An overview and comparison of strategies to address numerical errors in dynamics of constrained multibody systems

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Abstract

In this work, several strategies used to solve the equations of motion for dynamics of constrained multibody systems are examined in terms of computational performance. In this process, the formulation of the equations of motion under the framework of multibody systems is revisited. For that, the Newton-Euler methodology is used due to its simplicity and easiness to develop and implement, in which the algebraic constraint equations are incorporated in the formulation by employing the Lagrange multipliers technique. The resulting equations of motion are constituted by a set of differential algebraic equations that must be solved during the dynamic analysis of the multibody systems. This study aims at investigating the performance of approaches to control the constraint violations and strategies to handle the problems of redundant constraints and singularities [1-16]. Several multibody models are utilized to assess the performance of the different strategies and scenarios. This study can be useful in offering guidelines to select a suitable strategy to deal with the resolution of the equations of motion in multibody dynamics.

The formulation of the equations of motion utilized in the present study follows closely the Nikraveshs work [8], where absolute coordinates are considered to characterize the multibody systems configuration. Thus, the set of typically nonlinear holonomic constraints $\mathbf{\Phi}$ that represent the kinematic structure of a constrained multibody system can be written in the form,

$$\mathbf{\Phi} \equiv \mathbf{\Phi}(\mathbf{q}, t) = 0 \tag{1}$$

where $\mathbf{q}$ denotes the system vector of coordinates and t represents the time variable. Differentiating Eq. (1) twice with respect to time results in the constraint equations at the velocity and acceleration levels as

$$\dot{\mathbf{\Phi}} \equiv \mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} = -\mathbf{\Phi}_t = \boldsymbol{\nu} \tag{2}$$

$$\ddot{\mathbf{\Phi}} \equiv \mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} = -\left(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}}\right)_{\mathbf{q}}\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}} - \mathbf{\Phi}_{tt} = \boldsymbol{\gamma} \tag{3}$$

where $\mathbf{\Phi}_{\mathbf{q}}$ represents the Jacobian matrix of the constraints that contains the partial derivatives of the constraint equations with respect to the coordinates, $\dot{\mathbf{q}}$ denotes the system vector of velocities, $\boldsymbol{\nu}$ is the right-hand side of velocity, $\ddot{\mathbf{q}}$ is the vector of accelerations, and $\boldsymbol{\gamma}$ is the right-hand side vector of acceleration constraints that contains the quadratic terms in velocities in the equations of motion. The equations of motion for a constrained multibody system can be written in the form [8],

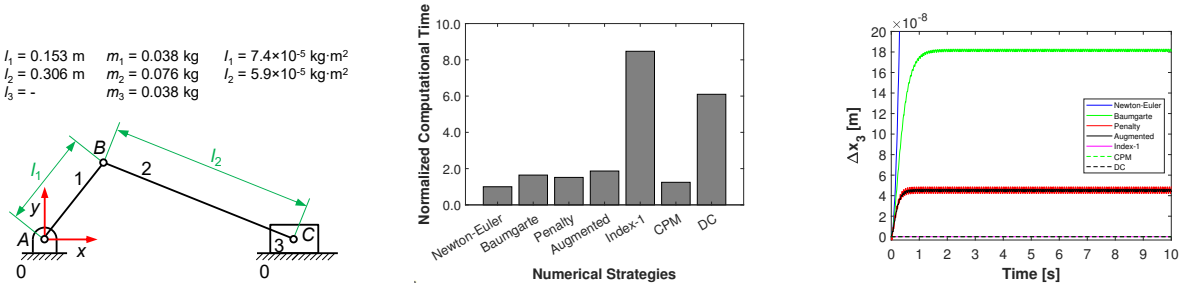
$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda} = \mathbf{g} \tag{4}$$

in which $\mathbf{M}$ denotes the system mass matrix, $\boldsymbol{\lambda}$ is the Lagrange multipliers vector relating to the joint reaction forces, and $\mathbf{g}$ is the vector of forces that contains the applied forces and moments. In forward dynamic analysis, a unique solution can be obtained when the algebraic constraint equations are joined to the differential equations of motion, and a proper set of initial conditions for positions and velocities are considered. Thus, Eq. (3) is appended to Eq. (4), resulting in a system of differential algebraic equations (DAE) as,

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_{\mathbf{q}}^{T} \\ \mathbf{\Phi}_{\mathbf{q}} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{Bmatrix} \tag{5}$$

Eq. (5) can be solved for $\ddot{\mathbf{q}}$ and $\boldsymbol{\lambda}$, utilizing any numerical technique suitable for handling systems of linear equations. Subsequently, the new positions and velocities at the next time step can be obtained using any suitable numerical integrator. This procedure is repeated until the final analysis time is reached. The slider-crank linkage of unequal rods, depicted in Fig. 1a, is chosen as a demonstrative application example. From the analysis of the plots in Figs. 1b and 1c, it can be observed that the Newton-Euler

approach with no stabilization is more efficient in terms of computational effort, but is exhibits a worse response in terms of accuracy, as it is visible in the blue line of Fig. 1c that grows indefinitely with time. This fact is clearly related to the cumulative constraint violations. In turn, the remaining methods, the Baumgarte technique, the penalty method, the augmented Lagrangian formulation, index-1 augmented Lagrangian method, coordinate partitioning method (CPM) and direct correction (DC) approach, are effective in what concerns the violation of constraints. However, with the index-1 augmented Lagrangian method and direct correction (DC) approach, the computational cost is significantly increased (see Fig. 1b). In terms of the accuracy of the results produced, namely when comparing the differences with the analytical solution (see Fig. 1c), the lowest differences found are for the coordinate partitioning method with an average difference for the simulated time of -7.0×10^{-13} m, then direct correction (DC) approach with 2.3×10^{-17} m and, finally, the index-1 augmented Lagrangian method with 2.1×10^{-17} m.



(a) Slider-crank mechanism. (b) Computational cost. (c) Computational accuracy.

Figure 1: Assessment of the strategies for control of constraint violations.

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References

[1] Baumgarte, J.: Stabilization of constraints and integrals of motion in dynamical systems. Computer methods in applied mechanics and engineering, 1(1), 1–16, 1972.

[2] Bayo, E.; Ledesma, R.: Augmented Lagrangian and mass-orthogonal projection methods for constrained multibody dynamics. Nonlinear dynamics, 9(1), 113–130, 1996.

[3] De Jalon, J. G.; Bayo, E.: Kinematic and dynamic simulation of multibody systems: the real-time challenge. Springer Science & Business Media, 2012.

[4] Ider, S. K.; Amirouche, F. M.: Coordinate reduction in the dynamics of constrained multibody systems-a new approach. Journal of Applied Mechanics, 55(4), 899–904, 1988.

[5] Kim, S. S.; Vanderploeg, M. J.: QR decomposition for state space representation of constrained mechanical dynamic systems. Journal of Mechanisms, Transmissions, and Automation in Design, 108(2), 183–188, 1986.

[6] Mani, N. K.; Haug, E. J.; Atkinson, K. E.: Application of Singular Value Decomposition for Analysis of Mechanical System Dynamics. ASME. J. Mech., Trans., and Automation, 107(1), 82–87, 1985.

[7] Marques, F.; Souto, A. P.; Flores, P.: On the constraints violation in forward dynamics of multibody systems. Multibody System Dynamics, 39(4), 385–419, 2017.

[8] Nikravesh, P. E.: Computer-aided analysis of mechanical systems. Prentice-Hall, Inc., 1988.

[9] Pennestrì, E.; Valentini, P.: Coordinate reduction strategies in multibody dynamics: A review. In Proceedings of the Conference on Multibody System Dynamics, 2007.

[10] Pennestrì, E.; Vita, L.; Falco, D.: An investigation of the influence of pseudoinverse matrix calculations on multibody dynamics simulations by means of the Udwadia-Kalaba formulation. Journal of Aerospace Engineering, 22:365–372, 2008.

[11] Udwadia, F. E.; Phohomsiri, P.: Explicit equations of motion for constrained mechanical systems with singular mass matrices and applications to multi-body dynamics. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 462(2071), 2097–2117, 2006.

[12] Udwadia, F.E.; Kalaba, R.E.: Analytical Dynamics a New Approach. Cambridge University Press, Cambridge, 1996.

[13] Walton, W. C.; Steeves, E. C.: A new matrix theorem and its application for establishing independent coordinates for complex dynamical systems with constraints (Vol. 326). National Aeronautics and Space Administration, 1969.

[14] Wang, J. T.; Huston, R. L.: Computational methods in constrained multibody dynamics: Matrix formalisms. Computers & structures, 29(2), 331–338, 1988.

[15] Wang, J. T.; Huston, R. L.: A comparison of analysis methods of redundant multibody systems. Mechanics research communications, 16(3), 175–182, 1989.

[16] Wehage, R. A.; Haug, E. J.: Generalized coordinate partitioning for dimension reduction in analysis of constrained dynamic systems.