to-source metric. As an example, regressive coefficients suggested for the estimation of the horizontal PGA and SA are shown in Table 5.5 and Table 5.6. The ITA10 model is applicable to a wide range of earthquake magnitudes, varying between 4.0 and 6.9, as in ITA08 model, and for a greater distance range, namely up to 200 km. A comparative summary of the characteristics of the three selected GMMs is given in Table 5.7.

Table 5.5: ITA10 attenuation law coefficients for the horizontal maximum PGA (cm/sec^2).

Coefficients values															
e_1	c_1	c_2	h	c_3	b_1	b_2	s_A	s_B	s_C	s_D	s_E	f_1	f_2	f_3	f_4
3.672	-1.94	0.413	10.322	1.34e-04	-0.262	-0.0707	0	0.162	0.240	0.105	0.570	-5.03E-02	1.05E-01	-5.44E-02	0

Table 5.6: ITA10 attenuation law coefficients for the horizontal maximum 5% damped SA (cm/sec^2) for different fundamental periods (natural frequencies) of the investigated structures.

		Coefficients values for ranges of vibration period of the structure															
T (s)	f (Hz)	e_1	c_1	c_2	h	c_3	b_1	b_2	s_A	s_B	s_C	s_D	s_E	f_1	f_2	f_3	f_4
25.00	0.04	3.725	-1.976	0.422	9.445	2.70E-04	-3.15E-01	-7.87E-02	0	0.161	0.24	0.06	0.614	-4.42E-02	1.06E-01	-6.15E-02	0
14.29	0.07	3.906	-2.050	0.446	9.810	7.58E-04	-3.75E-01	-7.73E-02	0	0.154	0.235	0.057	0.536	-4.54E-02	1.03E-01	-5.76E-02	0
10.00	0.10	3.796	-1.794	0.415	9.500	2.55E-03	-2.90E-01	-6.51E-02	0	0.178	0.247	0.037	0.599	-6.56E-02	1.11E-01	-4.51E-02	0
6.67	0.15	3.799	-1.521	0.320	9.163	3.72E-03	-9.87E-02	-5.74E-02	0	0.174	0.24	0.148	0.740	-7.55E-02	1.23E-01	-4.77E-02	0
5.00	0.20	3.750	-1.379	0.280	8.502	3.84E-03	9.40E-03	-5.17E-02	0	0.156	0.234	0.115	0.556	-7.33E-02	1.06E-01	-3.28E-02	0
4.00	0.25	3.699	-1.340	0.254	7.912	3.26E-03	8.60E-02	-4.57E-02	0	0.182	0.245	0.154	0.414	-5.68E-02	1.10E-01	-5.34E-02	0
3.33	0.30	3.753	-1.414	0.255	8.215	2.19E-03	1.24E-01	-4.35E-02	0	0.201	0.244	0.213	0.301	-5.64E-02	8.77E-02	-3.13E-02	0
2.86	0.35	3.600	-1.320	0.253	7.507	2.32E-03	1.54E-01	-4.37E-02	0	0.22	0.257	0.243	0.235	-5.23E-02	9.05E-02	-3.82E-02	0
2.50	0.40	3.549	-1.262	0.233	6.760	2.19E-03	2.25E-01	-4.06E-02	0	0.229	0.255	0.226	0.202	-5.65E-02	9.27E-02	-3.63E-02	0
2.22	0.45	3.55	-1.261	0.223	6.775	1.76E-03	2.92E-01	-3.06E-02	0	0.226	0.271	0.237	0.181	-5.97E-02	8.86E-02	-2.89E-02	0
2.00	0.50	3.526	-1.181	0.184	5.992	1.86E-03	3.84E-01	-2.50E-02	0	0.218	0.28	0.263	0.168	-5.99E-02	8.50E-02	-2.52E-02	0
1.67	0.60	3.561	-1.230	0.178	6.382	1.14E-03	4.36E-01	-2.27E-02	0	0.219	0.296	0.355	0.142	-5.59E-02	7.90E-02	-2.31E-02	0
1.43	0.70	3.485	-1.172	0.154	5.574	9.42E-04	5.29E-01	-1.85E-02	0	0.210	0.303	0.496	0.134	-4.61E-02	8.96E-02	-4.35E-02	0
1.25	0.80	3.325	-1.115	0.163	4.998	9.09E-04	5.45E-01	-2.15E-02	0	0.210	0.304	0.621	0.15	-4.57E-02	7.95E-02	-3.38E-02	0
1.11	0.90	3.318	-1.137	0.154	5.231	4.83E-04	5.63E-01	-2.63E-02	0	0.212	0.315	0.68	0.154	-3.51E-02	7.15E-02	-3.64E-02	0
1.00	1.00	3.264	-1.114	0.140	5.002	2.54E-04	5.99E-01	-2.70E-02	0	0.221	0.332	0.707	0.152	-2.98E-02	6.60E-02	-3.62E-02	0
0.80	1.25	2.896	-0.986	0.173	4.340	7.83E-04	5.79E-01	-3.36E-02	0	0.244	0.365	0.717	0.183	-2.07E-02	6.14E-02	-4.07E-02	0
0.67	1.50	2.675	-0.960	0.192	4.117	8.02E-04	5.75E-01	-3.53E-02	0	0.251	0.375	0.667	0.203	-1.40E-02	5.05E-02	-3.65E-02	0
0.57	1.75	2.584	-1.006	0.205	4.505	4.27E-04	5.74E-01	-3.71E-02	0	0.252	0.357	0.593	0.22	1.54E-03	3.70E-02	-3.85E-02	0
0.50	2.00	2.537	-1.009	0.193	4.373	1.64E-04	5.97E-01	-3.67E-02	0	0.245	0.352	0.54	0.226	5.12E-03	3.50E-02	-4.01E-02	0

Table 5.7: Summary of the characteristics of the GMMs considered in this study.

	SP96	ITA08	ITA10
Intensity measure	PGA, PGV, PSV	PGA, PGV, SA	PGA, PGV, SA
Unit	$g, cm/s, cm/s$	$cm/s^2, cm/s, cm/s^2$	$cm/s^2, cm/s, cm/s^2$
Magnitude	Surface-wave ($M \geq 5.5$) Local ($M < 5.5$)	Moment	Moment
Magnitude range	$4.6 \leq M \leq 6.8$	$4.0 \leq M \leq 6.9$	$4.0 \leq M \leq 6.9$
Distance	Joyner-Boore and Epicentral	Joyner-Boore and Epicentral	(*) Joyner-Boore or Epicentral
Distance range	$R \leq 100$ km	$R \leq 100$ km	$R \leq 200$ km
Soil characteristics	Shallow	Shallow	A
	Deep	Deep	B
	Stiff	Stiff	C
			D
			E
Fault rupture	Not included	(**) Normal Strike-Slip Reverse	Normal Strike-Slip Reverse Unknown

(*) No separated tables for the regressive coefficients are reported in (Bindi et al., 2011) to account for the distinct site-to-source metrics.

(**) Although fault classification is highlighted, its contribution is neglected and regression values have not been provided by (Bindi et al., 2010).

5.2.2 Comparative assessment

To identify the most robust attenuation law for estimating the SA at the tower sites, a strategy was implemented to account for uncertainties in the model parameters and the lack of recalibration of the model factors to the instrumental records. This approach began by validating the predicted PGAs from the three well-established GMMs identified in the previous Section (i.e., SP96, ITA08, and ITA10), at the sites of the 794 towers affected by the Central Italy seismic sequence, analysed in Chapter 4, against those provided by the Da.D.O. database. The model showing the best correspondence was considered the most accurate for the ground motion characteristics, thus, subsequently employed to predict the SA at the tower sites.

The estimation was iterated by considering the five main seismic shocks, in each case adopting the specific earthquake magnitude, the distance of the towers from the event epicentres, and assuming shallow soil (for SP96 and ITA08) and type B soil (for ITA10) at the tower sites. This latter assumption overlooks the local variability of the response due to stratigraphic conditions. However, based on previous studies (Aloisio et al., 2024; Miano et al., 2020), although the sites are characterised by a geolithological setting that includes several soil materials, such as coarse and fine alluvial deposits, travertine, and arenaceous flysch, among others, the attribution of the class B according to the EC8 was considered the most representative for the investigated area. In these existing studies, soil

characterisation was carried out using the Italian soil class map originally produced based on surface geology data (Di Capua et al., 2016) and, more recently, incorporating the shear-wave velocity measurements (Forte et al., 2019). Upon the identification of the geological units of the investigated area, the shallow soil was considered the most suitable soil class according to the classification adopted by the other GMMs, i.e., SP96 and ITA08. Missing the information on local stratigraphy in the original database, the error introduced by this simplification is considered acceptable for a preliminary evaluation of the ground motion through the GMMs. A further refinement of this estimate based on an accurate classification of the soil type is an important scope of future work. Furthermore, based on the information on the events provided by the INGV (Luzi et al., 2020), a normal fault rupture was considered in the ITA10 model.

The results of the comparison with the PGA values provided by the Da.D.O. database are illustrated from Figure 5.2 to Figure 5.6 and summarised through the R-squared as performance metric in Table 5.8.

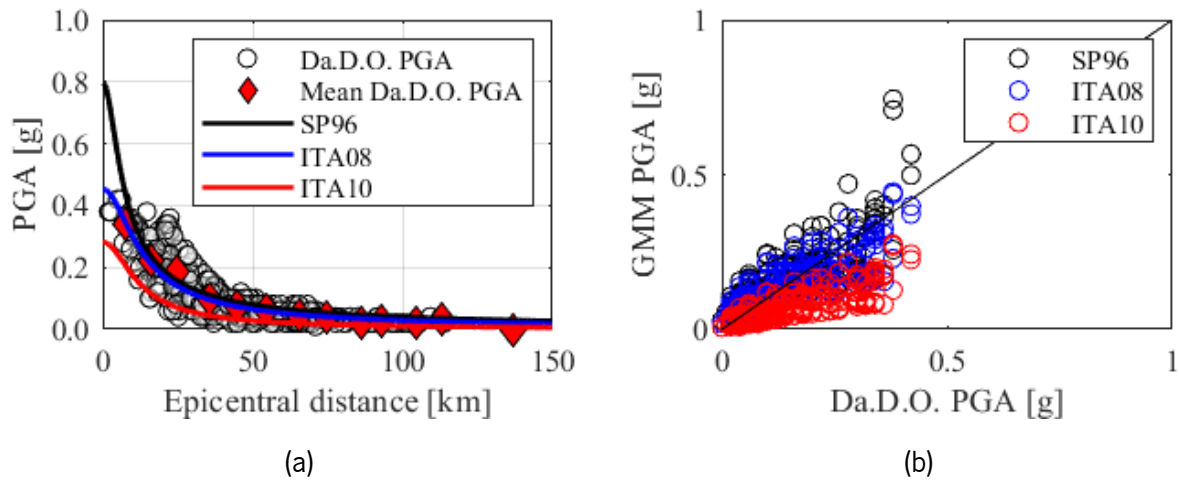


Figure 5.2: Comparison of GMMs predictions with Da.D.O. values for the M6.2 August 24, 2016 seismic shock.

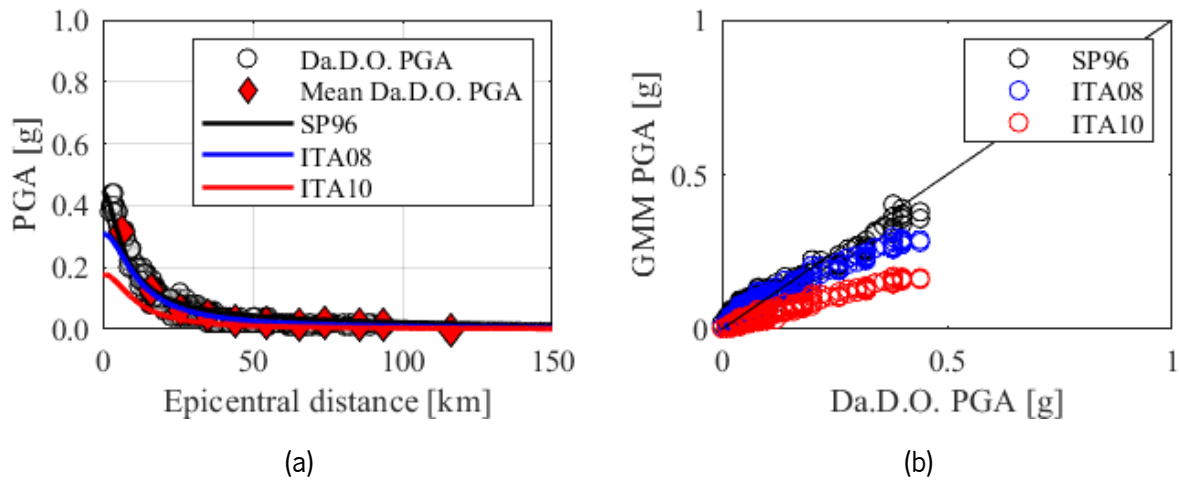


Figure 5.3: Comparison of GMMs predictions with Da.D.O. values for the M5.5 October 26, 2016 seismic shock.

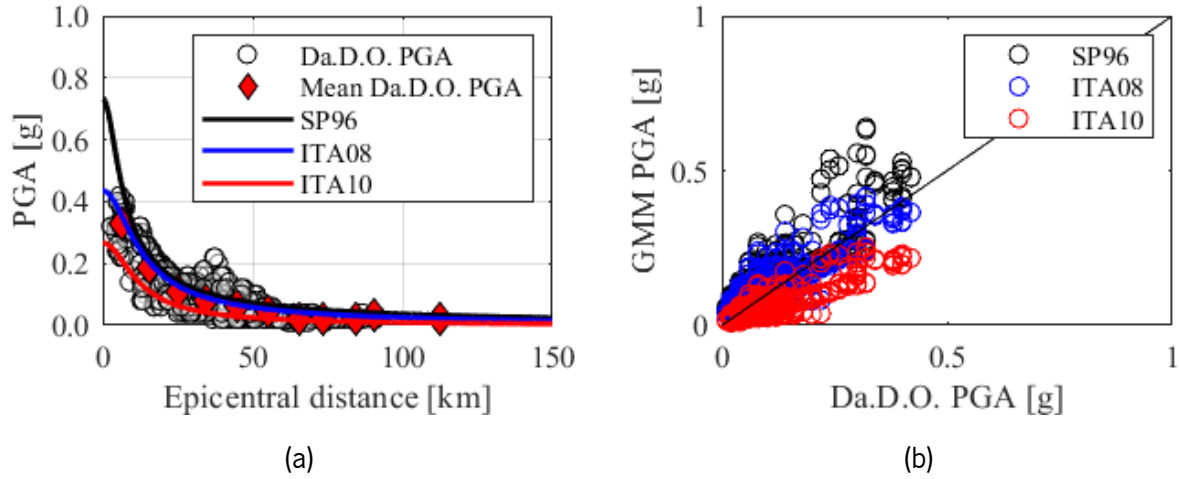


Figure 5.4: Comparison of GMMs predictions with Da.D.O. values for the M6.1 October 26, 2016 seismic shock.

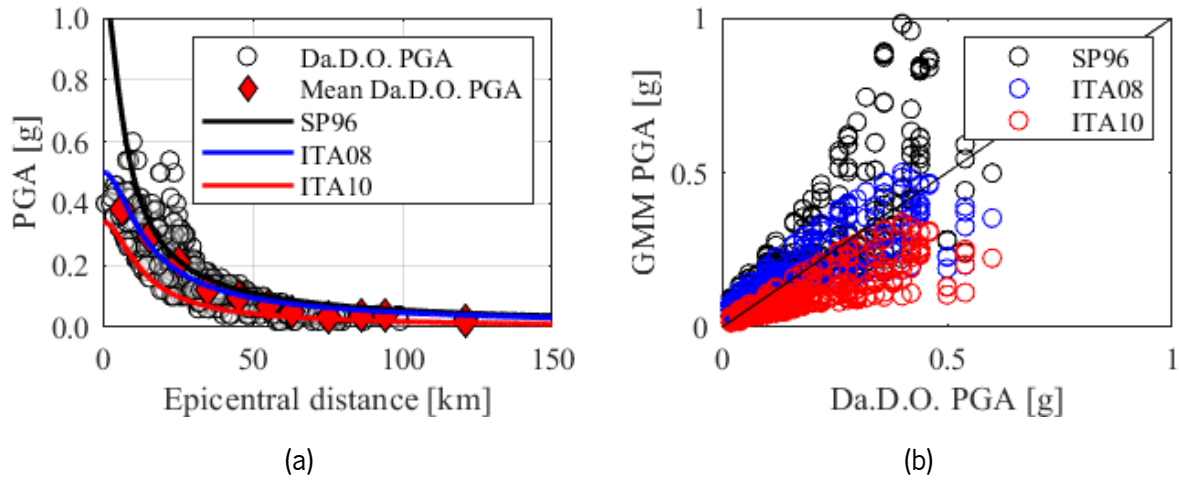


Figure 5.5: Comparison of GMMs predictions with Da.D.O. values for the M6.6 October 30, 2016 seismic shock.

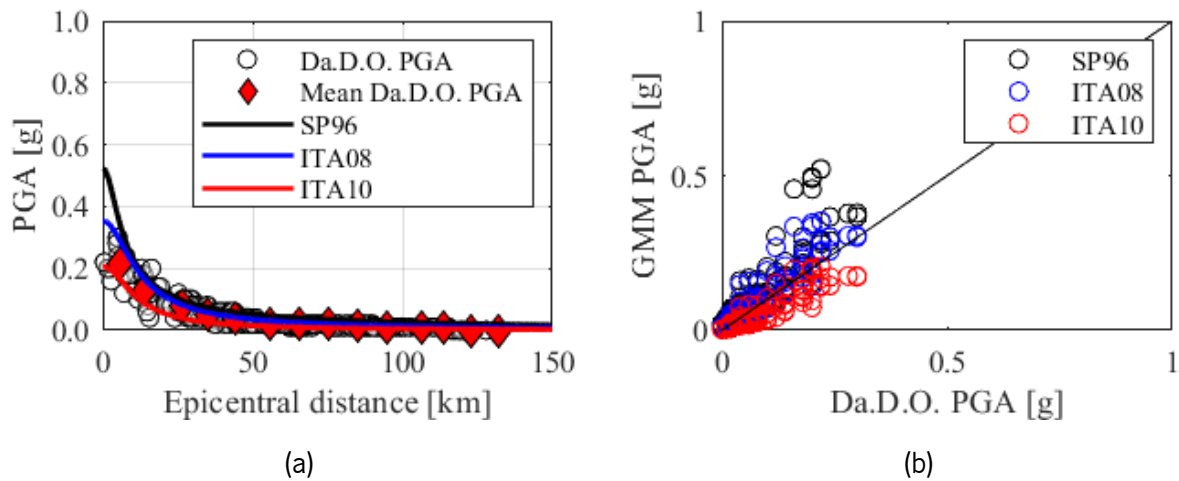


Figure 5.6: Comparison of GMMs predictions with Da.D.O. values for the M5.7 January 21, 2017 seismic shock.

Although the ITA10 is the current state-of-practice GMM for Italy, it consistently underestimated the PGA, irrespective of the earthquake event considered. This lower performance may be affected by a

larger sensitivity to the uncertainties in the model parameters. In contrast, SP96 and ITA08 exhibited a better agreement with the values reported in the Da.D.O. database. However, SP96 showed a slight tendency to overestimate the PGA at very small epicentral distances, resulting in a greater deviation from the Da.D.O. results in the plots. This becomes more pronounced as the earthquake magnitude increases (Figure 5.5). Based on this analysis, ITA08, proposed by (Bindi et al., 2010), demonstrated the best fit to the database values, as confirmed by the higher R-squared values.

Table 5.8: Summary of the R-squared values.

No. Event	SP96	ITA08	ITA10
1 st	0.65	0.71	0.11
2 nd	0.89	0.89	0.43
3 rd	0.48	0.74	0.42
4 th	-0.07	0.74	0.31
5 th	0.16	0.75	0.66

A further assessment and comparison of the predictive performance of the SP96 and ITA08 models was carried out by focusing, for each tower, on the events that caused the highest predicted PGA at the site prior to inspections, using again the Da.D.O. database values as reference for validation purposes. The results of this analysis are illustrated in Figure 5.7. These findings confirmed that the ITA08 model exhibits a very strong agreement with the database used in Chapter 4, consistently replicating the PGA values with high accuracy. Indeed, the R-squared values for the SP96 and ITA08 models are 0.46 and 0.77, respectively, indicating that the ITA08 model performs better than the SP96 model. To provide a clearer visualisation of the model performance, Figure 5.8 presents the correlation, for each tower, between the highest PGA values computed based on the values included in the Da.D.O. database and those predicted by SP96 and ITA08. This comparison highlights the superior predictive capability of the ITA08 model for the dataset at hand.

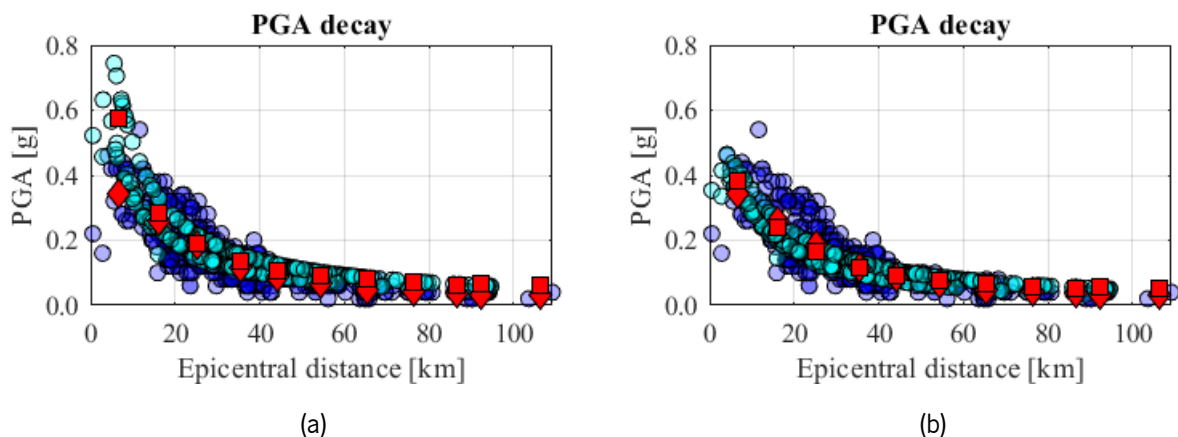


Figure 5.7: Correlations between the highest PGA prior to inspection and the corresponding epicentral distance. Da.D.O. values (dark blue circles) and GMM prediction (light blue circles) using: (a) SP96; (b) ITA08.

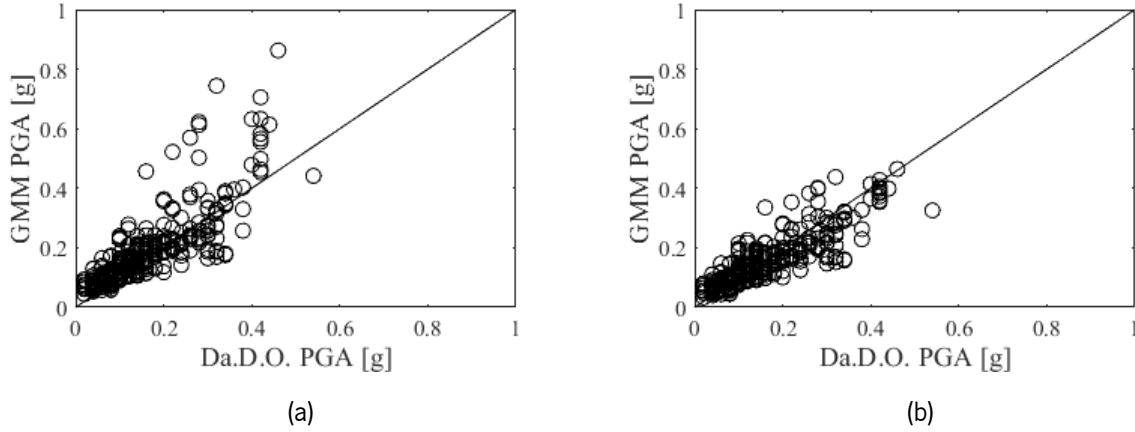


Figure 5.8: Correlation between the highest PGA prior to inspection computed based on Da.D.O. values and the highest PGA predicted using: (a) SP96; (b) ITA08.

In light of these considerations, the ITA08 model was selected in the present study to estimate the horizontal SA. For this estimation, attenuation laws require the knowledge of the fundamental period of vibration of the towers, as the model factors depend on this parameter (see Table 5.4). In the absence of an experimental characterisation of the fundamental period for all the investigated structures, empirical formulations from the literature are commonly employed. However, within the scope of the present thesis, as further discussed in the following Section, novel and robust formulations have been developed. These new formulations demonstrated superior performance compared to existing established methods, providing more accurate predictions of the fundamental period for historic towers.

5.3 EMPIRICAL FORMULATIONS

The fundamental period of a structure is a dynamic parameter that plays a key role in both seismic design and assessment procedures. While empirical equations for estimating the fundamental period or natural frequency of masonry towers exist and have shown promising results (see Section 5.3.1), several challenges remain unresolved. Specifically, these equations are based on limited datasets and have not been validated against distinct samples. Additionally, most proposed models rely on simple supervised machine learning methods, such as the power regression, incorporating only one or two independent variables, and without grouping the towers by similar structural behaviour.

In light of these considerations, novel empirical formulations for predicting the fundamental frequency of historic masonry towers have been derived in this thesis. A substantial dataset was compiled from the literature, gathering the largest database ever utilised in similar studies for a specific structural typology, i.e., 184 case studies (Section 5.3.2). This database focuses exclusively on civic towers and bell towers, excluding other historic slender structures such as minarets, lighthouses, chimneys, and

pagoda temples. Based on this database, reliable models were developed as functions of more geometric parameters (Section 5.3.3), assessing whether their inclusion, one by one, improves the predictive performance. Additionally, distinct clusters of towers with likely homogeneous behaviour were analysed, classifying the samples by their boundary conditions and their material, to provide tailored predictive equations. These models, obtained by means of power regression through Curve Fitting Toolbox included in MATLAB software (The MathWorks, 2012), were compared with the existing ones to evaluate their effectiveness. The is used for the power regression The prediction of higher vibration modes was left as future scope of the work, prioritising the first one, due to its importance for the tower dynamic response, the greater availability of supporting data in literature and its direct use in the GMMs. The collected database was divided into a training set and a validation set to ensure a fair assessment of the model performance against new samples unknown in the training stage. The accuracy and robustness of the developed models were assessed through two performance metrics, namely the coefficient of determination (R-squared):

$$R^2 = 1 - \frac{\sum_{i=1}^n (f_{i,exp} - f_{i,est})^2}{\sum_{i=1}^n (f_{i,exp} - f_{avg})^2} \quad (5.7)$$

and the Mean Squared Error (MSE):

$$MSE = \frac{\sum_{i=1}^n (f_{i,exp} - f_{i,est})^2}{n} \quad (5.8)$$

where $f_{i,exp}$ and $f_{i,est}$ are the i -th experimental and estimated frequencies, respectively, f_{avg} is the average of the experimental frequency values and n is the number of samples. In particular, the R-squared metric was employed to evaluate the models ability to fit the experimental data during the training phase, whereas the MSE metric was mainly adopted to assess the prediction accuracy for masonry towers not included in the training dataset (validation phase). The robustness of the models was analysed by comparing the MSE values for both the training and validation sets. Finally, a few formulations with the highest predictive performance were identified and commented. Among them, the most suitable for the estimation of the SA at the Central Italy tower sites through the ITA08 GMM was later used in the final Section of the Chapter.

5.3.1 Existing formulations

National design codes (NCSE, 2002; NTC, 2008) provide simplified empirical equations for the rapid estimation of the fundamental period of masonry structures. However, these equations were primarily developed for ordinary buildings, and their application to specialised masonry typologies, such as towers, may result in lower accuracy. In this context, recently, there has been growing scientific interest in developing similar formulations for distinct structural systems. Several alternative equations can be found in the literature addressing specific masonry typologies, including churches (Lopez et al., 2019), mosques (Ashayeri et al., 2021; Calik et al., 2020), and historic towers (Bartoli, Betti, Marra, et al., 2017; Diaferio et al., 2018; Faccio et al., 2011a; Rainieri & Fabbrocino, 2011; Shakya et al., 2016). Regarding the latter typology, most of these formulations provide the first natural frequency as a function of the total height (H_{tot}) of the tower only. Over time, the database of real-world case studies used to develop predictive equations has expanded from 24 to 59 case studies. These case studies encompass samples with likely different behaviour (e.g., different structural typologies, boundary conditions and material properties), introducing significant variability in the experimental results that cannot be easily explained by simplified formulations. To address this, recent studies have focused on specific structural systems, such as chimneys and minarets, developing targeted formulations (Bayraktar et al., 2022; Oliveira et al., 2022; Serhatoğlu & Livaoğlu, 2019; Shakya et al., 2016). Significant progress in estimating the first natural frequency has been made by distinguishing between isolated and bounded towers (Diaferio et al., 2018). For bounded towers, models based on the effective height (H_{eff}), defined as the difference between the height of the tower and of the adjacent structures, demonstrated to be more effective (Bartoli, Betti, Marra, et al., 2017; Diaferio et al., 2018). In contrast, few attempts have been made to combine more geometric properties, such as height and side length, as independent variables (Diaferio et al., 2018; Shakya et al., 2016) similarly to the (NCSE, 2002). A formula, reported in (Shakya et al., 2016), introduced the frequency as a function of the aspect ratio (H_{tot}/L_{min}) of the tower, namely the total height divided by the minimum side length (L_{min}). A recent contribution further refined predictive accuracy by incorporating the influence of openings (Bartoli et al., 2020). Attempts to include material parameters are even more limited (Bartoli, Betti, Marra, et al., 2017; Oliveira et al., 2022). For instance, (Bartoli, Betti, Marra, et al., 2017) proposed specific formulas for towers with a hollow square cross-section including additional predictors such as the material density and the Young's modulus, whereas (Oliveira et al., 2022) developed different empirical equations specifically for Ottoman minarets, classifying them by material type (stone, brick or mixed). While most of the previous results focused on the fundamental frequency, very few

formulations have been generated to predict higher vibration modes. In particular, the work of (Diaferio et al., 2018) included relationships to compute the second natural frequency for both isolated and bounded masonry towers, whereas the work of (Rainieri et al., 2014) provided equations for estimating the frequencies of historic masonry towers up to the 5th mode of vibration.

5.3.2 Database of historic masonry towers

Upon a literature review on dynamic identification and field-testing of masonry towers, a large database of 184 case studies was generated, collecting the following information: tower name and location, masonry material type (stone, brick or mixed), boundary condition (isolated or bounded), cross-section shape (rectangular, square, semi-circular, circular, elliptical, pentagonal, octagonal, dodecagonal), geometric dimensions (maximum height, effective height, side lengths, wall thickness and aspect ratio) and first experimental frequency value corresponding to a bending mode. Natural frequency values obtained through numerical simulations or other simplified formulations were not included in the database. The database includes for each tower the sources of the information and is freely accessible in an online repository (Testa et al., 2023a). The database revealed a significant prevalence of historic masonry towers with a square or rectangular cross-section. Their plan layout is often characterised by interactions with other structures such as churches, palaces and castles. The cross-section shape, wall thickness, minimum and maximum side lengths may vary along the height. Reported values refer specifically to the base of the tower (see Figure 5.9).

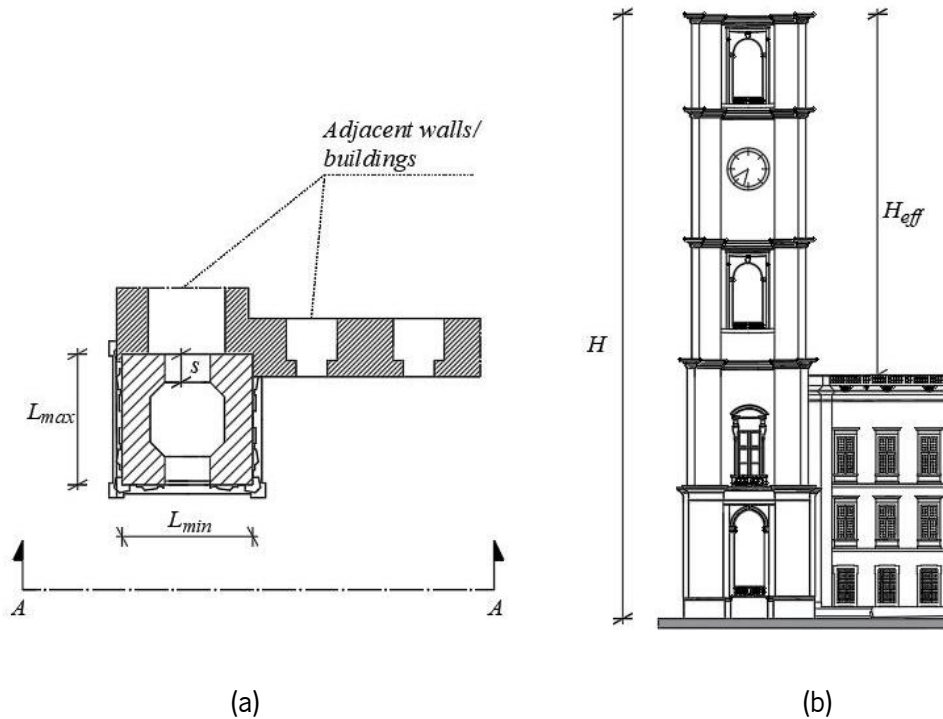


Figure 5.9: Geometric features: (a) plan view, thickness (s), maximum side length (L_{max}) and minimum side length (L_{min}); (b) elevation, total height (H_{tot}), effective height (H_{eff}).

Moreover, when the sources reported a variable wall thickness at the base, the database records the average value. Relying on the available sources, the complete set of information was obtained for 112 historic masonry towers, whereas for other 72 case studies geometric characteristics were partially unknown. This has driven the decision of considering the former, corresponding to 61% of the total database, as training set for the regression, and the latter, corresponding to 39% of the total database, as validation set. The statistics of the features collected in the database are presented in Table 5.9 and further discussed hereafter.

Table 5.9: Statistics of the entire masonry towers database, the training set and the validation set.

	Entire database					Training set					Validation set				
	μ	m	Min	Max	σ	μ	m	Min	Max	σ	μ	m	Min	Max	σ
H_{tot}	38.5	35.1	7.2	112.0	20.9	44.8	41.2	7.2	112.0	19.8	28.8	22.9	7.3	94.0	18.8
H_{eff}	30.4	27.5	7.1	103.7	16.0	30.9	27.5	11.0	103.7	16.2	27.7	20.0	7.1	50.0	15.3
L_{min}	7.3	7.1	1.5	16.0	2.6	7.2	7.0	3.0	14.5	2.4	7.4	7.2	1.5	16.0	3.0
L_{max}	7.6	7.2	1.5	16.0	2.7	7.5	7.2	3.4	14.5	2.4	7.7	7.3	1.5	16.0	3.1
H_{tot}/L_{min}	5.5	5.5	0.7	17.4	2.8	6.4	6.2	1.7	12.6	2.1	3.9	3.0	0.7	17.4	3.3
s	1.8	1.5	0.4	4.6	0.8	1.8	1.5	0.4	4.0	0.8	2.2	2.0	1.0	4.6	1.0
H_{tot}/s	26.6	25.0	5.8	66.3	11.1	27.6	25.6	6.6	66.3	11.0	19.3	17.8	5.8	34.2	9.3
H_{tot}/A	0.9	0.7	0.1	5.8	0.7	0.9	0.9	0.2	1.9	0.4	0.7	0.4	0.1	5.8	1.0

μ = mean; m = median; σ = standard deviation; A = base cross-section area

The analysis of the towers in the training set (Table 5.9) reveals a wide range of geometric and dynamic properties. The total height spans from 7.2 m (the Squat Masonry Tower of the Trani Castle) to 112.0 m (the Torrazzo in Cremona), the minimum side length varies from 3.0 m (the Bell Tower of S. Giorgio Church in Trignano) to 14.5 m (the Giotto Bell Tower), and the base wall thickness from 0.4 m (the Bell Tower of S. Giorgio Church in Trignano) to 4.0 m (the Bell Tower of St. Maria del Carmine Church). The aspect ratio ranges between 1.7 (the Squat Masonry Tower of the Trani Castle) and 12.6 (the Torre del Mangia in Siena). The fundamental frequencies of the towers included in the training set varies from 0.33 Hz (the Asinelli Tower) to 7.50 Hz (the Squat Masonry Tower of the Trani Castle), presenting a clear inverse non-linear trend for increasing values of the height. This relationship is visualised in Figure 5.10, where a colour map highlights the influence of different aspect ratios.

A similar inverse non-linear trend is observed between the fundamental frequency and the effective height (Figure 5.11), suggesting a likely good fitting with a power regression model. However, the correlation between the first natural frequency and the other geometric features (minimum and maximum side lengths and wall thickness) appears weaker. While these relationships also display a slight inverse non-linear trend, the scatter of the data is significantly larger, as illustrated in Figure 5.11.

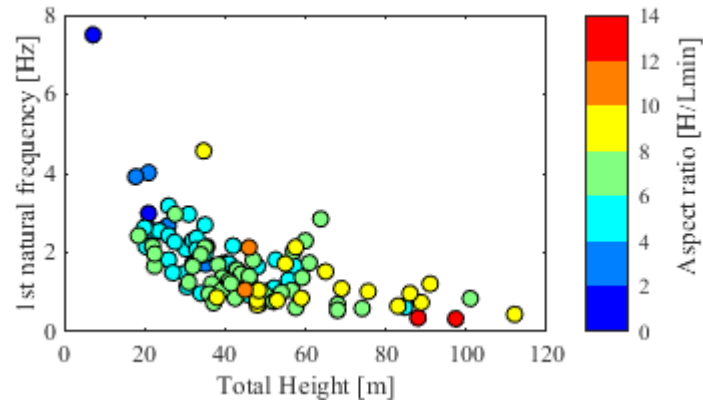


Figure 5.10: Fundamental frequency, total height and aspect ratio correlations of the training set.

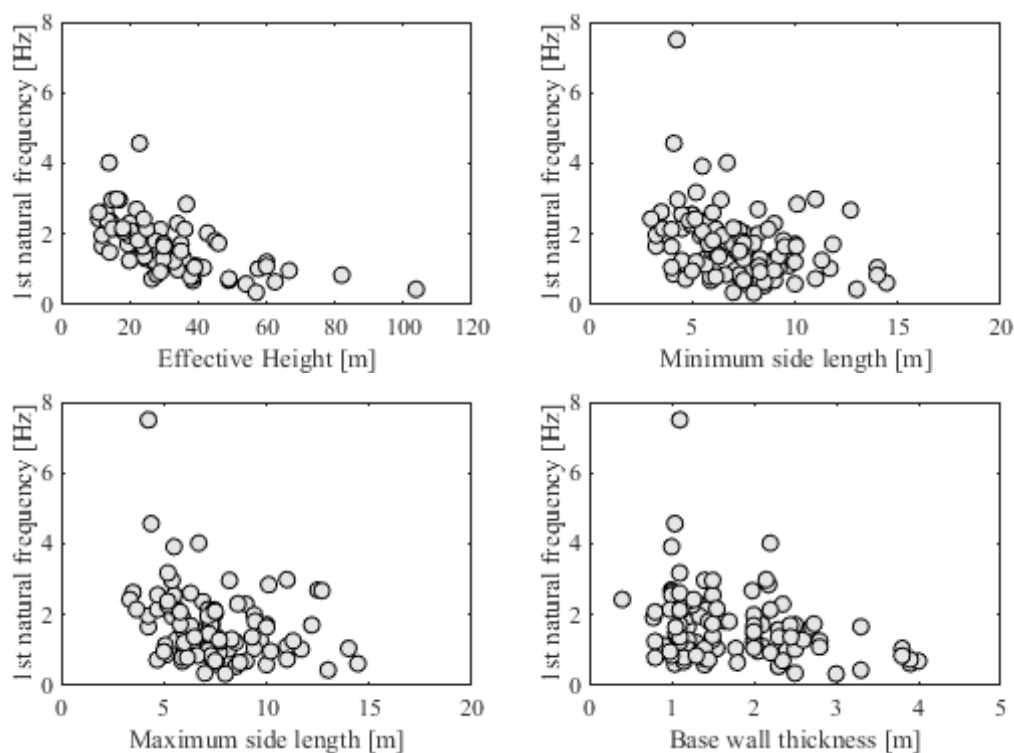


Figure 5.11: Correlations between first natural frequency and geometric characteristics of the training.

The histograms of the occurrences for the analysed parameters within the training and entire datasets are reported in Figure 5.12. The distributions are right skewed and, mostly, unimodal. In particular, referring to the interquartile range of these distributions for the entire dataset, the middle half of the tower height concentrates in the range of 22.3 to 50.0 m with a Coefficient of Variation (CoV) of 54%, and the effective height concentrates in the range of 20.0 to 38.1 m with a CoV of 52%. About the sides, the middle half of the minimum length is in the range of 5.5 to 8.5 m with a CoV of 36%, whereas the maximum length is in the range of 5.7 to 8.9 m with a CoV of 35%. For the wall thickness

distribution, a range of 1.2 to 2.2 m with a CoV of 44% is observed. Finally, the experimental frequency concentrates in the range of 1.20 to 2.66 Hz, with a CoV of 77%.

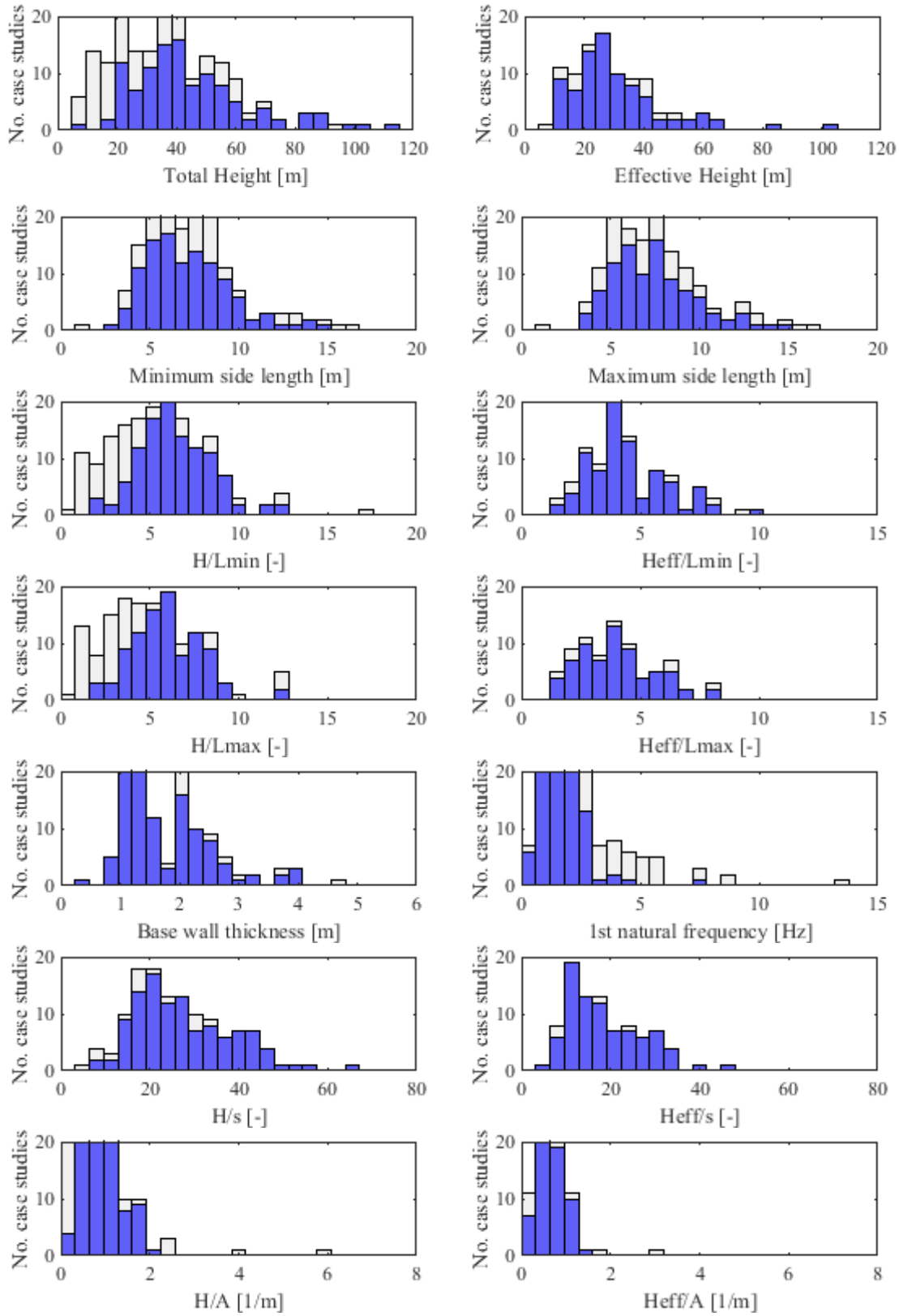


Figure 5.12: Histograms of the occurrence for the continuous variables of the entire database (white colour) and of the training set (blue colour).

The validation set was used to test the performance of the regression models against previously unseen data. Ideally, the distribution of samples between the training and validation sets should follow a statistical rationale. However, in this study, the availability of the information primarily determined the composition of the two sets. This approach introduced a slight bias in the process, as the validation set encompasses a large number of historic masonry towers with total height equal or less than 20.0 m (i.e., aspect ratio lower than 1.5), as also reflected in the statistics of Table 5.9 and the differences between the entire database and training set histogram distributions in Figure 5.12.

Although the total height of the towers ranges from 7.3 m (the Campanillas Tower in Malaga) to 94.0 m (the Arnolfo's Tower in Palazzo Vecchio) and the aspect ratio from 0.7 (the Moya Tower in Malaga) to 17.4 (the Arnolfo's Tower in Palazzo Vecchio, Florence), the mean values of these properties for the case studies belonging to the validation set are notably lower compared to those in the training set (Table 5.9). Consequently, the validation set includes a higher proportion of towers with large frequency values. This disparity is evident when comparing the distributions of the total dataset with those of the training set in Figure 5.12.

The training set lacks sufficient samples in the lower range of the height and, thus, in the higher range of the frequency. In contrast, other geometric properties, such as side lengths and wall thickness, are not significantly different when comparing the training and validation sets. In this latter, the minimum side length varies from 1.5 m (the Bell Tower of the Church "Madoninna della Neve" in Cuneo) to 16.0 m (the Torre de la Vela), while the thickness of the base masonry walls varies from 1.0 m (the Montale Tower) to 4.6 m (the Torre de la Vela), with mean values of 7.4 m and 2.2 m, respectively. Upon this analysis, three cases (i.e., the Campanillas Tower, Salto de la Mora Tower and Moya Tower) with aspect ratios of 1.0 or lower were excluded from the validation set, since they were expected to present a dynamic behaviour rather different from that of slender buildings.

A discrepancy between training and validation sets emerged also in the discrete parameters, namely the masonry material, the boundary condition and the cross-section shape, whose distributions in percentage are given in Figure 5.13. Although some unknown variables are highlighted as N/A, the validation set presents a relatively larger number of isolated and stone towers, as well as all non-rectangular cross-section shapes. This discrepancy and the related bias were duly considered in the evaluation of the predictive performance of the derived empirical formulations.

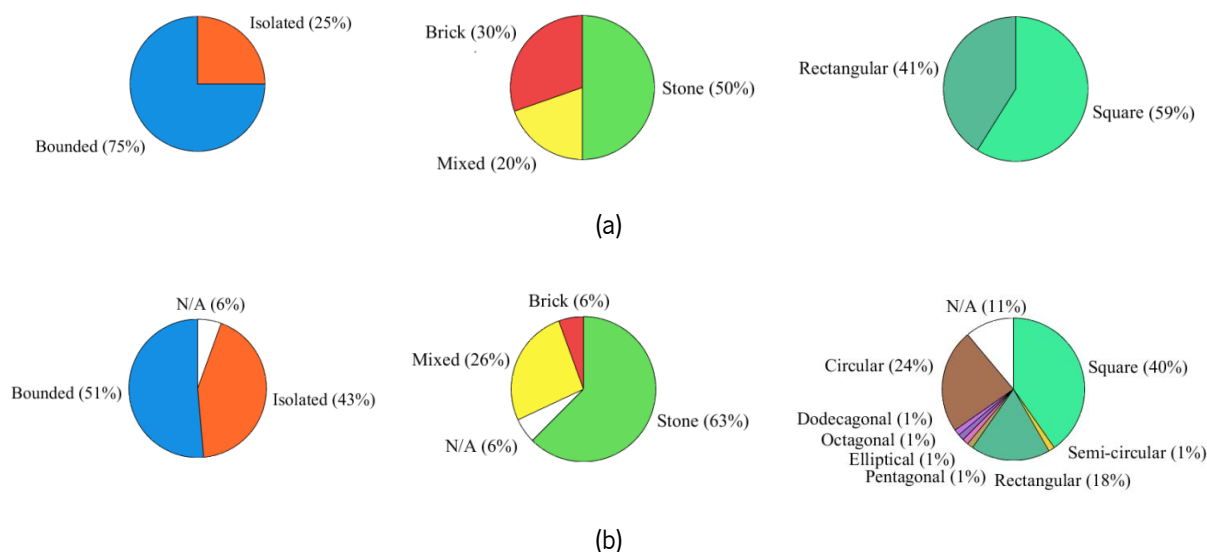


Figure 5.13: Distribution of the samples according to the plan configuration, masonry material and cross-section shape: (a) training set; (b) validation set.

5.3.3 Developed empirical formulations

An extensive set of 66 empirical formulations to predict the first natural frequency of historic masonry towers was developed via power regression, based upon the database described in the previous Section. These models consider one or more independent variables, including the total height or the effective height (the latter applicable only to bounded towers). Additional variables, such as minimum and maximum side lengths and wall thickness, were incrementally introduced to evaluate the predictive performance at progressively higher levels of knowledge regarding the tower characteristics. Thus, the following single independent variables were considered: (i) total height; (ii) effective height; (iii) aspect ratio. For two independent variables, the following combinations were considered: (i) total height and minimum side length; (ii) effective height and minimum side length. Equations depending on three independent variables were developed including these combinations: (i) total height, minimum side length, and wall thickness; (ii) effective height, minimum side lengths, and wall thickness; (iii) total height, minimum and maximum side length; (iv) effective height, minimum and maximum side lengths. Finally, in the case of four independent variables, the following combinations were investigated: (i) total height, wall thickness, minimum and maximum side lengths; (ii) effective height, wall thickness, minimum and maximum side lengths.

Beyond geometric parameters, boundary conditions and material properties also influence the natural frequency of the system. Accurately estimating these factors for historic masonry structures is inherently challenging. To address this, the prediction process was expedited by offering distinct formulas for clusters of towers classified using simple qualitative parameters, as boundary conditions

(bounded or isolated) and material properties (brick, stone, or mixed). For the sake of brevity, the complete set of 66 empirical models, further detailed in (Testa et al., 2023b), was included in Appendix C of this thesis. This Section focuses on describing a single equation, specifically the model used to estimate the SA in the last part of the Chapter. However, final remarks on the validation and comparison of all the formulations are also discussed.

The novel equation presented in greater detail is the one based on the total height:

$$f_1 = \frac{\alpha}{H_{tot}^\beta} \quad (5.9)$$

Various existing formulations in the literature follow this form. The calibrated parameter values of the proposed empirical law are reported in Table 5.10 together with the number of samples contained in the training and validation datasets and the performance measured in terms of R-squared and MSE. For a comparative purpose, five existing models, four specifically developed for masonry towers and one more generic, recommended in the Italian Code (NTC, 2008), are also included, with their performance evaluated against both the training and validation datasets. It should be noted that the existing formulas in Table 5.10, when extracted from original studies that include more than one model, are highlighted with a progressive number, e.g., (Bartoli, Betti, Marra, et al., 2017; Shakya et al., 2016; Testa et al., 2023b), ensuring that each one can be clearly distinguished.

Table 5.10: Equations based on the total height: existing and new formulations and their performance.

Training set	No. cases	Formulas	α	β	R2	MSE Training	No. Cases	MSE Validation
Entire	112	(Faccio et al., 2011a)	53.48	1.000	0.59	0.38	69	0.83
	112	(NTC, 2008)	20.00	0.750	0.42	0.55	69	1.78
	112	(Rainieri & Fabbrocino, 2011)	87.95	1.138	0.56	0.42	69	1.03
	112	(Shakya et al., 2016) (1)	66.23	1.080	0.54	0.43	69	0.89
	112	(Bartoli, Betti, Marra, et al., 2017) (1)	36.42	0.900	0.56	0.41	69	1.03
	112	Proposed law (1)	42.12	0.893	0.64	0.33	69	0.74

All the equations as functions of the total height are visualized in Figure 5.14, along with the training and validation datasets. Analysing the prediction error of the proposed formula a tendency to overestimate the frequency values in the lower range (i.e., below 1.0 Hz) and underestimate them in the higher range (i.e., above 2.5 Hz) emerges. The equation from the Italian Code, which was not specifically developed for slender masonry buildings, consistently underestimates the prediction

compared to the proposed formula. Similarly, the other existing equations also tend to underestimate the natural frequency, especially for tower heights above 20 m.

Although the plot of Figure 5.14 shows a clear non-linear relationship between experimental frequency and height, some scatter is evident, especially in the mid-range of the distribution (i.e., for towers between 20 and 60 m tall). A few outlier cases deviate significantly from the expected trend, being thus hardly predicted by the formulations.

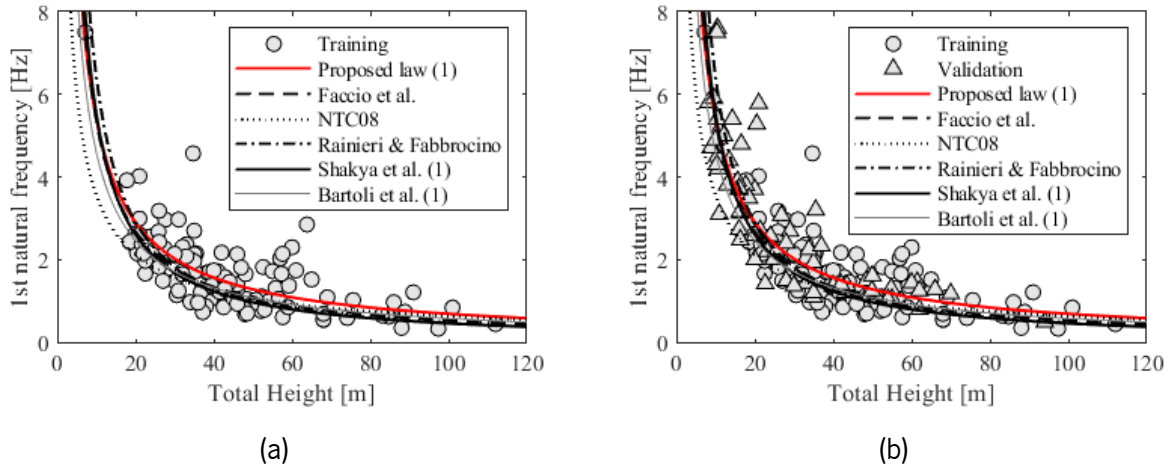


Figure 5.14: Prediction of the first natural frequency for the entire training set: (a) models and training samples; (b) models, training and validation samples.

The strong influence of the height on the dynamic behaviour of historic towers aligns with previous studies on natural frequency estimation, where this parameter is often the only predictor. However, comparing all the 66 empirical equations through R-squared values, it emerges that increasing the number of independent variables, including other geometric features, does not always improve the accuracy. This is likely due to the poorer correlation of such additional features with the fundamental frequency. Even when improvements do occur, they are generally almost negligible compared to the effect of classifying the towers based on boundary condition or material. Indeed, most of the variation in the response is explained by the height of the tower and the identification of subsets of towers with likely homogeneous behaviour. Among the analysed subsets, the class of isolated towers presents a very low scatter from the trend. Conversely, the high variability of the bounded towers is only partially explained by the use of the effective height instead of the total height and, in general, the prediction is less accurate. This suggests that geometric features alone do not fully characterise the dynamic behaviour of the bounded towers. A further subdivision according to the material potentially improves the estimation, although the limited number of cases used for training, especially for the mixed and stone towers, may still affect model reliability.

As shown in Table 5.10, the proposed height-based equation and similarly all the developed models provided a better fit to the training data as indicated by higher R-squared values and lower MSE compared to existing empirical laws from literature and codes. A key aspect of the present thesis is the use of an independent validation set to assess the accuracy of the formulations on unseen data. The novel equations consistently demonstrated improved predictive performance, with significantly lower MSE over the validation set compared to existing models. Moreover, although MSE was often higher for validation than training sets, it remained comparable in most cases, indicating the robustness of the proposed formulations.

Among all the developed formulas, the recommended expressions for a fast prediction of the fundamental frequency of historic towers are the following:

$$f_1 = 42.12H_{tot}^{-0.893} \quad (\text{all towers}) \quad (5.10)$$

$$f_1 = 58.57H_{tot}^{-1.026} \quad (\text{isolated towers}) \quad (5.11)$$

$$f_1 = 11.87H_{eff}^{-0.601} \quad (\text{bounded towers}) \quad (5.12)$$

$$f_1 = 190.90H_{tot}^{-1.378} \quad (\text{isolated brick towers}) \quad (5.13)$$

where Eq. (5.10) applies to any type of tower without the need for detailed information on its geometry, boundary conditions or materials. Therefore, this formulation is used in the next Section for estimating the SA at the tower sites through the GMM. In contrast, Eq. (5.11) to (5.13) are specific to certain tower classes, thus, require an increase of the knowledge of their characteristics.

5.4 VULNERABILITY ASSESSMENT: SPECTRAL ACCELERATION-BASED

The strategy, already adopted in Chapter 4, for the development of robust vulnerability models for the masonry towers affected by the Central Italy sequence (i.e., evaluation of DPMs, analysis and recalibration of the vulnerability functions and derivation of the fragility functions) was implemented in the present Section, with a focus on using SA as the intensity measure.

As described in Section 5.2, the ITA08 GMM, hereafter adopted for the estimation of the SA, requires a knowledge of the fundamental period of the structure. This was calculated using the proposed empirical formulation, Eq. (5.10). To this end, the analyses covered a dataset of 415 towers (GT0), further grouped into two subsets: 134 towers inspected between first and second shocks (GT1) and 237 towers inspected after last main shock (GT2). The reduction of the sample size, from the original 794 towers analysed in Chapter 4, is due to the lack of available height data in the database, which is required for the application of Eq. (5.10).

5.4.1 Estimation of fundamental period and Spectral Acceleration (SA)

The distribution of the fundamental periods for the entire dataset of 415 towers is given in Figure 5.15. Most towers exhibit a fundamental period ranging from 0.2 to 0.6 s, with the majority falling between 0.2 and 0.4 s. Fewer cases show longer periods, increasing up to approximately 0.9 ss. These values reflect the geometric characteristics of the towers, many of which are stocky and have heights under 30 meters, in contrast to the dataset used in Section 5.3.2 and in (Testa et al., 2023b) for developing the empirical predictive formulas. In that case, the first bending mode of the towers was mostly falling between 0.3 and 1.0 s.

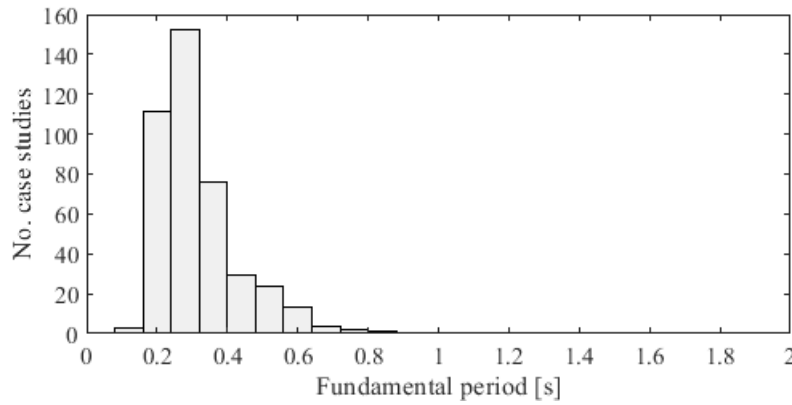


Figure 5.15: Histograms of the occurrence of the fundamental period for the analysed towers.

This difference can be visually verified by directly comparing the predicted results for the towers affected by Central Italy earthquake with those in the dataset described in Section 5.3.2 and in (Testa et al., 2023b). To this end, the fundamental periods (Figure 5.16a) and the natural frequencies (Figure 5.16b) are plotted against the height of the towers in the two sets, i.e., experimental and predicted. The graphs confirm the prevalence of short towers involved in the Central Italy events, while the use of the predictive formula reduces the scatter of the predicted periods.

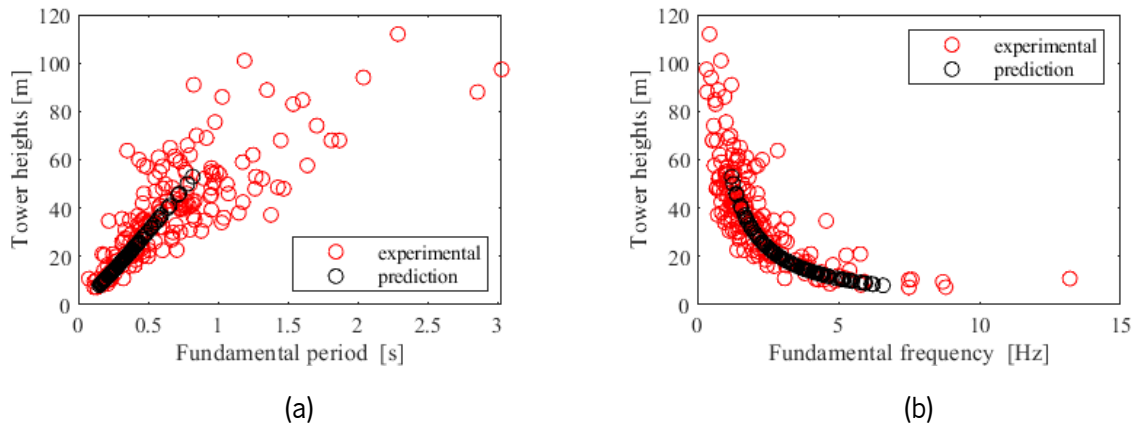


Figure 5.16: Correlations of the tower heights with the fundamental period (a), and with the fundamental frequency (b).

Once the fundamental periods of the analysed towers are determined, the SA can be estimated through the ITA08 GMM proposed by (Bindi et al., 2010). As detailed in Section 5.2, the estimation incorporates key inputs, including the earthquake magnitude, the epicentral distance and the soil characteristics. Here, the maximum estimated SA among the seismic events occurring before the survey date and the distance of the towers from the corresponding epicentres, and shallow soil at tower sites were considered. The SA estimated for the investigated towers are plotted as function of the fundamental period, T_1 , in Figure 5.17a. The results showed maximum values of about 0.90 g at short periods (at around 0.2 s), with values decreasing as the fundamental period increases, aligning with the expected trend. Indeed, most of the fundamental periods of the towers are between 0.2 and 0.4 s, a range typically associated to the plateaux of the acceleration spectra where the maximum amplifications occur. However, a significant number of towers characterised by short vibration periods features rather smaller SA. These values are explained by the epicentral distances of these towers (Figure 5.17b and Figure 5.18).

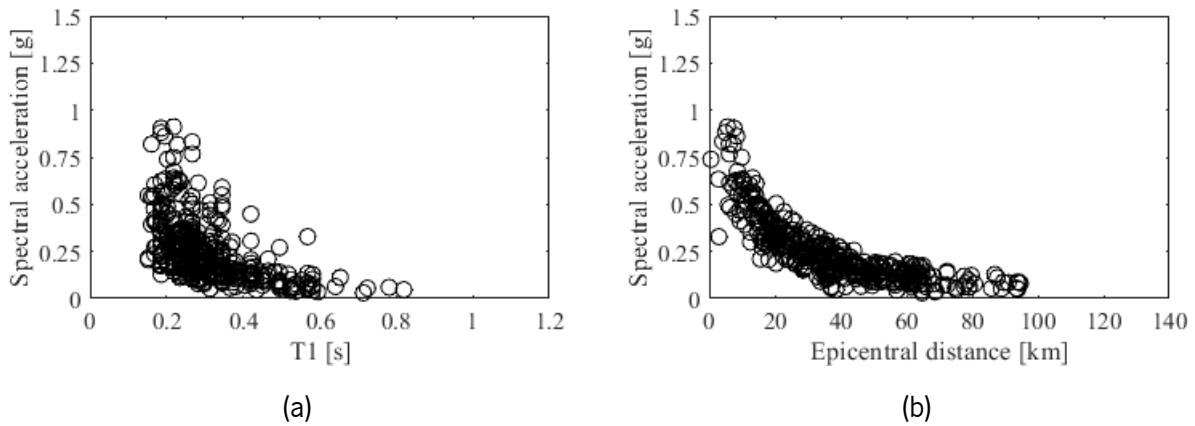


Figure 5.17: Spectral accelerations of the analysed towers as function of the fundamental period (a) and epicentral distance (b).

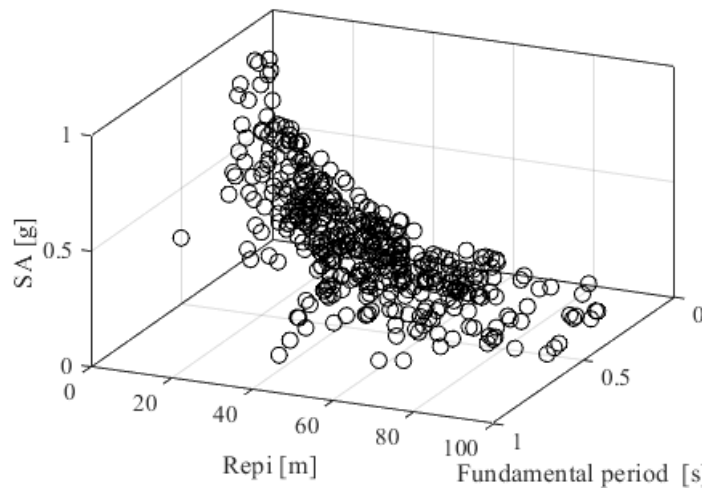


Figure 5.18: Spectral accelerations, epicentral distances, and fundamental period correlations.

The SA estimated through ITA08 for the analysed towers were validated against actual values, extracted from the ground motion time histories registered during the five main shocks of the 2016-2017 Central Italy seismic sequence considered in this study. In particular, for the five main shocks, already described in Chapter 4, the acceleration time histories featuring maximum horizontal PGA were examined, including both horizontal directions. More details are summarised in Table 5.11. The estimated SAs were compared with the 5% damped acceleration elastic spectra for the five main shocks, as illustrated in Figure 5.19.

Table 5.11: Seismic stations and ground motion time histories characteristics (source: INGV).

No. Event	M_w	Station	Lat.	Long.	Repi (km)	Soil (EC8)	Max PGA (cm/s ²)	Direction
1 st	6.2	Amatrice (AMT)	42.632	13.286	~8.5	B	850.804 368.387	X Y
2 nd	5.5	Campi (CMI)	42.850	13.093	~5.7	C	706.801 334.766	X Y
3 rd	6.1	Campi (CMI)	42.850	13.093	~7.7	C	638.312 302.557	X Y
4 th	6.6	Castelluccio di Norcia (CLO)	42.829	13.206	~6.8	C	418.616 571.424	X Y
5 th	5.7	Poggio Cancelli Base Diga (PCB)	42.558	13.338	~3.9	B	408.670 584.331	X Y

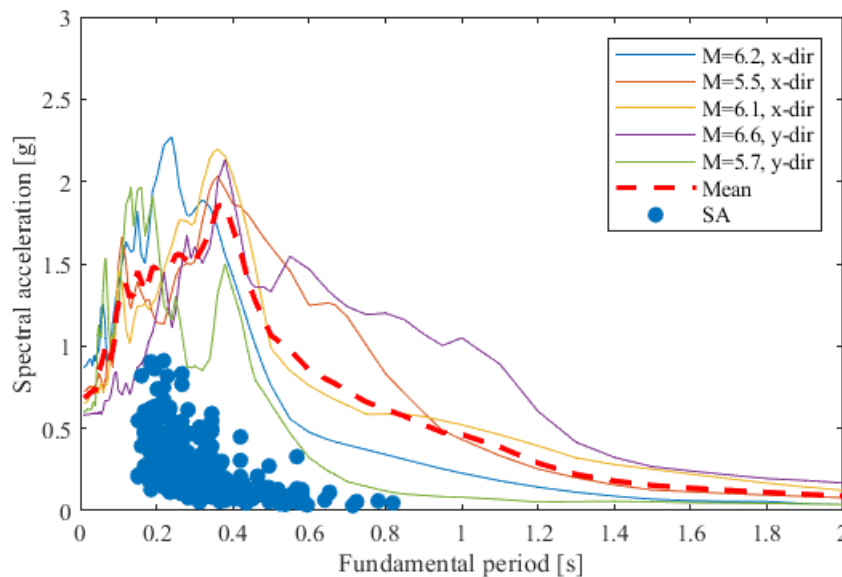


Figure 5.19: Comparison of the spectral accelerations with the response spectrum (source: INGV).

As it can be observed, the estimated SAs of the analysed towers are within the region of the response spectra where maximum amplifications occur. In particular, maximum instrumental SA values ranges between 1 and 2 g for fundamental periods varying between 0.2 and 0.4 s. The observed peak occurred at around 0.2 s for the first and fifth events and at around 0.4 s for the second, third and

fourth events. In most cases, the estimated SAs are significantly lower than the maximum instrumental values for each event, except when comparing the highest estimated values with the response spectra of the fourth and fifth seismic events. Indeed, a few towers show estimated SA of about 0.90 g at around 0.2-0.3 s which corresponds to the maximum recorded values for these events. However, it is worth mentioning that the differences between the estimated and actual values may be related to a series of uncertainties in the application of the GMM, with the most important being the assumed uniform soil type for all the towers. Additionally, the epicentral distance may contribute to these discrepancies, as most of the towers are located at epicentral distances rather larger than the seismic stations used for the analysis. For instance, the response spectra for the first and fourth events are extrapolated from records registered at approximately 7 and 10 km from the epicentre.

5.4.2 Damage levels and Damage Probability Matrices (DPMs)

After estimating the SAs for the towers included in the dataset, the methodology already adopted in Chapter 4 for developing vulnerability models, was followed. This involved first analysing the damage levels and calculating the DPMs. As previously mentioned, the analysis focused on the entire dataset of 415 towers (GT0), as well as on two subsets: one comprising 134 towers affected by a single main event (GT1) and another comprising 237 towers inspected after last main shock (GT2).

The damage to each tower was graded referring to the two individual mechanisms, namely A (shaft) and B (belfry), in addition to the overall macroelement condition, which combines the two mechanisms. In Figure 5.20, the damage grades are correlated with the distance to the epicentre of the event that caused the maximum SA estimated at its site prior to inspection. The results are also grouped into bins of 10 km and the average damage grades are computed and plotted against the average distance of the towers in each bin (represented by red diamonds in the graphs). Additionally, a plot of the maximum SA at the tower sites against the epicentral distance is provided together with the average value for each 10 km bin. The average values for very high distances (>80km) are considered statistically non-representative due to the limited samples available, thus ignored. A large scatter can be observed in the damage grades of different towers at the same range of distance from the event causing the maximum SA at their site, especially within few tens of kilometres. However, a clear overall decay emerges in all the average values that drop quickly within the first 20 km and go below D1 between 30 and 40 km (Figure 5.20a, b, and c). Interestingly, at the same distance, the maximum SA, which presents less scatter and a clear and constant reduction moving away from the epicentre, goes below 0.2g (Figure 5.20d). Only within the first 20-30 km, the average damage for mechanism B appears higher than the

one for mechanisms A. However, this difference in the average damage becomes quickly negligible for larger distances.

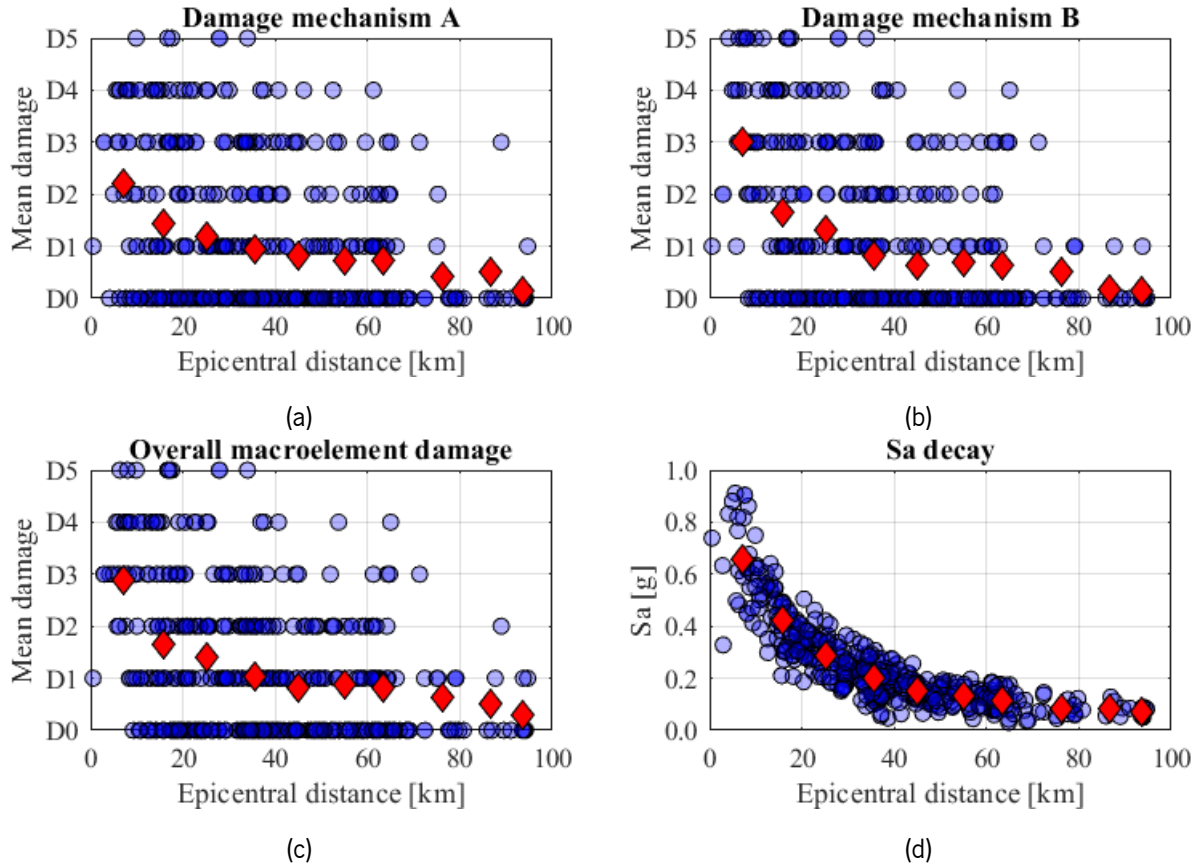


Figure 5.20: Decay of seismic effects with the epicentral distance (GT0): (a) damage grades for Mechanism A; (b) damage grades for Mechanism B; (c) damage grades for overall macroelement; (d) maximum SA estimated at the tower site. Scatter plot (blue points) and mean values for 10 km bins (red diamonds).

Notably, these results are comparable to those obtained using the PGA (see Chapter 4). However, the SA values are typically about twice as high as the PGA, reflecting an amplification effect likely caused by the short vibration periods obtained for the analysed towers.

Similarly to GT0, the maximum SA at the sites of the towers for GT1 and GT2 were correlated with the epicentral distance, together with the average values for increasing distance bins of 10 km. The results are presented in Figure 5.21. Values after 60 km are ignored due to the limited number of surveyed towers in this range, especially for GT1. While the maximum SA is higher in GT2, with several instances exceeding 0.6g, the average SA experienced by the towers in both cases is quite similar, showing a comparable decreasing trend that drops below 0.2g at 30-40 km from the epicentre.

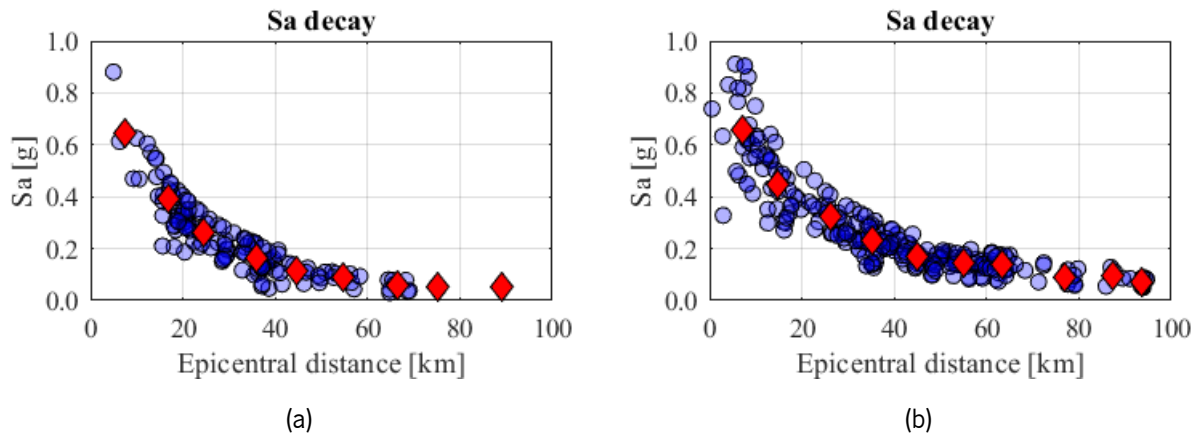


Figure 5.21: Decay of maximum SA with epicentral distance: (a) after the first event (GT1), and (b) after the last event (GT2). Scatter plot (blue points) and mean values for 10 km bins (red diamonds).

DPMs were developed for the entire dataset of 415 towers, grouped by the individual mechanisms and the overall macroelement damage, separating the towers in intervals of increasing SAs, as shown in Figure 5.22. The size of the SA bins was defined to ensure a statistically significant number of cases within each of them. Adopting a well-established mathematical notation, the ranges of SA are indicated as $[\cdot)$, to stress that the lower bound is included in the range while the upper bound is excluded.

The DPMs are well distributed, showing a rather good agreement with the expected behaviour. Indeed, while for lower seismic intensity the damage distribution is right-skewed generally unimodal showing peaks for lower damage extents, for higher seismic intensity the damage distribution is left-skewed showing peaks for greater damage extents. These trends are respected with a very few exceptions.

In particular, a high number of undamaged towers can be observed for lower seismic intensities, whereas the number of damaged towers grows for greater seismic intensities. This is also reflected in the values of mean damage which become significant for increasing intensity values. However, for the maximum values of the intensity measures, the observed damage is distributed quite uniformly or normally across the different damage grades. Similarly to what discussed in Chapter 4, only in the case of mechanism B a clear left-skewed distribution appears. This reflects the tendency for both the damage distributions and global mean damage values to increase more significantly for mechanism B than for mechanism A, starting from certain seismic intensity thresholds (e.g., 0.3g of SA). These results suggest a higher vulnerability of the belfry compared to the shaft of the towers, confirming the results showed in Chapters 3 and 4. For the case of SA, an increasing threshold is identified when compared to the case of PGA, due to the amplification effects induced by the very low values of the vibration periods of the investigated towers.

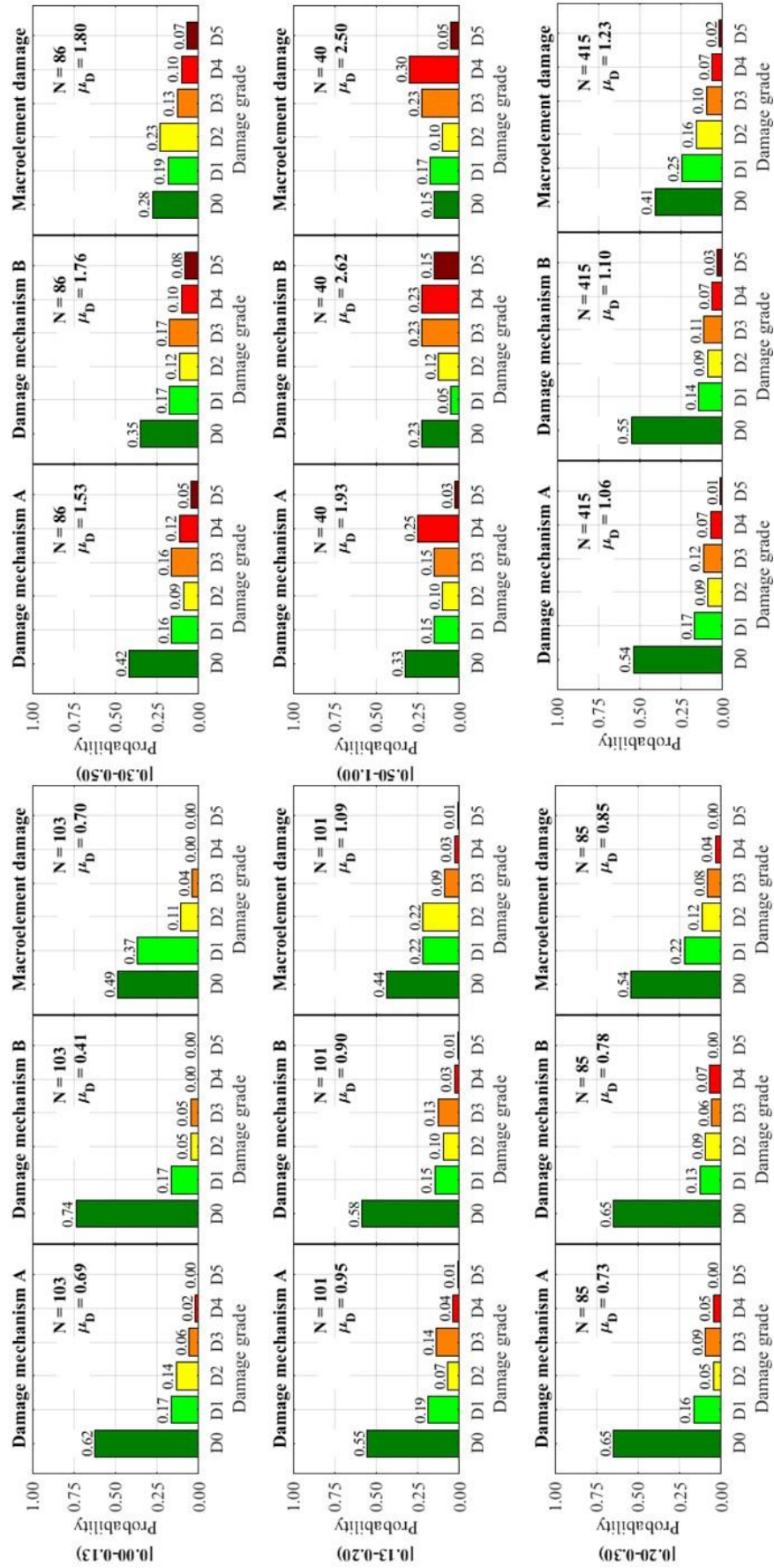


Figure 5.22: DPMs as functions of SA.

A condensed form of the DPMs as functions of the SA, for the individual mechanisms and the overall macroelement damage, is reported in Figure 5.23.

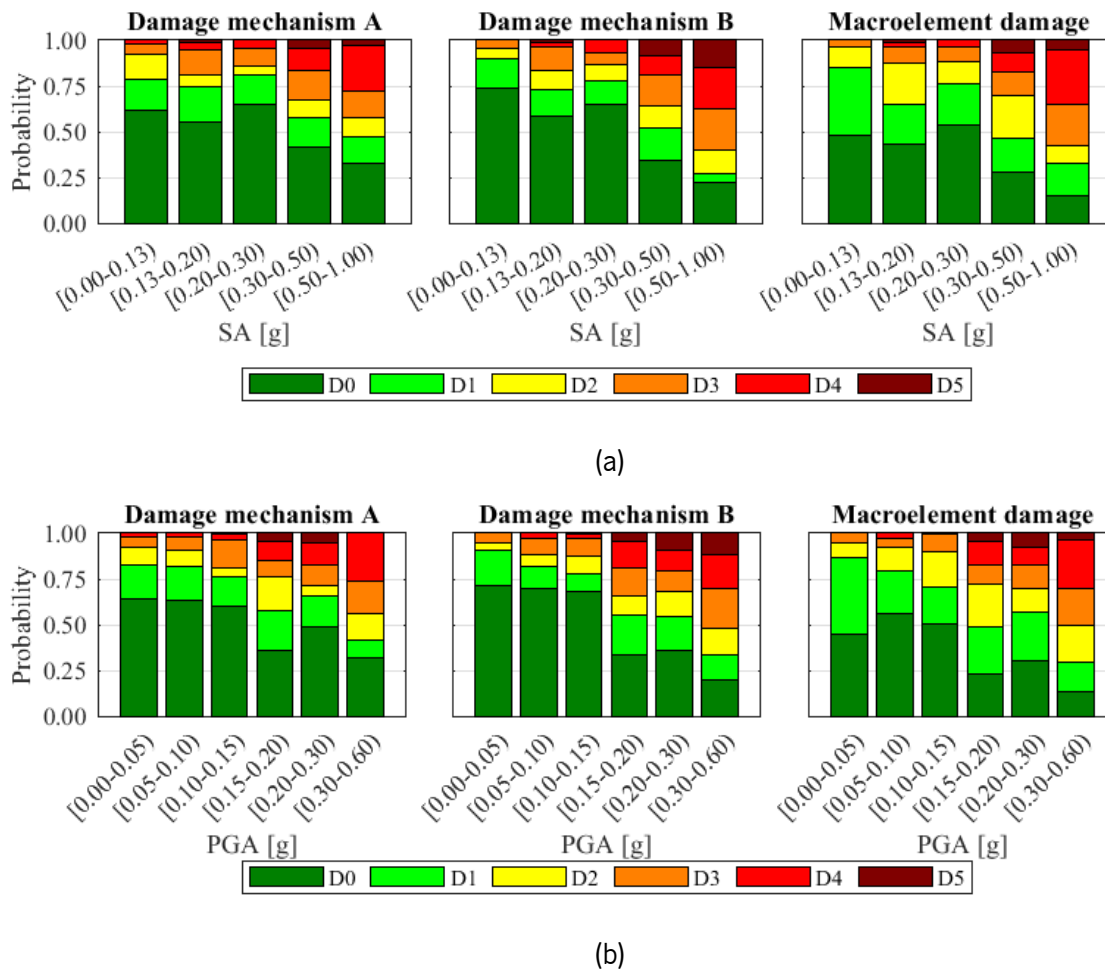


Figure 5.23: DPMs for the entire set using two distinct intensity measures: (a) SA and (b) PGA

The DPMs derived using the two different seismic intensities are very similar, except for the range values, which are higher in the case of SA, and despite the distinct number of bins adopted to ensure uniformly distributed sample size that is statistically significant. A clear shift from a right skewed distribution of the damage levels for small intensity to a left skewed for high intensity emerges in the case of mechanism B. Mechanism A, instead, is characterised by a lower percentage of collapses, even at the largest intensity, and a rather unimodal distribution with peaks in D3-D4. Finally, the maximum PGA was correlated with the maximum SA, both prior to the survey, for each tower, in Figure 5.24.

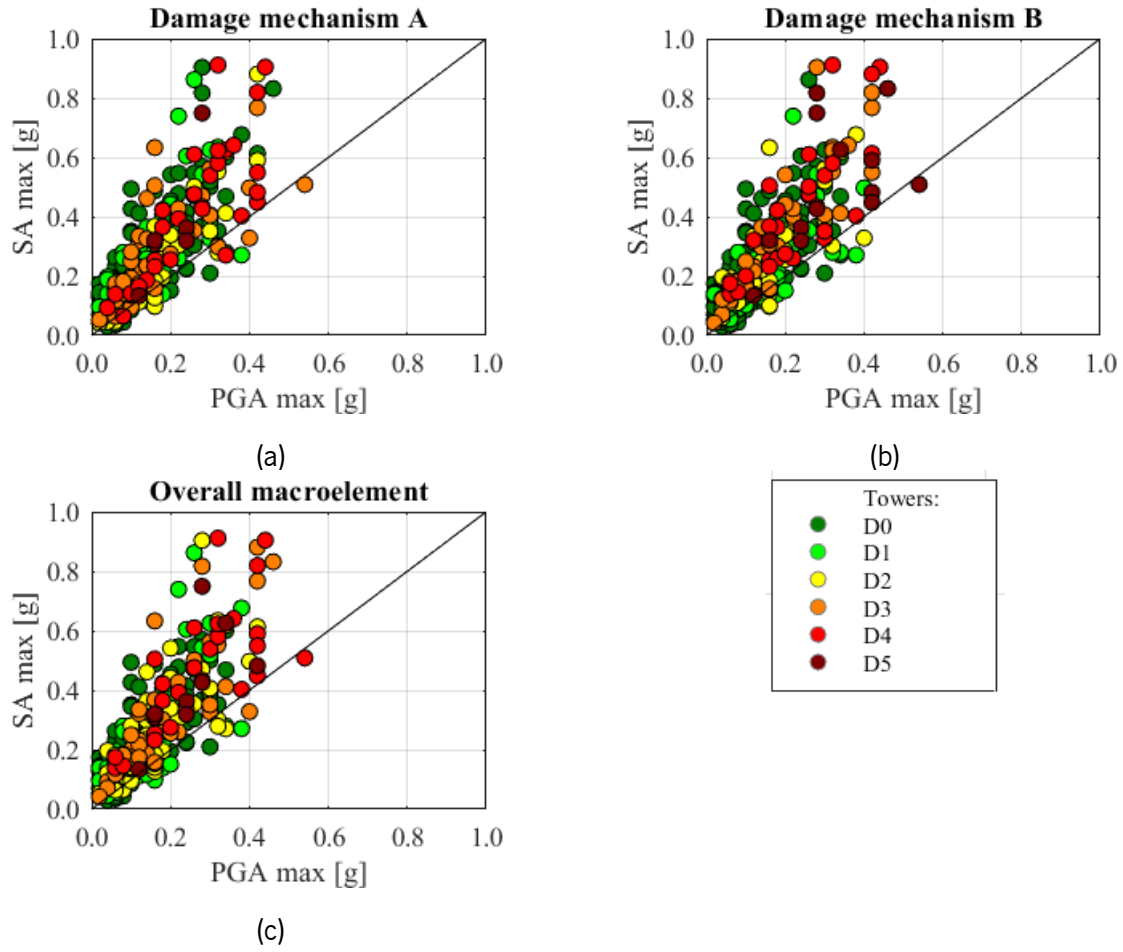


Figure 5.24: PGA-SA correlations for both individual damage mechanisms (a, b) and overall macroelement (c). Damage grades highlighted through the colour scale.

The results confirmed the amplification effects (rather than a de-amplification, as might have been expected for more slender structures) for most of the towers, regardless of the individual damage mechanism analysed. No clear trends or patterns in the damage grades as function of the intensity measures emerged. However, a tendency for higher damage levels at greater values of SA and PGA can be highlighted for damage mechanism B more than for mechanism A. This is reflected in the positive and larger values of the Pearson Correlation Coefficient, PCC (Table 5.12), here used to estimate the linear correlation between damage grade and intensity measure. The PCC is the ratio between the covariance of two generic variables, x_i and y_i , whose linear correlation is investigated, and the product of the their standard deviations (Montgomery, 2020):

$$PCC = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad (5.14)$$

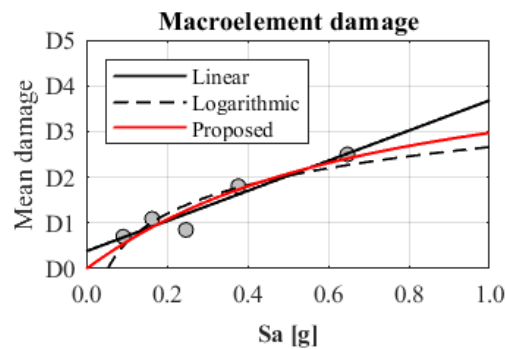
where n is the sample size, $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ and $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$. The PCC varies between -1 and $+1$, with these extreme values representing a perfect negative and positive linear correlation, respectively. Interestingly, a stronger correlation is observed with the PGA than with the SA, although this may be influenced by the evident non-linear relationship between the two measures (Figure 5.24).

Table 5.12: Correlation coefficients between the intensity measures and the damage levels.

	Mechanism A	Mechanism B	Overall macroelement	
	Damage grade	Damage grade	Damage index	Damage grade
SA	0.27	0.44	0.40	0.40
PGA	0.33	0.46	0.44	0.44

5.4.3 Vulnerability functions

In the absence of vulnerability curves specifically designed for historic masonry bell towers as functions of the SA, a comparison of potential models was carried out by fitting three distinct functions to the available data. In particular, the models used are expressed by Eq. (5.15), (5.16) and (5.17), which correspond to the linear, logarithmic and hyperbolic functions, respectively. The latter is similar to the formulation used in Chapter 4 and extensively discussed in Section 2.3.2. The three models, calibrated with the coefficients summarised in Table 5.13, are plotted together with the observational data in Figure 5.25, referring to the macroelement damage only for the sake of brevity.



Formulas:

$$\mu_D = a(SA) + b \quad (5.15)$$

$$\mu_D = \log(SA) a + b \quad (5.16)$$

$$\mu_D = 2.5 [1 + \tanh(a'' \log(SA) + b'')] \quad (5.17)$$

Figure 5.25: Calibration using Eq. (5.15), (5.16), (5.17), to fit the data.

Table 5.13: Calibration of vulnerability function coefficients and performance metrics results.

Function Type	a''	b''	MSE	R-squared
Linear	3.30	0.38	0.04	0.92
Logarithmic	2.09	2.66	0.08	0.82
Tanh	1.21	0.19	0.05	0.89

The linear and logarithmic models exhibit limitations due to the lack of constraints at both high and very low values. Indeed, the three models diverge mostly outside the range of the observational data while,

within the data range, they demonstrate strong agreement and a significant overlapping among them and with the observed mean damage values, as highlighted by the performance metrics in Table 5.13. In this light, Eq. (5.17) was considered the most suitable model and a recalibration of its parameters a'' and b'' was proposed, by fitting the global mean damage-SA pairs for the individual damage mechanisms (A and B) and the entire macroelement (Figure 5.26). The calibrated parameters of the novel formulation are reported in Table 5.14. It is worth mentioning that the use of the recalibrated functions is recommended for the available range of observation data, as the sample may not accurately represent the relationship between SA values above 1 g and higher damage levels (D4 and D5).

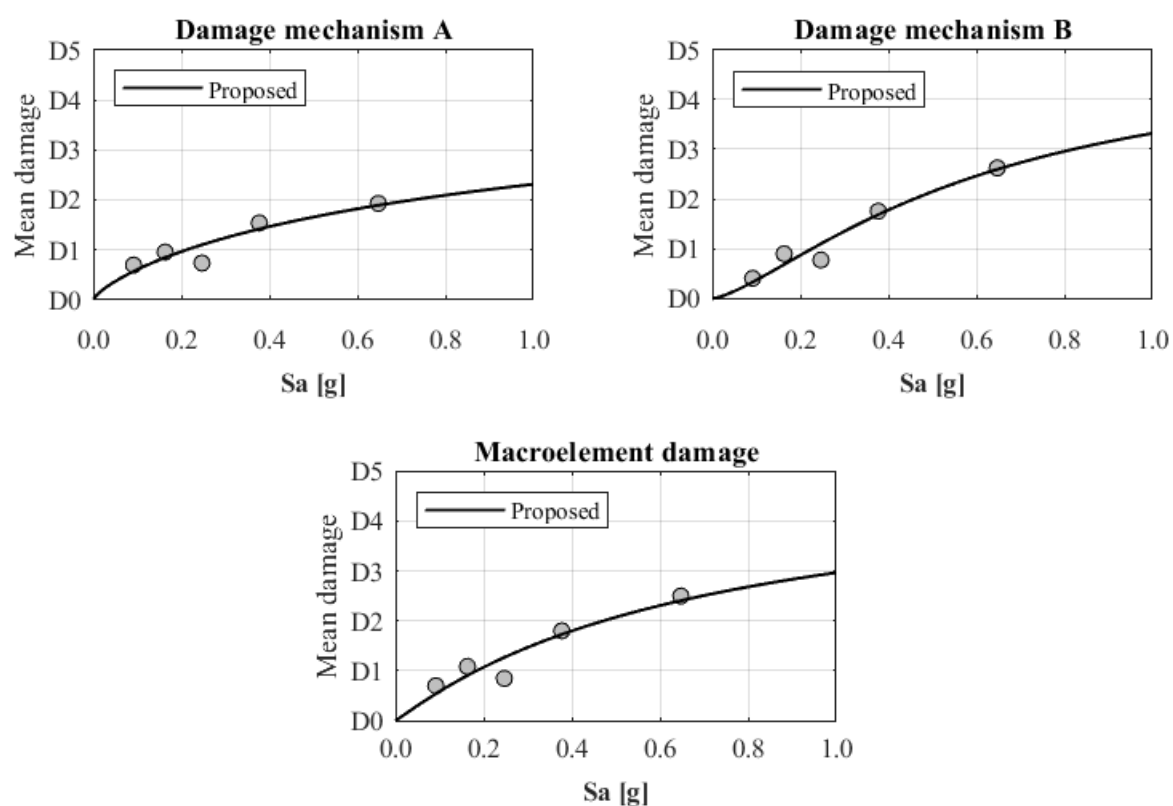


Figure 5.26: Observed mean damage, plotted with circles, compared with the developed vulnerability functions for the entire dataset (GT0) as functions of SA.

Table 5.14: Calibration of vulnerability function coefficients.

	a''	b''
Mechanism A	0.920	-0.076
Mechanism B	1.600	0.340
Macroelement damage	1.210	0.190

Several performance metrics were computed to assess the reliability of the proposed vulnerability models. These metrics helped to compare the correlation of different seismic intensity measures (i.e.,

SA and PGA) to the damage and identify the most robust model formulation. This approach follows the methodology introduced in (Rossetto & Elnashai, 2003).

In particular, the following metrics were calculated based on the error between the predicted and the actual mean damage for the entire dataset: mean of the error ($\mu(\varepsilon)$), variance of the error ($var(\varepsilon)$), R-squared, Mean Square Error (MSE), Residual Sum of the Square (RSS), and interquartile range of the error ($IQR(\varepsilon)$).

While R-Squared, MSE, and RSS have been already mentioned and employed previously, through Eq. (5.7), Eq. (5.8) and Eq. (3.8), respectively, the mean of the error, the variance of the error and the interquartile range of the error are here reported for the sake of clarity:

$$\mu(\varepsilon) = \frac{1}{N} \sum_{i=1}^N \varepsilon_i \quad (5.18)$$

$$var(\varepsilon) = \frac{1}{N-1} \sum_{i=1}^N (\varepsilon_i - \mu(\varepsilon))^2 \quad (5.19)$$

$$IQR(\varepsilon) = Q3 - Q1 \quad (5.20)$$

The mean refers to the average value of the error (ε), which was computed as the difference between the mean damage observed and predicted using the novel vulnerability functions. This metric is commonly used to evaluate models, systems, and processes, by providing a single value that represents the overall performance. A low mean generally indicates a model with smaller prediction errors. The variance measures how much the errors deviate from the mean, quantifying the spread of the data. It helps assess the consistency of the model performance: with low variance typically suggesting less variability. Finally, the IQR as well is a metric of the variability. It represents the range between the first quartile (Q1) and the third quartile (Q3), corresponding to the 25th and the 75th percentiles, respectively. This range contains the middle 50% of the error data. The use of this metric focuses on the central portion of the distribution, reducing the influence of extreme values. A low IQR indicates that the middle 50% of the errors are closely grouped, suggesting low variability, while a high IQR indicates a greater spread, thus high variability.

It should be noted that the parameters of the benchmarked PGA-based models, although not here reported, were recalibrated by considering the same dataset of the SA-based ones, reduced with respect to the one analysed in Chapter 4, to guarantee a fair comparison. Table 5.15 reports the

performance metrics obtained using the distinct intensity measures, namely the SA and the PGA, for both the individual mechanisms and the overall macroelement damage.

Table 5.15: Performance metrics of the vulnerability models using SA and PGA.

	μ (ε)	var (ε)	R-squared	MSE	RSS	IQR (ε)
SA						
Mechanism A	0.01	0.05	0.85	0.04	0.18	0.19
Mechanism B	0.02	0.04	0.95	0.03	0.16	0.17
Macroelement	0.02	0.06	0.89	0.05	0.25	0.22
PGA						
Mechanism A	0.01	0.03	0.90	0.02	0.14	0.32
Mechanism B	0.01	0.04	0.93	0.04	0.22	0.23
Macroelement	0.01	0.05	0.89	0.04	0.26	0.42

In general, both the SA and PGA models exhibited low mean errors, of around 0.01-0.02, and similar variance, ranging from 0.03 to 0.06. The fitting metrics also showed strong performance, with R-squared values around 0.85 to 0.90, and MSE values between 0.02 and 0.05. These results indicated that the vulnerability functions developed using both SA and PGA performed similarly well, demonstrating their effectiveness in fitting the observational data and their strong correlation with structural damage. Although no significant differences were observed in the overall comparison, a slightly better performance of the PGA-based model was noted for mechanism A (shaft), while the SA-based model showed a marginally better performance for mechanism B (belfry). However, when considering the results for the entire macroelement damage, the comparison yields inconclusive results, as the two models exhibit opposing trends for distinct metrics.

Finally, in order to gain a better understanding of the distinct behaviours of the towers subjected to a single initial shock (GT1) and multiple shocks (GT2), the observed mean damage was plotted against increasing SA values and a recalibration was conducted to propose new formulations. The fit of these new formulations to the observational data for both earthquake scenarios is shown in Figure 5.27, with the calibration coefficients provided in Table 5.16 using the same form of the previous equation, Eq. (5.17).

For GT1, the observed mean damage remains relatively constant across the range of SA values, with no significant variations. This led to inconsistent results for the mechanism A and thus its vulnerability function was discarded. In contrast, for GT2, the observed mean damage increases rapidly as the spectral acceleration rises, confirming the cumulative effects of damage. These findings are consistent with those included in Chapter 4 where the analysis focused on the use of the PGA.

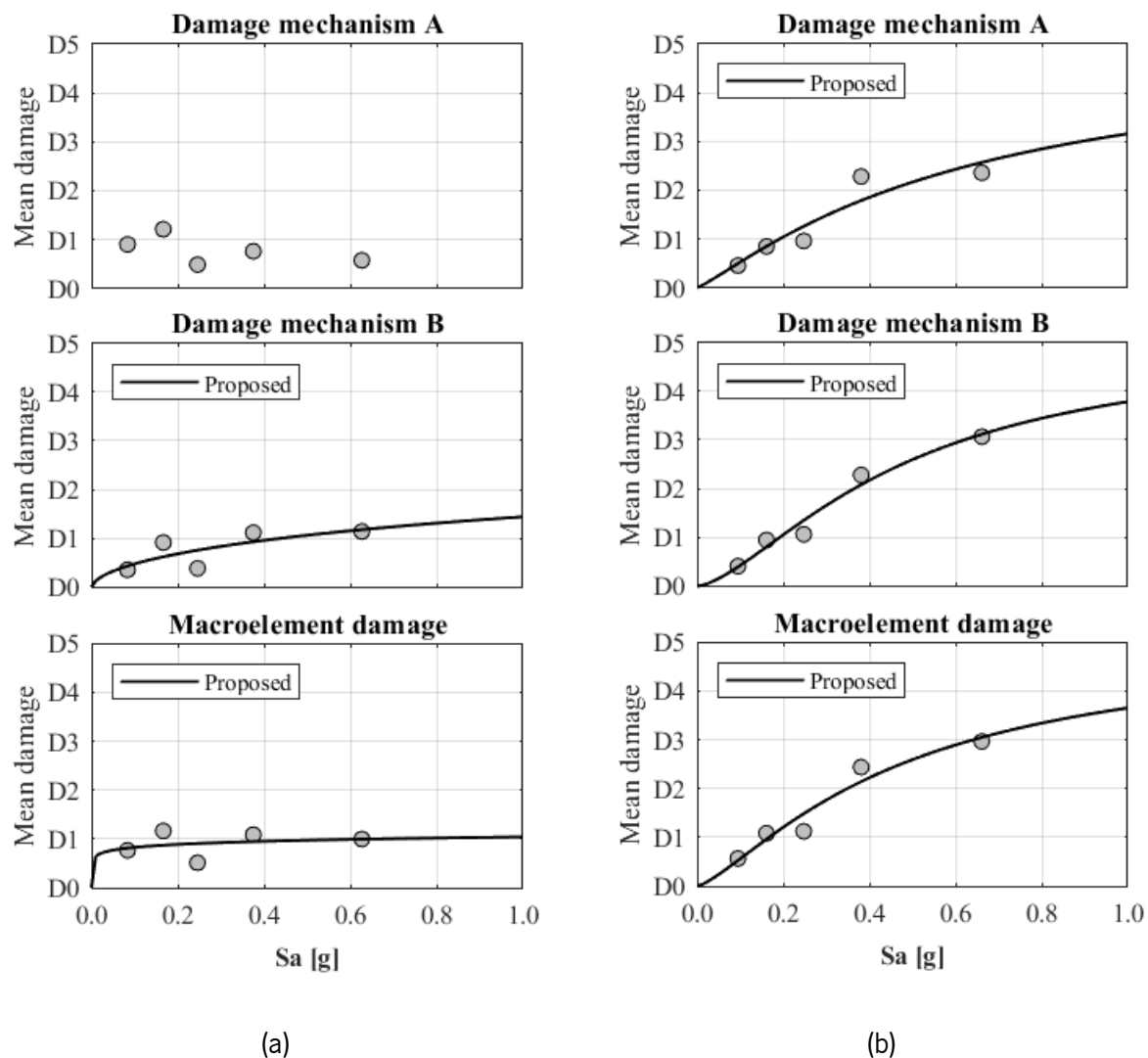


Figure 5.27: Observed mean damage as functions of the SA: (a) GT1 (b) GT2.

Table 5.16: Recalibration coefficients for the two distinct subsets and for the seismic intensity measures

		a'	b'
Mechanism A	GT1	-	-
	GT2	1.342	0.269
Mechanism B	GT1	0.671	-0.451
	GT2	1.757	0.567
Macroelement	GT1	0.141	-0.666
	GT2	1.534	0.500

5.4.4 Fragility functions

Finally, fragility functions were derived in this Section using the data fitting procedure previously applied in Chapter 4 and extensively described in Section 2.3.4. The method assumes a lognormal cumulative distribution function whose parameters are estimated by fitting the observed probability of exceedance

of the damage grade at multiple levels of intensity using the Maximum Likelihood Estimation (MLE) method (Baker, 2015). The fragility functions developed for the entire dataset of 415 towers (GTO) are presented in Figure 5.28. Considering the results discussed in Chapter 4, these functions were extrapolated through the optimised approach, proposed by (Porter, 2021), preventing the issue of crossing fragility curves.

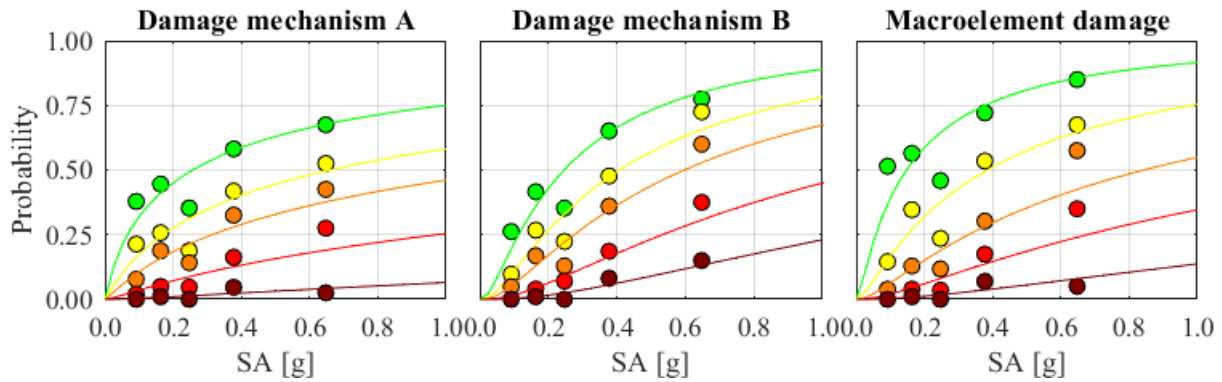


Figure 5.28: Fragility functions for GTO using SA.

Table 5.17 summarises the calibrated lognormal cumulative distribution parameters, namely median and standard deviation. Values are provided for the different damage states (from D1 to D5), according to the considered damage mechanisms. When comparing the results for the two damage mechanisms, differences emerge in the probabilities of exceeding the damage states as the SA increases. In particular, the belfry (mechanism B) consistently shows a lower median, indicating a tendency to suffer damage at lower SA values. When examining the tower as a macroelement, the fragility functions tend to display an intermediate behaviour for damage levels above D3. For lower levels of damage, instead, the probability of exceedance is either similar or higher than that of the individual mechanisms. This could indicate an overestimation of minor damage, as the method used to combine the individual mechanisms into the entire macroelement damage and to convert it from continuous level to discrete grade tends to overstate the damage, rather than averaging it, at lower damage extents.

Table 5.17: Median and standard deviation for the distinct damage grades, SA-based functions.

	Parameters	D1	D2	D3	D4	D5
Mechanism A	θ_{D_k}	0.25	0.66	1.22	3.77	21.03
	β	2.01	2.01	2.01	2.01	2.01
Mechanism B	θ_{D_k}	0.25	0.41	0.60	1.15	2.33
	β	1.15	1.15	1.15	1.15	1.15
Macroelement damage	θ_{D_k}	0.15	0.38	0.84	1.73	4.59
	β	1.39	1.39	1.39	1.39	1.39

The same procedure was applied to the same reduced dataset, using PGA as the intensity metric, for comparison purposes. The resulting fragility functions are illustrated in Figure 5.29, while the calibrated coefficients are reported in Table 5.18.

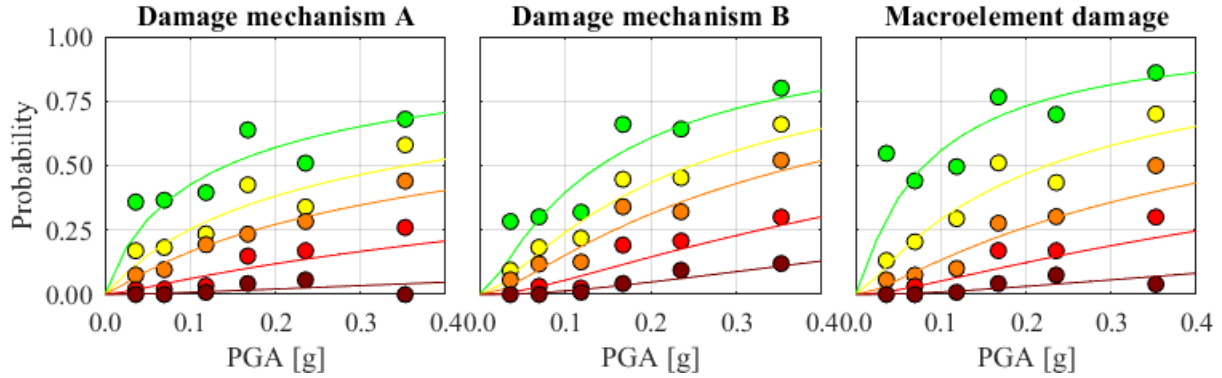


Figure 5.29: Fragility functions for GT0 using PGA.

Table 5.18: Median and standard deviation for the distinct damage grades, PGA-based functions.

	Parameters	D1	D2	D3	D4	D5
Mechanism A	θ_{D_k}	0.14	0.35	0.63	1.89	9.56
	β	1.90	1.90	1.90	1.90	1.90
Mechanism B	θ_{D_k}	0.14	0.25	0.38	0.78	1.71
	β	1.29	1.29	1.29	1.29	1.29
Macroelement damage	θ_{D_k}	0.08	0.22	0.51	1.10	3.09
	β	1.47	1.47	1.47	1.47	1.47

Comparing these fragility functions with those derived using SA highlights the amplification effects caused by the low periods of vibration of the investigated towers. In particular, the fragility functions for SA span a range of approximately 0.1 to 0.6 g, while those for PGA ranges from 0.1 to 0.4 g. Although the fragility functions exhibit similar overall behaviour and reach comparable probabilities, the latter occur at different seismic intensities. For instance, for the overall macroelement, the probabilities of reaching the five damage states were approximately 80%, 70%, 50%, 30% and 5% for the maximum bin of PGA (around 0.4g). Similar probabilities were observed for the maximum SA values, although at higher intensity (around 0.6g). This was confirmed by the median values of the two sets of fragility functions, generally similar, but slightly higher for the SA compared to the PGA. Thus, the use of SA causes a rightward shift in the fragility curve. Regarding the standard deviations of the two sets, similar values were observed as well. However, for Mechanism A, the SA produced smaller values than the PGA, indicating less variability. In contrast, Mechanism B and the overall macroelement showed higher standard deviations for the SA, suggesting greater variability. These results likely confirm the stronger correlation between PGA and damage mechanism A (shaft), as well as between SA and damage mechanism B (belfry), which was previously observed in the vulnerability functions.

To further compare the sets of fragility functions corresponding to the two seismic intensities, the Aikake Information Criterion (AIC) was computed. This metric, originally introduced by (Akaike, 1974), is commonly used in model selection to compare different statistical models fitted to the same dataset to choose the best model by balancing goodness of fit and complexity (Rossetto et al., 2014). Lower AIC values indicate better models. The AIC is defined as:

$$AIC = -2\log(L) + 2k \quad (5.20)$$

where L is the maximum likelihood of the model and k is the number of estimated parameters. The results are summarised in Table 5.19. It should be noted that the AIC, derived using the optimisation approach proposed by (Porter, 2021) to prevent crossing of the fragility functions, is a single value for all damage grades.

Table 5.19: AIC values for the fragility functions derived using the SA and the PGA.

Mechanism A		Mechanism B		Macroelement	
SA	PGA	SA	PGA	SA	PGA
12.31	12.49	12.31	12.59	12.59	12.74

Although the AIC of the fragility functions for the two intensity measures are almost coincident, the SA produced slightly smaller values than their counterparts for PGA, suggesting slightly better. In this case, having fragility models that use the same number of parameters (i.e., the same complexity), the AIC comparison depends solely on the likelihood function. As the likelihood is maximised (i.e., better fit to the observed data), the AIC becomes lower, thus slightly preferred.

5.5 CONCLUSIONS

This Chapter aimed to expand on the seismic vulnerability analysis of masonry towers at the territorial scale, focusing on the use of the Spectral Acceleration (SA) as a seismic intensity measure. The study leveraged the dataset of bell towers damaged during the Central Italy seismic sequence, as detailed in Chapter 4, to explore the potential of SA in vulnerability modelling.

A key part of this work was the identification of a suitable Ground Motion Model (GMM) for estimating the ground motion characteristics. To this end, three well-established methods (i.e., SP96, ITA08 and ITA10) calibrated for Italy using past earthquake data were analysed and compared. The optimal GMM, namely ITA08, was selected based on its accuracy in predicting PGA values, which were benchmarked against the Da.D.O. database at each tower location. Using this GMM, the SA was estimated for each tower site as a function of earthquake magnitude, epicentral distance, soil characteristics and the

fundamental frequency of the towers. This was obtained by employing a novel empirical predictive equation developed and validated within the scope of this thesis using the largest database, i.e., 184 case studies, ever utilised in similar studies for a specific structural typology.

A set of 415 towers affected by the Central Italy seismic sequence were finally employed to develop the vulnerability models (DPMs, vulnerability, and fragility functions) using SA and PGA as intensity measures. The present study constituted one of the first attempts to provide empirical vulnerability models as a function of the SA. Even though uncertainties may have influenced the estimation of the SA values, the analysis revealed that both intensity measures perform similarly in terms of various metrics, such as R-squared, MSE, and AIC. This suggests that SA and PGA models can be considered equally effective for this purpose. Still, a slightly stronger correlation between SA and the belfry damage mechanism was observed, suggesting that SA may provide additional insights, particularly when more precise data become available. Indeed, the uncertainties associated with SA estimation, particularly related to the soil characteristics and the determination of the fundamental period, suggest that future research incorporating experimental measurements could reduce these uncertainties and improve the accuracy of SA-based models.

CHAPTER 6

CONCLUSIONS

6.1 THESIS SUMMARY, MAIN CONCLUSIONS AND FINAL REMARKS

This thesis investigated the seismic vulnerability of historic masonry towers, with a particular focus on bell towers macroelements within churches. The main objectives and the original contribution of the thesis were to test existing empirical methods and to enhance them by developing novel vulnerability and fragility functions, relying upon post-earthquake survey data. The research adopted well-established methodologies for seismic risk assessment, including Damage Probability Matrices (DPMs), vulnerability functions, and fragility functions, while also addressing their limitations. DPMs are used to analyse datasets, identifying how the probability of specific damage levels to a class of assets evolves with varying earthquake intensities. Vulnerability functions quantify the relationship between expected mean damage to a class of assets and the seismic intensity required to induce it. Fragility functions, on the other hand, provide the probability of reaching or exceeding specific damage levels given a certain seismic intensity.

In the thesis, DPMs were initially estimated to provide an insight into the observed vulnerability and to identify potential biases in the data. Afterwards, existing vulnerability functions were tested, new ones calibrated and fragility functions derived to provide robust predictive tools suitable for territorial scale analysis of masonry towers.

The specific objectives of the research, as outlined in Chapter 1, included: (i) developing vulnerability models that employ multiple intensity measures, such as MCS macroseismic intensity, PGA, and SA; (ii) creating vulnerability models tailored to various tower classes and seismic scenarios, while assessing

the shaft and belfry separately as well as combining their behaviour within the tower's overall response as a macroelement (hereafter called macroelement approach).

The aim of these models was to provide a comprehensive tool for streamlining seismic vulnerability assessments at a territorial scale. The models offer a comparative analysis of the expected behaviour, enabling the prediction of damage and losses for specific scenarios, and allowing for the selection of the most appropriate models based on available information. Additionally, the research investigated whether different intensity measures improve correlations with observed damage, thereby enhancing the accuracy of predictions.

To achieve these objectives, several challenges typical of territorial scale risk assessment had to be addressed, including the lack or incompleteness of post-earthquake survey data, the validation of the few existing vulnerability functions for historic masonry towers, the calibration of new functions for previously underexplored scenarios, and the generation of a broad set of fragility functions for the scenarios under investigation. Developing these fragility functions involved a careful examination of available methodologies.

The thesis was structured into four main Chapters, plus introduction and conclusions, providing comprehensive insights into the research findings and the proposed models. The structure is as follows:

- (i) Chapter 1: Introduction to the research motivation and objectives.
- (ii) Chapter 2: Review of the current state of the field and identification of research gaps.
- (iii) Chapter 3: Preliminary validation and enhancement of existing vulnerability models.
- (iv) Chapter 4: Development of new vulnerability models to account for cumulative damage and various tower features that influence seismic behaviour.
- (v) Chapter 5: Further development of seismic vulnerability models by incorporating the SA as a seismic intensity measure.
- (vi) Chapter 6: Summary of research activities, discussion of main conclusions, and identification of potential future research directions.

In Chapters 3, 4 and 5 two distinct field survey datasets were gathered and analysed:

- (i) Dataset 1: Open dataset of post-earthquake damage

Dataset 1 contains data from 129 historic masonry bell towers struck by past earthquakes compiled from reports and scientific publications upon an extensive literature review. Each tower was affected by a single earthquake (single event or sequence). In particular, 31 towers were hit by the 1976 Friuli seismic event, 8 towers by the 1997 Umbria-Marche, 18 towers by the 2002 Molise, 6 towers by the 2009 L'Aquila, 55 towers by the 2012 Emilia, and the remaining 11 towers by the 2016 Central Italy earthquake. Ground motion was characterised resorting to the

macroseismic intensity, extracted from the Italian Macroseismic Database in terms of MCS and ranging from 5 to 10. The damage was classified according to the six-level EMS-98 damage scale, adopting a macroelement approach that differentiates between collapse mechanisms affecting the shaft and the belfry and then aggregates the results into the macroelement assessment. The database was made openly and freely available.

(ii) Dataset 2: 2016-2017 Central Italy dataset

Dataset 2 encompasses data from 794 historic masonry bell towers struck by five main shocks of the 2016-2017 Central Italy seismic sequence. It was extracted from the Database of Observed Damage (Da.D.O.), originally containing over 3000 masonry churches. The data includes the damage classified according to the six-level EMS-98 damage scale, adopting the same macroelement approach of the previous dataset, key tower features, such as maintenance state, geometry, typology, and location, and two seismic intensity measures: MCS (ranging from 5 to 10) and PGA (ranging from 0.02g to 0.54g).

Additionally, a third dataset was collected to define predictive formulas for the natural period of masonry towers, which were used to estimate the SA at their site:

(iii) Dataset 3: Open dataset of experimental periods of vibration

Dataset 3 comprises 184 case studies gathered through an extensive literature review of studies focused on the dynamic identification and field-testing of masonry towers. It includes: tower name and location, masonry material type (stone, brick or mixed), boundary condition (isolated or bounded), cross-section shape (rectangular, square, semi-circular, circular, elliptical, pentagonal, octagonal, dodecagonal), geometrical dimensions (maximum height, effective height, side lengths, wall thickness and aspect ratio) and the first experimental frequency value corresponding to a bending mode. Only data from experimental studies were included, with any estimates of natural frequency based on numerical simulations or simplified formulations excluded.

Dataset 1 was presented and discussed in detail within Chapter 3, where it was exploited to validate the existing vulnerability models. The compilation of Dataset 1 constituted a key contribution of the thesis, addressing the systematic lack of extensive open access information in the field. It offers an open and homogenised database of real-world data from masonry towers (accounting for mechanisms to the shaft and to the belfry) across various Italian seismic events. The database is expected to grow as more towers and additional attributes are incorporated, becoming an essential tool for advancing studies on the seismic vulnerability of this macroelement.

Based on this dataset, several preliminary analyses were conducted. Upon the combination of the damage levels for each mechanism (shaft and belfry), an overall damage level for the tower macroelement was computed. This distinction enabled the development of DPMs for increasing MCS intensity levels, applied to the entire dataset as well as distinct subsets of towers with likely homogenous behaviours. These subsets were defined by grouping towers according to the earthquake events they experienced and their typologies (isolated or bounded). This approach provided valuable insights into the susceptibility of different tower typologies to specific damage mechanisms and allowed for a detailed discussion on the differing likelihood of damage occurrence across mechanisms at higher seismic intensities. These first results confirmed a trend observed in the literature for historic towers, namely higher susceptibility to damage of the belfry rather than the shaft. A similar behaviour constantly emerged throughout the analyses conducted within the present thesis, irrespective of the dataset investigated. Next, the existing empirical vulnerability functions were tested against the database, using MCS intensity as the independent variable. The analysis revealed that while the functions proposed for individual damage mechanisms performed well, the formulation for the entire macroelement demonstrated a relatively lower predictive capability. As a result, a recalibration of the vulnerability function coefficients was carried out, leading to an improved formulation for the overall tower macroelement. Despite the significant size of the dataset, the analyses stressed potential limitations in the data collection, in particular: (i) an overestimation of the damage level for low intensity, due to missing information regarding slightly or non-damaged towers; (ii) an underestimation of the damage level for very strong intensity, due to statistically non-representative sample size; (iii) a non-representativeness of the samples for specific earthquakes. To mitigate these issues and address the uncertainties and biases inherent in the dataset, multiple statistical analyses were conducted. These analyses aimed to highlight the statistical relevance of the database and validate both the existing and newly recalibrated vulnerability functions. Therefore, they were tested against random subsets of various sample sizes, generated from the original dataset, following distinct sampling strategies. These analyses indicated that the recalibrated vulnerability curve derived from the dataset is sufficiently robust. Furthermore, the results suggested that the investigated vulnerability functions exhibit acceptable predictive performance.

In the final phase of this stage, fragility curves were developed using a widely adopted simplified approach, due to the limited available information. These curves were derived using the cumulative distribution expressed through the binomial probability density function, with the PGA retrieved from an

empirical correlation between PGA and macroseismic intensity. The outcome of this stage is a set of general models for the preliminary seismic vulnerability assessment of historic towers.

The validation of the predictive performance of both existing and newly developed models, along with strategies to assess and account for uncertainties and biases in the dataset, constitutes a significant contribution of the thesis. Nonetheless, the limitation identified in Dataset 1, prompted the collection of Dataset 2. The development and validation of vulnerability models, relying upon the Central Italy dataset, were presented adopting distinct intensity measures, first PGA and then SA.

In particular, in Chapter 4, the same approach fine-tuned in Chapter 3 was applied, but with DPMs, vulnerability and fragility functions, derived using the PGA. In the development of fragility functions, the previous simplified approach was replaced by a more robust method, leveraging the additional information provided in Dataset 2. This enabled the estimation of statistical fragility function parameters by maximising the likelihood function. Particular emphasis was given to the crossing behaviour of the fragility functions, which was tackled by resorting to an optimised approach that allow to respect the sequential damage states by simultaneously estimating the parameters for all the set of fragility functions.

Additionally, distinct models were generated to account for the effect of cumulative damage and certain features that may affect the tower vulnerability. Concerning the first aspect, the Central Italy dataset was separated into two subgroups: towers affected by the initial seismic shock and towers subjected to the entire seismic sequence. This distinction allowed to compare the tower behaviour under distinct seismic scenarios, namely single and multiple severe shocks. To this end, ranges of PGA for which the cumulative effect of subsequent shocks may be considered negligible were also identified. The results revealed that this classification is of paramount importance to produce more accurate predictive models of the seismic vulnerability. Therefore, tailored vulnerability and fragility functions were provided for each scenario. Regarding the second aspect, towers were grouped by state of maintenance, aspect ratio, typology (bounded towers classified as confined or integrated), and location (region and province). The analysis indicated that the state of maintenance is a significant indicator of vulnerability, highlighting the importance of considering this factor in empirical studies of historic towers.

Finally, in Chapter 5, based on the same Dataset 2, seismic vulnerability models were developed using SA as the seismic intensity measure. Since SA was not originally available in the database, it was estimated using GMM. Three established GMMs were compared for their ability to match the PGA values reported in the Da.D.O. database, with the ITA08 model selected due to its superior performance. SA was then computed based on this model, which required the knowledge of the

earthquake magnitude, epicentral distance, soil characteristics and fundamental frequency of the towers included in the dataset. While most parameters were known or could be inferred through reliable assumptions, the fundamental period required the use of predictive equations. To this end, a set of novel empirical formulations was developed, by training and validating them over the largest database ever collected before for this purpose, namely Dataset 3. The proposed formulations proved to outperform existing ones on both training and validation sets. Besides their applicability to support the GMM, these equations are in general of paramount importance in the field of seismic vulnerability assessment at various scale. Therefore, the collection of Dataset 3, which is openly accessible, editable, and usable, along with the development of these predictive formulas, further strengthens the contributions of this thesis. After computing SA, the seismic vulnerability models, namely DPMs, vulnerability and fragility functions were developed and compared with their counterparts, derived using the PGA. The comparison revealed that both intensity measures perform similarly. This suggests that SA and PGA models can be considered equally effective for this purpose. Still, a slightly stronger correlation between SA and the belfry damage mechanism as well as between the PGA and the shaft damage mechanisms was observed, suggesting that an informed selection of the intensity measure may provide additional insights and improve the predictive performance of the models.

6.2 FUTURE LINES OF RESEARCH

Based on the findings presented in this work, several areas remain open for further development. The analysis of Dataset 1 and Dataset 2 highlighted how data incompleteness, the lack of detailed asset characterisation, and errors in post-earthquake data collection or digitisation can hinder the development of reliable empirical methods for territorial-scale seismic assessment. To address these issues, compiling a verified and expanded dataset, building on Dataset 1, would provide researchers with a dependable, open-access resource free from errors or data gaps. In particular, more thorough post-earthquake surveys in areas characterised by low intensity and in general of the assets characterised by slight or no-damage are crucial to prevent biases and ensure the dataset is fully representative of the entire asset population. Additionally, a lack of data regarding highly destructive events, particularly severe damage and collapse, presents another challenge due to the limited information available from such extreme cases.

Furthermore, refining the classification of assets into more homogeneous groups could enhance the precision of vulnerability predictions. This would involve gathering additional characteristics during data collection, such as construction materials, recent repair or strengthening works, and the effects of local

site response. Including these features would allow for a clearer understanding of their impact on historic masonry tower vulnerability and the development of tailored assessment models.

Finally, this thesis made an initial effort to develop empirical vulnerability models based on SA. The estimation of this intensity measure, not commonly available in existing databases, was affected by uncertainties, particularly related to soil characteristics and the determination of the tower fundamental period. Future research incorporating experimental measurements or more robust estimation methods, along with improved local estimates (like those already available for PGA in existing databases), could reduce these uncertainties and improve the accuracy of SA-based models, which, despite these challenges, have shown promising results.

APPENDIX A

A NOVEL OPEN DATABASE OF OBSERVED DAMAGE

This Appendix presents information on historic masonry towers, gathered through an extensive literature review, compiled into a database of towers damaged by past earthquake events in Italy. In particular, the following details are provided: tower number, church or tower name, geo-localisation (latitude and longitude), tower typology (distinguished in isolated, I, and bounded, B), MCS intensity, damage level to the shaft and to the belfry (defined according to EMS-98), and the source of information. The database has been used to test earthquake risk models in Chapter 3.

Table A.1: A novel open database of structural damage observed in historic masonry towers

No.	Church/Tower Name	Lat.	Long.	Typology	I_{MCS}	d_1	d_2	Source
1	Event: the 1976 Friuli Earthquake							
1	Church in Arba ^A	46.163	12.812	B	8.5	4	0	(Curti et al., 2008)
2	Ognissanti Church	46.178	13.107	I	9	2	5	(Curti et al., 2008)
3	Santa Maria Assunta Cathedral	46.276	13.142	I	9.5	5	5	(Curti et al., 2008)
4	SS. Pietro and Paolo Church	46.184	13.071	I	9	5	5	(Curti et al., 2008)
5	Santa Elena Imperatrice Church ^A	46.184	13.071	I	9	3	2	(Curti et al., 2008)
6	Santo Spirito Church	46.405	13.189	B	9	3	3	(Curti et al., 2008)
7	Santo Stefano Church ^{A,B}	46.197	13.263	B	8.5	4	5	(Curti et al., 2008)
8	Santo Stefano Church	46.168	12.917	B	8.5	5	5	(Curti et al., 2008)
9	San Maria Church del Monte Castellano ^B	46.437	12.862	B	8	3	4	(Curti et al., 2008)
10	Sant'Antonio Abate Church	46.239	13.205	I	8	2	1	(Curti et al., 2008)
11	San Bartolomeo Apostolo Church	46.318	13.050	B	8.5	3	3	(Curti et al., 2008)
12	San Michele Arcangelo Church ^A	46.287	13.092	B	8.5	3	0	(Curti et al., 2008)
13	San Daniele Church	46.365	13.041	I	8.5	1	2	(Curti et al., 2008)
14	San Lorenzo Church	46.224	12.968	I	9.5	2	1	(Curti et al., 2008)
15	San Valentino Church	46.349	13.067	B	7.5	1	0	(Curti et al., 2008)
16	San Stefano Church	46.351	13.062	B	8.5	4	5	(Curti et al., 2008)
17	Santa Giuliana Church	46.222	13.005	I	8.5			(Curti et al., 2008)
18	San Martino Church	46.391	13.218	I	9	4	0	(Curti et al., 2008)
19	Santa Maria Maddalena Church	46.405	12.943	B	8			(Curti et al., 2008)
20	Church of the Beata Vergine ad Melotum	46.224	13.124	I	9.5			(Curti et al., 2008)
21	Santo Spirito Church	46.299	13.120	B	8.5	5	5	(Curti et al., 2008)
22	San Nicolò Church	46.223	12.974	I	9.5	3	3	(Curti et al., 2008)

No.	Church/Tower Name	Lat.	Long.	Typology	I_{MCS}	d_1	d_2	Source
23	SS. Gervasio and Protasio Church	46.200	13.275	I	8.5	3	4	(Curti et al., 2008)
24	San Leonardo Abate Church	46.119	13.535	B	6.5			(Curti et al., 2008)
25	Santa Maria Ad Nives Church	46.259	13.077	I	9.5			(Curti et al., 2008)
26	San Marco Church	46.140	12.913	I	7.5	2	4	(Curti et al., 2008)
27	SS. Trinità ^c	46.274	13.303	B	8.5		4	(Curti et al., 2008)
28	Church in Spilimbergo	46.132	12.902	B	7.5			(Curti et al., 2008)
29	San Biagio Church	46.215	13.206	B	8			(Curti et al., 2008)
30	San Pietro Apostolo	46.216	13.222	I	8.5	3	1	(Curti et al., 2008)
31	Church of Vito D'Asio	46.229	12.930	I	9			(Curti et al., 2008)
2	Event: the 1997 Umbria-Marche Earthquake							
32	San Salvatore Church ^c	42.983	12.930	B	7.5	4	4	(Sferrazza Papa & Silva, 2018)
33	Basilica of San Francesco ^c	43.075	12.605	B	6.5	0	2	(Spence et al., 1998)
34	Basilica of Santa Chiara ^c	43.069	12.617	B	6.5	0	0	(Spence et al., 1998)
35	San Rufino Cathedral ^c	43.071	12.618	B	6.5	0	0	(Spence et al., 1998)
36	Santa Maria Maggiore Church ^c	43.070	12.614	B	6.5	0	1	(Spence et al., 1998)
37	Civic Tower ^c	43.110	12.790	B	7.5	5	5	(Spence et al., 1998)
38	Santa Maria Assunta Cathedral ^c	43.110	12.791	B	7.5	2	1	(Spence et al., 1998)
39	San Filippo Church ^c	43.111	12.789	B	7.5	2	0	(Spence et al., 1998)
3	Event: the 2002 Molise Earthquake							
40	Church (S. Pietro in Vincoli Church) ^c	41.704	14.730	B	7	3	0	(Cifani et al., 2005)
41	San Giacomo Church ^c	41.711	14.990	B	7	0	5	(Cifani et al., 2005)
42	Sant'Antonio di Padova Church ^c	41.712	14.992	B	7	3	0	(Cifani et al., 2005)
43	Santa Maria Assunta Church ^c	41.689	14.807	B	7	0	0	(Cifani et al., 2005)

No.	Church/Tower Name	Lat.	Long.	Typology	I_{MCS}	d_1	d_2	Source
44	Sant'Alfonso dei Liguori Monastery °	41.663	14.968	B	6.5	3	3	(Cifani et al., 2005)
45	Sacro Cuore di Gesù Church °	41.681	14.722	I	5.5	1	5	(Cifani et al., 2005)
46	San Michele Arcangelo Church °	41.632	14.765	B	5.5	0	3	(Cifani et al., 2005)
47	Madonna Addolorata Church °	41.703	14.731	B	7	0	5	(Cifani et al., 2005)
48	San Giovanni Battista Church °	41.663	14.973	B	6.5	2	0	(Cifani et al., 2005)
49	S. Maria Assunta Cathedral °	41.802	14.793	B	6	0	1	(Cifani et al., 2005)
50	S. Francesco d'Assisi Church °	41.677	14.621	B	5.5	1	0	(Cifani et al., 2005)
51	Santa Maria Maggiore Church °	41.674	14.623	B	5.5	1	0	(Cifani et al., 2005)
52	Santa Maria Assunta Church °	41.763	14.733	B	5.5	2	3	(Cifani et al., 2005)
53	San Nicola di Mira Church °	41.593	14.911	B	5.5	0	2	(Cifani et al., 2005)
54	Santa Maria Assunta Church °	41.759	14.932	B	6	2	0	(Cifani et al., 2005)
55	San Giuliano Church °	41.684	14.964	B	8.5	3	4	(Cifani et al., 2005)
56	Sant'Elia Profeta Church °	41.621	14.874	B	5.5	2	0	(Cifani et al., 2005)
57	Santa Maria degli Angeli Church °	41.750	15.004	B	6	1	0	(Cifani et al., 2005)
4	Event: the 2009 L'Aquila Earthquake							
58	San Silvestro Church °	42.354	13.397	B	8.5	3	1	(Brandonisio et al., 2013)
59	San Pietro di Coppito Church °	42.353	13.396	B	8.5	4	5	(Brandonisio et al., 2013)
60	Margherita Palace °	42.351	13.398	B	8.5	3	0	(Lorenzoni et al., 2012)
61	Santa Gemma Church	42.081	13.773	B	7	2	1	(Ciber et al., 2015)
62	Santa Maria di Paganica °	42.353	13.400	B	8.5	5	5	(Augenti & Parisi, 2010)
63	San Bernardino da Siena Basilica °	42.350	13.402	B	8.5	4	5	(Augenti & Parisi, 2010)

No.	Church/Tower Name	Lat.	Long.	Typology	I_{MCS}	d_1	d_2	Source
5	Event: the 2012 Emilia Earthquake							
64	San Michele Arcangelo Church	44.765	11.485	B	5.5	3	NaN	(Sorrentino et al., 2014)
65	San Paolo Cathedral	44.826	11.461	I	6	0	0	(Paupério et al., 2012)
66	Sant'Agostino Church	44.792	11.386	I	6	3	3	(Sorrentino et al., 2014)
67	San Martino Church	44.808	11.344	I	5.5	4	4	(Ferrari, 2020)
68	Beata Vergine del Salice Church	44.815	11.307	I	5.5	1	0	(Decanini et al., 2012)
69	Sant'Anna Church	44.828	11.330	I	5	4	0	(Ferrari, 2020)
70	Santa Maria Maggiore Church	44.713	11.307	B	5.5	2	0	(Ferrari, 2020)
71	Duomo of SS Filippo & Giacomo	44.832	11.296	B	6.5			(Decanini et al., 2012)
72	Annunziata Church	44.830	11.295	B	6.5			(Sorrentino et al., 2014)
73	San Geminiano Vescovo Church	44.852	11.212	I	5.5	1	0	(Paupério et al., 2012)
74	San Lorenzo Church ^a	44.838	11.363	B	6.5	3	1	(Ferrari, 2020)
75	Visitazione Maria Vergine Church	44.833	11.319	B	5.5	4	0	(Sorrentino et al., 2014)
76	San Felice Church	44.839	11.140	B	7	5		(Sorrentino et al., 2014)
77	Natività of the S. Maria Santissima Church	44.841	11.169	I	6	3	0	(Sorrentino et al., 2014)
78	San Biagio Church ^a	44.862	11.118	B	7	5		(Sorrentino et al., 2014)
79	San Francesco Church	44.888	11.068	B	7	5	5	(Paupério et al., 2012)
80	San Sacramento Church	44.886	11.067	B	7			(Sorrentino et al., 2014)
81	Santa Maria Maggiore Church	44.886	11.066	B	7	4	3	(Sorrentino et al., 2014)
82	Beata Vergine della Porta Church	44.889	11.066	B	7	3	4	(Ferrari, 2020)
83	San Leonardo Limosino Church	44.875	11.129	B	6.5	5	5	(Sorrentino et al., 2014)
84	Madonna della Neve Church	44.923	11.096	I	5.5	2	2	(Sorrentino et al., 2014)
85	Beata Vergine Maria Church	44.959	11.088	B	5.5		1	(Sorrentino et al., 2014)

No.	Church/Tower Name	Lat.	Long.	Typology	I_{MCS}	d_1	d_2	Source
86	San Luca Evangelista Church ^a	44.863	11.077	B	5.5	4	5	(Ferrari, 2020)
87	SS Apostoli Giacomo and Filippo Church	44.861	11.058	I	5.5	4	3	(Paupério et al., 2012)
88	San Bartolomeo Church	44.834	11.061	B	6	5	5	(Sorrentino et al., 2014)
89	Church in Montalbano ^c	44.849	11.055	B	5.5	4	3	(Paupério et al., 2012)
90	Sant'Egidio Abate Church	44.837	11.031	I	7	4	4	(Ferrari, 2020)
91	San Giovanni Battista Church	44.859	11.000	B	5.5	5	5	(Sorrentino et al., 2014)
92	San Prospero Church	44.790	11.017	B	5		3	(Sorrentino et al., 2014)
93	San Michele Arcangelo Church	44.890	10.903	B	7	4	2	(Ferrari, 2020)
94	Santa Caterina d'Alessandria Church	44.841	10.956	B	7.5	3	3	(Sorrentino et al., 2014)
95	San Possidonio Church	44.892	10.996	B	6		5	(Sorrentino et al., 2014)
96	Conversione San Paolo Church	44.913	10.982	B	7		4	(Sorrentino et al., 2014)
97	San Giovanni Battista Church ^a	44.909	10.983	B	7	3		(Sorrentino et al., 2014)
98	Santa Caterina Church ^a	44.925	10.979	B	7		5	(Sorrentino et al., 2014)
99	Santa Maria Bianca Church	44.943	11.005	B	5.5	4	5	(Sorrentino et al., 2014)
100	San Pietro Apostolo Church	44.923	11.037	I	5.5	4		(Sorrentino et al., 2014)
101	San Tommaso Apostolo Church	44.944	10.857	I	6		2	(Sorrentino et al., 2014)
102	San Giacomo Maggiore Church	44.971	11.034	I	5.5			(Sorrentino et al., 2014)
103	Church in Bondeno ^c	44.887	11.414	B	5.5	3	2	(Paupério et al., 2012)
104	Sacro Cuore Church ^c	44.885	11.417	B	5.5	0	2	(Paupério et al., 2012)
105	San Benedetto Abate Church ^c	44.951	10.824	B	5	3	4	(Valente et al., 2017)
106	San Fiorentino Martire Church ^c	45.030	11.010	B	5	3	3	(Valente et al., 2017)
107	Santa. Maria Assunta Church ^c	44.984	11.351	B	5	3	3	(Valente et al., 2017)
108	Clock Tower ^c	44.832	11.294	I	6.5	5	5	(Acito et al., 2016)

No.	Church/Tower Name	Lat.	Long.	Typology	I_{MCS}	d_1	d_2	Source
109	Auditorium Ex Church of S. Lorenzo	44.728	11.292	B	6	2	4	(Ferrari, 2020)
110	San Filippo Neri Church	44.726	11.289	B	6	2	0	(Ferrari, 2020)
111	San Michele Arcangelo Church	44.772	11.075	B	5	4	3	(Ferrari, 2020)
112	Oratorio San Rocco	44.772	11.100	B	5	4	2	(Ferrari, 2020)
113	San Nicolò da Bari Church	44.728	11.043	B	5	0	2	(Ferrari, 2020)
114	Beata Vergine del Rosario Church	44.834	11.294	B	6.5	4	2	(Ferrari, 2020)
115	San Bartolomeo Church	44.831	11.296	B	6.5	1	1	(Ferrari, 2020)
116	SS. Senesio and Teopompo Martiri Church	44.848	11.071	B	5.5	3	4	(Ferrari, 2020)
117	Santa Maria Annunziata Church	44.802	10.695	B	6.5	0	0	(Ferrari, 2020)
118	Santa Maria Assunta Church	44.918	10.807	B	6.5	2	4	(Ferrari, 2020)
6	Event: the 2016 Central Italy Earthquake							
119	Sant'Agostino Church ^c	42.629	13.291	B	10.5	2	3	(Acito et al., 2021)
120	Civic Tower ^c	42.629	13.289	I	10.5	0	3	(Jain et al., 2020)
121	San Salvatore Church ^c	42.983	12.930	B	5	4	4	(Sferrazza Papa & Silva, 2018)
122	San Michele Arcangelo Church ^{B, c}	43.165	13.082	B	5	2	5	(De Matteis & Zizi, 2019)
123	Santa Maria del Rosario Church ^{B, c}	43.100	13.108	B	5	4	2	(De Matteis & Zizi, 2019)
124	Ex Church of San Carlo ^c	43.132	13.065	B	5	5	5	(De Matteis & Zizi, 2019)
125	Collegiata di Santa Maria ^c	42.931	13.087	B	5.5	3	2	(Giordano et al., 2019)
126	San Francesco Church ^c	42.943	13.343	B	5.5	2	0	(Clementi et al., 2020)
127	San Francesco Church ^c	42.727	13.687	B	5	3	0	(Ferracuti et al., 2022)
128	San Savino Church ^c	43.067	13.338	B	6	3	4	(Ferracuti et al., 2022)
129	San Massimo Oratorio ^c	42.677	13.288	B	6	0	4	(Ferracuti et al., 2022)

^a Coordinates related to the municipality and not to the exact tower location

^b MCS intensity related to the municipality and not to the tower location

^c Damage levels assigned in this work according to the gathered documentation

NaN not present in the tower

APPENDIX B

DAMAGE PROBABILITY MATRICES AND FRAGILITY FUNCTIONS FOR THE KEY FEATURES

In this Appendix, DPMs and fragility functions, derived by dividing the towers of the dataset analysed in Chapter 4 according to their most influential characteristics, are reported. In particular, the state of maintenance, the geometry, the tower typology and the location have been studied as key features, allowing to define several subsets of towers. For the state of maintenance, three subsets are identified, following three distinct levels, namely poor, acceptable and good. The geometry is investigated through the aspect ratio of the towers, and two subsets are defined by separating them based on a threshold equal to 4. The tower typology is analysed based on the distinction between integrated and confined towers. Finally, the location is explored according to the region and province where they stand.

For each key feature, two sets of fragility functions are presented, differentiated by the approach used (see Chapter 4): traditional and optimised. It should be noted that the latter approach prevents the curve crossing behaviour, an essential aspect while addressing multiple sequential damage levels. The parameter estimates obtained by applying these two fitting procedures are also included for each key feature considered.

B.1 Maintenance-based analysis

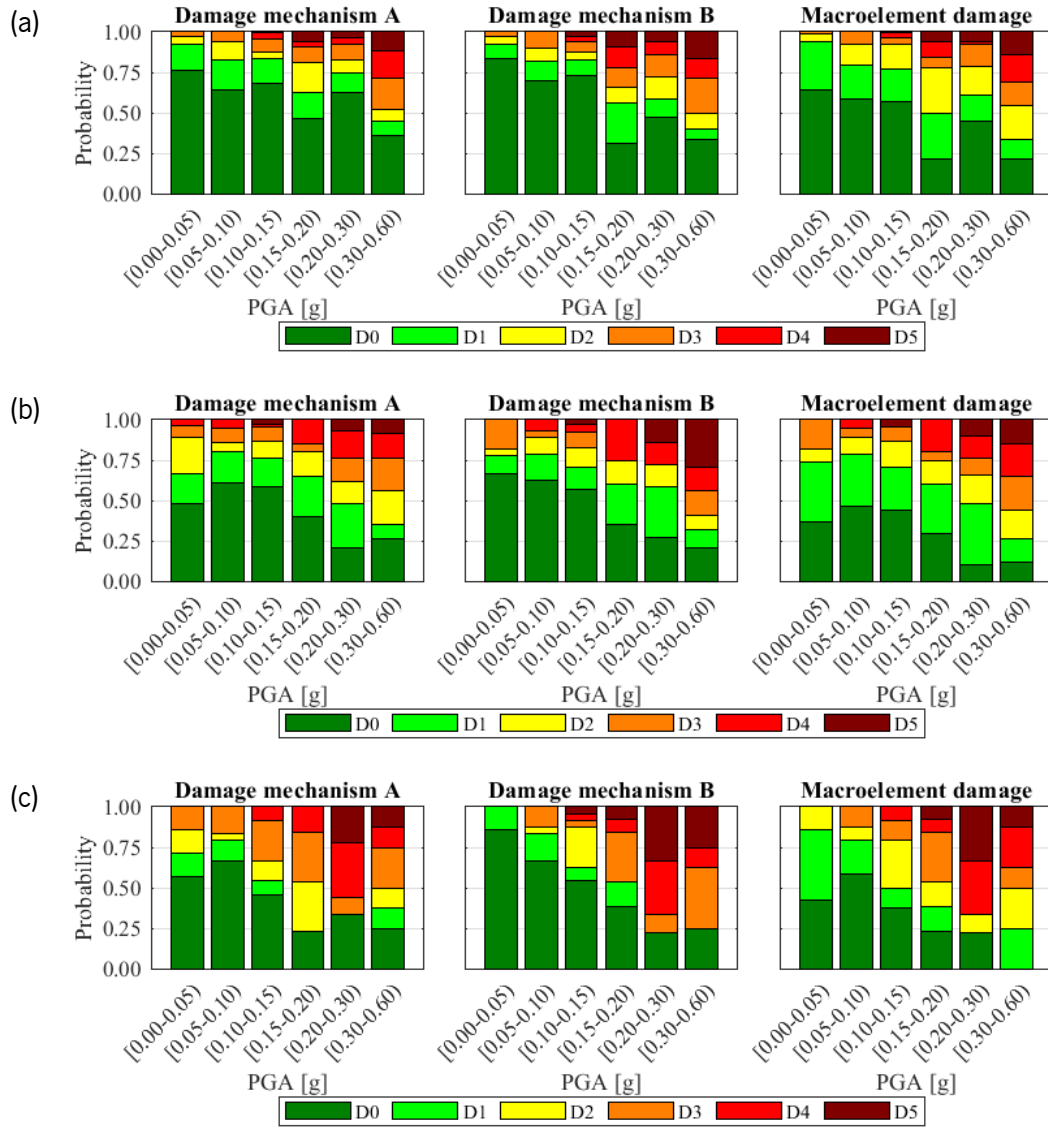


Figure B.1: DPMs as functions of the PGA: (a) Good; (b) Acceptable; (c) Poor.

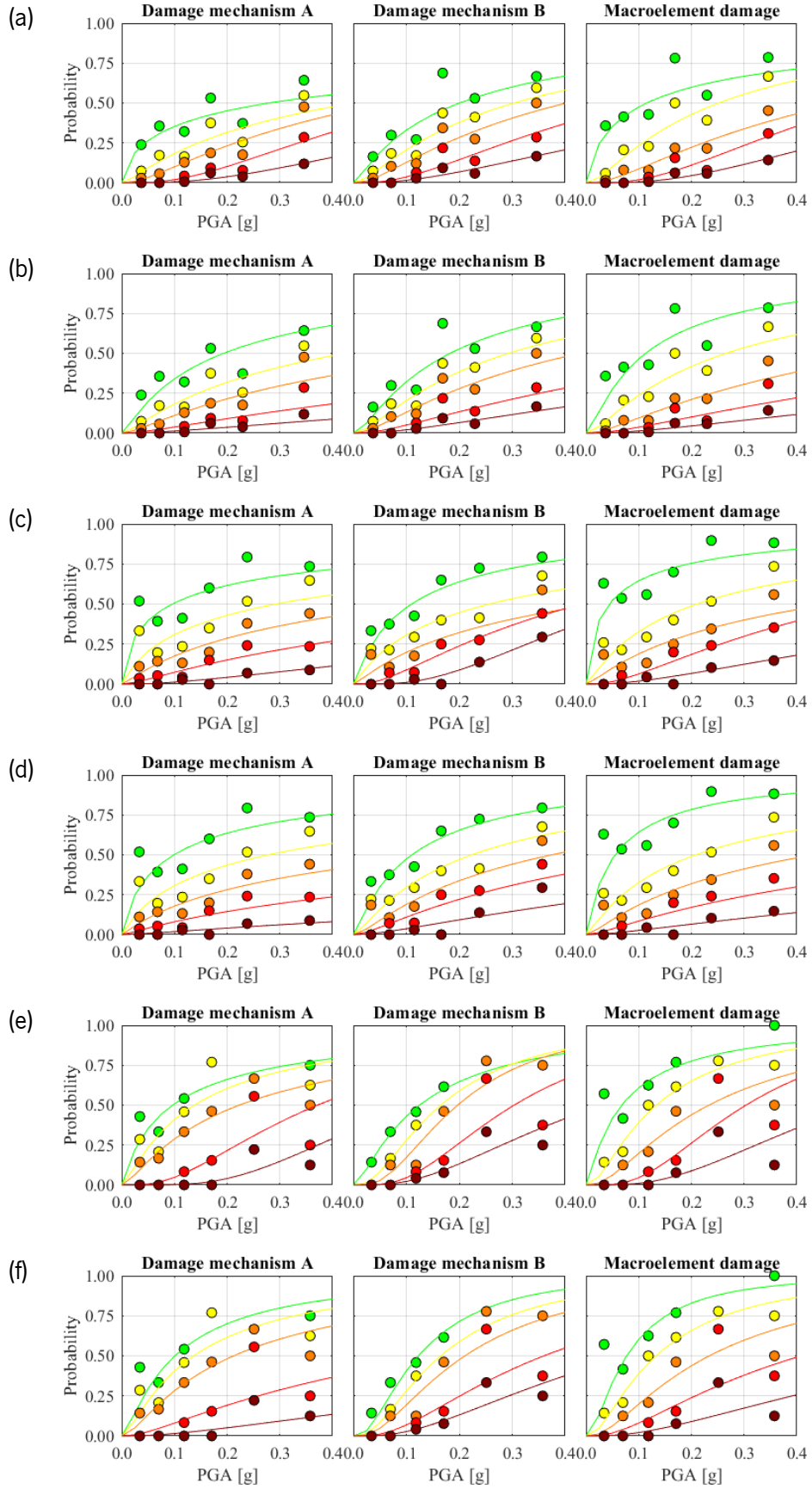


Figure B.2: Fragility functions for towers with good, acceptable and poor maintenance states: (a, c, and e) traditional approach; (b, d and f) optimised approach.

Table B.1: Median and standard deviation for damage states (from D1 to D5) of the distinct subsets accounting for the state of maintenance.

Approach Type	Parameters	State of Maintenance														
		Good					Acceptable					Poor				
		D1	D2	D3	D4	D5	D1	D2	D3	D4	D5	D1	D2	D3	D4	D5
Mechanism A																
Traditional	θ_{D_k}	0.28	0.44	0.51	0.60	0.96	0.10	0.29	0.58	1.13	2.05	0.10	0.14	0.22	0.37	0.55
	β	2.74	1.65	1.27	0.87	0.88	2.49	2.15	1.88	1.67	1.35	1.75	1.41	1.43	0.78	0.58
Optimised	θ_{D_k}	0.19	0.42	0.70	1.69	3.48	0.10	0.28	0.64	1.71	6.82	0.10	0.14	0.21	0.62	1.69
	β	1.60	1.60	1.60	1.60	1.60	2.03	2.03	2.03	2.03	2.03	1.30	1.30	1.30	1.30	1.30
Mechanism B																
Traditional	θ_{D_k}	0.20	0.25	0.40	0.54	0.93	0.11	0.26	0.46	0.43	0.54	0.12	0.16	0.20	0.30	0.47
	β	1.58	1.36	1.23	0.95	1.03	1.69	1.86	1.80	1.08	0.73	1.25	0.86	0.71	0.65	0.77
Optimised	θ_{D_k}	0.18	0.29	0.43	0.84	1.42	0.11	0.22	0.37	0.63	1.46	0.12	0.16	0.21	0.36	0.52
	β	1.30	1.30	1.30	1.30	1.30	1.50	1.50	1.50	1.50	1.50	0.86	0.86	0.86	0.86	0.86
Macroelement damage																
Traditional	θ_{D_k}	0.12	0.25	0.49	0.54	0.79	0.04	0.21	0.47	0.54	1.18	0.07	0.13	0.23	0.30	0.52
	β	2.21	1.27	1.16	0.81	0.81	2.20	1.67	1.74	1.10	1.18	1.43	1.04	1.00	0.65	0.70
Optimised	θ_{D_k}	0.11	0.27	0.60	1.14	2.03	0.05	0.21	0.43	0.95	2.37	0.08	0.13	0.23	0.41	0.78
	β	1.37	1.37	1.37	1.37	1.37	1.64	1.64	1.64	1.64	1.64	1.00	1.00	1.00	1.00	1.00

B.2 Geometry-based analysis

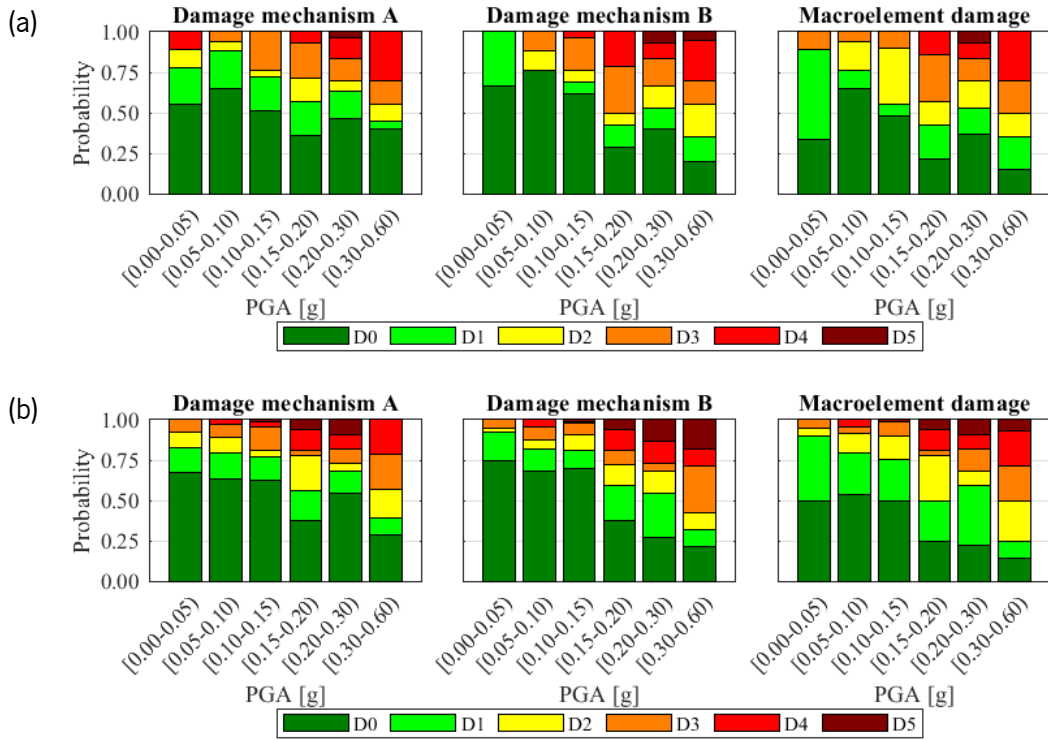
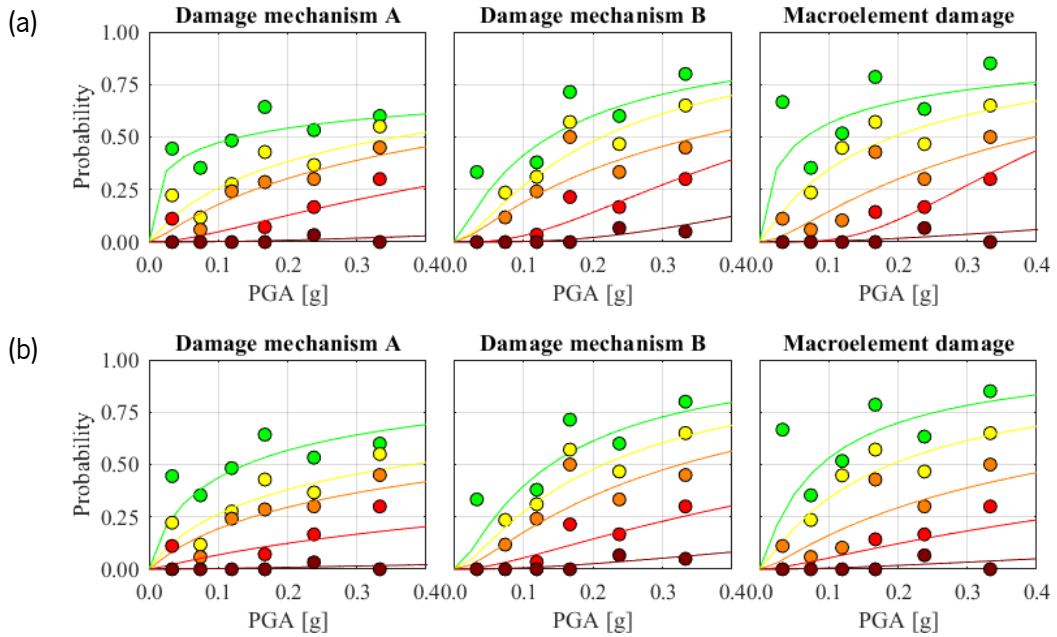

 Figure B.3: DPMs as functions of the PGA: (a) $H_{tot}/L_{min} \leq 4$; (b) $H_{tot}/L_{min} > 4$.


Figure B.4: Fragility functions for towers characterised by aspect ratio equal or lower than 4: (a) traditional approach; (b) optimised approach.

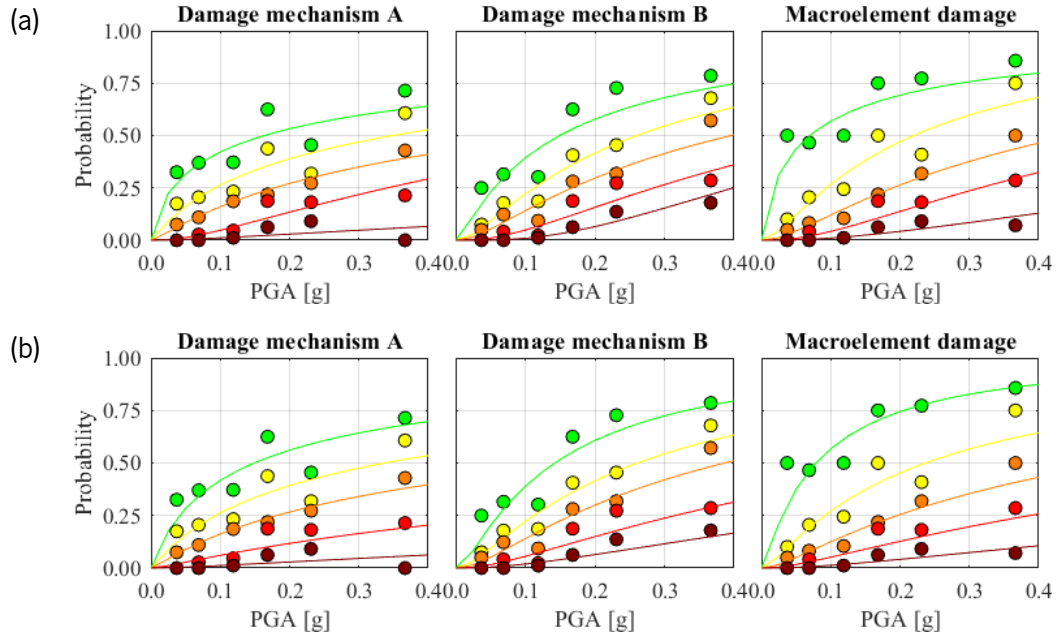


Figure B.5: Fragility functions for towers characterised by aspect ratio greater than 4: (a) traditional approach; (b) optimised approach.

B.3 Interaction-based analysis

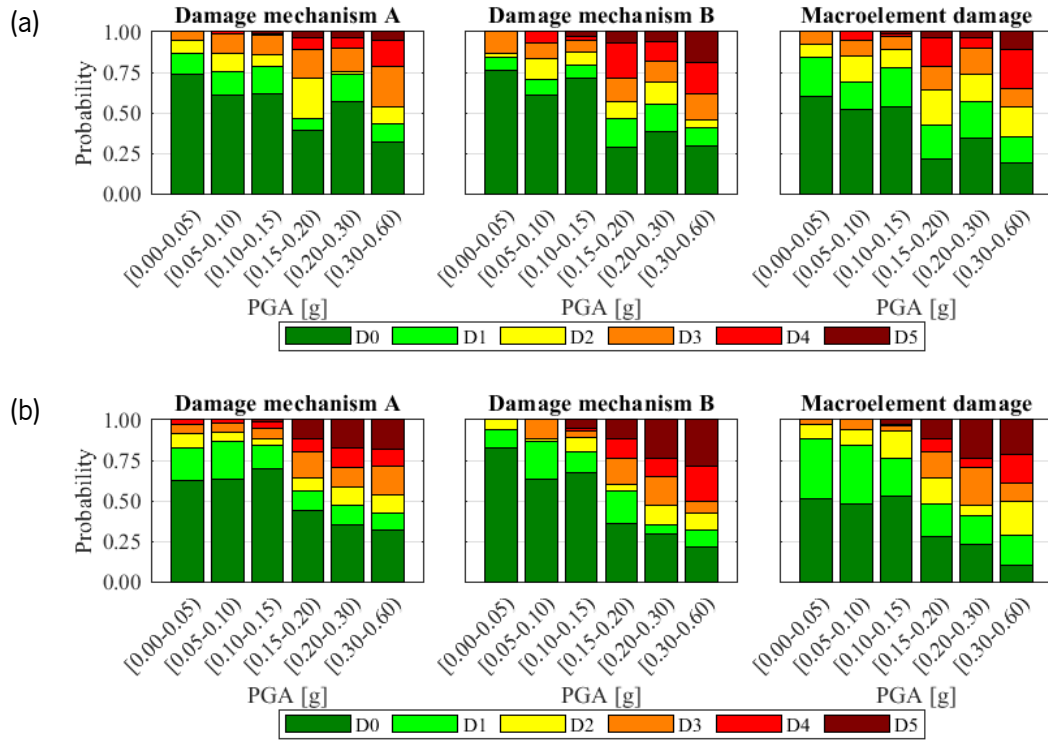


Figure B.6: DPMs as functions of the PGA: (a) Confined; (b) Integrated.

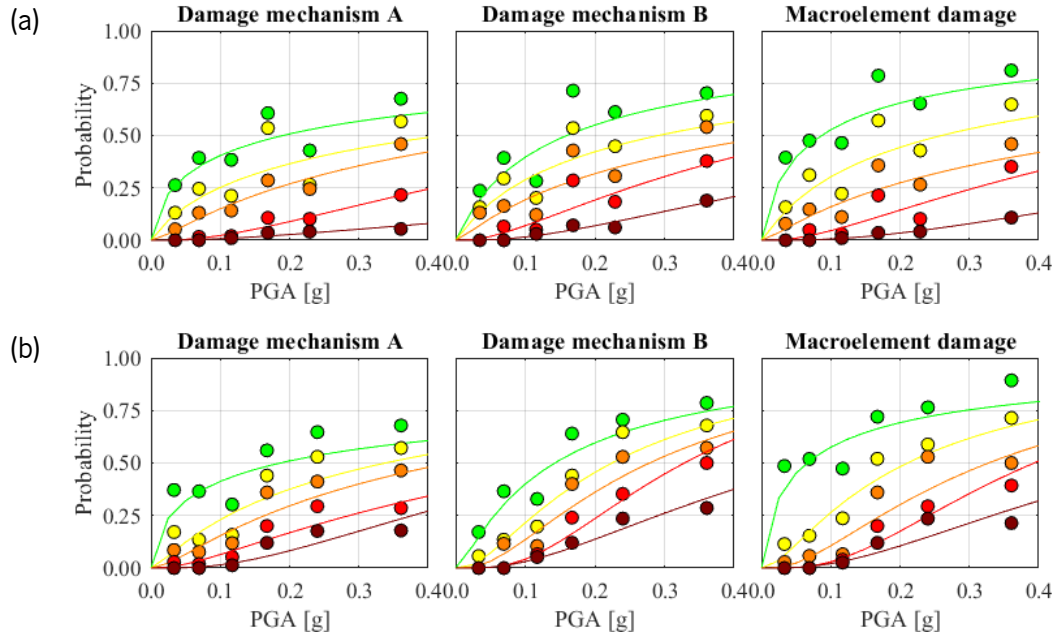


Figure B.7: Fragility functions for confined towers: (a) traditional approach; (b) optimised approach.

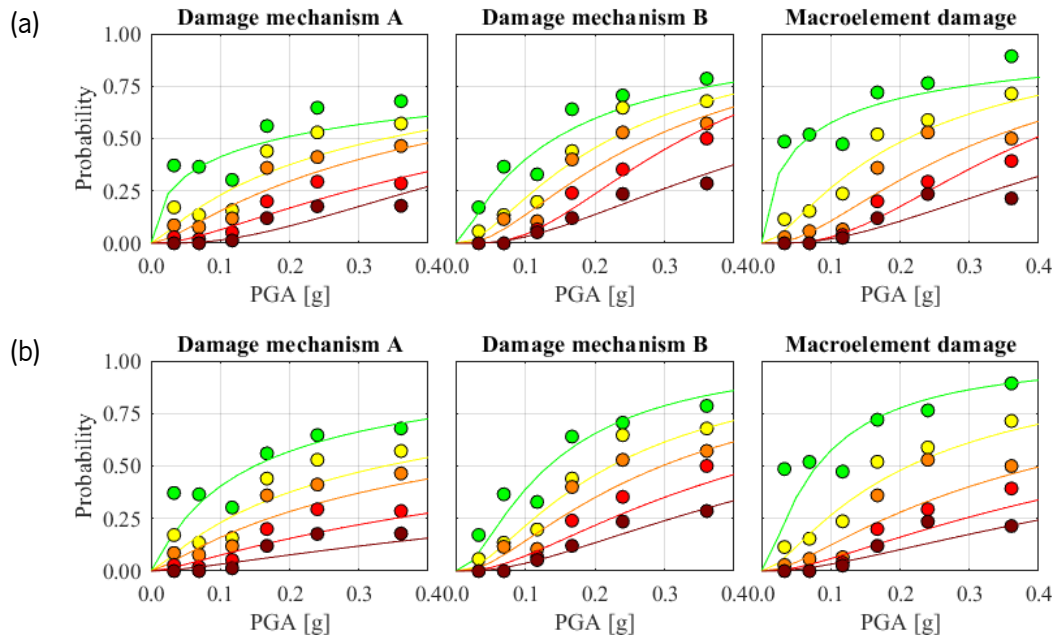


Figure B.8: Fragility functions for integrated towers: (a) traditional approach; (b) optimised approach.

Table B.2: Median and standard deviation for damage states (from D1 to D5) of the distinct subsets accounting for the geometry and interactions.

Approach Type	Parameters	Geometry and Interactions														
		Aspect Ratio					Confined					Integrated				
		H/L ≤ 4					H/L > 4									
		D1	D2	D3	D4	D5	D1	D2	D3	D4	D5	D1	D2	D3	D4	D5
Mechanism A																
Traditional	θ_{D_k}	0.13	0.36	0.49	0.92	5.90	0.16	0.35	0.60	0.78	6.03	0.19	0.42	0.55	0.83	2.62
	β	4.00	1.97	1.72	1.34	1.43	2.49	1.98	1.81	1.24	1.80	2.66	2.15	1.65	1.06	1.34
Optimised	θ_{D_k}	0.14	0.38	0.62	2.29	28.44	0.15	0.34	0.66	1.95	7.59	0.17	0.37	0.69	2.88	8.14
	β	2.11	2.11	2.11	2.11	2.11	1.91	1.91	1.91	1.91	1.91	1.92	1.92	1.92	1.92	1.92
Mechanism B																
Traditional	θ_{D_k}	0.14	0.21	0.35	0.50	1.02	0.15	0.26	0.40	0.58	0.69	1.16	0.29	0.46	0.54	0.93
	β	1.46	1.22	1.42	0.86	0.81	1.48	1.24	1.28	1.07	0.82	1.79	1.91	1.77	1.14	1.04
Optimised	θ_{D_k}	0.14	0.21	0.33	0.77	2.33	0.14	0.26	0.39	0.73	1.34	0.16	0.26	0.41	0.89	2.25
	β	1.27	1.27	1.27	1.27	1.27	1.25	1.25	1.25	1.25	1.25	1.61	1.61	1.61	1.61	1.61
Macroelement damage																
Traditional	θ_{D_k}	0.07	0.19	0.40	0.44	2.98	0.07	0.22	0.45	0.66	1.50	0.09	0.26	0.57	0.65	1.23
	β	2.54	1.65	1.31	0.64	1.29	2.10	1.26	1.28	1.09	1.17	2.10	1.85	1.75	1.11	1.00
Optimised	θ_{D_k}	0.09	0.19	0.47	1.23	5.19	0.08	0.24	0.51	1.01	2.35	0.09	0.24	0.54	1.33	4.32
	β	1.56	1.56	1.56	1.56	1.56	1.42	1.42	1.42	1.42	1.42	1.70	1.70	1.70	1.70	1.70

B.4 Location-based analysis

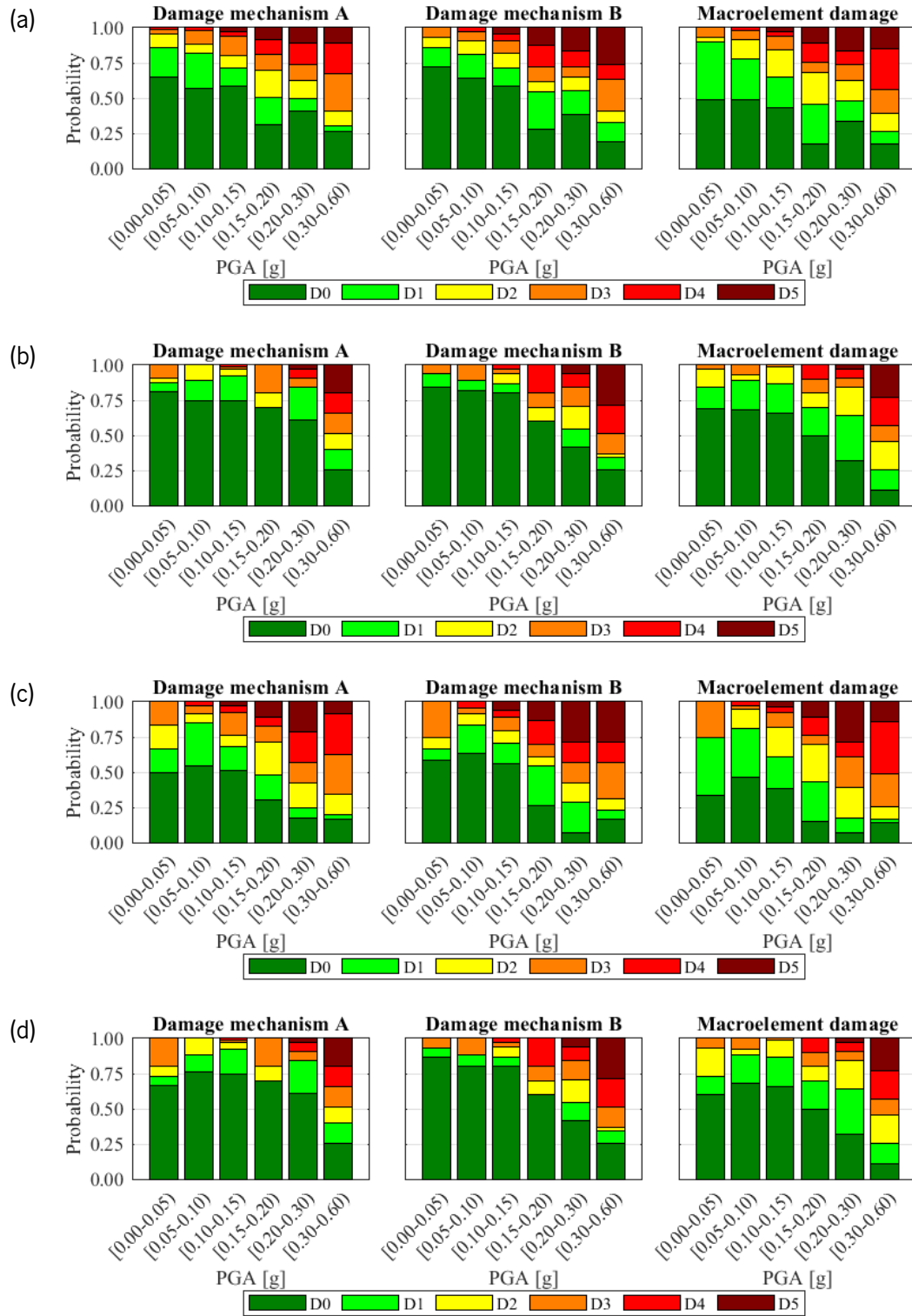


Figure B.9: DPMs as functions of the PGA: (a) Marche; (b) Umbria; (c) Macerata; and (d) Perugia.

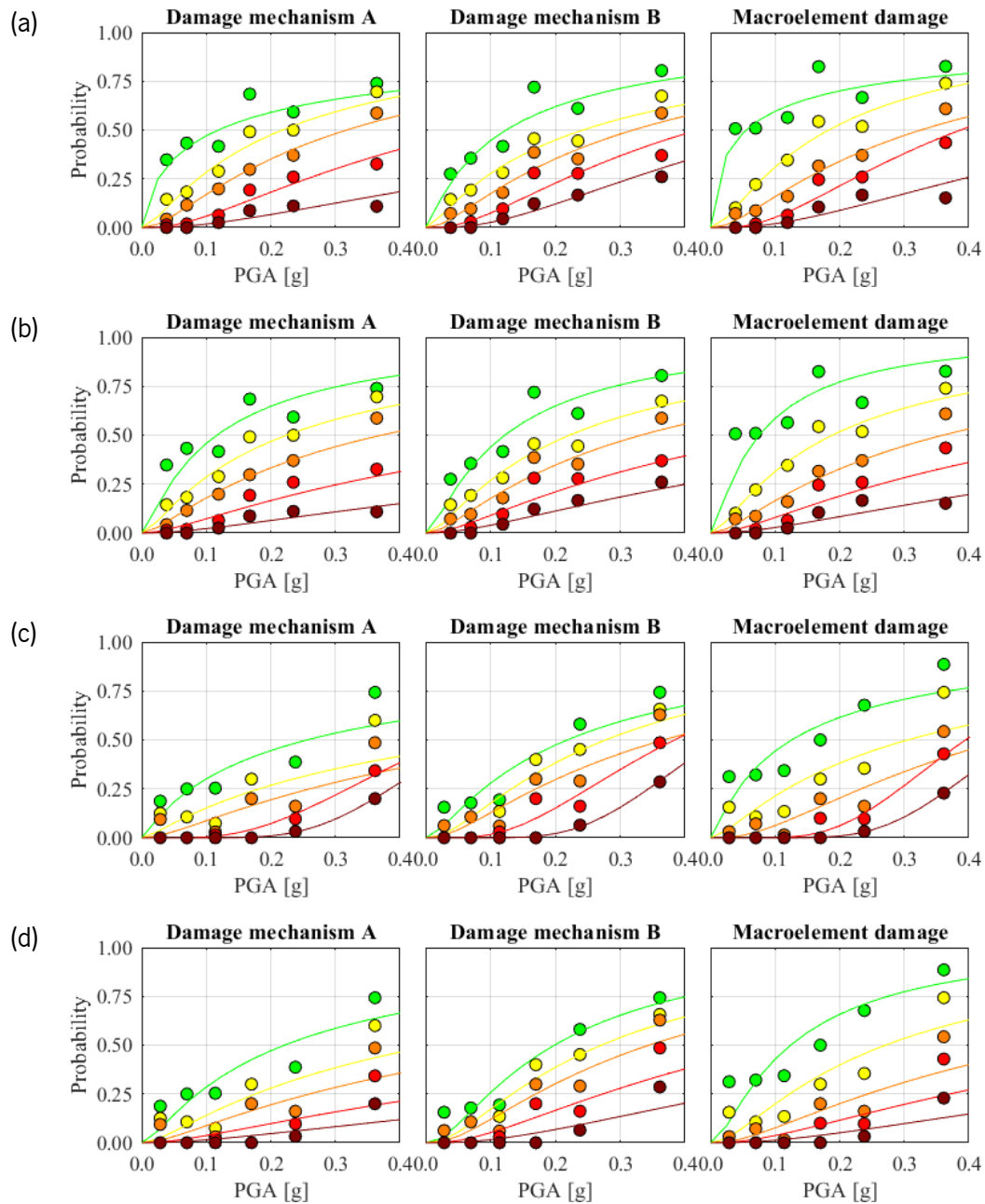


Figure B.10: Fragility functions for towers located in Marche and in Umbria: (a and c) traditional approach; (b and d) optimised approach.

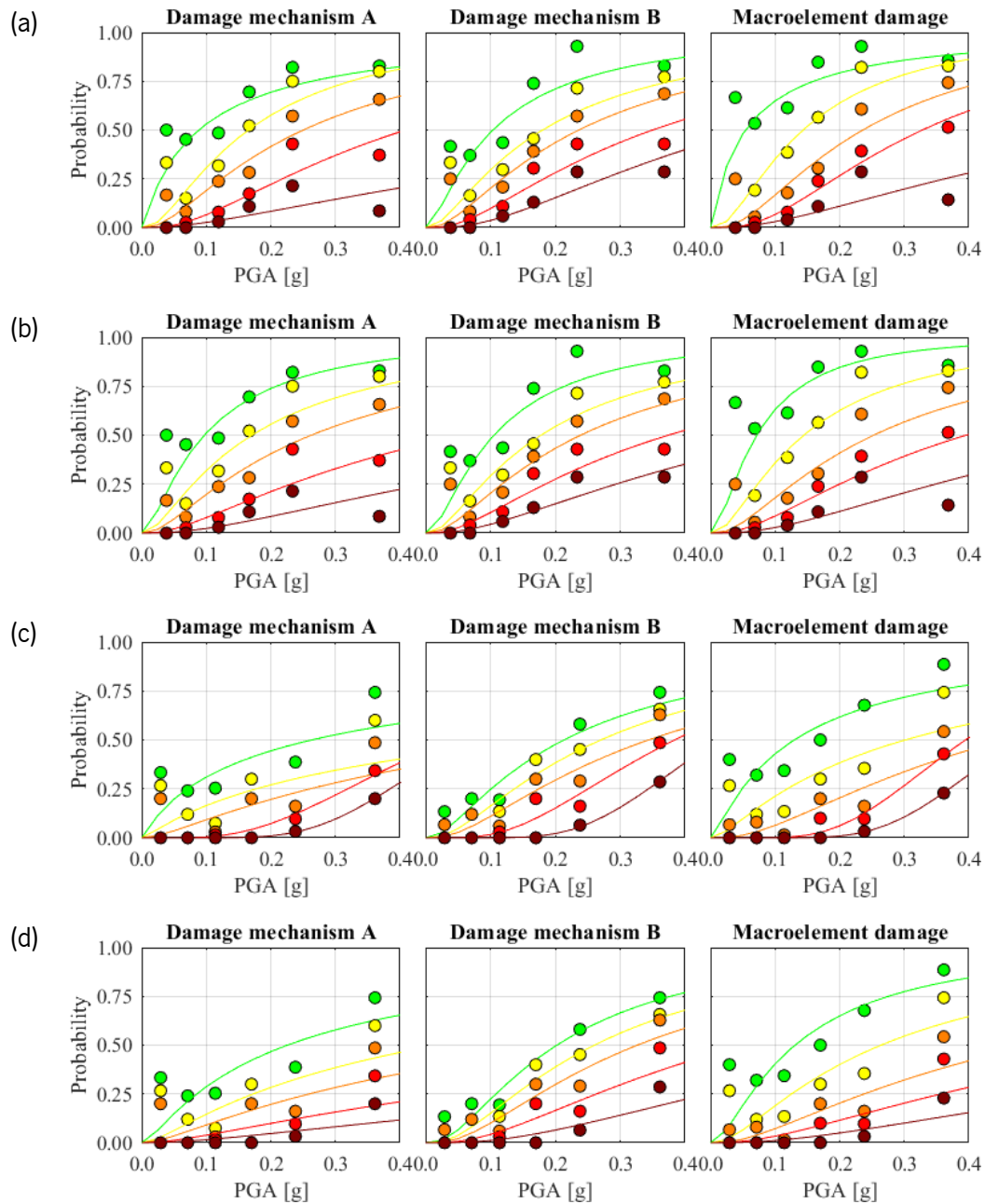


Figure B.11: Fragility functions for towers located in Macerata and in Perugia: (a and c) traditional approach; (b and d) optimised approach.

Table B.3: Median and standard deviation for damage states (from D1 to D5) of the distinct subsets accounting for the location (region and province).

Approach Type	Parameters	Location																			
		Region					Province														
		Marche					Umbria					Macerata					Perugia				
		D1	D2	D3	D4	D5	D1	D2	D3	D4	D5	D1	D2	D3	D4	D5	D1	D2	D3	D4	D5
Mechanism A																					
Traditional	θ_{D_k}	0.12	0.22	0.32	0.52	1.14	0.26	0.57	0.67	0.48	0.50	0.09	0.16	0.25	0.41	1.13	0.26	0.64	0.70	0.48	0.50
	β	2.29	1.36	1.21	1.05	1.16	1.81	1.70	1.40	0.60	0.39	1.62	1.00	1.05	0.93	1.25	1.96	1.90	1.47	0.59	0.39
Optimised	θ_{D_k}	0.12	0.22	0.37	0.80	1.76	0.22	0.45	0.67	1.22	2.09	0.10	0.17	0.26	0.50	0.95	0.22	0.46	0.68	1.28	2.24
	β	1.43	1.43	1.43	1.43	1.43	1.40	1.40	1.40	1.40	1.40	1.13	1.13	1.13	1.13	1.13	1.44	1.44	1.44	1.44	1.44
Mechanism B																					
Traditional	θ_{D_k}	0.12	0.24	0.32	0.42	0.58	0.22	0.28	0.37	0.38	0.45	0.10	0.18	0.24	0.35	0.50	0.21	0.27	0.34	0.38	0.45
	β	1.57	1.49	1.24	1.00	0.91	1.33	1.10	1.14	0.63	0.40	1.18	1.10	1.02	0.97	0.90	1.10	1.00	1.00	0.63	0.40
Optimised	θ_{D_k}	0.12	0.22	0.33	0.56	0.96	0.20	0.27	0.34	0.55	0.96	0.10	0.18	0.24	0.37	0.60	0.20	0.26	0.33	0.49	0.81
	β	1.29	1.29	1.29	1.29	1.29	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	0.92	0.92	0.92	0.92	0.92
Macroelement damage																					
Traditional	θ_{D_k}	0.05	0.18	0.32	0.39	0.76	0.13	0.31	0.45	0.39	0.47	0.05	0.14	0.23	0.33	0.75	0.13	0.30	0.45	0.39	0.47
	β	2.46	1.22	1.18	0.83	0.99	1.59	1.41	0.98	0.44	0.36	1.60	0.94	0.89	0.79	1.08	1.45	1.38	1.00	0.44	0.36
Optimised	θ_{D_k}	0.07	0.19	0.36	0.64	1.23	0.12	0.27	0.54	0.82	1.37	0.07	0.14	0.25	0.40	0.69	0.13	0.26	0.50	0.75	1.23
	β	1.32	1.32	1.32	1.32	1.32	1.18	1.18	1.18	1.18	1.18	1.01	1.01	1.01	1.01	1.01	1.10	1.10	1.10	1.10	1.10

APPENDIX C

TOWER FREQUENCY PREDICTIVE EQUATIONS

In this Appendix, novel empirical formulations to help the prediction of the fundamental frequency of historic masonry towers are presented. Three models rely on a single independent variable: (i) total height; (ii) effective height; (iii) aspect ratio. Two models adopt two independent variables: (i) total height and minimum side length; (ii) effective height and minimum side length. Four models include three independent variables: (i) total height, minimum side length, and wall thickness; (ii) effective height, minimum side lengths, and wall thickness; (iii) total height, minimum and maximum side length; (iv) effective height, minimum and maximum side lengths. Finally, two models employ four independent variables: (i) total height, wall thickness, minimum and maximum side lengths; (ii) effective height, wall thickness, minimum and maximum side lengths. Additionally, distinct formulations are provided for clusters of towers with likely homogeneous behaviour, classifying the samples based on their interaction with surrounding buildings (isolated and bounded), and their material (brick, stone and mixed). These models obtained by means of power regression are validated against new samples unknown in the training stage. Moreover, they are compared with similar equations existing in literature to evaluate their effectiveness. The accuracy and robustness of the developed models are assessed through two performance metrics, namely the coefficient of determination (R-squared) and the Mean Squared Error (MSE).

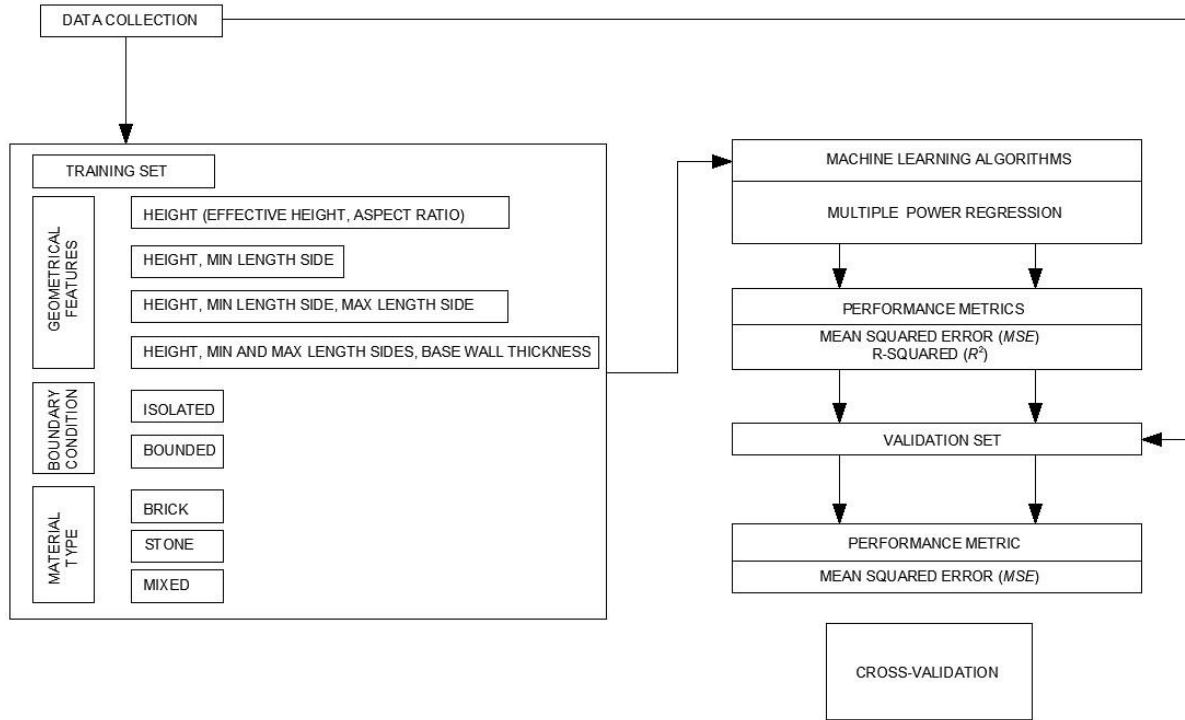


Figure C.1: Flowchart methodology for the development and validation of empirical formulation for predicting the first natural frequency of historic masonry towers.

Table C.1: Summary of the existing and novel empirical formulations proposed for predicting the frequency of historic masonry towers.

Model	Proposed law	H_{tot}	H_{eff}	H/L_{min}	L_{min}	L_{max}	s
		α	β	β	γ	θ	δ
1 independent variable	from (1) to (9)	x	x				
	from (10) to (13)	x		x			
	(14)	x		x			
2 independent variables	from (15) to (23)	x	x		x		
	from (24) to (27)	x		x	x		
	from (28) to (36)	x	x		x		x
3 independent variables	from (37) to (40)	x		x	x		x
	from (41) to (49)	x	x		x	x	
	from (50) to (53)	x		x	x	x	
4 independent variables	from (54) to (62)	x	x		x	x	x
	from (63) to (66)	x		x	x	x	x

C.1 One independent variable models

$$f_1 = \frac{\alpha}{H_{tot}^\beta} \quad (C.1)$$

$$f_1 = \frac{\alpha}{H_{eff}^\beta} \quad (C.2)$$

$$f_1 = \frac{\alpha}{\left(\frac{H_{tot}}{L_{min}}\right)^\beta} \quad (C.3)$$

Table C.2: Summary of single independent parameters formulas, according to Eq. (C.1), Eq. (C.2) and Eq. (C.3).

<i>H_{tot}</i>								
Training set	No. cases	Formulas	α	β	R2	MSE Training	No. Cases	MSE Validation
Entire	112	(Faccio et al., 2011a)	53.48	1.000	0.59	0.38	69	0.83
	112	(NTC, 2008)	20.00	0.750	0.42	0.55	69	1.78
	112	(Rainieri & Fabbrocino, 2011)	87.95	1.138	0.56	0.42	69	1.03
	112	(Shakya et al., 2016) (1)	66.23	1.080	0.54	0.43	69	0.89
	112	(Bartoli, Betti, Marra, et al., 2017) (1)	36.42	0.900	0.56	0.41	69	1.03
Isolated	112	Proposed law (1)	42.12	0.893	0.64	0.33	69	0.74
	28	(Diaferio et al., 2018) (1)	135.34	1.320	0.79	0.43	28	3.61
	28	Proposed law (2)	58.57	1.026	0.92	0.17	28	1.37
Bounded	84	(Diaferio et al., 2018) (2)	28.35	0.830	0.19	0.38	37	1.30
	84	Proposed law (3)	21.64	0.695	0.38	0.35	37	0.40
Brick – Isolated	11	Proposed law (4)	190.9	1.378	0.87	0.07	2	0.80
Brick – Bounded	23	Proposed law (5)	35.34	0.869	0.59	0.18	2	0.13
Stone – Isolated	10	Proposed law (6)	57.44	1.023	0.95	0.21	18	1.88
Stone – Bounded	46	Proposed law (7)	16.01	0.590	0.29	0.42	23	0.17
Mixed – Isolated	7	Proposed law (8)	872.3	1.73	0.89	0.08	8	52.30
Mixed – Bounded	15	Proposed law (9)	17.49	0.651	0.38	0.24	11	0.82
<i>H_{eff}</i>								
Training set	No. cases	Formulas	α	β	R2	MSE Training	No. cases	MSE Validation
Bounded	84	(Bartoli, Betti, Marra, et al., 2017) (2)	19.54	0.790	0.34	0.37	13	0.28
	84	(Diaferio et al., 2018) (3)	12.96	0.686	0.26	0.42	13	0.36
	84	Proposed law (10)	11.87	0.601	0.40	0.34	13	0.20
Brick – Bounded	23	Proposed law (11)	16.05	0.735	0.61	0.17	1	0.01
Stone – Bounded	46	Proposed law (12)	9.40	0.504	0.29	0.42	6	0.14
Mixed – Bounded	15	Proposed law (13)	10.91	0.592	0.46	0.21	5	0.46
<i>H/L_{min}</i>								
Training set	No. cases	Formulas	α	β	R2	MSE Training		
Entire	112	(Shakya et al., 2016) (2)	3.58	0.570	0.14	0.80		
	112	Proposed law (14)	7.40	0.860	0.38	0.58		

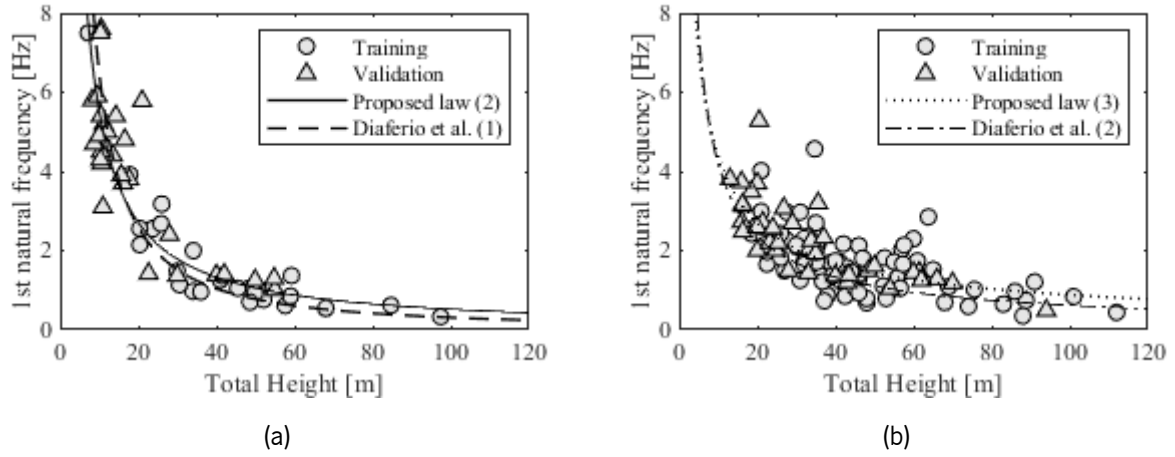


Figure C.2: Prediction of the first natural frequency: (a) isolated; (b) bounded towers.

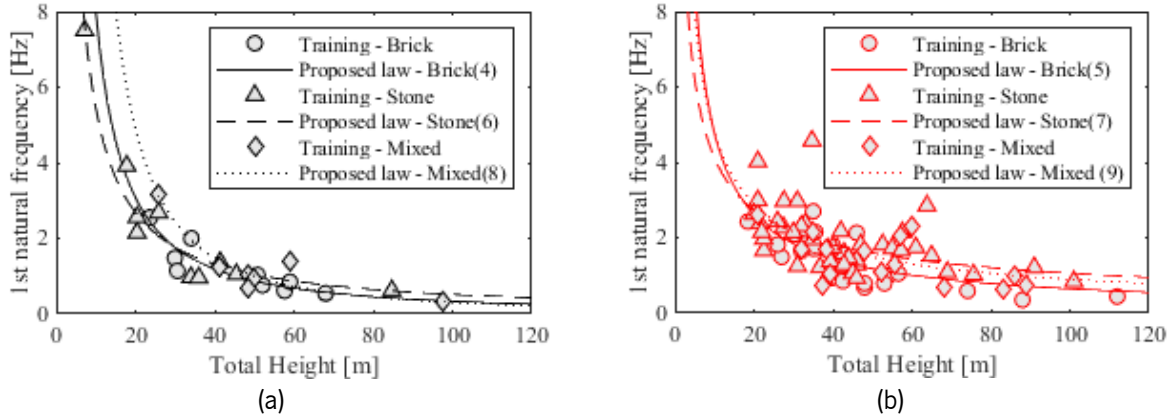


Figure C.3: Prediction of the first natural frequency for brick, stone and mixed historic towers according to their boundary condition: (a) isolated; (b) bounded.

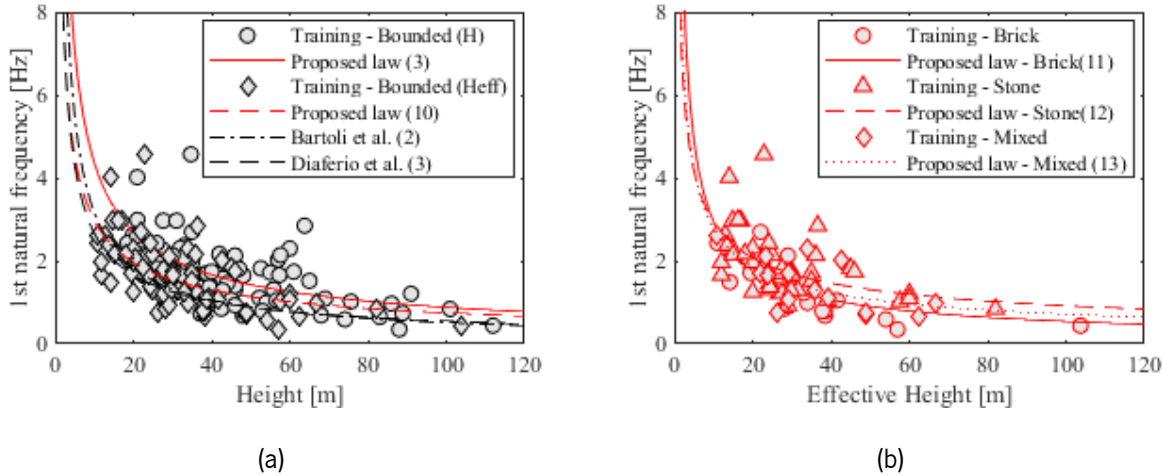


Figure C.4: (a) Comparison between the prediction based on the total height and effective height for bounded towers; (b) Prediction of the first natural frequency for brick, stone and mixed bounded towers.

C.2 Two independent variables models

$$f_1 = \alpha H_{tot}^{\beta} L_{min}^{\gamma} \quad (C.4)$$

$$f_1 = \alpha H_{eff}^{\beta} L_{min}^{\gamma} \quad (C.5)$$

Table C.3: Summary of two independent parameters formulas, according to Eq. (C.4) and Eq. (C.5).

Training set	No. cases	Formulas	<i>H_{tot} L_{min}</i>			R2	MSE Train	No. cases	MSE Valid
			α	β	γ				
Entire	112	(Bartoli, Betti, Marra, et al., 2017) (3)	47.29	-1.08	0.22	0.61	0.37	44	0.85
	112	Proposed law (15)	36.74	-0.96	0.20	0.66	0.32	44	0.75
Isolated	28	Proposed law (16)	49.36	-1.09	0.21	0.92	0.16	15	1.39
Bounded	84	Proposed law (17)	21.96	-0.80	0.21	0.40	0.34	28	0.48
Brick – Isolated	11	Proposed law (18)	231.17	-1.69	0.48	0.88	0.06	2	1.22
Brick – Bounded	23	Proposed law (19)	40.13	-1.00	0.21	0.61	0.18	1	0.39
Stone – Isolated	10	Proposed law (20)	44.20	-1.14	0.35	0.96	0.15	7	1.99
Stone – Bounded	46	Proposed law (21)	16.18	-0.74	0.23	0.33	0.40	19	0.27
Mixed – Isolated	7	Proposed law (22)	847.60	-1.67	-0.10	0.89	0.08	6	55.16
Mixed – Bounded	15	Proposed law (23)	13.81	-0.77	0.33	0.41	0.23	7	1.50

Training set	No. cases	Formulas	<i>H_{eff} L_{min}</i>			R2	MSE Training	No. cases	MSE Validation
			α	β	γ				
Bounded	84	(Bartoli, Betti, Marra, et al., 2017) (4)	33.97	-1.42	0.81	0.02	0.55	10	0.71
	84	Proposed law (24)	11.04	-0.70	0.21	0.42	0.33	10	0.29
Brick – Bounded	23	Proposed law (25)	15.65	-0.77	0.08	0.61	0.17	1	0.01
Stone – Bounded	46	Proposed law (26)	9.16	-0.68	0.32	0.33	0.40	4	0.14
Mixed – Bounded	15	Proposed law (27)	8.40	-0.65	0.22	0.47	0.20	4	0.66

C.3 Three independent variables models

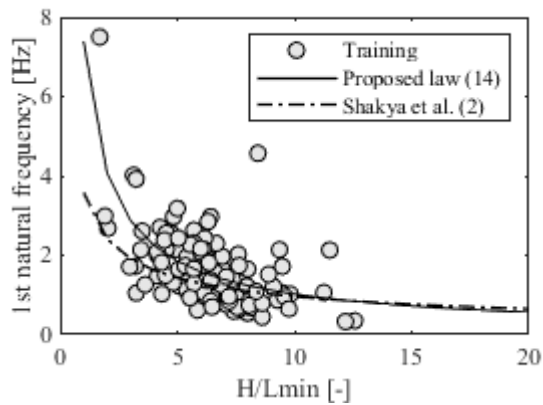
$$f_1 = \alpha H_{tot}^{\beta} L_{min}^{\gamma} s^{\delta} \quad (C.6)$$

$$f_1 = \alpha H_{eff}^{\beta} L_{min}^{\gamma} s^{\delta} \quad (C.7)$$

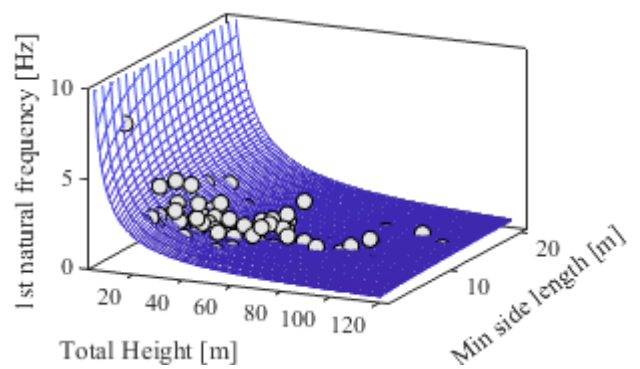
Table C.4: Summary of three independent parameters formulas, according to Eq. (C.6) and Eq. (C.7).

Training set	No. cases	Formulas	H_{tot}		L_{min}		s		R2	MSE Training	No. cases	MSE Validation
			α	β	γ	δ	δ	δ				
Entire	112	Proposed law (28)	38.93	-0.96	0.16	0.05	0.66	0.32	10	0.49		
Isolated	28	Proposed law (29)	49.84	-1.09	0.21	0.01	0.92	0.16	5	0.28		
Bounded	84	Proposed law (30)	21.08	-0.80	0.23	-0.03	0.40	0.34	5	0.70		
Brick – Isolated	11	Proposed law (31)	209.25	-1.70	0.45	0.60	0.97	0.02	2	0.65		
Brick – Bounded	23	Proposed law (32)	27.68	-1.01	0.43	-0.20	0.62	0.17	-	-		
Stone – Isolated	10	Proposed law (33)	16.63	-1.31	1.20	-1.07	0.92	0.29	1	0.26		
Stone – Bounded	46	Proposed law (34)	13.05	-0.68	0.35	-0.17	0.33	0.40	1	1.26		
Mixed – Isolated	7	Proposed law (35)	437.89	-1.55	0.09	-0.24	0.90	0.07	2	0.04		
Mixed – Bounded	15	Proposed law (36)	2.88	-0.57	0.97	-0.71	0.52	0.19	4	1.51		

Training set	No. cases	Formulas	H_{eff}		L_{min}		s		R2	MSE Training	No. cases	MSE Validation
			α	β	γ	δ	δ	δ				
Bounded	84	Proposed law (37)	10.12	-0.70	0.26	-0.06	0.42	0.33	3	0.70		
Brick – Bounded	23	Proposed law (38)	10.16	-0.78	0.35	-0.22	0.64	0.16	-	-		
Stone – Bounded	46	Proposed law (39)	7.51	-0.66	0.44	-0.18	0.34	0.39	-	-		
Mixed – Bounded	15	Proposed law (40)	1.95	-0.49	0.89	-0.68	0.57	0.17	3	1.25		



(a)



(b)

Figure C.5: Prediction of the first natural frequency based on the aspect ratio (a) and on the total height and minimum side length (b) for the entire training set.

$$f_1 = \alpha H_{tot}^{\beta} L_{min}^{\gamma} L_{max}^{\theta} \quad (C.8)$$

$$f_1 = \alpha H_{eff}^{\beta} L_{min}^{\gamma} L_{max}^{\theta} \quad (C.9)$$

Table C.5: Summary of three independent parameters formulas, according to Eq. (C.8) and Eq. (C.9).

Training set	No. cases	Formulas	<i>Htot</i> <i>Lmin</i> <i>Lmax</i>			θ	R2	MSE Training	No. cases	MSE Validation
			α	β	γ					
Entire	96	Proposed law (41)	37.66	-1.01	-0.14	0.41	0.68	0.33	43	0.88
Isolated	28	Proposed law (42)	48.09	-1.14	-0.84	1.13	0.93	0.15	15	2.04
Bounded	68	Proposed law (43)	23.97	-0.87	-0.02	0.29	0.42	0.38	28	0.53
Brick – Isolated	11	Proposed law (44)	70.51	-1.24	-2.49	2.69	~1.00	~0.00	2	0.51
Brick – Bounded	20	Proposed law (45)	44.14	-1.15	-0.78	1.16	0.82	0.09	1	0.12
Stone – Isolated	10	Proposed law (46)	44.35	-1.13	0.75	-0.42	0.96	0.15	7	1.90
Stone – Bounded	33	Proposed law (47)	15.55	-0.67	0.48	-0.29	0.25	0.51	19	0.26
Mixed – Isolated	7	Proposed law (48)	761.48	-1.61	0.09	-0.25	0.89	0.08	6	38.24
Mixed – Bounded	15	Proposed law (49)	13.77	-0.77	0.60	-0.26	0.41	0.23	7	1.44

Training set	No. cases	Formulas	<i>Heff</i> <i>Lmin</i> <i>Lmax</i>			θ	R2	MSE Training	No. cases	MSE Validation
			α	β	γ					
Bounded	68	Proposed law (50)	11.26	-0.76	0.17	0.11	0.43	0.37	10	1.21
Brick – Bounded	20	Proposed law (51)	12.78	-0.82	-1.09	1.29	0.82	0.09	1	0.99
Stone – Bounded	33	Proposed law (52)	10.46	-0.69	0.78	-0.49	0.28	0.49	4	0.18
Mixed – Bounded	15	Proposed law (53)	8.56	-0.66	0.96	-0.72	0.48	0.20	4	1.49

C.4 Four independent variables models

$$f_1 = \alpha H_{tot}^{\beta} L_{min}^{\gamma} L_{max}^{\theta} S^{\delta} \quad (C.10)$$

$$f_1 = \alpha H_{eff}^{\beta} L_{min}^{\gamma} L_{max}^{\theta} S^{\delta} \quad (C.11)$$

Table C.6 Summary of four independent parameter formulas, according to Eq. (C.10) and Eq. (C.11).

Training set	No. cases	Formulas	<i>Htot</i> <i>Lmin</i> <i>Lmax</i> <i>s</i>				α	β	γ	θ	δ	R2	MSE Train.	No. cases	MSE Valid.
Entire	96	Proposed law (54)	39.66	-1.01	-0.18	0.41	0.05	0.68	0.33	10	0.53				
Isolated	28	Proposed law (55)	40.61	-1.16	-1.07	1.52	-0.19	0.93	0.15	5	0.33				
Bounded	68	Proposed law (56)	24.26	-0.88	-0.03	0.30	0.01	0.42	0.38	5	0.80				
Brick – Isolated	11	Proposed law (57)	68.53	-1.23	-2.58	2.78	-0.02	0.98	~0.00	2	0.50				
Brick – Bounded	20	Proposed law (58)	34.43	-1.17	-0.40	0.97	-0.15	0.82	0.08	-	-				
Stone – Isolated	10	Proposed law (59)	18.75	-1.34	0.13	1.14	-1.13	0.98	0.07	1	0.23				
Stone – Bounded	33	Proposed law (60)	13.74	-0.65	0.63	-0.36	-0.13	0.25	0.51	1	0.99				
Mixed – Isolated	7	Proposed law (61)	435.26	-1.56	0.07	0.04	-0.25	0.90	0.07	2	0.04				
Mixed – Bounded	15	Proposed law (62)	1.62	-0.54	4.16	-2.85	-0.97	0.57	0.17	4	1.55				
Training set	No. cases	Formulas	<i>Heff</i> <i>Lmin</i> <i>Lmax</i> <i>s</i>				α	β	γ	θ	δ	R2	MSE Train.	No. cases	MSE Valid.
Entire Bounded	68	Proposed law (63)	10.43	-0.75	0.23	0.10	-0.06	0.43	0.37	3	0.70				
Brick – Bounded	20	Proposed law (64)	7.92	-0.88	-0.28	0.88	-0.29	0.83	0.08	-	-				
Stone – Bounded	33	Proposed law (65)	8.94	-0.68	1.00	-0.58	-0.22	0.29	0.48	-	-				
Mixed – Bounded	15	Proposed law (66)	1.14	-0.48	4.50	-3.23	-0.96	0.63	0.14	3	1.74				

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