

Full length article

Extending the Finite Area Method for enhanced simulation of deformable membranes and its application to extrusion blow moulding

Wagner de Campos Galuppo^{a,*}, Pedro Santana^b, Francisco Alves^b, João Miguel Nóbrega^a

^a Institute for Polymers and Composites, University of Minho, Campus Azurém, Guimarães, 4800-058, Braga, Portugal

^b Logoplaste Innovation Lab, Estrada da Malveira, 900, Cascais, 2750-834, Lisbon, Portugal

ARTICLE INFO

Keywords:

Finite strain theory
Membrane formulation
Hyperelasticity
Finite area method
Extrusion blow moulding

ABSTRACT

This paper investigates the mechanics of the Extrusion Blow Moulding (EBM) process, focusing on the clamp and inflation phases. The simulation framework employed in this study has been developed using OpenFOAM[®] and extends the available Finite Area Method with a vertex-centred approach. This cutting-edge developments integrated a corotational formulation with a quasi-static approximation. The computational code and algorithm are verified and validated through comparisons between analytical solutions, experimental measurements and simulation results, ensuring the accuracy and reliability of the proposed approach. The implemented code is verified by checking fundamental principles of hyperelastic materials behaviour, demonstrated through equibiaxial deformation of spherical balloons. The use of neo-Hookean and Mooney–Rivlin material models provides crucial insights into material responses under different inflation conditions. The studies begin employing a conceptual mould within the EBM process that allows examining the effects of mould clamping on parison deformation. Then the study extends to analysing a real bottle manufactured via EBM, employing Stereolithography (STL) digitization and thickness mapping to quantify the thickness distribution. The simulation results closely replicates experimental observations, capturing the inherent non-linearity of these process stages, including large deformations and displacements, hyperelasticity and parison-mould contacts. This interdisciplinary research enhances understanding of the EBM process phases and analogous processes, providing valuable insights for plastic manufacturing. It improves the design process for meeting specific requirements and offers guidelines to avoid potential failures. The findings underscore the importance of such studies in advancing sustainable and efficient plastic manufacturing practices, with applications ranging from inflatable structures to EBM processes.

1. Introduction

Efforts are currently underway to enhance industrial process efficiency, optimizing material usage, and minimizing waste generation through recycling and direct improved resources' utilization, such as material, and indirect, such as energy and transportation [1–3]. These endeavours are expected to yield significant environmental, social and economic benefits, particularly in a society grappling with the challenged of reducing plastic consumption [4,5], which has been growing exponentially since 1950 as presented in Fig. 1.

Computational numerical simulations have seen increasing application and utilization across diverse industries and fields, providing valuable insights to guide processes design, products development, and performance improvements. Mathematical modelling and numerical

simulations are also instruments in predicting potential failures or performance issues prior to the manufacture of the real final product or device [8–16].

Among the manufacturing processes in the packaging industry, the blow moulding stands out for its versatility and usage. It is a manufacturing process that produces hollow plastic parts by inflating a tubular shape until it conforms to a mould, creating a geometry closely aligned with the desired final shape [17–19]. There are three main different techniques for the blow moulding process, namely, Extrusion Blow Moulding (EBM) [20]; Injection blow moulding [21]; Injection stretch blow moulding [22]. EBM stands as the largest of the three, in the use of total produced plastic material, followed by stretch blow moulding and injection blow moulding [23].

* Corresponding author.

E-mail addresses: wagnergaluppo@gmail.com (W.d.C. Galuppo), Pedro.Santana@logoplaste.com (P. Santana), francisco.alves@logoplaste.com (F. Alves), mnobrega@dep.uminho.pt (J.M. Nóbrega).

URLs: <https://www.linkedin.com/in/galuppo> (W.d.C. Galuppo), <https://www.linkedin.com/in/pmfsantana/> (P. Santana), <https://www.linkedin.com/in/francisco-alves-6a75562/> (F. Alves), <https://crheo.org/miguel-nobrega/> (J.M. Nóbrega).

<https://doi.org/10.1016/j.tws.2024.112184>

Received 20 March 2024; Received in revised form 31 May 2024; Accepted 1 July 2024

Available online 4 July 2024

0263-8231/© 2024 The Author(s). Published by Elsevier Ltd. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

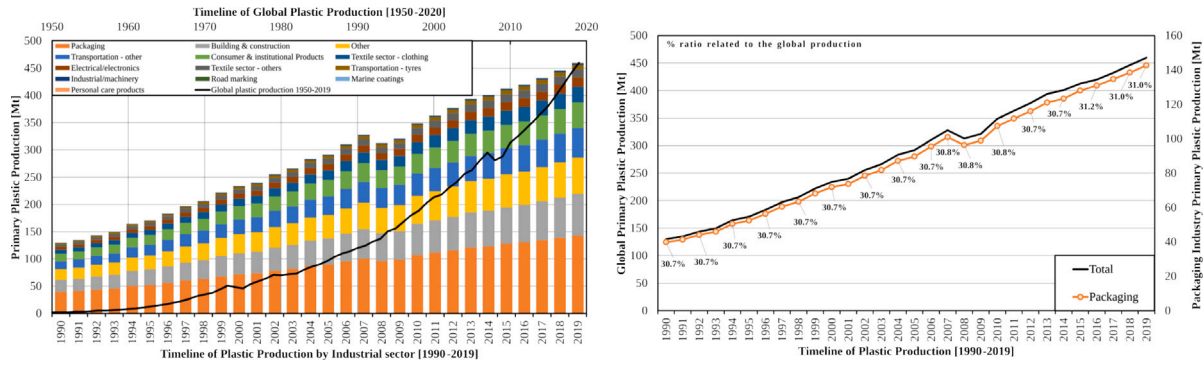


Fig. 1. Left-hand side figure is the global primary plastics production from 1950 to 2019 and by industrial sector from 1990 to 2019. Right-hand side figure is the timeline between 1990 and 2019 of all primary plastic production and packaging industry [6,7].

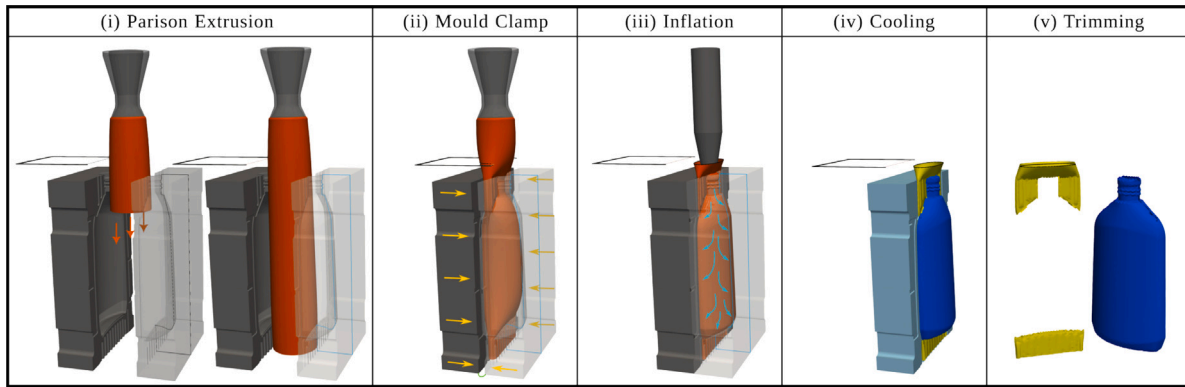


Fig. 2. Schematic illustration of the main EBM process phases.

The EBM is a cyclic process that encompasses distinct phases that are depicted in Fig. 2 and can be described as follows: [21,23,24].

1. **Parison Extrusion (Phase I):** The process cycle starts by melting the plastic material and pushing it through an extrusion set formed by a mandrel and die, which shapes it into a parison, i.e., a tube-like structure made up of the molten plastic.
2. **Mould Clamping (Phase II):** The parison is subsequently clamped into a mould. This phase ensures that the parison conforms to the desired alignment with the mould geometry. It often involves leaving a single opening, typically at the top, to facilitate the next phase procedures. It is common to cut the parison with a hot wire during this processing phase, and the moulds move with the clamped parison to another place to continue with the next phase, while the parison is continuously extruded.
3. **Inflation (Phase III):** During this phase, compressed air is blown into the inner cavity of the parison through the opening left on the previous phase. This action results in a significant increase in pressure on the inner surface of the parison. The pressure difference between the inner and outer surfaces creates a resultant force that deforms the parison and progressively expands to conform to the cavity in which it is confined, thus shaping it to the desired final shape.
4. **Cooling (Phase IV):** Once the parison makes contact with the mould walls, the cooling process begins, mainly driven by the lower mould temperature. The mould temperature is controlled via internal water circuits to extract the heat, which is transferred from the hot parison to the mould by conduction. This cooling process objective is to, as quickly as possible, solidify the molten plastic, and it is at this phase that the final shape and properties are effectively set.
5. **Trimming and Ejection (Phase V):** After reaching the appropriate stiffness, the solidified part is then ejected from the moulds.

The excessive plastic, called flash, are trimmed away. The end of this phase marks the conclusion of the EBM process cycle.

Phases I to IV are of outstanding importance in shaping the final properties of the manufactured part. Therefore, these phases hold exceptional significance when considering numerical modelling for the process simulation. From those, this work covers Phases II and III, of the envisaged developments that aim to simulate all the relevant EBM process phases.

Due to the similarity of the two processes, the state-of-the-art presented hereafter, covers not only work on the mould clamping and inflation phases of the EBM process, but also developments related to the forming phase of the thermoforming process. Furthermore, it is worth noting that these processes share similarities, not only in their simulation aspects, but also with regard to materials used for processing, as highlighted by prior studies [25–27].

The pioneering work found was conducted by Khayat et al. [28], which primarily focused on modelling the forming phase in the thermoforming and EBM processes. A simplified assumption was made to solve the fluid (air) behaviour within a cavity, considering it as a three-dimensional potential flow driven by the displacement of an isotropic homogeneous membrane. This membrane was discretized to represent the parison of arbitrary shape, with confined air between parison moulds released through strategically positioned vents. The mechanical behaviour of the membrane considered a hyperelastic Mooney–Rivlin material model. The study covered an investigation into the inflation of a square-shaped container and explored custom positioning strategies for the air vents. Furthermore, the back pressure effects on the membrane's trajectories were rigorously addressed. When considering scenarios where the pressure inside the membrane remains constant or follows a time-dependent preset function, their model accounted for the resulting pressure variations across both time and location on the membrane's surface.

Followed by the previous study, the investigation conducted by Debbaut et al. [29] involved the comparison of experimental results with numerical predictions. This research focused on an industrial blow-moulded part featuring a complex geometrical shape of a bottle with a handle. The governing equations describing the motion of the body were represented using a Lagrangian approach, moreover they employed a membrane-based modelling approach. Simulations were carried out under isothermal conditions, utilizing the viscoelastic Kaye–Bernstein–Kearsley–Zapas (K-BKZ) constitutive model. While the study reported minor discrepancies between experimental data and numerical predictions at specific locations, these deviations were attributed to factors such as the initial parison shape and thickness, sagging effects, temperature influence, and potential slippage.

In the study by Nam et al. [30], simulations of the thermoforming process were conducted both by utilizing a three-dimensional solid element analysis and also a membrane approximation. The investigation focused on the free inflation of a rectangular sheet, employing a hyperelastic material model. They performed two distinct analyses, which exhibited consistent behaviour for sheets with lower thicknesses, but deviated as the sheet thicknesses were increased. To measure the proximity of results, the dimensionless parameter $I^* = (W/H)$ was introduced, where W represents the sheet width and H denotes its thickness. The findings of the study revealed that non-isothermal analysis yielded numerical results more closely aligned with the experimental data, where at the extremities of the sheet is noticed the largest differences with the highest difference of 17%, but the overall profile is similar for the parts thermoformed without plug assistance. Additionally, accounting for the initial temperature distribution across the sheet emerged as a crucial factor for achieving more reliable simulations of the thermoforming process. It was noticed that when opting for the membrane approximation over the shell, a closer alignment between results of these two approaches were verified, particularly for cases where the parameter I^* exceeded 100.

Marckmann et al. [31], reported the development of a computational code for simulating the EBM and thermoforming of thermoplastic hollow parts. Their approach considered the principle of Virtual Work associated with a hybrid (Eulerian–Lagrangian) method, accounting for inertia effects. The parison material was assumed isothermal, nonlinear hyperelastic Mooney–Rivlin constitutive model, and with the membrane approach. The interaction between the parison and the mould was modelled as perfectly sticky. Moreover, the study implemented adaptive mesh refinement techniques, significantly enhancing performance, particularly for capturing fine details near the corners. The research presented results from four simulation cases that illustrated the benefits achieved with the improvements made. To validate the developed tools, numerical results were rigorously compared with both experimental data and numerical data obtained in existing literature. The accuracy of the simulations was notably commendable for both blow-moulding and thermoforming processes. Identified areas for potential improvement included refining constitutive models, particularly by considering the influence of temperature.

There are several applications where thin structures with membrane formulations and membrane shell are used in real and simulations [32–42]. The work of Danielson [32] on explicit methods using lumped-mass for simulating solids contact problems presented an interesting formulation of lumped-mass explicit analysis, where inertial, contact and internal forces are used to describe the mechanical behaviour of thin structures like shell membranes. This work associated with the content published in the work of Cardiff [43] enlightened the developments of this research group for these approaches, where the parison is approximated to a thin structure as a shell membrane like, and deformed using the predominant action forces, like weight, pressure, etc., instead of using just internal, inertial and contact forces as was done in Danielson [32].

In this study, an innovative computational framework is developed to numerically simulate the clamp and inflation phases of the EBM

process. The work performed, allows for evaluating the effectiveness of employing a lumped-mass explicit method that incorporates a novel artificially damped velocity scheme, as a computational framework for simulating the deformation of finite strain membrane structures. The approach utilizes a novel corotational formulation with a quasi-static approximation. Additionally, the Finite Area Method for discretizing the overall fields available in the OpenFOAM® computational library finite area class, which is face-centred [44], is modified to a vertex-centred arrangement, as it is more suitable for the adopted quasi-static approximation. This new arrangement utilizes non-overlapping cells centred at the vertices of the finite area mesh [43] to evaluate mass, force, and displacement fields. The material behaviour is modelled using neo-Hookean and Mooney–Rivlin hyperelastic constitutive models, assuming of isothermal and adiabatic conditions.

This paper is organized as follows: Section 2 introduces the theoretical background, covering the material description and the mathematical model, as well as the details of the implemented numerical algorithm. It also focuses on the discretization of membranes within the context of finite strain theory, alongside the constitutive models and contact condition. Section 3 presents the evaluation examples, beginning with a simplified geometrical configuration that facilitates the analytical validation, and two additional complex cases studied both experimental and numerically. In Section 4, the results obtained on the simulations of those evaluation cases are analysed, being the numerical results compared with the analytical and experimental data, for validation purposes. The paper concludes in Section 5 that synthesizes the findings and implications of this research.

2. Theoretical formulation

This work considers the matter at a macroscopic length scale level to its analysis as stated by the continuum mechanics field. Furthermore, this document adheres to the nomenclature used in continuum mechanics [45,46], using capital letters and small letters to represent material and spatial descriptions, respectively. It is pertinent to note that, in certain contexts for convenience, the subscript zero ($_0$) is used to designate a material description, following the same approach of the following books [45,46]. A particle denoted as X is an element of a body B , expressed as $X \in B$. A body B itself is defined by a set of such particles, i.e., $B = \{^1X, ^2X, ^3X, \dots, ^nX\}$, where each particle is distributed in space and time. The spatial region occupied by the body B at any given time defines its configuration, denoted as κ . This body, at time $t = 0$, occupies its initial configuration, frequently referred to as Base or undeformed configuration κ_0 , in which a particle X occupies position $\mathbf{X}$ in the Euclidean space, referred to a reference frame of right-handed system in material coordinates. When the continuum experience external actions, e.g. external forces, it responds by changing its geometric shape into a new configuration state at time t , designated by κ_t . The particles' distribution for the body at this state is designated by current configuration, or deformed configuration. At this time, the particle X changes its original position $\mathbf{X}$ to a new position $\mathbf{x}$ in the deformed configuration, as illustrated in Fig. 3. The mathematical description for these configurations of a deformable body can follow two approaches: the well known Material/Lagrangian description and the Spacial/Eulerian description. The present study follows the Material description formulation associated with the finite strain theory, due to the intrinsic geometrical nonlinearities for simulating the EBM mould clamping and inflation phases, where large displacements, large rotations and large strains, and material nonlinearities are considered with respect to EBM processing materials and their inherent condition during these phases.

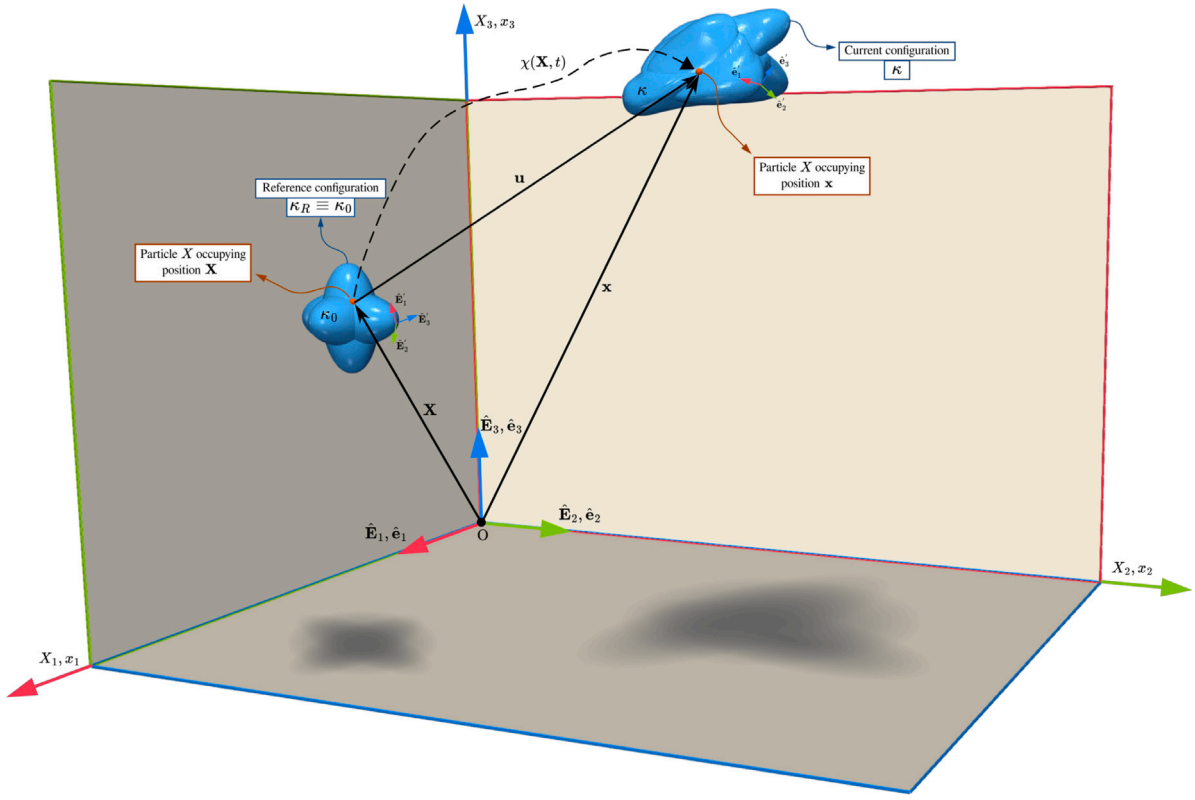


Fig. 3. Configurations for two instants of a body by a Lagrangian description of motion.

2.1. Material description

The motion of a body describes how each point moves from its position in the reference configuration κ_R to its current position. This reference can be the initial body configuration, which is usually an unformed/stress-free state referred as $\kappa_R \equiv \kappa_0$. On the other hand, it can also be related to any known intermediate state of its configuration. This gives the rise to two distinguishable approaches to formulate the discrete equations, the Updated Lagrangian approach, and the Total Lagrangian approach. The former updates the reference κ_R to a previous known state, and the latter uses the original configuration at time $t = 0$ as a reference configuration $\kappa_R \equiv \kappa_0$. The Total Lagrangian was the chosen option to be used for discretizing the governing equations. Thus, the current configuration is related to the reference body in its initial configuration, as described by:

$$\mathbf{x} = \chi(\mathbf{X}, t) \quad , \quad \mathbf{X} = \chi(\mathbf{X}, 0), \quad (1)$$

and a generic variable φ of the body in a configuration is given by:

$$\varphi = \varphi(\mathbf{x}(\mathbf{X}, t), t) = \varphi(\mathbf{X}, t), \quad (2)$$

where χ is the mapping function/deformation mapping function for the body from its reference configuration to a given instant current configuration at time t . This function depends on time and a referential body configuration, for instance at time $t = 0$, i.e. $\kappa_0(B) = \chi(\mathbf{X}, 0) = \mathbf{X}$. Thus, the motion is the mapping between the reference and the current configurations. The displacement is a vector $\mathbf{u}$ formed by connecting the particle positions from its reference configuration $\mathbf{X}$ to their current configuration $\mathbf{x}$, as follows:

$$\mathbf{u} = \mathbf{x} - \mathbf{X} \quad , \quad \mathbf{u}(\mathbf{X}, t) = \mathbf{x}(\mathbf{X}, t) - \mathbf{X}. \quad (3)$$

When all the deformed configuration particles are connected to their reference configuration the displacement field is found and the relative displacements sensed by the continuum is its deformation, which is the key field for determining stresses, strengths, fatigues,

and more. The continuum mechanics fundamental field that relates the body from its reference and deformed configurations to strains is called the deformation gradient $\mathbf{F}$, and is used to remove rigid body translation and rotation from deformations, the principal key for the mathematical modelling covered in the next chapter. The deformation gradient is a tensor related to two-point material from reference to current configuration, thus it has its basis in both configurations, and is better visualized in index notation as follows:

$$\mathbf{F}(\mathbf{X}, t) = F_{i,J} \hat{\mathbf{e}}_i \hat{\mathbf{E}}_J \quad , \quad F_{i,J} = \frac{\partial x_i}{\partial X_J} = \begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{bmatrix}. \quad (4)$$

where $\hat{\mathbf{e}}_i$ and $\hat{\mathbf{E}}_J$ are the unit orthonormal base vectors, presented in Fig. 3, for the spatial coordinates (Current Configuration) and material coordinates (Reference Configuration) directions, respectively. Furthermore, a local coordinate system using $\hat{\mathbf{e}}'_i$ and $\hat{\mathbf{E}}'_J$, presented in Fig. 3 and also as unit orthonormal base vectors, follows the same material and spacial orientations as the previous Global Coordinate system used as the basis of the deformation gradient tensor. There is a linear transformation between these local and global coordinate system that transforms one into the other and vice versa with the inverse of the transformation matrix.

By isolating $\mathbf{x}$ from Eq. (3) and replacing in Eq. (4), the deformation gradient can be written based on displacements as follows:

$$\mathbf{F} = \frac{\partial}{\partial \mathbf{X}}(\mathbf{X} + \mathbf{u}) = \frac{\partial \mathbf{X}}{\partial \mathbf{X}} + \frac{\partial \mathbf{u}}{\partial \mathbf{X}} = \mathbf{I} + \frac{\partial \mathbf{u}}{\partial \mathbf{X}} = (\mathbf{I} + \nabla_0 \mathbf{u})^T. \quad (5)$$

where $\nabla_0 \mathbf{u}$ is the material displacement gradient tensor, material due to the partial derivatives being taken with respect to $\mathbf{X}$ in the material coordinates.

2.2. Mathematical model

The EBM process mould clamping and inflation phases are governed by the continuity (Eq. (6)) and linear momentum balance (Eq. (8)) equations. Considering the rapid processing times that these phases undergo, the simulations are simplified to assume isothermal condition, which is in line with previous works covering the same process [29,31], and all the elastic deformation upon the body due to work is perfectly stored as elastic energy as postulated in Green elastic materials [47], which were successfully assessed in previous works [28–31].

Based on the above, the continuity conservation equation is expressed as follows:

$$\int_{\Omega_0} \rho_0 d\mathbf{X} = \int_{\Omega} \rho d\mathbf{x} \quad , \quad d\mathbf{x} = J d\mathbf{X} \quad \rightarrow \quad \int_{\Omega_0} (\rho_0 - J\rho) d\mathbf{X} = 0. \quad (6)$$

where J is the Jacobian of the motion, and it is obtained by calculating the determinant of the deformation gradient tensor $\mathbf{F}$ [45], Ω is the body volume, ρ is its specific mass, and each property taking the subscripts ($_0$) refers to the variable being evaluated at the reference configuration, the ones at current configuration does not take subscripts.

A particular situation happens for the case of polymer materials that are used under EBM processing conditions, in which the predominance of incompressibility is assumed [48], thus the Jacobian is unitary, $J = 1$. Also, many polymers largely deform without changing its volume [48] under this processing conditions typical to EBM, consequently mass and volume are preserved. These conditions correspond to an isochoric deformation, in which $\rho_0 = \rho$.

The second equation that rules the mechanical behaviour for the polymer processed with the EBM process concerns the momentum balances, in which the applied loads can be grouped into surface forces and body forces. The considered predominant surface forces are the forces that arise from the pressure difference between the inner and outer parison surface $\mathbf{f}_p$, the force that takes action within the membrane plane $\mathbf{f}_m$ due to strains and its material stiffness, and the contact forces $\mathbf{f}_c$ after the body being in contact with the mould surface. The body forces taken into account are the inertial resulting force $\mathbf{f}_r$ and the weight of the body, attributed to gravitational acceleration $\mathbf{f}_g$. During the parison extrusion phase of the EBM process, the weight force vector of the parison is primarily utilized to orient the extrusion direction and aid by straightening the parison. In a quasi-static approach, the solution is reached when the sum of these forces are approximately zero, as stated in

$$\mathbf{f}_p + \mathbf{f}_m + \mathbf{f}_c + \mathbf{f}_g = \mathbf{f}_r \Rightarrow \mathbf{f}_r \approx 0. \quad (7)$$

These forces are succinctly described via the linear momentum balance equation that obeys the following expression:

$$\frac{\partial}{\partial t} \int_{\Omega} \rho \mathbf{v} d\Omega = \oint_{\Gamma} \hat{\mathbf{n}} \cdot \boldsymbol{\sigma}^T d\Gamma + \int_{\Omega} \rho \mathbf{b} d\Omega, \quad (8)$$

where the left-hand term accounts for inertial forces, the first term in the right-hand respects to surface forces, while the second is related to body forces. Furthermore, $\mathbf{v}$ is the velocity vector, calculated as the time derivative of the displacements $\mathbf{v} = \partial \mathbf{u} / \partial t$, $\hat{\mathbf{n}}$ is the body surface area unity normal vector, $\boldsymbol{\sigma}$ is the Cauchy/True stress tensor, Γ is the body closed surface and this first term from the right-hand side of Eq. (8) is obtained from the divergence theorem, and expressed as a closed integral, and, finally, $\mathbf{b}$ is the body force vector per unit mass. Mainly, two approaches are possible for the solution of the previous equations, by following an implicit method or an explicit method. This work uses the latter by a modified version of the leap-frog algorithm [49] that does the integration of the Newton's second law of motion, associated with a lumped mass method.

The default algorithm known as kick-drift-kick [49], which is a version of Leap-frog algorithm, also known as velocity Verlet algorithm, is presented in Algorithm 1 [49].

Algorithm 1 leap-frog algorithm

(Step 1) Calculate the resulting force vector for each vertex at time t_i using Eq. (7).

(Step 2) Calculate acceleration of each vertex based on the configuration at time t_i

$$\mathbf{a}_i = \mathbf{f}(\mathbf{x}_i) = \frac{1}{m} \sum \mathbf{f}_{r,i} \quad (9)$$

(Step 3) Calculate velocity at time $t_{i+1/2}$

$$\mathbf{v}_{i+1/2} = \mathbf{v}_i + \mathbf{a}_i \frac{\Delta t}{2} \quad (10)$$

(Step 4) Calculate the new body position vector at time $t = i + 1$

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \mathbf{v}_{i+1/2} \Delta t \quad (11)$$

(Step 5) Update position vector to this new body configuration and calculate new acceleration vector at time $t = i + 1$

$$\mathbf{a}_{i+1} = \mathbf{f}(\mathbf{x}_{i+1}) = \frac{1}{m} \sum \mathbf{f}_{i+1} \quad (12)$$

(Step 6) Calculate velocity at time $t = i + 1$

$$\mathbf{v}_{i+1} = \mathbf{v}_{i+1/2} + \mathbf{a}_{i+1} \frac{\Delta t}{2} \quad (13)$$

(Step 7) Loop back to the calculation of Step 3

In this work, a modified version of the presented algorithm is considered. The modification is related to the calculation of velocity field corresponding to time step t_{i+1} performed in Eq. (13). The new calculation is formulated in such a way that an artificial damping factor is inserted to the dynamic response of the explicit algorithm. Since a quasi-static approach is being considered, the intermediate solutions, which include a non-physical damping to increase stability, do not affect the final result, for which a zero net force is achieved. In fact, the solution is considered converged when the force reaches a value lower than a predefined tolerance. Thus, the modification considers a linear variation between the velocity in the time interval t_i and t_{i+1} , as follows:

$$\Delta \mathbf{x} = \int_{t_i}^{t_{i+1}} \mathbf{v}(t) dt, \quad \text{Assuming a linear variation for: } \mathbf{v}(t) \quad (14)$$

$$= \alpha(t) \mathbf{v}_{i+1} + (1 - \alpha(t)) \mathbf{v}_i,$$

where,

$$\alpha(t) = \frac{t - t_i}{t_{i+1} - t_i} = \frac{t - t_i}{\Delta t}. \quad (15)$$

Consequently, $\alpha(t)$ varies linearly between 0 and 1, while t varies between t_i and t_{i+1} . Substituting them in Eq. (14):

$$\Delta \mathbf{x} = \int_{t_i}^{t_{i+1}} \frac{t - t_i}{\Delta t} \mathbf{v}_{i+1} + \frac{t_{i+1} - t}{\Delta t} \mathbf{v}_i dt, \quad (16)$$

which results in:

$$\mathbf{v}_{i+1} = \frac{2\Delta \mathbf{x}}{\Delta t} - \mathbf{v}_i. \quad (17)$$

With this expression, numerical simulations presented instabilities, thus an artificially damped coefficient, that goes in the range of $0 \leq \zeta < 1$ has been added to the first term of the right-hand side of Eq. (17) as follows:

$$\mathbf{v}_{i+1} = \zeta \frac{2\Delta \mathbf{x}}{\Delta t} - \mathbf{v}_i. \quad (18)$$

Consequently, the kick-drift-kick Leap-frog algorithm is achieved when $\zeta = 1$. This results in a default periodic behaviour, which occurs because of the conservation of the system's energy and the transformation between the different forms of energy involved, namely kinetic and potential. By including a damping coefficient (ζ) between 0 and 1 in the calculation, the oscillatory behaviour of the undamped system is

dissipated, allowing the forces to reach an equilibrium state. This makes the quasi-static solution possible when $\sum \mathbf{f} \approx \mathbf{0}$.

In the default undamped algorithm, the system oscillates, showing the dynamics of a periodic steady state. However, it can quickly diverge when applied to undamped or stiff linear systems due to its inherent numerical instability under certain conditions. The addition of the damping coefficient is one strategy used to mitigate instabilities and reduce the oscillations. It dissipates the energy and errors, promoting stability [50,51]. As the system approaches a quasi-static state, the convergence criterion for equilibrium is met when the maximum resulting force is less than a defined threshold value. At this point, the configuration is saved, and the external loads are updated to the next step values. This iterative process continues, ensuring that force balance is achieved for each step.

The quasi-static approximation improves calculation stability by ensuring that the system approaches equilibrium at each calculation. This avoids residual oscillations and associated errors that are more prone to occur in dynamic processes. The implemented calculation for the proposed algorithm is schematically presented in Algorithm 2 for the mould clamping phase, and in Algorithm 3 for the inflation phase.

Algorithm 2 Mould clamping phase

```

Load parison and moulds meshes, fields, boundary conditions and
simulation setup.
Initialize incremental displacements list to displace the moulds
parts towards closing.
Initialize list of free vertices which are free to move.
while Moulds not touching each other do
    Displace the moulds to the new position.
    if mould hit any free vertices from the parison then
        Update the hit vertices list.
        Displace the vertices to the touching point at the moulds
        surface.
    end if
    Calculate for this new configuration all the relevant fields,
    stresses, strains, thicknesses etc.
    while maximum resultant force ( $f_r$ )  $\leq f_{convergence}$  do
        Iterate as Algorithm 1, but taking Eq. (18) in place of
        Eq. (13).
        Displace free vertices to the new configuration and calculate
        the new resulting force field.
        if displaced vertices of parison hit the moulds then
            Remove vertices' IDs from the free vertices to move list.
            Add vertices' IDs to the touched vertices list.
            Displace touched vertices to the touched location in the
            mould surface.
        end if
    end while
    Save converged configuration and updated fields.
end while
end

```

2.3. Membrane formulation

The implemented membrane formulation uses the OpenFOAM®'s finite area library classes [44] for the parison's discretization, which is a locally two-dimensional surface, placed in the three-dimensional Euclidean space as a curved surface made up of flat triangle as finite area cells, i.e., each cell has coplanar centroid and vertices. Differently from the originals developments as having fluid phases in the surroundings of interfacial finite area surface [44], here these fluid phases neighbouring the finite area mesh, for instance the blown air during the inflation, are not simulated, and a boundary conditions is

Algorithm 3 Inflation phase

```

Load closed mould and clamped parison meshes, fields, boundary
conditions and simulation setup.
Create a list containing all the values for the incremental blown
pressures.
for each pressure in the blown pressures list do
    Update pressure value for each finite area cell
    Calculate the resulting force for all free vertices to move.
    while maximum resultant force ( $f_r$ )  $\leq f_{convergence}$  do
        Run Algorithm 1, but taking Eq. (18) in place of Eq. (13).
        Displace free vertices to the new positions and calculate the
        new resulting force field.
        if displaced vertices of parison hit the mould then
            Remove vertices' IDs from the free vertices to move list.
            Add vertices' IDs to the touched vertices list.
            Displace touched vertices to the touched location in the
            mould surface.
        if free vertices list is empty then
            Print "MOTIONLESS CONDITION REACHED!"
            Save latest configuration and updated fields.
        end
    end if
    end while
    Save latest configuration and updated fields.
end for
end

```

applied for that instead. This was the basis for extending the further developments required to simulate the mould clamping and inflation phases from the EBM process.

The Fig. 4 shows the mesh of a typical parison, generated by the meshing software gmsh [52], which uses triangular face area cells to discretize the parison surface. The created mesh is then converted to OpenFOAM® mesh format before using it as a finite area mesh. Fig. 4 also illustrates a typical vertex (black dots) stencil in the centre of its surrounding finite area cells (translucent purple triangular faces), notice that more vertices are illustrated because the neighbouring finite area cells for the stencil vertex contains its own three vertices each. The implemented discretization resorts to a vertex-centred explicit method for the governing equations, namely continuity (6) and linear momentum balance (8) equations, where these finite area mesh cells' vertices are displaced linearly towards the resulting force direction obtained via Eq. (7), specifically in the Step 4 in Algorithm 1 with Eq. (11). The finite area cells' edges are the orange lines, and edges' centroids are the smaller purple points. The cells are modelled as constant strain triangles (CST) in a corotational formulation.

To create a corotational formulation, a local coordinate system is constructed and updated as the simulation progresses and the membrane deforms. This is achieved using the coordinate system class available in OpenFOAM® and creating the own local reference frame for each finite area cell.

The local coordinate system for each cell is placed at its centroid, as indicated by the yellow points in Fig. 4. The local x -axis (red arrows), denoted as $\hat{\mathbf{x}}_i$, is a unit vector that points from the cell centroid towards the first vertex index among the three vertices that make up the cell's vertices list. The local z -axis (blue arrows), denoted as $\hat{\mathbf{z}}_i$, is the unit normal vector of the face, oriented from the higher pressure side to the lower pressure side. Essentially, it points from the internal region towards the external region of the parison surface. The local y -axis (green arrows) is determined by taking the cross product of the $\hat{\mathbf{z}}_i$ and $\hat{\mathbf{x}}_i$ vectors, resulting in $\hat{\mathbf{y}}_i = \hat{\mathbf{z}}_i \times \hat{\mathbf{x}}_i$, which is also a unit vector.

The resultant local reference frame is a right-handed orthonormal basis, located on all membrane cells. Furthermore, the fields for

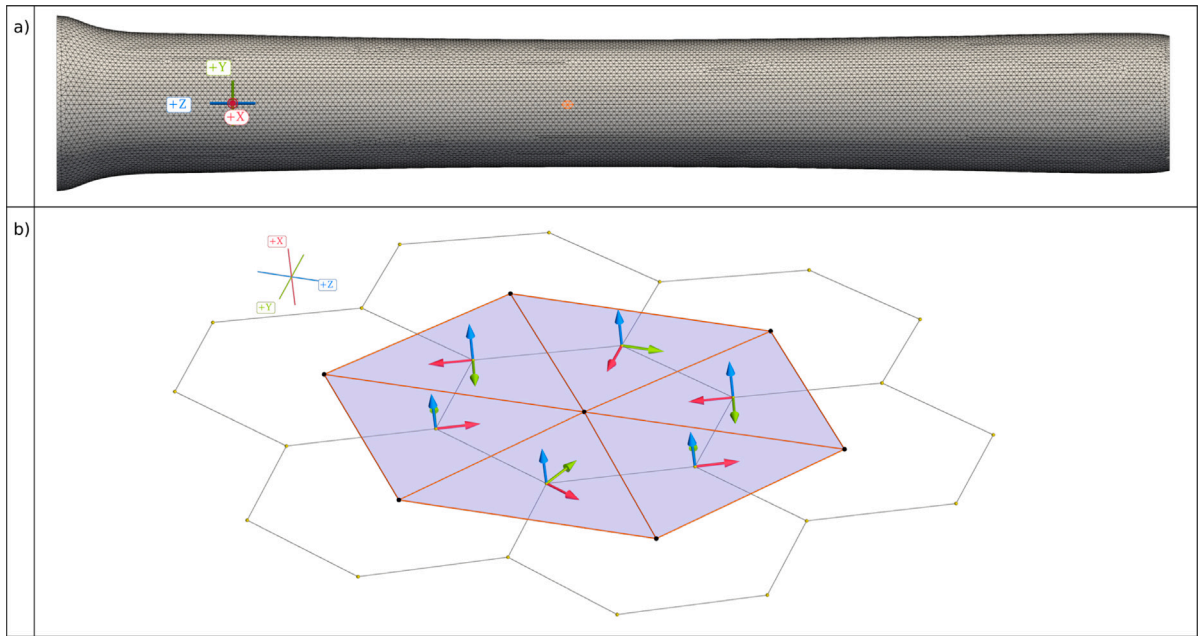


Fig. 4. Parison Mesh (a) and the stencil for the vertex-centred method (b).

stresses, strains, pressures, thicknesses, areas, and volumes are discretized as finite Area Fields, where these properties are evaluated at the surface faces' centroids.

2.4. Constitutive models

The behaviour of matter in the EBM process is governed by constitutive equations, which provides a mathematical description of material response. Given the extensive existing range of materials and substances, various models have been deployed to represent their behaviour. Among these models, two groups of materials, namely, viscoelastic and hyperelastic are commonly used for describing the inflation and EBM processes materials behaviour [29,30]. However, these models present different complexities due to the inherent nonlinearities of these materials behaviours. Viscoelastic models are computationally more expensive when compared to the hyperelastic counterparts, but they offer increased accuracy when the relaxation times from the viscoelastic spectrum are sufficiently small in relation to the overall time frame considered for evaluation and observation [53–55].

The mould clamping and inflation phases, among the five mentioned in Section 1, are performed within a short time span compared to entire time to perform all these EBM processing phases. Considering the relaxation time of the materials under these processing conditions of mould clamping and inflation phases, it is reasonable to assume their behaviour to resemble that of a hyperelastic materials [56]. For constitutive modelling, the Saint Venant–Kirchhoff, neo-Hookean, and Mooney–Rivlin models are considered, progressively, these model increases the complexity and ability to capture the materials nonlinearities.

Hyperelastic materials are characterized by having a load path work independence, i.e., the work done is independent of the deformation path [45–48,57]. Furthermore, the potential function for the stresses in these models postulates an existing Helmholtz free-energy continuous function ψ , per unit reference volume. This function is referred to as the strain–energy function and is a scalar-valued function of a tensor, which can be calculated based on different strain tensors, e.g. $\mathbf{F}$ the deformation gradient tensor, $\mathbf{C} = \mathbf{F}^T \mathbf{F}$ the right Cauchy–Green deformation tensor, or $\mathbf{E} = (1/2)(\mathbf{F}^T \mathbf{F} - \mathbf{I}) = (1/2)(\mathbf{C} - \mathbf{I})$ the Green–Lagrangian strain tensor [45,46,58]. Another commonly used deformation tensor used to obtain the potential function ψ is the left Cauchy–Green deformation

tensor, which has a spatial orientation and is obtained as $\mathbf{b} = \mathbf{F} \mathbf{F}^T$. Thus, a stress rises from the derivation of the stored energy potential functions, as follows.

$$\mathbf{S} = 2 \frac{\partial \psi(\mathbf{C})}{\partial \mathbf{C}} = \frac{\partial w(\mathbf{E})}{\partial \mathbf{E}}, \quad (19)$$

where $\mathbf{S}$ is the Second Piola–Kirchhoff stress tensor [45,46,58]. There are different ways to measure stress tensors, and these different stress tensors can be converted into one type or another via the deformation gradient tensor, where the first relation is reached through Nanson's formula [45,46,58], that relates areas in the reference and deformed configurations, and then the first relation is between Cauchy stress $\boldsymbol{\sigma}$ and nominal stress $\mathbf{P}$ tensors, as:

$$\mathbf{P} = J \boldsymbol{\sigma} \cdot \mathbf{F}^{-T}. \quad (20)$$

The Saint Venant–Kirchhoff is the simplest hyperelastic model where large deformations are due to rotations and involves small strains. It models the material behaviour analogous to the way that the linear elastic materials are modelled in the infinitesimal strain theory, but in the finite strain theory it is stated as in Eq. (21), where $\mathbb{C}$ is the fourth-order tensor of elastic moduli for the Kirchhoff materials [45,46,58]. A fourth-order tensor has 81 independent parameters, and due to symmetries for the stress tensor $\mathbf{S}$ and strain tensor $\mathbf{E}$, that rules the linear map, associated with the intrinsic preservation of energy in hyperelasticity leads to this fourth order tensor in being also symmetric [45,46,58], then the independent parameters are reduced into 21. Finally, by considering the isotropic behaviour, the independent parameters are reduced into 2 independent parameters, also known as material constants, as follows

$$\mathbf{S} = \mathbb{C} : \mathbf{E} = 2\mu \mathbf{E} + \lambda (tr(\mathbf{E})) \mathbf{I}, \quad (21)$$

where the material constants are the shear modulus μ and the Lamé's first parameter λ , which both are also known as Lamé's parameters and have the dimensions of pressure. These constants can be expressed in terms of the Young's modulus E , that has the dimensions of pressure, and the dimensionless Poisson's ratio ν . Notice that the scalar E and the tensor $\mathbf{E}$ represent different quantities.

The relationship between the Lamé's parameters with Young's modulus and Poisson's ratio, associated with the use of a corotational formulation, as described in Section 2.3 for plane stress conditions of

a membrane, the general constitutive relation from Eq. (21) can be simplified into the form in an orthonormal basis local coordinate system given by:

$$\mathbf{S} = \mathbb{C} : \mathbf{E} \rightarrow \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{Bmatrix} E_{11} \\ E_{22} \\ 2E_{12} \end{Bmatrix}, \quad (22)$$

where the stresses are obtained by multiplying a compliance matrix with the strains. In the particular situation of the plane stress, the deformation tensors, right Cauchy–Green $\mathbf{C}$, Green–Lagrangian $\mathbf{E}$ and left Cauchy–Green $\mathbf{b}$ can be simplified as follows:

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix}, \quad (23)$$

$$\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{21} & E_{22} & 0 \\ 0 & 0 & E_{33} \end{bmatrix}, \text{ and} \quad (24)$$

$$\mathbf{b} = \begin{bmatrix} b_{11} & b_{12} & 0 \\ b_{21} & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}. \quad (25)$$

In addition to the previous compliance matrix from Eq. (22), in order to calculate the deformation in the normal direction of the membrane plane, it is assumed that the material can deform freely in this direction, thus the term E_{33} can be calculated by:

$$E_{33} = \frac{\nu}{E} (S_{11} + S_{22}). \quad (26)$$

Eq. (26) can be further simplified assuming the referred incompressibility, which is considered in this work, as stated in Section 2.2. The finite area cells are planar, and their volumes are obtained by multiplying the area by the thickness. With the incompressibility assumption, the cells deform with an isochoric deformation and the deformation in the third direction from the local coordinate system of a cell, E_{33} , can be obtained with the following relation:

$$\Omega_0 = \Omega \Rightarrow \Gamma_N H = \Gamma_n h \Rightarrow \frac{\Gamma_N}{\Gamma_n} = \frac{h}{H}, \quad (27)$$

then

$$C_{33} = (F_{33})^2 = \left(\frac{h}{H}\right)^2 = \left(\frac{\Gamma_N}{\Gamma_n}\right)^2 \text{ and} \quad (28)$$

$$E_{33} = \frac{1}{2} \left[\left(\frac{h}{H}\right)^2 - 1 \right] = \frac{1}{2} \left[\left(\frac{\Gamma_N}{\Gamma_n}\right)^2 - 1 \right],$$

where Γ_N and Γ_n represent the normal area of the cell membrane in the reference and current configurations, respectively, and H and h represent the cell thickness in the reference and current configurations, respectively. Taking into account incompressibility, since the areas and thicknesses of the reference configuration and the area of the deformed configuration are known, the thickness of the deformed configuration can be determined by $h = (\Gamma_N H) / \Gamma_n$.

Eq. (19) can also be simplified for the case of incompressible materials, as follows:

$$\mathbf{S} = 2 \frac{\partial \psi(\mathbf{C})}{\partial \mathbf{C}} - p \mathbf{J} \mathbf{C}^{-1}, \quad (29)$$

where p denotes the cell internal pressure, an unknown function to be determined by the incompressibility condition.

The neo-Hookean material model extends the linear law of the Saint Venant–Kirchhoff model Eq. (21), valid for large deformations of large rotations and small strains [58–60]. For the neo-Hookean model the restriction of small strains is dropped, since it is the simplest model for dealing with a limited range of large strains (up to 50%) with good accuracy [61–63]. The strain–energy function ψ for these materials are given in terms of the isochoric part of $\mathbf{C}$, as follows:

$$\psi(\mathbf{C}) = c_{10} (I_{\mathbf{C}} - 3), \quad (30)$$

$$\boldsymbol{\sigma} = 2c_{10} \mathbf{b} - p \mathbf{I}, \quad (31)$$

where $\mathbf{I}$ is the second order identity tensor.

The Mooney–Rivlin material model is the most comprehensive model covered in this study. There are several other hyperelastic models such as Polynomial, Ogden, Arruda–Boyce, etc., but the more complex they become the computational costs tends to increase as well. The Mooney–Rivlin material model with two constants was chosen for the present work due to its versatile applicability and because it covers a wide range of deformation with good accuracy (up to 200%) [61–63], which was relevant to numerically model the EBM process mould clamping and inflation phases. In the same way as in the neo-Hookean material, the Mooney–Rivlin postulates its strain–energy function based on the right Cauchy–Green deformation tensor and its invariants. The strain–energy function for this material model is defined as follows:

$$\psi(\mathbf{C}) = c_{10} (I_{\mathbf{C}} - 3) + c_{01} (II_{\mathbf{C}} - 3), \quad (32)$$

where $II_{\mathbf{C}} = \text{tr}(\mathbf{C}\mathbf{C}) = \mathbf{C}:\mathbf{C}$ is the second invariant of the right Cauchy–Green deformation tensor, and $c_{10} = \mu_{10}/2$, $c_{01} = \mu_{01}/2$ are material constants empirically obtained via material characterization, furthermore the shear modulus is $\mu = \mu_{10} - \mu_{01}$ [48,64]. The stress calculated by the Mooney–Rivlin material model can also be determined and presented in terms of the Cauchy stress tensor $\boldsymbol{\sigma}$ and the left Cauchy–Green deformation tensor $\mathbf{b}$ as follows:

$$\boldsymbol{\sigma} = 2(c_{10} + c_{01}) \mathbf{b} - 2c_{01} \mathbf{b}^2 - \frac{2}{3} (c_{10} I_{\mathbf{C}} + 2c_2 II_{\mathbf{C}}) \mathbf{I}. \quad (33)$$

2.5. Contact model

There are several contact models available in the literature that can be used, including perfect sticky, penalty method, and frictional contact models, among others. Each model has its own specialities, advantages, and disadvantages, depending on the type of contact and the specific requirements of the simulation analysis [65]. The chosen contact model was the perfectly sticky model that assumes adhesion between the parison and the mould upon contact. This selection is logical because the moulds are kept at a lower temperature than the parison, and their surfaces are usually rough to aid in air escape. This roughness induces a non-slip condition between the parison and the mould upon contact. Therefore, as the moulds move to clamp the parison and the parison deforms due to its stiffness and/or the pressure forces during the inflation process, a mapping is established between the vertices of the cells and the surfaces of the moulds to capture these contact points. When a cell vertex intersects the surface of the mould, it is moved to the point of intersection. In subsequent calculation steps, it moves along with the mould surface.

3. Evaluation examples

In this section, two evaluation examples are included. Firstly, an evaluation and assessment case for the actual EBM process using a conceptual mould, which was conceived to allow the measurement of the parison at the end of the mould clamping phase. Secondly, the last evaluation corresponds to a case where a real container, manufactured by the EBM process, is studied. This involves comparing results from numerical simulation with experimentally obtained data, after the mould clamping and inflation phases.

In addition to these evaluation examples, a straightforward evaluation of an analytical study is conducted and presented in Appendix, where simulated results are compared with the analytical solution to verify the implemented formulation. This is achieved by using the classic problem of free inflation of a spherical balloon, which is schematically presented and verified in A.1 and A.2, respectively. The numerical results obtained from the implemented code, in conjunction with the

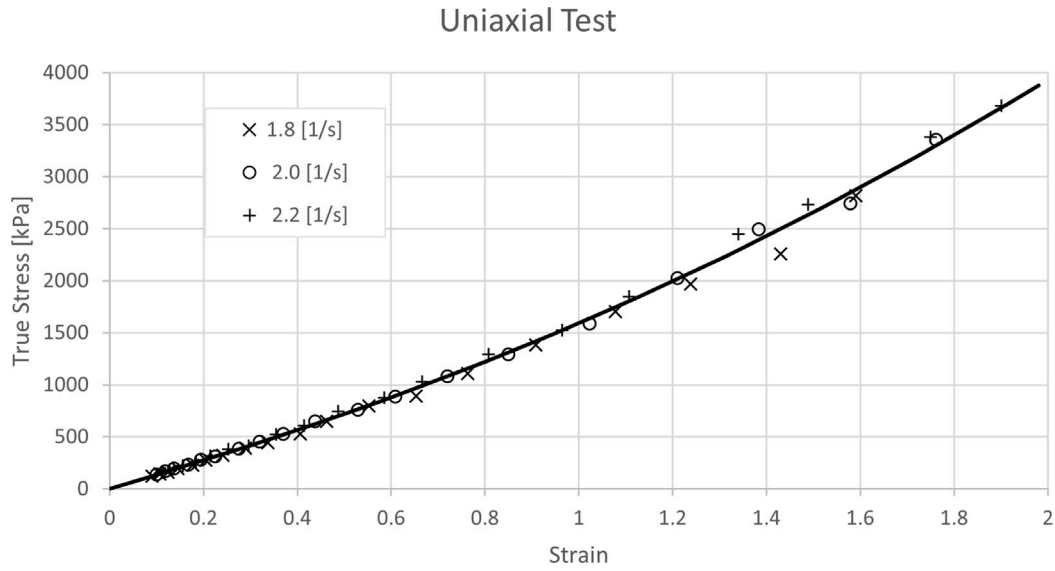


Fig. 5. Experimental data collected from uniaxial elongational tests at different rates (points) and the corresponding two-parameter Mooney–Rivlin model fitting (line).

incorporated constitutive models, successfully replicate the actual analytical behaviour observed during the free inflation of the spherical balloon.

Given the similarities to the real process, the gradual increase in internal pressure studied in Appendix was chosen as an appropriate method to simulate the inflation phase of these two EBM process evaluation examples presented in this chapter.

3.1. Material characterization

The material used in the experiments, HDPE 5502, was characterized using uniaxial tests performed at the processing temperature ($T = 190^\circ\text{C}$) with an AR-G2 rheometer with the Sentmanat Extensional Rheometer (SER) platform [66]. The tests were conducted at three different elongational rates, $\dot{\epsilon} = \{1.8, 2.0, 2.2\} [1/s]$, which represent the rates experienced by the material during the inflation stage. The obtained data was fitted to the two-parameter Mooney–Rivlin hyperelastic model, being the results are presented in Fig. 5.

The fitting that minimized the quadratic error for the experimental points resulted in the constants $c_{10} = 226.58 \text{ kPa}$ and $c_{01} = 1.01 \text{ kPa}$ for the two-parameter Mooney–Rivlin model, given by Eq. (33).

3.2. Conceptual mould

This evaluation example aims at comparing simulated and experimental results during the clamp phase of the EBM process, which is used to produce a real part. To facilitate the experimental analysis, a conceptual mould was devised to enable visualization and recording of the parison deformation during the mould clamping phase. The conceptual mould is depicted in Fig. 6, where one can observe the recording cameras location, the frontal camera on the right mould and the side camera on the left mould (highlighted in orange).

The primary objective of this evaluation is to compare the experimental dimensions of the parison's geometry at the final stage of the mould clamping phase with the corresponding numerical results.

The material used to extrude the parison was raw virgin High-Density Polyethylene (HDPE) 5502, provided by TotalEnergies Petrochemicals. This material possesses a specific mass of $\rho = 954 \text{ kg/m}^3$ and a thermal expansion coefficient of $\beta = 1.1 \times 10^{-4} \text{ K}^{-1}$ [67].

The extrusion tool setup employed a divergent configuration with a constant die gap opening, with a mass flow rate of $\dot{m} = 3.73 \text{ g/s}$, at a processing temperature of $T = 190^\circ\text{C}$. The EBM cycle lasted 20.4 s per parison cut. Moreover, the moulds' closure took place in 0.33 s which

corresponds just to 1.62% of the cycle time. The parison dimensions, at the beginning of the mould clamping phase, were measured from two view points, front and side, using the above-mentioned cameras. These measurements were essential for generating the discretized initial parison geometry to be used in the calculations.

To determine the parison's dimensions, each view point was approximated to a cylinder with a varying diameter throughout its axial length, ranging from the top to the bottom. Since at a same axial position there are two different values for the diameters, the parison cross-section was approximated by an ellipse. Accordingly, the front diameter regarding the parison's width, i.e., the ellipse's major axis, while the side diameter corresponds to the parison's depth, i.e., the ellipse's minor axis. The methodology used to measure the parison in the reference configuration is depicted in Fig. 7. This process involved the use of a grid scale (Fig. 7a) and superimposing grid images onto the parison images. This approach was adopted to minimize out-of-focus distortions that can occur in camera-captured images.

The previous measurements approximated the intermediate variation between each measurement plane from the top to the bottom as a quartic function. This function equates the measured diameter along the Z coordinate, where $Z = 0.0 \text{ mm}$ corresponds to the die gap, and its value increases along the extrusion direction, i.e. the parison length. Since the diameters differ depending on the view point, one quartic function was obtained for each view-point, as follows:

$$D(Z)_{\text{front}} = -5.048 \times 10^{-9} Z^4 + 3.762 \times 10^{-6} Z^3 - 9.461 \times 10^{-4} Z^2 + 0.138 Z + 47.170 \text{ mm, and} \quad (34)$$

$$D(Z)_{\text{side}} = -1.411 \times 10^{-8} Z^4 + 8.894 \times 10^{-6} Z^3 - 2.006 \times 10^{-3} Z^2 + 0.206 Z + 47.170 \text{ mm.} \quad (35)$$

The measured values used to obtain the above equations, which were used to create the virtual model, are presented in the graph between the two views of the virtual parisons in Fig. 7(b). The parison surface was built by using Blender [68], a 3D computer graphics software tool.

The initial thickness distribution of the real parison could not be directly measured. Therefore, it was made a simplifying assumption of a constant initial thickness, uniformly distributed throughout the entire parison length. This thickness distribution was calculated in a way that ensures mass conservation, in addition, due to the incompressibility condition that applies to extruded polymers, volume conservation is also guaranteed. Furthermore, this uniformly distributed approximation was simple but reasonable, considering that the parison was

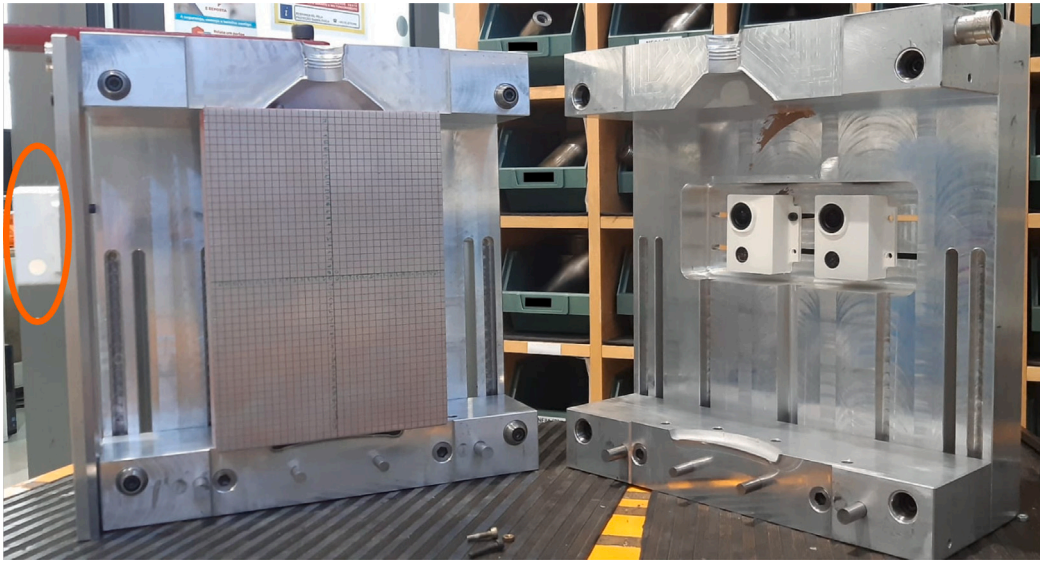


Fig. 6. Conceptual moulds used for the experimental measurements.

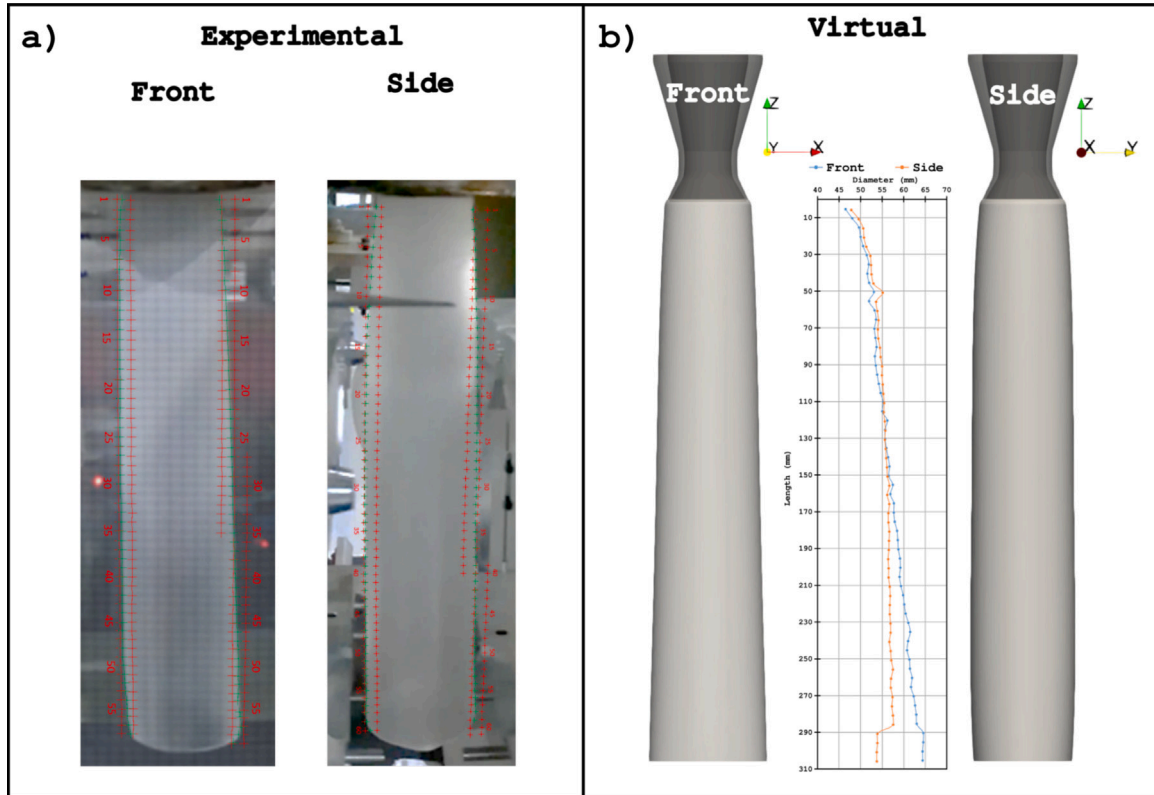


Fig. 7. (a) Real parison and (b) virtual built parison in front and side perspective views.

extruded with a constant die gap. However, as the extrusion time increases, a non-uniform thickness distribution may occur due to the viscoelastic response of the material under the extrusion conditions, as well as the effects of drawdown and sagging. Although these differences could be obtained, they were neglected in this work.

For this example, the total weight of the parison, including the flash, i.e. the clamped material, was $m = 76.8$ g. Utilizing the previously determined specific mass, the volume as $V = 79.45$ cm³. The processing temperature was approximately 153 K higher than the room temperature $T = 25$ °C, at which the previous measurements were conducted. Assuming negligible variations in the thermal expansion

coefficient of Total 5502 within this temperature range, the corrected volume calculated is $V = 81.206$ cm³ for the section of the parison below the knife, which was positioned at 54.0 mm below the extrusion die gap.

The area of the parison downstream from the knife is $A = 457.68$ cm². Dividing the total polymer volume by this area, an average uniform thickness for the entire parison of $H = 1.774$ mm is obtained.

3.3. Real bottle

For this last evaluation study, a real part was in fact produced by the EBM process. All the material and process definitions used for

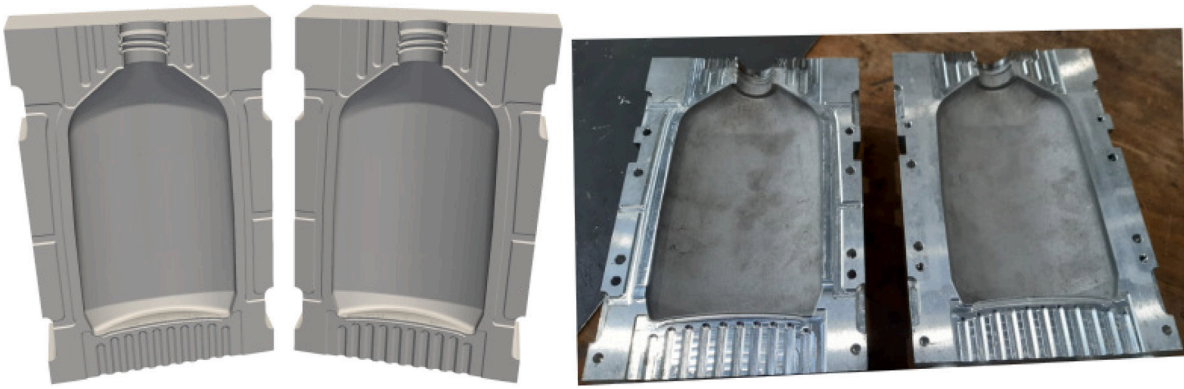


Fig. 8. Moulds used to produce the bottle.

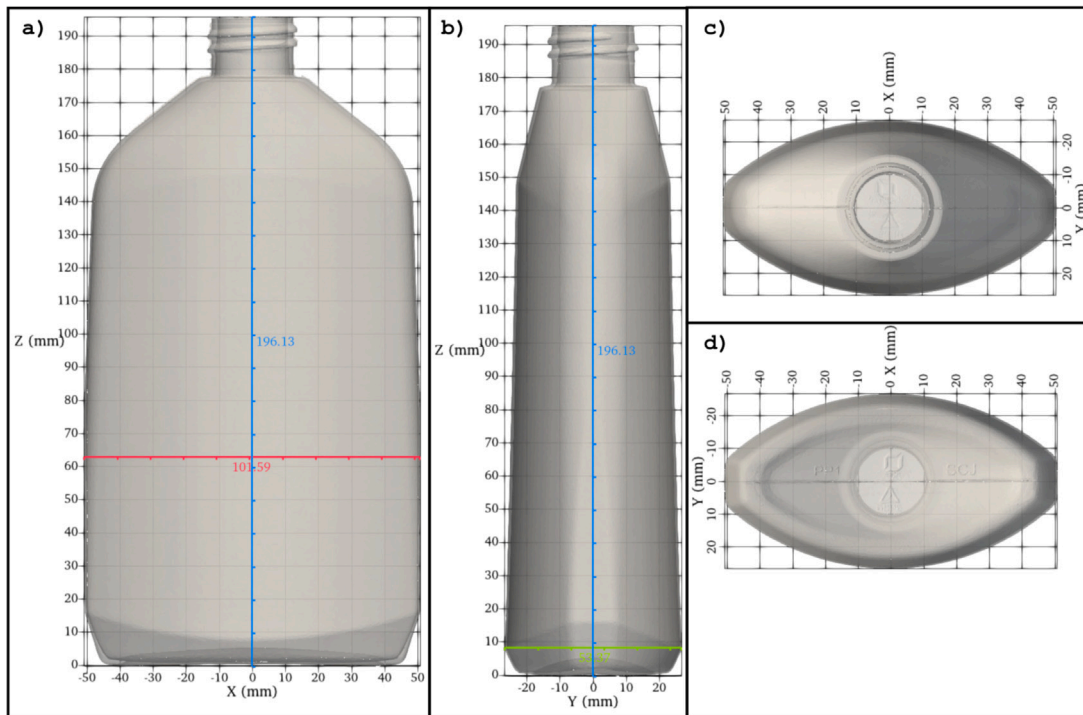


Fig. 9. Several views of the real bottle produced by the EBM process and digitalized via X-ray microtomography. (a) frontal view, (b) lateral view, (c) top view, and (d) bottom view.

manufacturing this part were the same as those covered in Section 3.2, likewise the definitions and setups for preparing and build the virtual data to perform the simulation. The difference in here regard the moulds' geometry, and the simulation of both phases, i.e., the parison clamping and inflation.

The moulds are depicted in Fig. 8, together with the computer-aided design (CAD) drawings, left and right sides, respectively. This mould was designed to produce the final part depicted in Fig. 9.

The produced bottle sample, used to compare with the numerical simulation results, was scanned by using an X-ray microtomography from the RX Solutions, model Easytom S. The scanned bottle is presented in Fig. 9 in several perspectives. Moreover, the microtomography allows obtaining the thickness distribution in the entire bottle.

Finally, the virtual setup used to simulate the mould clamping and inflation phases for this assessment evaluation example is presented in Fig. 10.

4. Results and discussions

In this section, the results of the two cases presented in Section 3 are analysed and discussed.

4.1. Conceptual mould

The experimental measurements for the parison followed the approach described in Section 3.2. The behaviour of a cylinder or a ring when subjected to transversal compression or tension, due to the material's stiffness, a reaction occurs in an axis perpendicular to the plane direction of applied forces. When under tension/traction the circle becomes ovalized with the major axis aligned with the applied forces, on the other hand, when under compression, the major axis switches to axis perpendicular to the applied forces, as verified in other studies [69]. Accordingly, the analysis in this section is undertaken, encompassing both qualitative and quantitative dimensions, where the just effects of the compression forces induced by the mould closure are responsible for the observed parison deformation.

First, examining the parison dimensions in a frontal view perspective, the experimental data, as shown on the left side of Fig. 11, revealed that the parison stretched outward in the plane of mould partition plane. This observation is consistent with expectations, as the parison is compressed in the perpendicular direction when the moulds are closed.

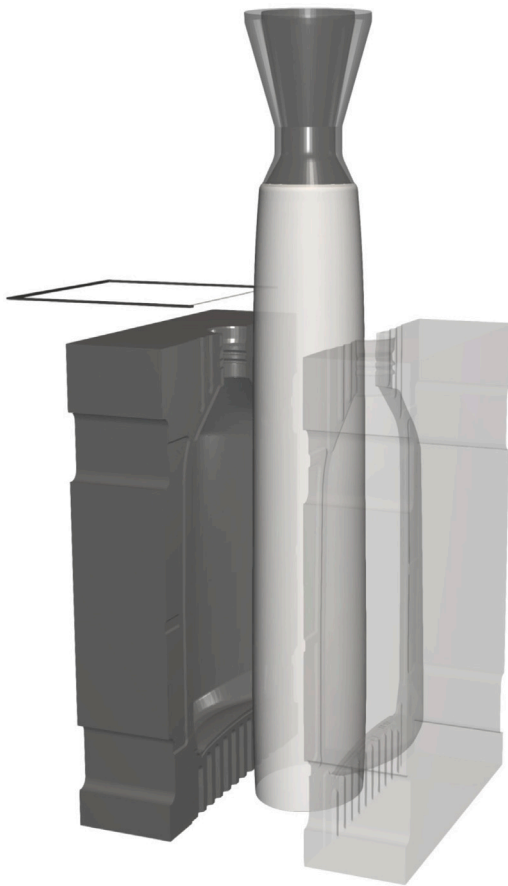


Fig. 10. Setup for the numerical simulation for the mould clamping and inflation phases in the EBM process.

Regions of the parison that are not affected by the moulds deform due to the stiffness of the material, although there is less deformation expected in those regions. Furthermore, the stiffer the material of the parison, the more noticeable the displacements are expected to be in the unaffected regions.

Based on the information provided above, it is expected that the mould clamping phase will result in an elliptical cross-section. The axis ratio, which is the ratio of the minor axis to the major axis, decreases as the mould closes. This behaviour was observed in both the experimental images and the numerical simulation results, as shown in Fig. 11.

These measurements were taken from the uppermost plane adjacent to the bottle neck, extending towards the bottom within the visible portion of the conceptual mould. The length of the position was standardized by dividing the actual length by the total length. This yielded a consistent dimensionless scale known as the Downstream Dimensionless Length, ranging from 0 at the top to 1 at the bottom. The Downstream Dimensionless Length serves as the abscissa axis for Figs. 12(a) and (b).

Comparing the numerical results for the width of the parison's major axis to the corresponding experimental measurements, in Fig. 12(a), there is a close correspondence observed along the length of the parison. Notably, the numerical profile exhibits a slightly more convex shape than the experimental one. In contrast, when viewed from the side in Fig. 12(b), the numerical profile appears slightly larger and more concave compared to the experimental measurements.

Among the variables that could affect these profiles, the most significant factor is the support pressure applied during the mould closure, which occurs simultaneously with the parison cut. These phenomena

cause an oscillating pressure evolution and a corresponding alteration in the dimensions of the parison, which are observed as incremental and decremental changes in the camera images. Thus, although the measured and numerical results are not an exact match, this evaluation yields reliable findings that closely approximate real measured data. These results establish a strong basis for further evaluation in more intricate scenarios, especially in the subsequent example, which involve comparing real produced parts with their corresponding simulations.

4.2. Real bottle

In this final evaluation example, the material distribution for a finished part is compared with the results of numerical simulations. The experimental data was digitized by reconstructing the bottle as a Stereolithography (STL) file object, and the thickness was mapped from 3759 measurement points uniformly distributed across the external surface of the bottle. Consequently, the colour map on the part surface, shown in Fig. 13(a), represents interpolated values between these measured points. This is consistent with the numerical results in which the surface distribution is interpolated from calculated points.

For a qualitative comparison, both the experimental and simulated bottles, viewed from the two principal directions, are juxtaposed in a colour gradient scale map presented in Fig. 13. The Fig. 13(a) displays the experimental data, while the simulated data is presented in Fig. 13(b). It should be noted that the geometries on the left, most side of Fig. 13, feature spare chords, which are not part of the actual bottle. These spare chords locations are visually represented in a separate bottle positioned alongside, and they are marked by black lines. Their diameter has been intentionally exaggerated to some extent, yet it varies proportionally with the actual part thickness. The black meridian line, in analogy with the geographic coordinate system (GCS), namely line ML in Fig. 13, is taken from the prime meridian and anti-meridian lines in the geometry, splitting the geometry in two pieces a west and east halves. This is the longest profile taken in the geometry. Furthermore, following the same analogy as used in GCS, another three latitudinal intersection planes were taken to evaluate the thicknesses profiles, namely lines LQ1, LQ2 and LQ3 as indicated in Fig. 13, for comparing the results. These latitudinal profiles were taken at 1/4, 2/4 and 3/4 of the bottle total height, going from the bottom to the neck.

In these results, it is evident the overall good agreement between the measured and calculated data. Notably, a thicker region is observed at the central area of the bottle. This thickness increase is attributed to the proximity between the parison and the mould in this region, which is the first zone to touch the mould at the inflation phase. This causes the thickness to decrease with the distance from this central area. As the parison deforms, it stretches and becomes thinner until touching the moulds.

A distinct thicker region is also noticeable at the partition surface, where the parison surfaces come into contact with each other. This thicker region, a result of the closing process, is commonly referred to as the "welding edge". It plays a crucial role as it provides to the part added thickness enhancing its mechanical strength in this region, which is important during the detachment and removal of excess plastic, known as flash, from the final part. Moreover, it also serves to prevent failures during service, since weld regions are a region with a decreased mechanical strength and prone to failure. Furthermore, certain areas of maximum thickness are observed at the corners of these welding edges due to the moulds' compression implied in the parison's surfaces during the mould clamping phase. These thicker regions are consistently observed in both the experimental and numerical data.

A symmetrical pattern in the thickness distribution map is observed between the primary rectangular orthonormal planes XY, XZ, and YZ in the numerical results. In contrast, the results display slight variations between the two halves, i.e., the meridian and anti-meridian thickness profiles, in the case of the experimental data, which is expected in the context of real parts, where perfect symmetry does not occur.

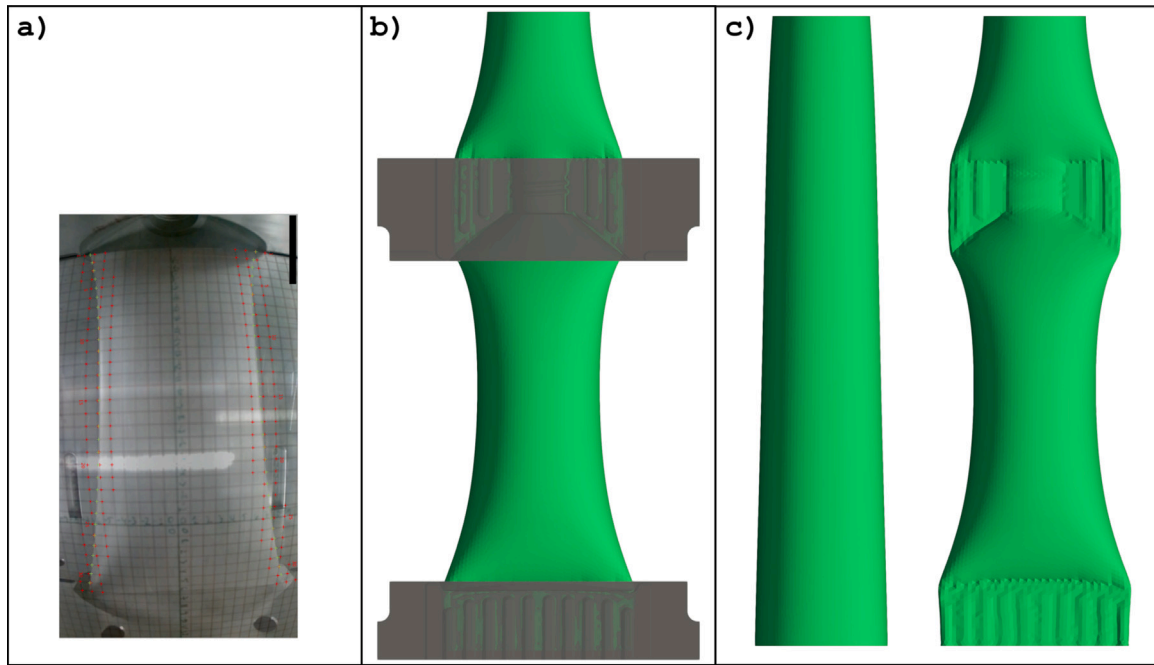


Fig. 11. (a) Experimental and (b) numerical geometry configurations at the end of mould clamping phase, in (c) is the parison before clamping at left, and after clamp at right.

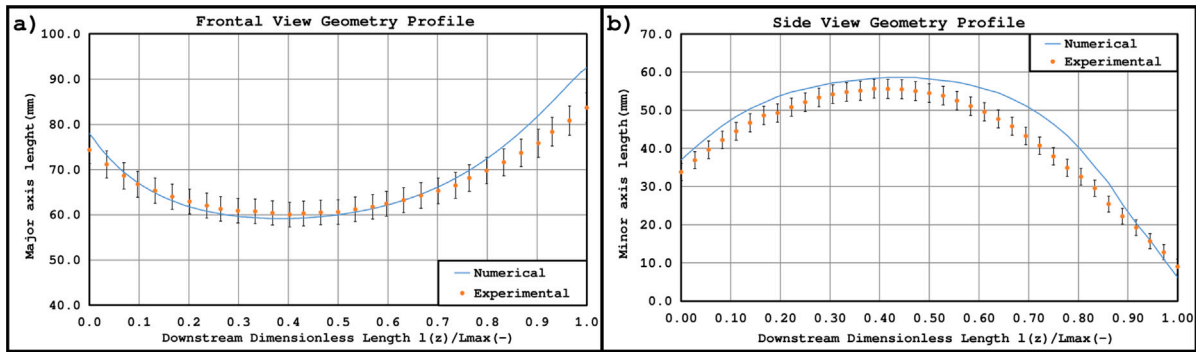


Fig. 12. Experimental and numerical profiles for the parison at the end of mould clamping phase for a frontal view at (a) and side view at (b).

When considering the profiles obtained by the latitudinal intersect planes, i.e., lines LQ1, LQ2 and LQ3, there are no significant differences in thickness distribution map between the halves, but a minor difference becomes apparent when considering the prime meridian bisect plane, line ML of the bottle's height. In this latter case, LM profile, it can be observed, primarily in the thicker region, that this thicker region is slightly wider at LQ1 profile than the LQ2 when compared to the same region, which, in turn, is slightly wider than the region at LQ3 profile. This asymmetry might be attributed to an initial non-uniform thickness distribution in the parison before the mould clamping phase.

As mentioned before, the effects of elapsed time for extrusion and own weight, can result in a non-uniform thickness distribution, despite setting a constant die gap opening during extrusion. Thus, considering this source of thickness deviation in the parison prior to the mould clamping phase, a phenomenon known as “necking” effect, results in a central region with lower thickness than the bottom region. This was not considered, thus was simplified, when discretizing the initial thickness field for the virtual parison.

The observed difference in the thickness distribution map between the halves is more prominent when considering the XZ bisect plane. This discrepancy could potentially arise from minor irregularities in the parison, such as slight tortuosity or deviations from perfect straightness in its axial central line at the end of extrusion. These characteristics

were simplified and assumed as perfectly straight in the virtual parison reconstruction. This experimental asymmetries resulted in different contact places with the mould, thus the thicker side may come into contact with the moulds earlier, resulting in less stretching when compared to the opposite side.

It is also noticed that the neck region of the bottle, as observed in the experimental measurements, exhibits a significantly greater thickness when compared to the same region in the simulated results. The discrepancy arises from the fact that the simulations do not consider the calibration performed by the blown air nozzle in the real process. This is achieved by repositioning the nozzle to a lower position and physically compressing the polymer into the mould cavity in this region of the bottle's neck. This operation aims to shape the neck in the final part, particularly with regard to the screw design and intricate details in this region, thus creating a thicker wall in this place. These phenomena were not considered in the numerical model.

For a quantitative comparison, the thickness profiles under scrutiny corresponding to the positions delineated by the black lines in Fig. 13. These profiles are classified according to the orthonormal planes on which they are situated, as well as their specific locations along the axes where these planes intersect. The thickness profiles across the four cross-sections are depicted in Fig. 14.

The graph, in Fig. 14(a) illustrates the thickness profile along for the line ML of Fig. 13, along the dimensionless perimeter on the abscissa.

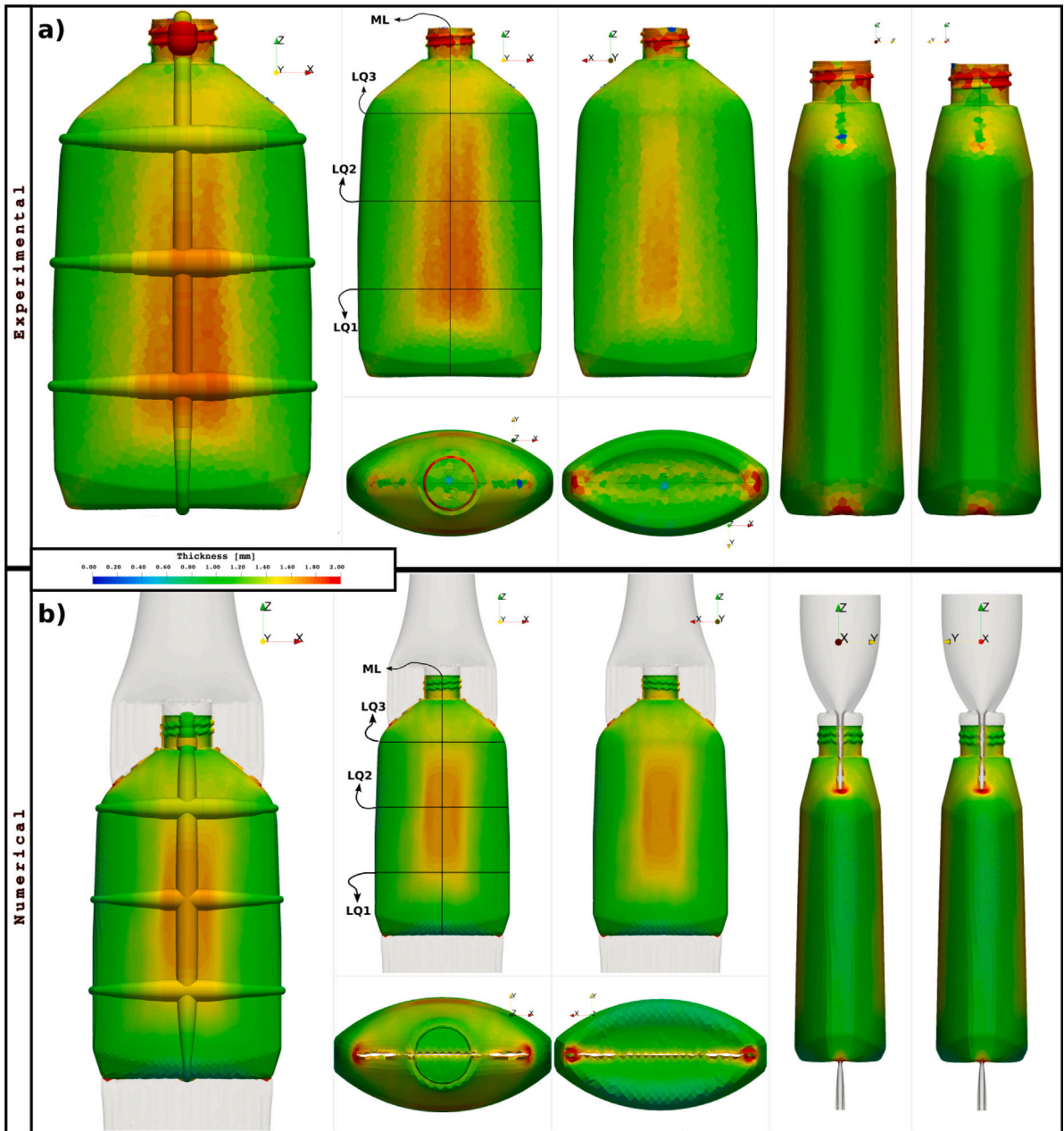


Fig. 13. All views of the experimental results at (a), and numerical results at (b), for the bottles.

This line extends from the anti-meridian (the rear side of the bottle's neck) towards the prime meridian (the front part of the bottle's neck). In Fig. 14(b), (c), and (d), the thickness profiles for lines LQ1, LQ2, and LQ3 (see Fig. 13) are depicted, respectively. In these graphs, the abscissa represents the "Profile Perimeter Angle ($^{\circ}$)", defined as the angle in the XY plane measured counter-clockwise, ranging from 0° to 360° , where 0° is aligned with the $X+$ axis from the rectangular orthonormal basis. It is notable that, in the experimental measurements, the region spanning from 0.0 to 0.5, corresponding to the bottle's rear, is thinner compared to the region between 0.5 and 1.0, representing the front part, which is consistent with the graph shown in Fig. 14(a).

In both profiles, i.e., meridional and latitudinal cross-section profiles, the thickness experiences a rapid increase around the dimensionless profile value of 0.5. Notably, the welding edge at the bottom of the bottle intersects this profile at the dimensionless perimeter of 0.5. It is worth mentioning that some measured points in this region exhibited a significant decrease in thickness. Accordingly, detailed measurement verification was conducted in this area, revealing that surface

irregularities in this region adversely affected the device's measurements. As a result, the thickness in this region was confirmed to be 1.47 ± 0.067 mm, which is in good agreement with the numerical results, and the measurements' uncertainty was provided with the device used for that. Further insights into this verification are available in Fig. 15, offering both a comprehensive and zoomed-in perspectives views for additional clarity.

The other three plots, i.e., Fig. 14(b), (c), and (d), across all profiles from 0° to 180° (the rear side of the bottle), it is evident that the measured thickness is lower than the front side of the profile, i.e., from 180° to 360° , corroborating the asymmetry observed during the qualitative analysis. The concurrence and correspondence between these profiles are in agreement between numerical simulations and measured data, with the profile from Fig. 14(d) exhibiting the most congruence.

A noteworthy observation is the presence of a plateau region characterized by constant thickness, spanning the angular range from 60° to 120° and from 240° to 300° , in the numerical simulation results. This phenomenon arises because these regions of the parison came into

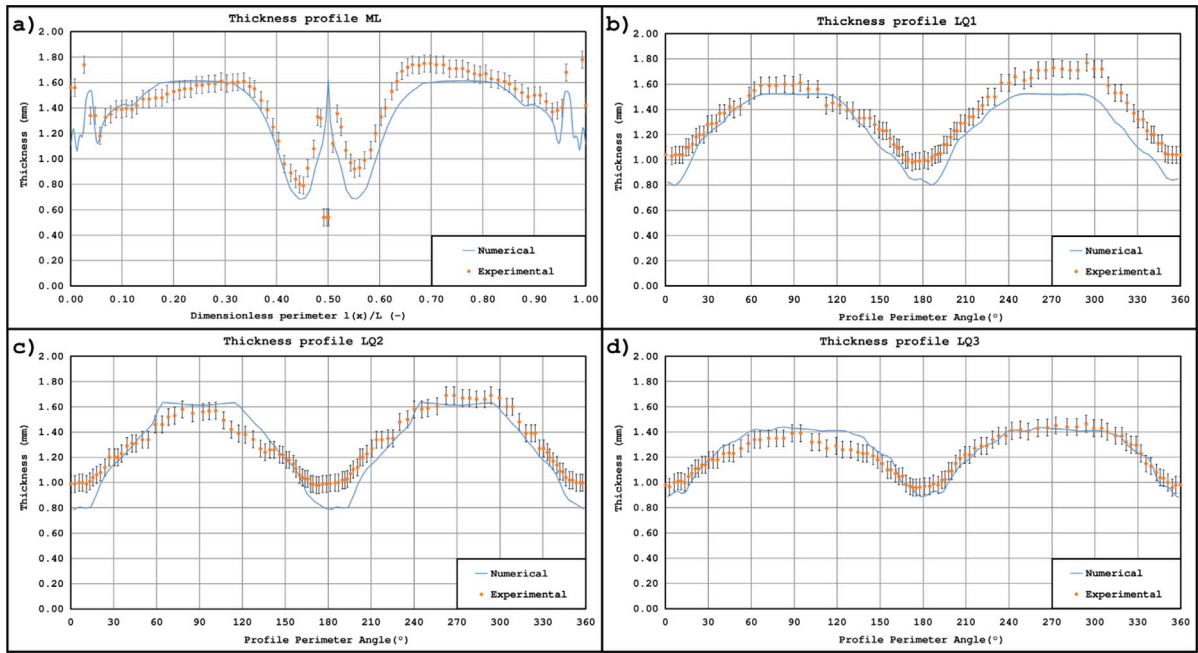


Fig. 14. Profiles for the thickness distributions in all four cross-sections.

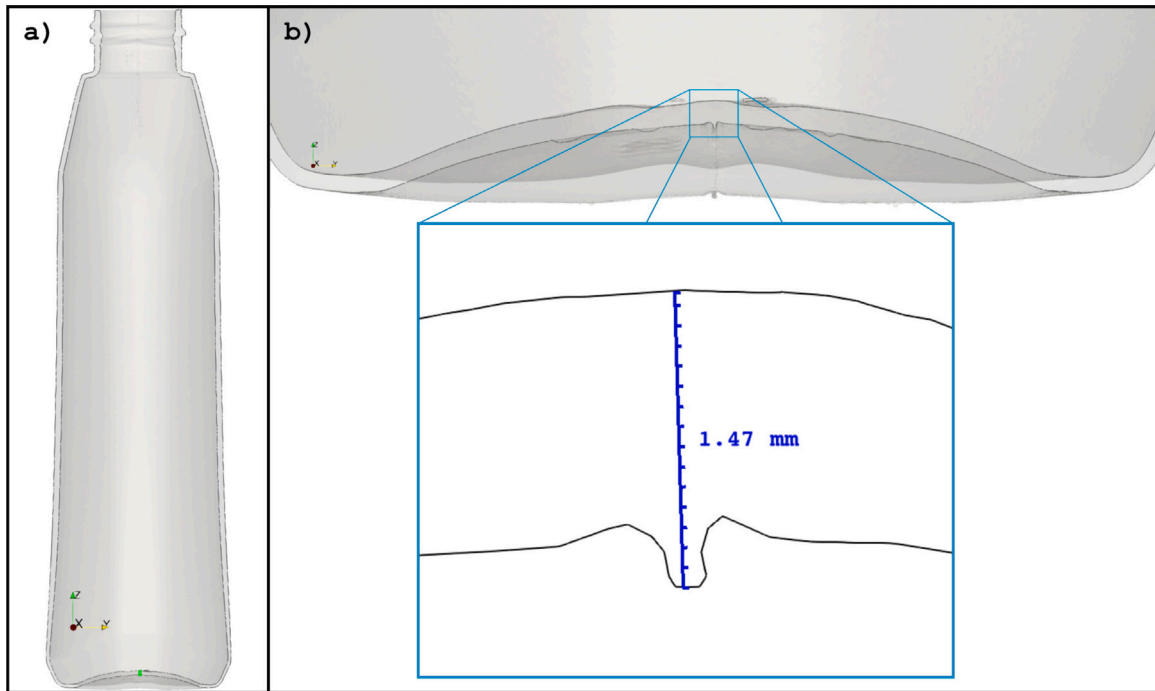


Fig. 15. Measurement verification for the thickness profile of plane YZ at the welding edge region. (a) full bottle slice and (b) zoom at the mould partition plane.

contact with the moulds during the mould clamping phase, resulting in what is commonly referred to as a “frozen zone” within the parison. In experimental processing, it can be noticed a plateau in the same region for the bottle, but it did not afford a capability to assess, capture and validate this phenomenon, nor verified by the same behaviour in their thickness profiles.

5. Conclusions

This research work covers the development and assessment of a modelling framework for the complex mechanics of the clamp and

inflation phases in the Extrusion Blow Moulding (EBM) process. This was accomplished by implementing a novel Finite Area Method in the OpenFOAM® computational library. The developed tools incorporate a membrane approach with hyperelastic materials, in a vertex centred arrangement, following a corotational formulation, and adopting a quasi-static approximation. The result is a comprehensive, innovative computational model that can simulate the deformation of membranes, which encompasses the envisaged phases of the EBM process.

The validity and reliability of the developed tools have been thoroughly confirmed through comparisons of the numerical results with analytical solutions and experimental measurements.

The developed framework has been validated through a comprehensive study of a renowned problem in the literature regarding the inflation of a spherical balloon. This study has yielded invaluable insights into the behaviour of hyperelastic materials under different inflation conditions. The obtained results confirm the proper implementation of the code.

Subsequently, the code was first evaluated in the EBM process, with a specific emphasis on the mould clamping phase. For that, a novel conceptual mould was devised and used to collect relevant experimental data that is not accessible with conventional moulds. The good correlations obtained between the numerical results and experimental data demonstrate that the developed simulation framework can accurately capture the most relevant details of the parison deformation, induced during the mould clamping phase.

Then, the final validation study involved the production of a real bottle, where the predicted final thickness distribution by the code is compared with experimental data measured using X-ray microtomography. The collected data is analysed in detail both qualitatively and quantitatively. Again, an excellent match is achieved between the experimental and numerical results, indicating that the developed code can accurately capture the most important aspects of the thickness distribution in a real part.

Based on the results obtained, it can be concluded that the implemented simulation framework is an excellent tool for guiding the design of the EBM process, tools, and parts. In a society that prioritizes sustainability and performance in a world dependent on plastic products, this research and development can assist in designing EBM processes to meet specific requirements and predict potential failures in advance. This provides valuable guidance for the continuous evolution of the industry.

CRedit authorship contribution statement

Wagner de Campos Galuppo: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Pedro Santana:** Writing – review & editing, Visualization, Validation, Resources, Project administration, Investigation, Formal analysis, Data curation, Conceptualization. **Francisco Alves:** Writing – review & editing, Visualization, Validation, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **João Miguel Nóbrega:** Writing – review & editing, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

This work was supported by National Funds through the FCT - Portuguese Foundation for Science and Technology under the PhD grant 2020.08395.BD, with the DOI (<https://doi.org/10.54499/2020.08395.BD>), and the projects UID/CTM/50025/2019 and UIDB/04436/2020. Additional funding was provided by the ANI - Agência Nacional de Inovação S. A. via EBMSimulator project (POCI-01-0247-FEDER-047166). We extend our gratitude to Professor José Luís Carvalho Martins Alves for his precious support and guidance on the understanding of the hyperelastic constitutive models behaviour, and to the computational

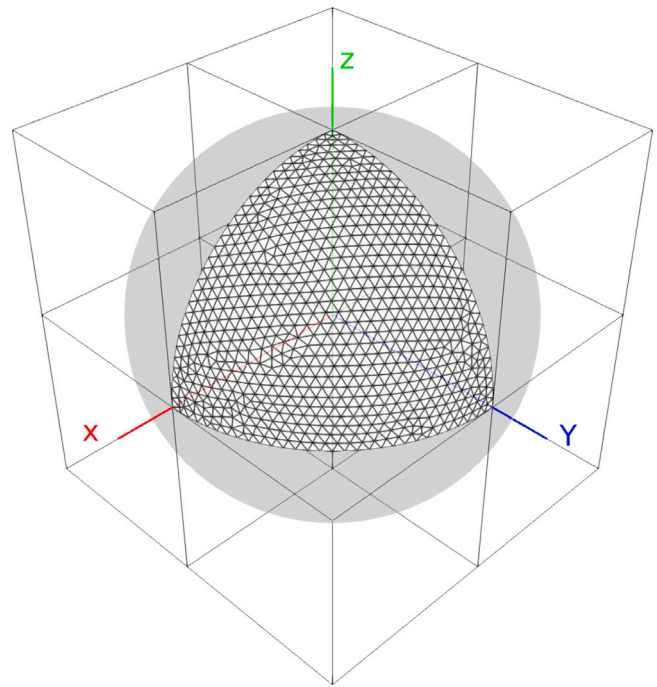


Fig. 16. Discretized spherical balloon.

resources provided by SeARCH-On2 and the Minho Advanced Computing Center (MACC).

Appendix. Spherical balloon

A.1. Problem description

Due to its similarities with the physics of the inflation phase in the EBM process, the well-known process of free inflation of a spherical balloon has been investigated with the developed calculation tools. A reference case has been defined, as stated in Holzapfel [48], by using a sphere with a constant initial dimensionless radius $R^* = R/H = 100$. One can see in Fig. 16 the discretized finite area mesh for 1/8 of an entire sphere centred at the origin and placed in the first octant of the three-dimensional Euclidean space. A symmetry boundary condition is defined for each of the Cartesian planes formed by the principal rectangular axis. The mesh has 30 cells per boundary edge and, 1378 triangular cells in total.

When positive pressure is applied to the inside surface of the spherical balloon, it deforms to reach an equilibrium state where the radius increases. This particular geometry chosen allows an analytical solution that equates equilibrium conditions between the internal pressure and the internal forces induced in the membrane due to its stiffness.

Two alternative approaches are analysed. The first involves by imposing a state of deformation on the spherical balloon, in order to calculate the induced stress field. The second approach involves gradually increasing the internal pressure and seeking the equilibrium condition for the resulting forces. The latter resembles the approach implemented numerically for simulating the mould clamping and inflation phases of EBM process.

It is well-established that every direction within the plane of the membrane for a perfect sphere corresponds to a principal direction [48]. Consequently, under the inflation pressure, the sphere undergoes an equibiaxial deformation in the two angular directions of the spherical coordinate system. This leads to the following stretch ratios: $\lambda_\theta = \lambda_\phi = \lambda$ and $\lambda_r = 1/\lambda^2$ [48], where λ_θ and λ_ϕ are the stretches in

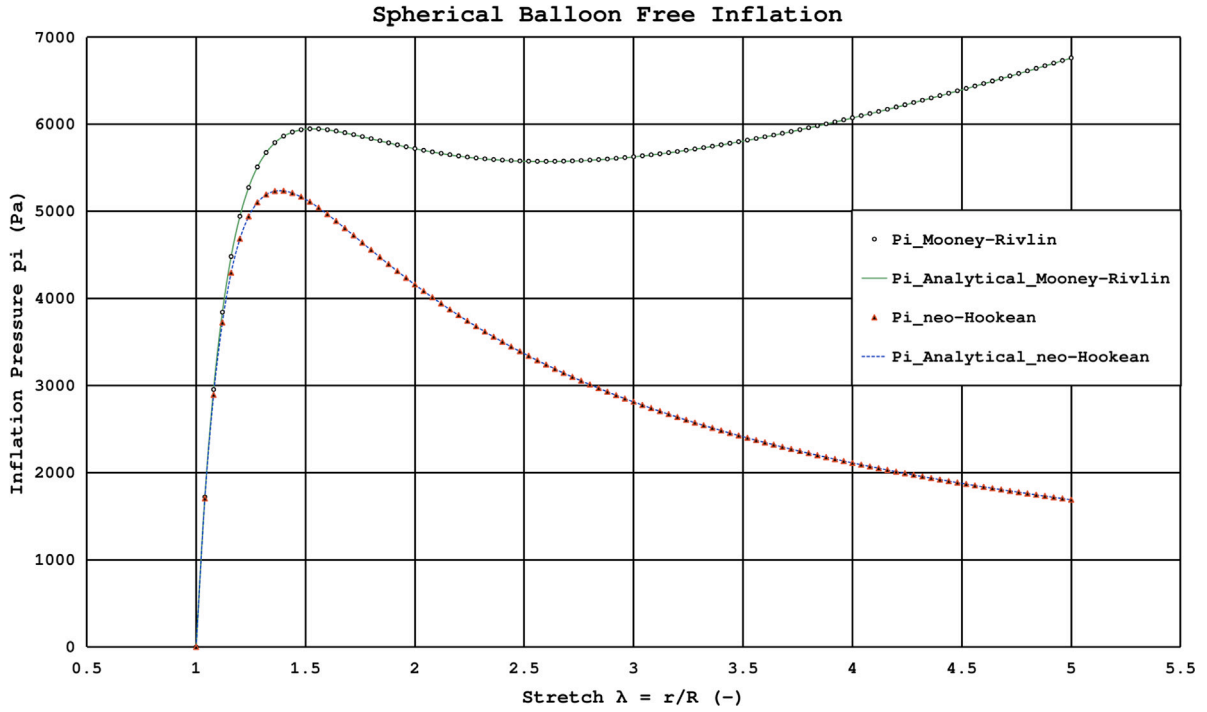


Fig. 17. Inflation pressure by stretches for the spherical balloon with the imposed deformation.

the angular basis directions and λ_r is in the radial basis direction in the spherical orthonormal coordinate system. The corresponding Cauchy stresses can be expressed in a unified form, as

$$\sigma = \sum_{p=1}^N \mu_p (\lambda^{\alpha_p} - \lambda^{-2\alpha_p}), \quad (36)$$

which encompasses both hyperelastic material models employed in this work, the neo-Hookean and Mooney–Rivlin models [48]. Here, μ_p represents the shear moduli, and N is a positive integer determining the number of terms in the strain energy function, and α_p is a dimensionless model parameter. With $N = 1$ and $\mu_1 = 2c_1$ Eq. (36) corresponds to the neo-Hookean material model [48]. For $N = 2$, $\mu_1 = 2c_{10}$ and $\mu_2 = -2c_{01}$ the evolution corresponds to the two term Mooney–Rivlin material model [48].

For the equilibrium condition, the analytical solution for internal pressure is a function of current radius r , thickness h , and stress σ , is given by:

$$p_i = 2 \left(\frac{h}{r} \right) \sigma. \quad (37)$$

The incompressibility condition implies that $r = R\lambda$ and $h = H/\lambda^2$. By substituting these into Eq. (37) and incorporating σ from Eq. (36), the following analytical expression is derived for the inflation pressure:

$$p_i = 2 \frac{H}{R} \sum_{p=1}^N \mu_p \left(\lambda^{(\alpha_p-3)} - \lambda^{(-2\alpha_p-3)} \right) \quad (38)$$

Simplifying Eq. (38) for the neo-Hookean material model yields:

$$p_i = 4 \frac{H}{R} \left[c_1 \left(\frac{1}{\lambda} - \frac{1}{\lambda^7} \right) \right] \quad (\text{neo-Hookean}), \quad (39)$$

while for the Mooney–Rivlin the relation is the following:

$$p_i = 4 \frac{H}{R} \left[c_{10} \left(\frac{1}{\lambda} - \frac{1}{\lambda^7} \right) + c_{01} \left(\lambda - \frac{1}{\lambda^7} \right) \right] \quad (\text{Mooney-Rivlin}). \quad (40)$$

A.2. Results

This analysis encompasses two types of simulations, as described in A.1. The first simulation involves an imposed state of deformation,

whereas the second entails a gradual increase in internal pressure until equilibrium among all the forces is reached.

In the first simulation, the perfect spherical geometry simplifies the analysis of the balloon's deformed state. Specifically, the current configuration of the balloon in spherical coordinates only changes its radius, denoted as $\kappa_i(r, \theta, \phi) = \kappa_0(\lambda R, \Theta, \Phi)$. As the implemented numerical code uses the rectangular coordinate system, a transformation is required to express the imposed deformation in this spherical coordinates. Utilizing the transformation identities ($x = r \sin(\theta) \cos(\phi)$, $y = r \sin(\theta) \sin(\phi)$, and $z = r \cos(\theta)$), the imposed state of deformation in rectangular coordinates is given by:

$$\begin{aligned} \kappa_i(x, y, z) &= \kappa_0(\lambda R \sin(\Theta) \cos(\Phi), \lambda R \sin(\Theta) \sin(\Phi), \lambda R \cos(\Theta)) \\ &= \kappa_0(\lambda X, \lambda Y, \lambda Z). \end{aligned} \quad (41)$$

The simulations use a constant shear modulus, $\mu = 422.5$ kPa, consistent with Holzapfel [48]. The neo-Hookean material model is defined by a constant $c_1 = 2 \mu$, while the Mooney–Rivlin model incorporates constants $c_{10} = 0.4375 \mu$ and $c_{01} = 0.0625 \mu$ following the material parameters provided in Anand [64].

The evolution of inflation pressure concerning the imposed stretch λ is depicted in Fig. 17 for both analytical and simulation results. Simulation results were computed using Eq. (37), with the von Mises stress σ_{vm} calculated by

$$\sigma_{vm} = \sqrt{\frac{1}{2} \left[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 \right] + 3 \left(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2 \right)}. \quad (42)$$

Using the von Mises stress in Eq. (37) is an appropriate choice, obviating the need for complex transformations between coordinate systems. Moreover, it provides an effective measure of stress in the material and is widely employed in solid mechanics analysis. Furthermore, due to the chosen shape of a spherical geometry, the stress state in the spherical coordinate system aligns with the principal stresses, namely, $(\sigma_1 = \sigma_r = 0)$, $(\sigma_2 = \sigma_\theta)$, $(\sigma_3 = \sigma_\phi)$. As a result, $\sigma_{vm}(x, y, z) = \sigma_{vm}(r, \theta, \phi) = \sigma_\theta = \sigma_\phi = \sigma$ as calculated from Eq. (36).

In the context of the second type of simulation for the stated problem, the discussion is centred on the Mooney–Rivlin material model.

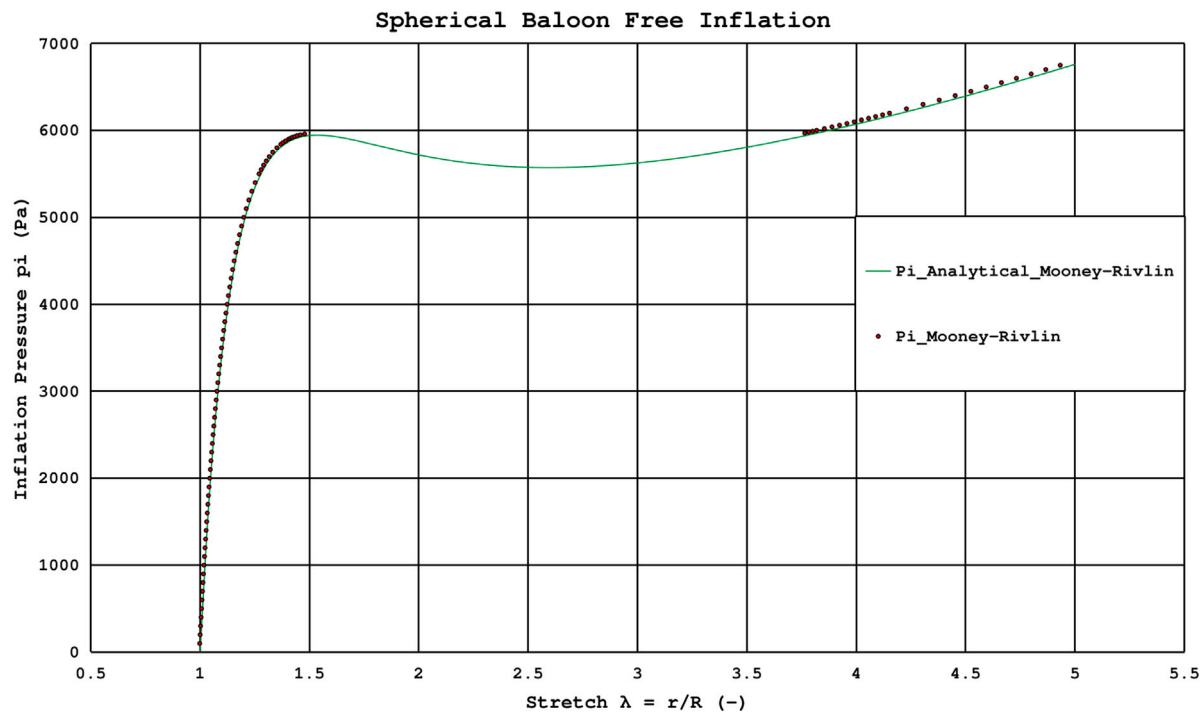


Fig. 18. Inflation pressure by stretches for the spherical balloon with the imposed internal pressure.

This choice was deliberate because this simulation method, characterized by a gradual, step-by-step increase in internal pressure, reveals a distinct behaviour. Specifically, at a stretch value of approximately $\lambda \approx 1.5$, both material models experience a local maximum pressure. Beyond this point, the material “snaps through” [48], and a new equilibrium position is only attained at $\lambda \approx 3.8$ for the Mooney–Rivlin model just, within the studied range. This phenomenon does not manifest in the case of the neo-Hookean material model. The progression of stretches necessary to reach equilibrium under applied internal pressure is illustrated in Fig. 18.

References

- [1] Plastics Europe - the facts 2020, 2021, URL <https://plasticseurope.org/knowledge-hub/plastics-the-facts-2020/>. (Accessed 25 November 2023).
- [2] Plastics Europe - the facts 2021, 2022, URL <https://plasticseurope.org/knowledge-hub/plastics-the-facts-2021/>. (Accessed 25 November 2023).
- [3] Plastics Europe - the facts 2022, 2023, URL <https://plasticseurope.org/knowledge-hub/plastics-the-facts-2022/>. (Accessed 25 November 2023).
- [4] H. Pilz, J. Schweighofer, E. Kletzer, The contribution of plastic products to resource efficiency, 2005, URL <https://www.teppfa.eu/wp-content/uploads/HSEGUAFinalreportplasticsresourceefficiencyupdateJan2005.pdf>. (Accessed 26 November 2023).
- [5] J. Rankin, European parliament votes to ban single-use plastics, *Guardian* 27 (2019).
- [6] R. Geyer, J.R. Jambeck, K.L. Law, Production, use, and fate of all plastics ever made, *Sci. Adv.* 3 (2017) e1700782, <http://dx.doi.org/10.1126/sciadv.1700782>.
- [7] Organisation for economic co-operation and development (OECD), 2023, URL https://stats.oecd.org/viewhtml.aspx?datasetcode=PLASTIC_USE_10&lang=en. (Accessed 25 November 2023).
- [8] S.S. Ajeesh, S.A. Jayachandran, Computation of shear buckling stress of thin-walled sections using constrained spline finite strip method, *Thin-Walled Struct.* (2024) 111813, <http://dx.doi.org/10.1016/j.tws.2024.111813>, URL <https://www.sciencedirect.com/science/article/pii/S0263823124002568>.
- [9] L. Li, F. Jia, L. Liu, R. Yu, Q. Zhang, L. Li, Response of aluminium honeycomb sandwich panels under combined shock and impact loading: Experimental and numerical investigations, *Thin-Walled Struct.* 193 (2023) 111256, <http://dx.doi.org/10.1016/j.tws.2023.111256>, URL <https://www.sciencedirect.com/science/article/pii/S0263823123007346>.
- [10] E. Burman, M.A. Fernández, S. Frei, F.M. Gerosa, A mechanically consistent model for fluid–structure interactions with contact including seepage, *Comput. Methods Appl. Mech. Engrg.* 392 (2022) 114637, <http://dx.doi.org/10.1016/j.cma.2022.114637>, URL <https://www.sciencedirect.com/science/article/pii/S0045782522000457>.
- [11] P.V. Rodrigues, B. Ramoa, A.V. Machado, P. Cardiff, J.M. Nóbrega, Assessing the compressive and impact behavior of plastic safety toe caps through computational modelling, *Polymers* 13 (24) (2021) <http://dx.doi.org/10.3390/polym13244332>, URL <https://www.mdpi.com/2073-4360/13/24/4332>.
- [12] F. Klein Florentin, B. Oliveira, J. Pereira, J. Correia, A. De Jesus, F. Berto, Fatigue behavior of metallic components obtained by topology optimization for additive manufacturing, *Frattura ed Integrità Strutturale* 15 (2020) 119–135, <http://dx.doi.org/10.3221/IGF-ESIS.55.09>.
- [13] S. Chahardoli, A.A. Nia, Investigation of mechanical behavior of energy absorbers in expansion and folding modes under axial quasi-static loading in both experimental and numerical methods, *Thin-Walled Struct.* 120 (2017) 319–332, <http://dx.doi.org/10.1016/j.tws.2017.09.013>, URL <https://www.sciencedirect.com/science/article/pii/S0263823117308121>.
- [14] P. Nørtoft, J. Gravesen, Isogeometric shape optimization in fluid mechanics, *Struct. Multidiscip. Optim.* 48 (2013) 909–925.
- [15] B. Schafer, Z. Li, C. Moen, Computational modeling of cold-formed steel, *Thin-Walled Struct.* 48 (10) (2010) 752–762, <http://dx.doi.org/10.1016/j.tws.2010.04.008>, URL <https://www.sciencedirect.com/science/article/pii/S0263823110000704>. Coupled Instabilities in Metal Structures CIMS 2008.
- [16] M. Assato, M.J.S. de Lemos, Turbulent flow in wavy channels simulated with nonlinear models and a new implicit formulation, *Numer. Heat Transf., Part A: Appl.* 56 (4) (2009) 301–324, <http://dx.doi.org/10.1080/10407780903244346>.
- [17] J. Goff, T. Whelan, D. DeLaney, *The Dynisco Extrusion Processors Handbook*, 2000, p. 284, Edition 2.
- [18] M. Ghafur, B. Koziey, J. Vlachopoulos, Simulation of thermoforming and blowmolding—Theory and experiments, *Rheol. Fundam. Polym. Process.* (1995) 321–383.
- [19] L.F. Ramos-De Valle, Principles of polymer processing, in: *Handbook of Polymer Synthesis, Characterization, and Processing*, Wiley Online Library, 2013, pp. 451–461.
- [20] S.L. Belcher, *Practical Extrusion Blow Molding*, vol. 54, CRC Press, 1999.
- [21] S.L. Belcher, *Practical Guide to Injection Blow Molding*, vol. 71, CRC Press, 2007.
- [22] O. Brandau, *Stretch Blow Molding*, William Andrew, 2016.
- [23] M. Kutz, *Applied Plastics Engineering Handbook: Processing and Materials*, William Andrew, 2011.
- [24] M. Thielen, *Extrusion Blow Molding*, Carl Hanser Verlag GmbH & Company KG, 2021.
- [25] J.L. Throne, *Understanding Thermoforming*, Hanser Gardner Publications Cincinnati, OH, 2008.
- [26] P.W. Klein, *Fundamentals of Plastics Thermoforming*, Springer International Publishing, 2009, <http://dx.doi.org/10.1007/978-3-031-02392-7>.
- [27] S.A. Ashter, *Thermoforming of Single and Multilayer Laminates*, William Andrew Publishing, 2014, <http://dx.doi.org/10.1016/C2012-0-02821-9>.
- [28] R.E. Khayat, P. Raducanu, A coupled boundary/finite-element approach for the three-dimensional simulation of air venting in blow molding and thermoforming, *Int. J. Numer. Methods Eng.* 43 (1) (1998) 151–174.

- [29] B. Debbaut, O. Homerin, N. Jivraj, A comparison between experiments and predictions for the blow molding of an industrial part, *Polym. Eng. Sci.* 39 (9) (1999) 1812–1822.
- [30] G.J. Nam, J.W. Lee, K.H. Ahn, Three-dimensional simulation of thermoforming process and its comparison with experiments, *Polym. Eng. Sci.* 40 (10) (2000) 2232–2240.
- [31] G. Marckmann, E. Verron, B. Peseux, Finite element analysis of blow molding and thermoforming using a dynamic explicit procedure, *Polym. Eng. Sci.* 41 (3) (2001) 426–439.
- [32] K.T. Danielson, Curved node-to-face contact schemes for higher-order finite elements in lumped-mass explicit methods, *Comput. Methods Appl. Mech. Engrg.* 395 (2022) 115056, <http://dx.doi.org/10.1016/j.cma.2022.115056>, URL <https://www.sciencedirect.com/science/article/pii/S0045782522002675>.
- [33] G.A. Holzapfel, R. Eberlein, P. Wriggers, H.W. Weizsäcker, Large strain analysis of soft biological membranes: Formulation and finite element analysis, *Comput. Methods Appl. Mech. Engrg.* 132 (1) (1996) 45–61, [http://dx.doi.org/10.1016/0045-7825\(96\)00999-1](http://dx.doi.org/10.1016/0045-7825(96)00999-1), URL <https://www.sciencedirect.com/science/article/pii/S0045782596009991>.
- [34] E. Oñate, B.-H. Kröplin, *Textile Composites and Inflatable Structures*, vol. 3, Springer Science & Business Media, 2005.
- [35] S. Yang, B. Zhao, W. Chen, The added mass of a biaxial tensioned membrane in still air, *Thin-Walled Struct.* 195 (2024) 111531, <http://dx.doi.org/10.1016/j.tws.2023.111531>, URL <https://www.sciencedirect.com/science/article/pii/S026382312301008X>.
- [36] Q. Han, C. Wu, M. Liu, H. Wu, A corotational isogeometric assumed natural strain shell element in updated lagrangian formulation for general geometric nonlinear analysis of thin-walled structures, *Thin-Walled Struct.* 194 (2024) 111311, <http://dx.doi.org/10.1016/j.tws.2023.111311>, URL <https://www.sciencedirect.com/science/article/pii/S0263823123007899>.
- [37] M. Pellicciari, S. Sirotti, A. Aloisio, A.M. Tarantino, Analytical, numerical and experimental study of the finite inflation of circular membranes, *Int. J. Mech. Sci.* 226 (2022) 107383, <http://dx.doi.org/10.1016/j.ijmecsci.2022.107383>, URL <https://www.sciencedirect.com/science/article/pii/S0020740322002909>.
- [38] R. Zorrilla, R. Rossi, R. Wüchner, E. Oñate, An embedded finite element framework for the resolution of strongly coupled fluid–structure interaction problems. Application to volumetric and membrane-like structures, *Comput. Methods Appl. Mech. Engrg.* 368 (2020) 113179, <http://dx.doi.org/10.1016/j.cma.2020.113179>, URL <https://www.sciencedirect.com/science/article/pii/S0045782520303649>.
- [39] M. Remesíková, M. Šagát, P. Novýsedlák, Discrete Lagrangian algorithm for finding geodesics on triangular meshes, *Appl. Math. Model.* 76 (2019) 396–427, <http://dx.doi.org/10.1016/j.apm.2019.06.013>, URL <https://www.sciencedirect.com/science/article/pii/S0307904X19303750>.
- [40] Y. Yang, S. Shen Qian, J. Hai Gong, G. Zhi Qiu, Design method for precise-forming of air-supported structures considering weld seams, *Thin-Walled Struct.* 145 (2019) 106397, <http://dx.doi.org/10.1016/j.tws.2019.106397>, URL <https://www.sciencedirect.com/science/article/pii/S0263823119305142>.
- [41] J. Rodriguez, G. Rio, J. Cadou, J. Troufflard, Numerical study of dynamic relaxation with kinetic damping applied to inflatable fabric structures with extensions for 3D solid element and non-linear behavior, *Thin-Walled Struct.* 49 (11) (2011) 1468–1474, <http://dx.doi.org/10.1016/j.tws.2011.07.011>, URL <https://www.sciencedirect.com/science/article/pii/S0263823111001777>.
- [42] E. Verron, G. Marckmann, An axisymmetric B-spline model for the non-linear inflation of rubberlike membranes, *Comput. Methods Appl. Mech. Engrg.* 190 (46) (2001) 6271–6289, [http://dx.doi.org/10.1016/S0045-7825\(01\)00227-4](http://dx.doi.org/10.1016/S0045-7825(01)00227-4), URL <https://www.sciencedirect.com/science/article/pii/S0045782501002274>.
- [43] P. Cardiff, I. Demirdžić, Thirty years of the finite volume method for solid mechanics, *Arch. Comput. Methods Eng.* 28 (5) (2021) 3721–3780, <http://dx.doi.org/10.1007/s11831-020-09523-0>.
- [44] Z. Tukovic, H. Jasak, Simulation of free-rising bubble with soluble surfactant using moving mesh finite volume/area method, in: *Proceedings of 6th International Conference on CFD in Oil & Gas, Metallurgical and Process Industries*, No. CFD08-072, 2008, pp. 1–11.
- [45] J.N. Reddy, *An Introduction to Continuum Mechanics*, Cambridge University Press, 2013.
- [46] T. Belytschko, W.K. Liu, B. Moran, K. Elkhodary, *Nonlinear Finite Elements for Continua and Structures*, John Wiley & Sons, 2014.
- [47] R. Ogden, *Non-linear elastic deformations*, in: *Dover Civil and Mechanical Engineering*, Dover Publications, 1997.
- [48] G. Holzapfel, *Nonlinear Solid Mechanics: A Continuum Approach for Engineering*, Wiley, 2000.
- [49] W.C. Swope, H.C. Andersen, P.H. Berens, K.R. Wilson, A computer simulation method for the calculation of equilibrium constants for the formation of physical clusters of molecules: Application to small water clusters, *J. Chem. Phys.* 76 (1) (1982) 637–649, <http://dx.doi.org/10.1063/1.442716>, arXiv:https://pubs.aip.org/aip/jcp/article-pdf/76/1/637/11237336/637_1_online.pdf.
- [50] B. Fornberg, On the instability of leap-frog and crank-nicolson approximations of a nonlinear partial differential equation, *Math. Comp.* 27 (121) (1973) 45–57.
- [51] P. Gurfil, I. Klein, Stabilizing the explicit Euler integration of stiff and undamped linear systems, *J. Guid. Control Dyn.* 30 (6) (2007) 1659–1667.
- [52] C. Geuzaine, J.-F. Remacle, *Gmsh*, 2020, URL <http://gmsh.info/>. (Accessed: 06 June 2023).
- [53] B. Koziey, M. Ghafur, J. Vlachopoulos, F. Mirza, Chapter 3 computer simulation of thermoforming, in: *Composite Sheet Forming*, in: *Composite Materials Series*, vol. 11, Elsevier, 1997, pp. 75–89, [http://dx.doi.org/10.1016/S0927-0108\(97\)80005-7](http://dx.doi.org/10.1016/S0927-0108(97)80005-7), URL <https://www.sciencedirect.com/science/article/pii/S0927010897800057>.
- [54] A. Weizel, T. Distler, R. Detsch, A. Boccaccini, L. Bräuer, F. Paulsen, H. Seitz, S. Budday, Hyperelastic parameter identification of human articular cartilage and substitute materials, *J. Mech. Behav. Biomed. Mater.* 133 (2022) 105292, <http://dx.doi.org/10.1016/j.jmbbm.2022.105292>, URL <https://www.sciencedirect.com/science/article/pii/S1751616122002053>.
- [55] W. Zhang, J. Jilberto, G. Sommer, M.S. Sacks, G.A. Holzapfel, D.A. Nordsletten, Simulating hyperelasticity and fractional viscoelasticity in the human heart, *Comput. Methods Appl. Mech. Engrg.* 411 (2023) 116048, <http://dx.doi.org/10.1016/j.cma.2023.116048>, URL <https://www.sciencedirect.com/science/article/pii/S004578252300172X>.
- [56] M.O. Ghafur, B. Koziey, J. Vlachopoulos, Simulation of thermoforming and blowmolding — Theory and experiments, in: *Rheological Fundamentals of Polymer Processing*, Springer Netherlands, Dordrecht, 1995, pp. 321–383, http://dx.doi.org/10.1007/978-94-015-8571-2_15.
- [57] S. Wiesheier, J. Mergheim, P. Steinmann, Discrete data-adaptive approximation of hyperelastic energy functions, *Comput. Methods Appl. Mech. Engrg.* 416 (2023) 116366, <http://dx.doi.org/10.1016/j.cma.2023.116366>, URL <https://www.sciencedirect.com/science/article/pii/S0045782523004905>.
- [58] E.B. Tadmor, R.E. Miller, R.S. Elliott, *Numerical solutions: The finite element method, Continuum Mechanics and Thermodynamics: From Fundamental Concepts To Governing Equations*, Cambridge University Press, 2011, pp. 277–316, <http://dx.doi.org/10.1017/CBO9781139017657.011>.
- [59] K.B. Sautter, M. Meßmer, T. Teschemacher, K.-U. Bletzinger, Limitations of the st. Venant–Kirchhoff material model in large strain regimes, *Int. J. Non-Linear Mech.* 147 (2022) 104207, <http://dx.doi.org/10.1016/j.ijnonlinmec.2022.104207>, URL <https://www.sciencedirect.com/science/article/pii/S0020746222001810>.
- [60] K.K. Žur, N. Firouzi, T. Rabczuk, X. Zhuang, Large deformation of hyperelastic modified Timoshenko–Ehrenfest beams under different types of loads, *Comput. Methods Appl. Mech. Engrg.* 416 (2023) 116368, <http://dx.doi.org/10.1016/j.cma.2023.116368>, URL <https://www.sciencedirect.com/science/article/pii/S0045782523004929>.
- [61] B. Kim, S.B. Lee, J. Lee, S. Cho, H. Park, S. Yeom, S.H. Park, A comparison among Neo-Hookean model, Mooney–Rivlin model, and Ogden model for chloroprene rubber, *Int. J. Precis. Eng. Manuf.* 13 (5) (2012) 759–764, <http://dx.doi.org/10.1007/s12541-012-0099-y>.
- [62] S. Rossi, N. Abboud, G. Scovazzi, Implicit finite incompressible elastodynamics with linear finite elements: A stabilized method in rate form, *Comput. Methods Appl. Mech. Engrg.* 311 (2016) 208–249, <http://dx.doi.org/10.1016/j.cma.2016.07.015>, URL <https://www.sciencedirect.com/science/article/pii/S0045782516307563>.
- [63] Y. Gorash, T. Comlekci, R. Hamilton, CAE-based application for identification and verification of hyperelastic parameters, *Proc. Inst. Mech. Eng., Part I: J. Mater.: Des. Appl.* 231 (7) (2017) 611–626, <http://dx.doi.org/10.1177/1464420715604004>.
- [64] L. Anand, Moderate deformations in extension-torsion of incompressible isotropic elastic materials, *J. Mech. Phys. Solids* 34 (3) (1986) 293–304, [http://dx.doi.org/10.1016/0022-5096\(86\)90021-9](http://dx.doi.org/10.1016/0022-5096(86)90021-9), URL <https://www.sciencedirect.com/science/article/pii/S0022509686900219>.
- [65] P. Wriggers, T.A. Laursen, *Computational Contact Mechanics*, vol. 2, Springer, 2006.
- [66] M.L. Sentmanat, Miniature universal testing platform: From extensional melt rheology to solid-state deformation behavior, *Rheologica Acta* 43 (6) (2004) 657–669.
- [67] Technical datasheet 5502, 2022, URL <https://polymers.totalenergies.com/5502>. (Accessed 18 July 2023).
- [68] Blender Online Community, *Blender - A 3D Modelling and Rendering Package*, Blender Foundation, Stichting Blender Foundation, Amsterdam, 2018, URL <http://www.blender.org>.
- [69] M. Lillie, J. Gosline, Mechanical properties of elastin along the thoracic aorta in the pig, *J. Biomech.* 40 (10) (2007) 2214–2221, <http://dx.doi.org/10.1016/j.jbiomech.2006.10.025>.