

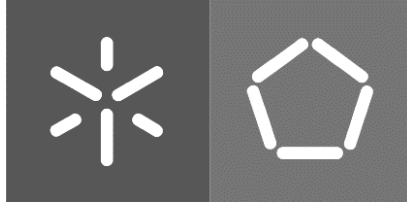


Universidade do Minho
Escola de Engenharia

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Oblique Detonation Wave Engine

October 2024



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Escola de Engenharia

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Oblique Detonation Wave Engine

Specialization in Aerospace Engineering Design

Work carried out under the guidance of the
Professor Dr. Gustavo Rodrigues Dias
Professor Dr. Francisco Brojo

October 2024

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AGRADECIMENTOS

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INTEGRITY STATEMENT

I declare that I have acted with integrity in the preparation of this academic work and confirm that I have not resorted to the practice of plagiarism or any form of misuse or falsification of information or results in any of the stages leading to its preparation.

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Motor de detonação oblíqua

RESUMO

Nos últimos anos a indústria espacial e da aviação tem procurado soluções para resolver os principais problemas, principalmente ao nível dos motores utilizados. Alguns desses problemas estão ligados ao peso da aeronave e ao impulso específico para elevados números de Mach. A hypersonic airbreathing propulsion oferece diferentes soluções para este tipo de problemas nomeadamente a diminuição do peso, uma vez que não necessita de transportar no interior da aeronave o oxidante usado na combustão para além de também oferecer um impulso específico superior para elevados número de Mach. O principal tipo de motor utilizado deste tipo de propulsão foi durante muitos anos, e ainda o é, o scramjet embora cada vez mais sejam realizados estudos procuram descobrir novos tipos de configurações, que apresentem melhor desempenho, surgindo então os motores de detonação oblíqua.

Esta dissertação, apresenta o desenvolvimento de um código, que procura calcular e comparar a performance dos motores scramjet com motor de detonação oblíqua. Para efetuar essa comparação, é utilizada uma configuração de área variável para o caso do motor scramjet, ao contrário do que é mais comum, área constante. A comparação entre a área variável e a área constante permitirá perceber se o motor de detonação oblíqua apresenta vantagens a nível de performance, sobre as configurações de scramjet apresentadas.

O estudo será efetuado através da análise dos diversos sistemas de um motor, nomeadamente a admissão e consequente compressão do ar, a combustão e finalmente a respetiva expansão, sendo também definidos parâmetros que procuram definir as condições de operação deste mesmo motor, principalmente a altitude de operação e o número de Mach.

PALAVRAS-CHAVE

Detonações, motor de detonação oblíqua, propulsão hipersónica, scramjet de área variável

Oblique detonation wave engine

ABSTRACT

In recent years, the space and aviation industry has been looking for solutions to solve the main problems, especially in terms of the engines used. Some of these problems are linked to the weight of the aircraft and the specific thrust for high Mach numbers. Hypersonic airbreathing propulsion offers different solutions to this type of problem, namely weight reduction, since it does not need to carry the oxidizer used in combustion inside the aircraft, in addition to also offering a higher specific impulse at high Mach number. The main type of engine used in this type of propulsion was for many years, and still is, the scramjet, although more and more studies are being carried out to discover new types of configurations, which present better performance, then the ODWE (Oblique Detonation Wave Engine) emerged.

This dissertation presents the development of a code, which seeks to calculate and compare the performance of scramjet engines with oblique detonation engines. To make this comparison, a variable area configuration will be used for the case of the scramjet engine, as opposed to what is more common, constant area. The comparison between the variable area and the constant area will allow us to understand if the oblique blasting engine has performance advantages over the scramjet configurations presented.

The study will be carried out through the analysis of the various systems of an engine, namely the intake and consequent compression of the air, the combustion and finally the respective expansion, and defined parameters that seek to define the operating conditions of this same engine, mainly the operating altitude and the number of Mach.

KEYWORDS

Detonations, hypersonic propulsion, oblique detonation wave engine, variable area scramjet

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LIST OF SYMBOLS, GREEK LETTERS, SUBSCRIPTS AND ABBREVIATIONS

Symbol	Description	Units
A	Area	m^2
a	Speed of sound	$m.s^{-1}$
C_p	Specific heat	$J.kg^{-1}.K^{-1}$
F	Uninstalled thrust	N
$\frac{F}{\dot{m}_0}$	Specific thrust	$N.s^{-1}.kg^{-1}$
f	Fuel/air ratio	-
g	Acceleration of gravity	$m.s^{-2}$
g_0	Acceleration of gravity on the surface of the Earth	$m.s^{-2}$
h	height	m
h_f	Absolute sensible enthalpy of fuel at the entrance of the combustor	$J.kg^{-1}$
h_{pr}	Heat of combustion	$J.kg^{-1}$
I_{sp}	Specific impulse	s
M	Mach number	-
$\dot{m}$	Mass flow rate	$kg.s^{-1}$
p	Absolute pressure	Pa
Q	Energy added per unit mass	$J.kg^{-1}$
$\tilde{q}/\tilde{Q}$	ODW heat flux	-
r	Radius	m
R	Gas constant	$J.kg^{-1}.K^{-1}$
S	Specific fuel consumption	$kg.s^{-1}.N^{-1}$
S_a	Stream thrust function	$N.s.kg^{-1}$
T	Absolute temperature	K
T_{ig}	Self-ignition temperature	K
V	Absolute velocity	$m.s^{-1}$

Greek Letter	Description	Units
β	ODW angle	deg
γ	Ratio of specific heat	-
η	Efficiency	-
θ	Wedge angle	deg
λ	Lapse rate	$K.m^{-1}$
π_c	Ratio of total pressure	-
π_2	Pressure ratio	-
ρ	Density	$kg.m^{-3}$
Σ	Coefficient of isolator total pressure recovery	-
T	-	-
Φ	Fuel/air equivalence ratio	-
ϕ	Equivalence ratio of the engine	-

Subscript	Description
b	Combustion process
c	Compresion process
Ch.J.	Chapman-Jouguet restriction
e	Exhaust
f	Fuel
KE	Kinetic Energy
p	Propulsive
o	Overall
Ratio	Ratio of conditions
st	Stoichiometric condition
t	Total or Stagnation
th	Thermal
0	Freestream
1	Upstream
1_n	Normal component of upstream
2	Downstream
2_n	Normal component of downstream
3	Burner entrance
4	Burner exit
5	Nozzle exit

Abbreviations**Description**

ODW

Oblique Detonation Wave

ODWE

Oblique Detonation Wave Engine

1. INTRODUCTION

Interest in hypersonic airbreathing propulsion has been growing in recent years, since this technology allows several advantages over traditional space and aviation engines, namely the possibility of flying at speeds greater than five times the speed of sound, but also the fact that it allows faster access to space and faster commercial flights.

The reason why this work has been carried out, as well as what will be carried out in it, will be referred to in the Motivation and Objectives Chapters.

1.1 Motivation

The most used type of Hypersonic airbreathing propulsion engines is the scramjet, which has been the target of intense studies for several years, with the aim of enhancing their performance. This type of motors has the advantage of having a high specific impulse, in addition to not having to transport the oxidizer, compared to scramjet engines. For this reason, it is possible to carry more payload weight, increasing cost efficiency.

Although the scramjet has several advantages, it also has some disadvantages, namely the complexity of the combustion system being high. Due to this reason, the concept of oblique detonation engines emerged, which replaces the diffusion process in the combustion of scramjets, by oblique detonations, reducing the geometry and complexity of the combustion system.

Even so, several studies will still be needed to make the construction and use of this type of engine viable.

1.2 Objectives

The objective of this dissertation is to describe the process of developing a numerical tool, which aims to compare the performance of an oblique detonation engine with a scramjet engine. For this, the results obtained in this work has been compared with the results of other works, namely with works that perform combustion at constant pressure and constant area, since this project will address, in the case of the scramjet, variable area.

The numerical tool should be able, considering the inputs given, and the geometry of the chosen inlet, to calculate the performance for a given range of Mach numbers.

In the end, and after the results are obtained by the numerical tool, they will be compared with the results of other studies, to understand the advantages and disadvantages of its use.

2. BIBLIOGRAPHIC REVIEW

In this chapter, the preliminary concepts will be addressed, which will serve as a basis for the concepts and formulas, and the decisions made, in the methodology chapter, and the conclusions that will be drawn from the results obtained in the numerical tool.

2.1 Hypersonic Airbreathing Propulsion

With the aim of making hypersonic respiratory propulsion more accessible and efficient, this technology has been increasingly studied, although there has been no practical evolution in the last 40 years in terms of propulsive systems [1], at least, in terms of vehicles.

Therefore, the air-breathing propulsion systems most used now are ramjet and scramjet, which differ in the way in which the air flow enters the combustor, causing combustion in each case to be subsonic and supersonic, respectively [2].

In addition to these configurations, others have been proposed, such as oblique wave detonation engines (ODWE), rotary detonation engines (RDE), and pulse detonation engines (PDE) [2]. In the case of ODWE, its main peculiarity is the fact that combustion occurs by detonation and not by deflagration.

The main disadvantage of ODWEs, and of scramjets, is the fact that they have a high initial Mach number, which makes them dependent on other engines for take-off and acceleration, up to the desired initial Mach number, increasing the complexity of the propulsive system. For this reason, much study is still needed in this area until its application is viable.

2.2 Scramjet

To define what a scramjet engine is, we first must understand the bases of a gas turbine and a ramjet engine. The main difference between the two is the air compression process before combustion, where in the gas turbine this is done through a compressor, while in the ramjet, this is done through shock waves, which are normally caused by the front body of the vehicle. When looking more specifically at the operation of the ramjet engine, the air flow is slowed down to subsonic speeds after compression, making combustion also subsonic. According to Heiser and Pratt et al. [3], the operation of the ramjet engine is efficient for Mach numbers in the range 3-6. As the Mach number exceeds this range, the use of the ramjet engine is no

longer efficient, as it is no longer advantageous to reduce the speed of the air flow to subsonic speed. Therefore, it was necessary to create an engine that did not require this deceleration, and could perform supersonic combustion, originating the scramjet engine.

Although this configuration offers this advantage, when compared to the ramjet engine, and has been the subject of numerous studies and evolutions over the years, there are still problems that need to be overcome to make this engine more efficient and viable, in addition to the high initial Mach number. Since the operating environment has high temperatures and pressures, which jeopardizes the structural integrity of the engine and vehicle, the need to guarantee stable and efficient mixing and combustion in supersonic regime, in addition to a combustion chamber of appropriate dimensions, are some of the sizing problems of this type of engine [3].

In the Figure 2.1, it is possible to visualize a two-dimensional diagram of the structure of a scramjet engine.

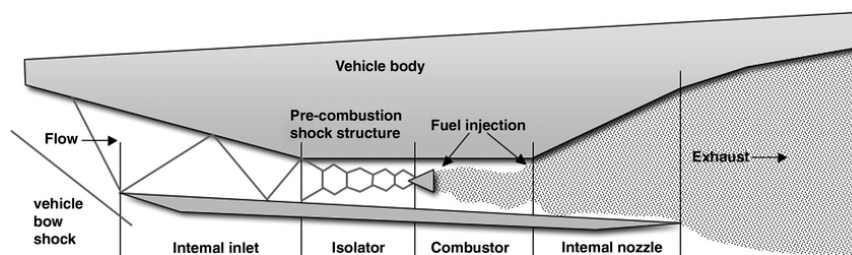


Figure 2.1: Diagram of a scramjet engine [4]

2.3 Oblique Detonation Wave Engine

As seen previously, the engines most used in the construction of hypersonic airbreathing propulsion vehicles are the scramjet, which are characterized by using deflagration as a combustion process. According to Pratt et al [5], the use of normal detonation waves was proposed by Roy in 1946 [6], and later studies proved its feasibility, although it was found difficult to control the detonation wave, as well as the complex geometry of the chamber, which this process would require.

The use of oblique detonation waves instead of normal detonations was proposed by Dunlap et al [1], since as happens in a scramjet engine, it allows the flow that passes through the burner to be supersonic.

This meant that it was not necessary to implement a variable geometry to stabilize the wave, since this stabilization would be carried out by a wedge [7]. If we look at the geometries of a

scramjet and an ODWE, we see that the difference between them is the existence of this wedge in the combustor. As will be seen later, the existence of a wedge is necessary for the stabilization of shock waves, unlike what happens in scramjet engines. The position of the wedge depends on the type of compression that will be chosen for the work, representing great influence mainly on the level of drag generated. The typical model of an oblique detonation engine can be observed in the Figure 2.2.

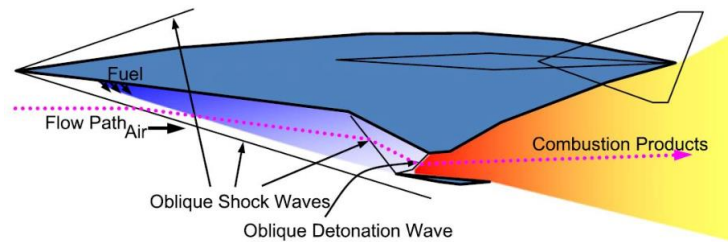


Figure 2.2: Diagram of an Oblique Detonation Wave Engine [8]

Citing Heiser and Pratt et al. [3], “The immediate advantages of the ODW process are that the drag, convection heating, length, weight, cost, and maintenance of the combustor are almost entirely eliminated”. In this way, different studies were carried out to increase the efficiency and viability of ODWEs. One of them was the study carried out by Valorani et al. [9], which sought to find a solution to the high initial Mach number, with the possibility of this being reduced. This would result in a reduction in operating weight, since less oxidant would be needed to be transported, since the lower the Mach number, the smaller the amount of oxidant used, for shorter time. But it would also allow the use of shorter burners, when compared to those used in scramjets. Another advantage is the simplicity of the compression system, especially when compared to Ramjet engines. This is because, the temperature variation throughout the compression process is smaller, due to the existence of higher temperatures and pressures in the burner due to detonation, since the existence of shock waves, causes an increase in pressure and temperature, without using more fuel.

One of the most important studies was carried out by Ashford et al. [10], where the performance of an oblique wave detonation engine was compared with that of a diffusive scramjet engine. Some considerations were made, such as constant area and constant pressure, as well as the fact that it is an ideal and perfect gas. For Mach 10, at which the study was carried out, it was concluded that the use of ODWEs can reduce the length of the engine up to 50%.

To replace diffusive burning, Jiang et al [11], proposed a prototype of a standing oblique detonation ramjet engine. The engine was successfully developed, concluding that the model works steadily, and confirming that oblique detonation can be stationary and controllable in the burner. This model can be viewed in Figure 2.3

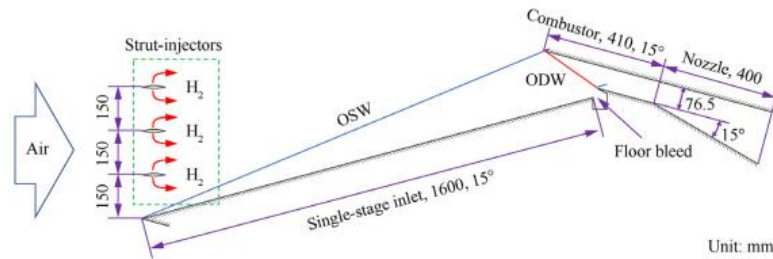


Figure 2.3: Schematic of the Sodramjet proposed by Jiang [11]

2.3.1 Combustion

To understand the combustion process, it is important to define deflagration and detonation, to understand their differences.

Starting with deflagration, a combustion process characteristic of scramjet engines, this can be defined as the passage of a flame front as a subsonic wave through a fuel mixture, according to Diéguez et al [12].

According to Wolański et al. [13], the detonation process was first described by Berthelot, Vieille, Mallard and Le Chatelier in 1881. It was only twenty years later that Chapman and Jouguet presented the zero-dimensional theory of detonation.

Detonation is a combustion wave that propagates through a gaseous mixture of fuel and oxidizer, causing a reaction.

This can be classified according to the Mach number that occurs after detonation. If the Mach number is subsonic, we have a detonation called overdriven, if it is sonic, we are facing the Chapman-Jouguet detonation (referred to in the following chapter), and finally, if the Mach number is supersonic, the detonation is called underdriven.

As with oblique shocks, oblique detonations can have a weak or strong shock solution, for the same wedge angle. To evaluate the angle of the oblique detonation wave as a function of the angle of deflection and heat addition, a diagram was created, based on the upstream conditions, and considering the different possible conditions downstream. This diagram can be seen in Figure 2.4.

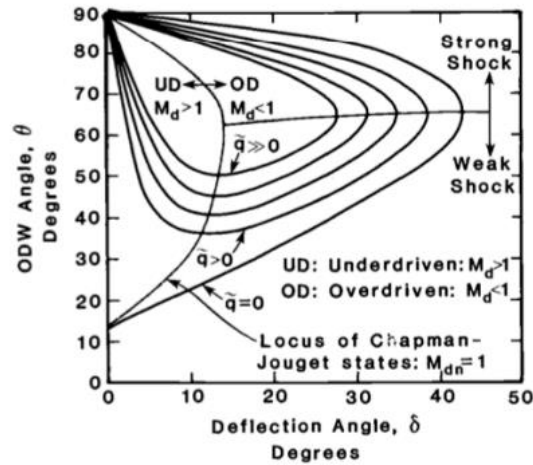


Figure 2.4: Oblique detonation wave angle as function of deflection angle and heat addition [3]

According to the diagram, we can have strong or weak overdriven oblique detonation waves. In the case of strong overdriven detonations, associated increases in static pressure are so high that they cause detachment and are therefore unnatural. In the case of weak overdriven detonation waves, these are restricted by the Chapman-Jouguet angle and the detachment angle, which is why this is the region of interest for ODW engines. The Chapman-Jouguet angle is connected to the Chapman-Jouguet locus, which is the point of minimum wave angle at each locus of states for a given $\tilde{q} > 0$, as can be seen in Figure 2.4. Detonations are not considered for these engines due to the downstream Mach number being supersonic, and it needs to be subsonic or sonic, to ensure that the oblique detonation wave is attached and stable.

Upon detonation, the front propagates at speeds in the order of km/s in the air-fuel mixture, which produces a significant increase in pressure. According to Wolański et al [13], if the detonation speed is above 1.8 km/s, there is a pressure increase of more than ten times. If the detonation speed reaches 3km/s, a pressure increase of up to twenty times is obtained.

During deflagration, the speed of the flame is in the order of dozens of m/s, so the combustion will have to be organized in a stoichiometric ratio, at a higher burning speed, which results in high combustion temperatures and production of high NO_x concentration. Due to these phenomena, in common aviation engines, which have a turbine, as is the case with turbofans, turbojets, among others, it is necessary to mix more air before the turbine, due to the high temperature, increasing the complexity of the system.

In detonation engines this complexity does not exist, since the combustion temperature is lower, in addition to not using ignition devices [1].

According to Rosato et al. [14], detonation can effectively increase the efficiency of the thermodynamic cycle by up to 20%, when compared to typical cycles that are based on diffusive burning, since the greater the speed of burning or conversion of the "material", typically tens of thousands of times faster than engines that use deflagration, resulting in more compact and efficient systems.

2.3.2 Chapman-Jouguet Theory

As we saw previously, Chapman and Jouguet established the first theory about detonations, the zero-dimensional theory of detonation. This described the detonation wave as a hydrodynamic discontinuity, where energy would be released, assuming that it was stable, planar, and one-dimensional.

Another important topic for this work is the Rankine-Hugoniot conditions, which describe the relationship between the states on both sides of a shock wave or combustion wave (deflagration or detonation), for one-dimensional flows in fluids or for one-dimensional deformations in solids.

In the equation (2.1), Rankine-Hugoniot conditions are described, where the prefixes 1 and 2 refer to the state before and after detonation, respectively. Therefore, h_1 and h_2 represent, respectively, the specific enthalpies before and after detonation, p_1 and p_2 , the pressures before and after detonation and, finally, ρ_1 and ρ_2 , the flow densities before and after detonation.

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \quad (2.1)$$

Using these conditions, it is possible to plot the Rankine Hugoniot curve, where it will be possible to determine the conditions of state 2, considering the conditions of state 1 that are provided. One of these curves can be visualized in Figure 2.5, where regions I and II correspond to supersonic waves, hence detonations, while regions IV and V correspond to subsonic waves, also called deflagrations.

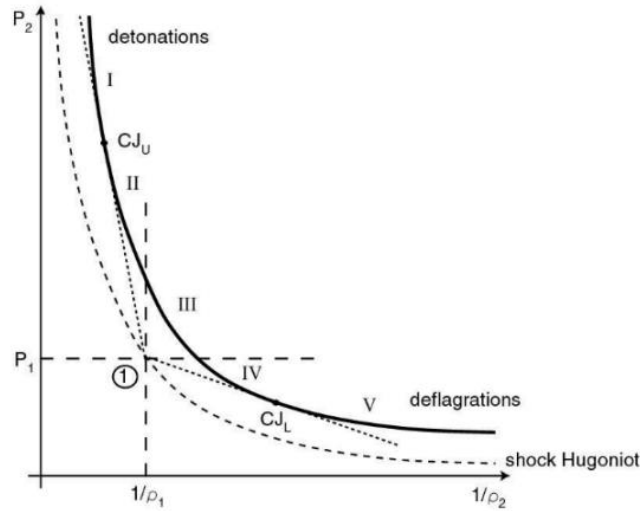


Figure 2.5: Rankine-Hugoniot curve [15]

In the case of detonation, considerations of gas dynamics are sufficient to predict the propagation speed of the wave, through the shock wave, while in deflagration it is necessary to know the structure of the wave, in addition to the transport processes, whether turbulent or diffusive.

Chapman and Jouguet suggested that detonations travel at the lowest speed for all solutions in the detonation branch. Observing Figure 2.5, we can see that there are two branches, which correspond to the Chapman-Jouguet points, the upper point, CJ_u , and the lower point, CJ_L . The lower one is in the deflagration branch, while the upper one is in the detonation branch, which corresponds to the point used for ODWEs, since this type of engine uses detonations and not deflagrations, in addition to corresponding to the point of lowest entropy and total pressure lost.

2.4 Synthesis

According to the points addressed during the bibliographic review, it is perceived that there is a lack of knowledge of oblique detonation engines, especially in the possibility of them replacing the most used hypersonic engines today, the scramjet.

These reasons led to this work, with variable area configuration, but also the work carried out by Pereirinha et al. [16], with constant area configuration, to be carried out. In them, a comparison is made between oblique detonation engines and different configurations of

scramjet engines, to understand if the performance of the oblique detonation engine is superior or inferior to scramjet engines.

3. METHODOLOGY

In this chapter, the process used to develop the intended program has been described. The results of this work will later be compared with those obtained by Pereirinha et al. [16]. The main differences between the two works are the fact that this work opts for a variable area and not a constant area, like Pereirinha's work.

To obtain a reliable comparison, the main considerations will be maintained, namely the operating altitude, the operating Mach number, but unlike Pereirinha, this project will include an isolator, to facilitate calculations of the variable area.

This way, the geometric representation of the engine will also be the same, which is displayed in the Figure 3.1.

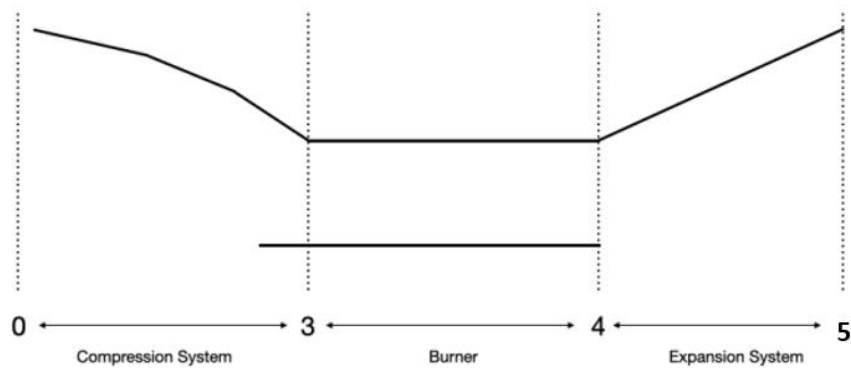


Figure 3.1: Representation of engine geometry

Analysing the Figure 3.1, it is possible to perceive the distinction between the 3 dimensioned systems, for the configurations of the oblique detonation engine and scramjet. Between point 0 and 3, the compression system, where points 1 and 2, represent, respectively, the input and output of the insulator, for variable area configurations. Between points 3 and 4 the combustion system/burner, and finally between points 4 and 5, the expansion system.

3.1 Atmosphere Model

To carry out the intended study and evaluate its performance, mainly at the propulsive level, it is necessary to start by choosing the simulation conditions for the different desired altitudes. In this way, the International Standard Atmosphere model is implemented, which is recognized and accepted internationally, having been proposed in 1952 by the International Civil Aviation Organization (ICAO).

According to this model, it is important to describe the processes for obtaining different parameters for different altitudes, such as temperature, pressure, speed of sound and gravitational acceleration.

To determine the properties as altitude varies, it is necessary to apply the equations related to the ISA model [17].

To determine the temperature within the troposphere, the equation (3.1) is used,

$$T = T_0 - 6,5 \frac{h(m)}{1000} \quad (3.1)$$

where T_0 is 288.15 K. As altitude increases, from the end of the troposphere to an altitude of 20 km, the temperature remains constant, with a value of 216.65 K. From 20 km and up to 32 km altitude, the temperature can be calculated using the equation (3.2)

$$T = T_{20km} + (h(m) - 20000)10^{-3} \quad (3.2)$$

where T_{20km} , and as previously mentioned, is 216.65 K.

Next, looking at the pressure, which within the troposphere can be calculated, according to the equation used below:

$$p = p_0 \left(\frac{T}{T_0} \right)^{\frac{-g_0}{R\lambda}} \quad (3.3)$$

where p_0 is 101325 Pa, R is $287.0531 \text{ J kg}^{-1} \text{ K}^{-1}$, λ is $-6.5 \times 10^{-3} \text{ km}^{-1}$ and g_0 is 9.8067 ms^{-2} . When reaching 11 km, and up to 20 km altitude, the pressure will have to be calculated differently. This can be obtained through equation (3.4).

$$p = p_{11km} e^{\frac{-g_0(h-11000)}{RT_{11km}}} \quad (3.4)$$

where is p_{11km} 22632.06 Pa and T_{11km} is 216.65 K.

For altitudes between 20 and 32 km, the pressure can be calculated using the equation (3.5).

$$p = p_{20km} \left(\frac{T}{T_{20km}} \right)^{\frac{-g_0}{R\lambda}} \quad (3.5)$$

The next property to be analysed is the speed of sound. To calculate this, the following equation is used:

$$a = \sqrt{\gamma RT} \quad (3.6)$$

where $\gamma=1.4$.

Finally, to calculate the gravitational acceleration for a specific altitude, the following equation is applied:

$$g = g_0 \left(\frac{6371000}{6371000 + h} \right)^2 \quad (3.7)$$

Where g_0 is 9.8067 ms^{-2} .

3.2 Compression System

The next system to be analysed is the compression system. This represents an extremely important system for the engine's performance, and it is therefore necessary of being capable of meeting certain requirements, namely the need to ensure that the desired cycle temperature ratio is achieved, with maximum compressive efficiency and a minimum increase in entropy. In this chapter, several factors will be discussed, namely the number of shock waves, as well as the type of waves, the type of inlet to be implemented and the type of combustion.

To evaluate the performance of this system, certain assumptions were made in its design. The flow will be one-dimensional, the air will behave like a perfect, inviscid gas, as well as the neglect of heat transfer to or from the wall.

The next topics represent the important factors in sizing the compressive system.

3.2.1 Inlet Type

To begin sizing the compressive system, the type of inlet to be used was chosen. To guarantee the most reliable comparison with the work carried out by Pereirinha et al. [16], the choice focused on the same type of inlet used by this author. This choice is based on the work of Quan et al. [18].

To understand the reasons for choosing the type of inlet, it is necessary to understand the different types of inlets according to the possible types of “compression”, internal, external, and mixed.

In the case of an external compression inlet, the flow must enter the combustor at a certain angle in relation to the external flow, to ensure that all oblique waves are located outside the throat inlet. For this to occur, the cowl inlet cannot be placed parallel to the freestream flow, taking an increase in the cowl drag generated, making of the non-parallel wedge an uncommon choice.

In the case of the internal compression inlet, it also ends up generating a lot of drag, due to its shape, in addition to being quite difficult to integrate into the rest of the engine and vehicle. To resolve this uncertainty, the solution involves a mixed inlet, being the most common and popular choice for applications in hypersonic airbreathing propulsion engines. Due to this reason, this ended up being the chosen inlet, which can be seen in the Figure 3.2.

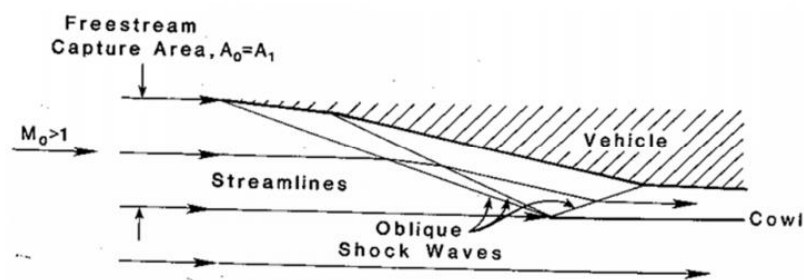


Figure 3.2: Mixed compression inlet [18]

A favourable point of this type of inlets is the fact that the compression and the cowl angle are, in a certain way, independent, which allows the cowl to be parallel to the freestream flow.

3.2.2 Isolator

According to Cardoso et al. [19], an isolator is a constant area diffuser placed between the compression system and the combustion chamber. Inside, the pressure increases in a controlled manner, through consecutive shock waves that interact with the boundary layer, which allows the flow to adjust to the pressure imposed by the downstream components.

The main care to be taken is that the isolator must be long enough to contain all shock waves and prevent the inlet from unstating.

The isolator model chosen was based on the work of Gu et al. [20], where a tubular isolator is placed before the combustor, and after the combustor, the exhaust. For this work, a radius of 80 mm was chosen for the isolator entrance.

When choosing the isolator, one of the most important parameters, is the pressure ratio that exists between its inlet and outlet. This can be described by the equation (3.8).

$$\pi_2 = \frac{P_2}{P_1} \quad (3.8)$$

According to Gu et al. [20], the values of this pressure ratio can vary between 1 and 2.5. In this project, initially 1 has been used.

Next, it is necessary to determine the properties of the flow at the exit of the isolator. These can be obtained through the equations for the conservation of mass, momentum and energy, represented by the equations (3.9), (3.10) and (3.11), respectively.

$$\dot{m}_1 = \dot{m}_2 = \rho_1 A_1 u_1 = \frac{p_1 A_1 u_1}{RT_1} \quad (3.9)$$

$$p_2 A_2 + \dot{m}_2 u_2 = p_1 A_1 + \dot{m}_1 u_1 \quad (3.10)$$

$$T_2 = T_1 \frac{1 + \frac{\gamma_1 - 1}{2} M_1^2}{1 + \frac{\gamma_2 - 1}{2} M_2^2} \quad (3.11)$$

By combining the equations (3.9), (3.10) and (3.11), it is possible to generate the equation (3.12), that allows calculating the Mach number at the isolator exit.

$$M_2 = \frac{u_2}{\sqrt{\gamma_2 RT_2}} = \sqrt{\frac{C^2}{1 - \frac{\gamma_2 - 1}{2} C^2}} \quad (3.12)$$

where,

$$C = \frac{M_1 \sqrt{\gamma_1 RT_1} + (1 - \pi_2) \frac{RT_1}{M_1 \sqrt{\gamma_1 RT_1}}}{\sqrt{(\gamma_2 RT_1) \left(1 + \frac{\gamma_1 - 1}{2} M_1^2\right)}} \quad (3.13)$$

where γ_1 and γ_2 are the specific heat ratios of the gas at the corresponding cross-section, and R is the gas constant.

Then, the area at the isolator exit/combustor inlet can be calculated using the equation (3.14).

$$A_2 = \frac{A_1}{\sqrt{\gamma_2 R T_2}} \left[\frac{1}{\pi_2} (1 + \gamma M_1^2) - 1 \right] \quad (3.14)$$

Finally, the total pressure at the exit of the isolator can be calculated, as well as the recovery coefficient of the total pressure of the isolator. These can be calculated using the equations (3.15) and (3.16), respectively.

$$P_2 = P_1 \left[\frac{\left(1 + \frac{\gamma_2 - 1}{2} M_2^2 \right)^{\frac{\gamma_2}{\gamma_2 - 1}}}{\left(1 + \frac{\gamma_1 - 1}{2} M_1^2 \right)^{\frac{\gamma_1}{\gamma_1 - 1}}} \right] \quad (3.15)$$

$$\sigma_2 = \frac{P_2}{P_1} \quad (3.16)$$

3.2.3 Number of Oblique Shocks

After choosing the type of inlet to use, it is necessary to determine the number of oblique shocks that will be generated by the inlet, according to the desired conditions, namely the desired cycle temperature ratio. The number of shocks must guarantee that this ratio is efficient and the increase in entropy is kept to a minimum, in addition to providing an acceptable performance.

In this project, the sizing of the compression system has been based on the cycle temperature ratio T_3/T_0 , representing the temperature ratio between the burner inlet and the outside, as shown by the Figure 3.1.

It is also important to ensure that all oblique shock waves transmit an equal amount of geometric rotation of the flow, in addition to ensuring that the shock-on-lip condition occurs [3].

According to Smart et al [21], the greater the number of shocks, the more the optimal total pressure recovery will increase, although this decreases with the increase in Mach number. This effect can be seen in Figure 3.3, that describes how the optimal total pressure recovery varies with the Mach number, for a different number of shocks.

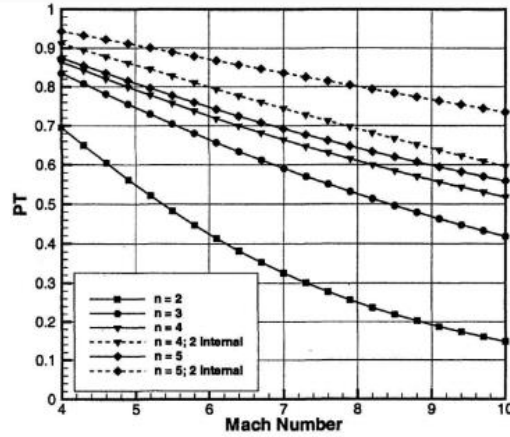


Figure 3.3: Maximum total pressure recovery for two-dimensional scramjet inlets with up to five shocks [21]

In Figure 3.4, it is analysed how the compressive efficiency varies with the number of Mach, for a different number of oblique shock waves.

Observing the Figure 3.3 and Figure 3.4, it is concluded that a system of 4 oblique shocks is the most efficient choice, since a higher number of shocks leads to complications in system sizing, while a lower number leads to a reduction in the optimum total pressure recovery.

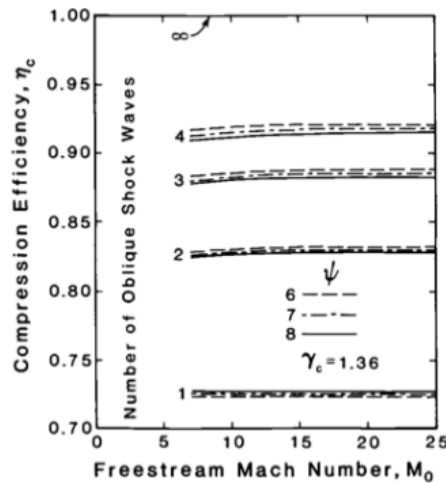


Figure 3.4: Adiabatic compression efficiency as a function of freestream Mach number, static temperature ratio, and number of oblique shock waves [3]

To calculate the flow properties after each shock, the oblique shock equations has been applied. In this way, equations (3.17), (3.18), (3.19), (3.20) and (3.21), taken from Pereirinha et al. [16], were implemented in the project code.

$$p_{ratio} = 1 + \frac{2\gamma_0}{\gamma_0 + 1} (M^2 \sin^2 \beta - 1) \quad (3.17)$$

$$p_{ratio} = \frac{(\gamma_0 + 1)M^2 \sin^2 \beta}{2 + (\gamma_0 - 1)M^2 \sin^2 \beta} \quad (3.18)$$

$$T_{ratio} = \frac{p_{ratio}}{\rho_{ratio}} \quad (3.19)$$

$$\tan \theta = \frac{2 \cot \beta (M^2 \sin^2 \beta - 1)}{M^2 (\gamma_0 + \cos(2\beta)) + 2} \quad (3.20)$$

$$M_{ratio} = \frac{1}{M} \left(\frac{2}{\gamma_0 - 1} + M^2 \sin^2 \beta \right)^{1/2} \left(\frac{2\gamma_0 M^2 \sin^2 \beta}{\gamma_0 - 1} - 1 \right)^{1/2} \quad (3.21)$$

In order to determine compressive efficiency and the kinetic energy (η_c and η_{KE} , respectively), the equations (3.22), (3.23) and (3.24), taken from [3], are applied.

$$\pi_c = \frac{p_{t3}}{p_{t0}} = \frac{p_3}{p_0} \left(\frac{1}{\varphi} \right)^{\gamma_c/(\gamma_c-1)} \quad (3.22)$$

$$\eta_c = \frac{\varphi - \left(\frac{1}{\pi_c} \right)^{(\gamma_c-1)/\gamma_c}}{\varphi - 1} \quad (3.23)$$

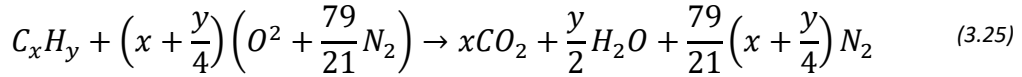
$$\eta_{KE} = 1 - \frac{2}{(\gamma_c - 1)M_0^2} \left[\left(\frac{1}{\pi_c} \right)^{(\gamma_c-1)/\gamma_c} - 1 \right] \quad (3.24)$$

where $\varphi = T_3/T_0$.

3.3 Combustion System

The next thing to pay attention to is the combustion study, where it is considered that it starts with the air and fuel already homogeneously mixed.

To understand how combustion takes place, it can be described using a chemical equation (3.38), called the complete stoichiometric equation, described in the equation (3.25).



In this case it represents the complete combustion of air and hydrocarbon fuel.

Next, it is necessary to calculate the stoichiometric fuel/air ratio. This can be obtained through the equation (3.26).

$$f_{st} = \frac{36x + 3y}{103(4x + y)} \quad (3.26)$$

For the case of non-stoichiometric mixtures, it is important to define another property, called equivalence ratio, which represents the ratio between the real fuel/air ratio and the stoichiometric fuel/air ratio. The representation of the equivalence ratio is made in the equation (3.27).

$$\phi = \frac{f}{f_{st}} \quad (3.27)$$

The equivalence ratio, according to Heiser and Pratt [3], must be in the range of 0.2 to 2, so that combustion occurs within a reasonable time scale, which is the range considered in this work.

Another important point for the project is the selection of fuel. For this purpose, several fuels were compared, in different aspects, as can be seen in the Table 3.1, where p is for standard conditions and T_{ign} for standard conditions and 1 atm.

The choice ended up being H_2 , to obtain a more reliable comparison between the results of the works. When observing the Table 3.1, it can be seen that H_2 has the highest heat of combustion value, in addition to a higher auto-ignition temperature. The main drawback of

using H_2 is the need for a lot of storage space due to its very low density, although several studies and advances have been carried out to counter this problem.

Table 3.1: Properties of several fuels [22]

Fuel	$h_{pr}[MJ/kg]$	$\rho[kg/m^3]$	$T_{ign}[K]$	f_{st}
H_2	119.96	0.08	845.15	0.02913
CH_4	50.01	0.65	810.15	0.05825
C_2H_6	47.49	1.22	745.15	0.06241
C_3H_8	46.3	1.79	743.15	0.06408
C_4H_{10}	45.74	2.36	693.15	0.06497

3.3.1 Post-combustion

It is now necessary to calculate the properties of the flow after combustion. Since we are dealing with area sections, and since the radius of the section after combustion is different from the inlet entrance radius, this difference must be considered. For this project, a section radius of 101.8 mm will be considered.

To calculate these properties, it is necessary to apply the equations (3.28), (3.29) and (3.30), taken from the work of Gu et al. [20]. The variables represented with point 2 are calculated at the exit of the isolator.

$$M_4 = \frac{M_2}{\sqrt{\tau(r) \left(\frac{1 + \gamma_{4(r)} - 1}{2} M_2^2 \right) - \left(\frac{\gamma_{4(r)} - 1}{2} M_2^2 \right)}} \quad (3.28)$$

$$P_3 = \frac{P_2}{\left[1 + \frac{\gamma_{4(r)} - 1}{2} M_2^2 \left(1 - \frac{1}{\tau(r)} \right) \right]^{\frac{\gamma_{4(r)}}{\gamma_{4(r)} - 1}}} \quad (3.29)$$

$$T_{3(r)} = T_2 \tau(r) \quad (3.30)$$

where,

$$\tau(r) = \frac{\mathbb{Q}(r)h_{pr}}{34.32c_{p2}T_2} \quad (3.31)$$

To simplify the calculations, certain values taken from other works were assumed. In case of $\mathbb{Q}(R)$, a value of 0.2 was assumed, according to Gu et al.[20], and for $\gamma_{4(R)}$ a value of 1.286 was assumed, according to Yang et al. [23].

3.3.2 Oblique Detonation Wave Engine

Previously, in Chapter 2, it was said that the types of detonations that could be applied in hypersonic airbreathing propulsion were weak overdriven detonations and Chapman-Jouguet detonations, although the latter correspond to those of minimum total pressure loss and the point of minimum entropy increase. Considering the types of detonations that can be used in this project, the developed program must be able to determine the wedge angle that generates the Chapman-Jouguet detonation, while for the remaining Mach numbers, the weak overdriven detonation must be guaranteed.

To ensure that pre-ignition does not occur, in the specific case of burning H_2 , below stoichiometric conditions, the temperature recorded before detonation must be below 1000 K, while to guarantee ignition, the temperature after detonation must be greater than 1000 K. If these conditions are not met, the program will warn the user of the error that has occurred.

To determine the Chapman-Jouguet detonation angle for the Mach number considered, the equation (3.32) must be applied.

$$(M_{1n}^2 - 1)^2 - 2(\gamma + 1)M_{1n}^2 \tilde{Q} = 0 \quad (3.32)$$

where,

$$M_{1n} = M_1 \sin \beta \quad (3.33)$$

$$\tilde{Q} = \frac{Q_{nb}}{c_p T} \quad (3.34)$$

To find the wedge angle that generates the Chapman-Jouguet detonation under the desired conditions, the equation (3.35) must be solved.

$$\theta_{CH.J.} = \beta_{CH.J.} - \tan^{-1} \left[\frac{1 + \gamma M_{1nCH.J.}^2}{(\gamma + 1) M_{1nCH.J.}^2 \sqrt{(M_1/M_{1nCH.J.}^2)^2 - 1}} \right] \quad (3.35)$$

For the remaining Mach numbers of the range under study, and in order to guarantee weak overdriven detonation, the detonation angle will be calculated using the equation (3.36), since the wedge angle has already been defined in the previous equation (3.35).

$$\tilde{Q} = -\frac{\gamma + 1}{2} X^2 M_1^2 \sin^2 \beta + (1 + \gamma M_1^2 \sin^2 \beta) X - \left(1 + \frac{\gamma - 1}{2} M_1^2 \sin^2 \beta \right) \quad (3.36)$$

The equations (3.32) and (3.35) were taken from Murthy et al.[24], and X has been obtained through the equation (3.39).

3.3.3 Post-Detonation Properties

After calculating the conditions before detonation, mainly the detonation angles for each Mach number in the desired range, it is necessary to determine the conditions after detonation. For this it is assumed that the air and fuel are homogeneously mixed [5], although this is not completely real.

Since the influence of the fuel temperature is neglected, the air temperature after the compression system is equal to the temperature of the air/fuel mixture.

To calculate the pressure after detonation, the equation (3.37) is used,

$$p_2 = p_1 + \rho_1 u_{1n}^2 (1 - X) \quad (3.37)$$

and the detonation temperature ratio can be calculated using equation (3.38).

$$\frac{T_2}{T_1} = 1 + \frac{u_{1n}^2}{2C_p T_1} (1 - X)^2 + \frac{Q_{\eta b}}{C_p T_1} \quad (3.38)$$

while X is calculated by

$$X \equiv \frac{\rho_1}{\rho_2} = \frac{u_{2n}}{u_{1n}} = \frac{\tan(\beta - \theta)}{\tan \beta} \quad (3.39)$$

where,

$$u_{1n} = u_1 \sin \beta \quad (3.40)$$

$$u_{1t} = u_1 \cos \beta \quad (3.41)$$

$$u_{2n} = u_2 \sin(\beta - \theta) \quad (3.42)$$

$$u_{2t} = u_2 \cos(\beta - \theta) \quad (3.43)$$

3.4 Expansion System

The last system to be assessed is the expansion system, whose main objective is to accelerate the flow to produce the maximum amount of thrust. It can operate in different ways, whether over-expanded, under-expanded or ideally, depending on the ratio between the engine inlet and outlet pressures. If this ratio is 1, we are facing an ideal configuration; while if it is greater than 1, we will have an over-expanded nozzle configuration, allowing a lower weight in the nozzle configuration; Finally, if the ratio is less than 1, we are dealing with an under-expanded nozzle, which causes a decrease in the thrust produced and an increase in the nozzle geometry and consequently in its weight.

In this work an ideal nozzle will be considered, where the pressure at inlet of the engine and on the outlet is the same, making the ratio between them 1. The expansion process will be adiabatic and isentropic, in addition to being one-dimensional and calorically perfect flow.

To calculate the properties of the expansion system, the equations (3.44), (3.45) and (3.46) will have to be applied.

$$T_5 = T_4 \left[1 - \eta_e \left(1 - \left(\frac{p_5 p_0}{p_0 p_4} \right)^{\frac{R}{c_{pe}}} \right) \right] \quad (3.44)$$

$$V_5 = \sqrt{V_4^2 + 2c_{pe}(T_4 - T_5)} \quad (3.45)$$

$$\frac{A_5}{A_0} = (1 + f) * \frac{p_0 T_5 V_0}{p_5 T_0 V_5} \quad (3.46)$$

3.4.1 Performance

Finally, the last step is the definition of the equations that will evaluate the performance of the designed engine, which were taken from the work of Pratt et al. [3]. As previously mentioned, assumptions were made that are far from what happens in reality, at least it is necessary to understand and know how to interpret the results obtained in accordance with the assumptions made previously.

The first performance factor to be evaluated is specific thrust, which is defined by the equation (3.47).

$$\frac{\text{Uninstalled thrust}}{\text{Inlet air mass flow rate}} = \frac{F}{\dot{m}_0} \quad (3.47)$$

When using the stream thrust function, there is another way to determine the specific thrust, from velocities and temperatures obtain at the entrance and exit of the engine, defined in the equation (3.48).

$$\frac{F}{\dot{m}_0} = (1 + f)Sa_5 - Sa_0 - \frac{R_0 T_0}{V_0} \left(\frac{A_5}{A_0} - 1 \right) \quad (3.48)$$

where,

$$Sa_5 = V_5 \left(1 + \frac{RT_5}{V_5^2} \right) \quad (3.49)$$

$$Sa_0 = V_0 \left(1 + \frac{RT_0}{V_0^2} \right) \quad (3.50)$$

Sa represents the stream thrust function at inlet and exit of the engine (Point 0 and 5, respectively), depending on the flow velocities and temperatures recorded in these sections. Based on the equations described, it is possible to obtain the flow rate of the air mass, which can be obtained through the derivative of equation (3.48), which indicates not only the uninstalled thrust required for the operation, but also the specific thrust of the developed engine.

The next factor to consider is the specific fuel consumption, which can be defined by the equation (3.51).

$$\frac{\text{Fuel mass flow rate}}{\text{Uninstalled thrust}} = S = \frac{\dot{m}_f}{F} \quad (3.51)$$

The specific fuel consumption can also be defined through the specific thrust, using the equation (3.52).

$$S = \frac{f}{\frac{F}{\dot{m}_0}} \quad (3.52)$$

Next, is one of the most important performance parameters to evaluate the performance of an engine, the specific impulse, representing the efficiency with which the engine produces thrust.

This is defined by the equation (3.53).

$$\frac{\text{Uninstalled thrust}}{\text{Fuel weight flow rate}} = I_{sp} = \frac{F}{g_0 \dot{m}_f} \quad (3.53)$$

Next, it will be the propulsive efficiency, which represents the ratio of thrust power to the mechanical power of the engine, being defined by the equation (3.54), which can be applied due to the fact that an ideal nozzle has been selected, where the inlet and outlet pressure of the engine are the same.

$$\eta_p = \frac{2 \left[(1 + f) \frac{V_e}{V_o} - 1 \right]}{(1 + f) \left(\frac{V_e}{V_o} \right)^2 - 1} \quad (3.54)$$

If the fuel/air ratio is too high, and according to the equation (3.54), the propulsive efficiency may be greater than 1, making the propulsive efficiency over 100%.

Therefore, as the stoichiometric ratio of fuel/air is 1, this term ends up "disappearing", giving rise to another equation. This is represented in the equation (3.55).

$$\eta_p = \frac{2}{\frac{V_e}{V_o} + 1} \quad (3.55)$$

The next factor is thermal efficiency, which represents the ratio of the engine's mechanical power to chemical energy rate. This can be defined by the equation (3.56).

$$\eta_{th} = \frac{(1 + f) \frac{V_e^2}{2} - \frac{V_o^2}{2}}{f h_{pr}} \quad (3.56)$$

Finally, overall efficiency, which tells us about the efficiency with which the internal combustion engine can extract the energy from a fuel to provide to the vehicle. This is defined by the equation (3.57).

$$\text{Overall efficiency} = \eta_o = \frac{\text{Thrust power}}{\text{Chemical energy rate}} = \frac{F V_o}{\dot{m}_f h_{pr}} \quad (3.57)$$

A more direct way to calculate overall efficiency is through the product between thermal efficiency and propulsive efficiency, being defined by equation (3.58).

$$\eta_o = \eta_{th} \eta_p \quad (3.58)$$

4. RESULTS

In this chapter, the results obtained by the numerical tool are represented, as well as the necessary previous studies, to validate the tool developed.

For the comparison between the work developed in this dissertation and the work of Pereirinha et al. [16], the studies carried out for model validation and the parametric studies has been the same, so in the next two subchapters, the preliminary results obtained by Pereirinha et al. [16], will be discussed.

4.1 Model Validation

Before starting the implementation and analysis of the developed tool, it is necessary to test it and compare the results with the available bibliography. These allows the validation of the implemented model and discover what possible obstacles may exist in the implementation of the tool.

4.1.1 Compression System

The analysis begins with the compression system, according to the design conditions. Primarily, the wedge angle is found, which is the same for all shocks. In this case, only weak shocks will be considered, to ensure that the flow is supersonic throughout the engine, and four oblique shock waves are be considered. The inputs used in this test are described in the Table 4.1, where the subscript c, represents the compression system.

Table 4.1: Inputs for the validation of the compression system [16]

Property	Assigned Value
M_0	10
T_0	288.15 K
T_{ratio}	2
γ_c	1.362

The results obtained in the validation of the compressive system can be seen in the Table 4.2.

Table 4.2: Results of validation of the compression system [16]

Wedge Angle [°]	T_3 [K]	T_{ratio}	Error [%]
3.18	575.7	1.998	0.1019

The numerical tool finds a wedge angle, by iteration, where the difference between the desired T_{ratio} and the actual T_{ratio} was less than 1%. Considering result obtained, the value of T_{ratio} turns out to be acceptable, given the computational time spent and the precision presented.

4.1.2 Scramjet

Now it is necessary to validate the scramjet engine implemented in the numerical tool, which will then be compared with values from Pratt [3]. The Table 4.3 represents the values that were used to validate the model.

Table 4.3: Input for validation of the scramjet engine [3]

Property	Assigned Value
V_0	3048 m/s
T_0	222 K
V_3	2530 m/s
T_3	1556 K
$\eta_b = \eta_e$	0.9
f	0.04
$\frac{V_{fx}}{V_3} = \frac{V_f}{V_3}$	0.5
$C_f \frac{A_w}{A_3}$	0.1
C_{pb}	1.51 kJ/kgK
C_{pe}	1.51 kJ/kgK
fh_{pr}	3510 kJ/kg
h_f	0
T°	222 K
$\gamma_b = \gamma_e$	1.238
R	289 (m/s) ² /K
$\frac{p_4}{p_0}$	258
$\frac{p_5}{p_0}$	1.4

If we look at the conditions imposed on the scramjet engine, namely, constant pressure, it is also necessary to validate this same condition. Therefore, tests were carried out to obtain the validation of this condition.

Table 4.4: Comparison of constant pressure results by the model and Pratt et al.[3]

Property	Constant Pressure [3]	Constant Pressure	Error [%]
V_4	2360 m/s	2359.71 m/s	0.012
T_4	3740 K	3730.86 K	0.244
V_5	3460 m/s	3460.65 m/s	0.019
T_5	1610 K	1609.05 K	0.059
$\frac{F}{\dot{m}_0}$	590 N s/kg	591.1 N s/kg	0.185
η_{th}	0.450	0.451	0.185
I_{sp}	1504	1506.86 s	0.190

According to the Table 4.4, none of the analysed properties has an error greater than 1%, so the constant pressure scramjet model can be considered validated, since the objective is to validate the code, so the values obtained should be similar, hence the relative errors are small.

4.1.3 Oblique Detonation Wave Engine

Now it is necessary to validate the mathematical model for the ODWE case. The results obtained were once again compared with Pratt's results et al. [3].

One of the factors considered was the analysis of the Chapman-Jouguet locus, where through an online program, and with points taken from Figure 4.1, it was possible to obtain the same curve, underlined in the Figure 4.2.

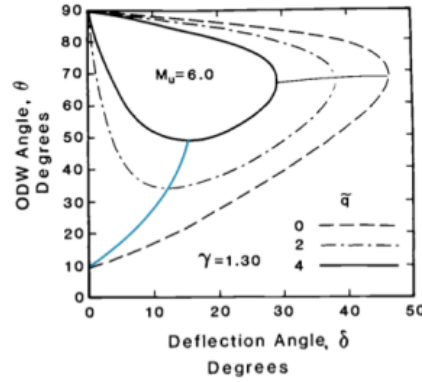


Figure 4.1: ODW angle as function of deflection angle and heat addition [3]

We noticed that the curves almost overlapped, starting to have a relatively low separation for deflection angles above 6°. As this separation is very small, we can consider the results obtained by the tool acceptable.

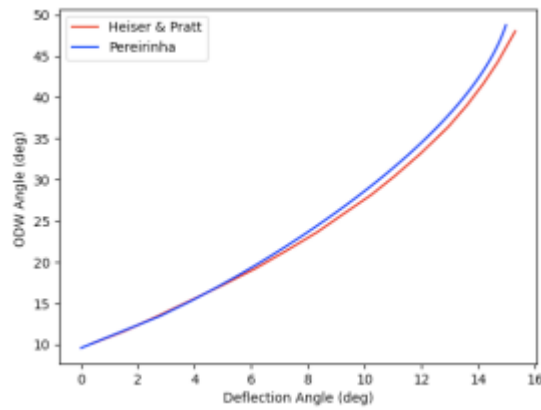


Figure 4.2: Comparison of Chapman-Jouguet locus[16]

To validate the model, it is necessary to determine the properties after the Chapman-Jouguet detonations. Thus, a computer program developed by NASA, called CEA (Chemical Equilibrium and Applications), was used. CEA is a program which calculates chemical equilibrium product concentrations from any set of reactants and determines thermodynamic and transport properties for the product mixture. For the test carried out, air was selected as an oxidizer and hydrogen for fuel. The results obtained by this program were later compared with the results obtained by the developed tool.

To carry out this study, the inputs used can be visualized in the Table 4.5.

Table 4.5: Input data [16]

Property	Assigned Value
T_0	298 K
p_0	101325 Pa
ϕ	1
M_{cj}	4.8335
a_{cj}	406.6 m/s
ρ_{ratio}	1.8042
γ_0	1.4015

The results obtained in the simulation, and their comparison with the results obtained by the CEA program, are visible in Table 4.6.

We realize that the error obtained is again less than 1%, so we can validate the mathematical model developed, and it is now possible to determine the properties after the Chapman-Jouguet detonations.

Table 4.6: Comparison of the results obtained by the model and CEA [16]

Property	CEA	Model	Error [%]
p_{ratio}	15.595	15.594	0.0054
T_{ratio}	9.878	9.923	0.4569

4.2 Parametric Studies

To study different components of the engine individually, to understand their behaviour and the influence they have on each other, several functions were defined for implementation in the code.

The results obtained in Pereirinha's work also served as a basis for the inputs chosen for the implementation of the case study.

4.2.1 Compression System

Once again, the study begins with the compression system. The first study carried out seeks to demonstrate how the T_{ratio} , influences the efficiency of the compression system and kinetic energy. This influence can be seen in the Figure 4.3 and Figure 4.4, respectively.

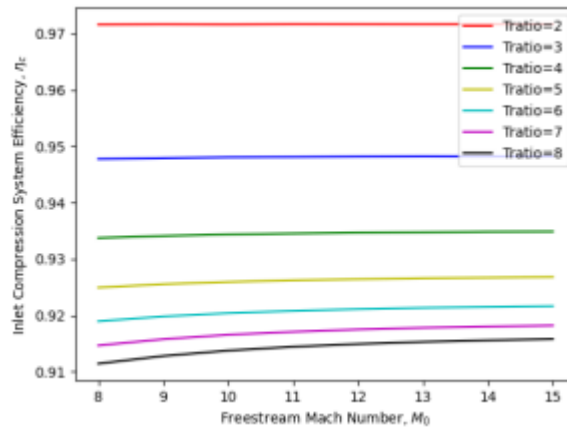


Figure 4.3 : Inlet compression system efficiency as function of M_0 [16]

Starting with Figure 4.3, we noticed that as the T_{ratio} increases, there is a decrease in the efficiency of the compression system.

On Figure 4.4, we noticed that the behaviour of the efficiency of kinetic energy is very similar to the behaviour of the efficiency of the compression system, since when T_{ratio} increases, there is a decrease in the efficiency of kinetic energy.

Therefore, when choosing the T_{ratio} , the negative impact that it has on both efficiencies should be considered, although a low value of T_{ratio} , negatively influences the overall performance of the engine, so it is necessary to choose the value of T_{ratio} .

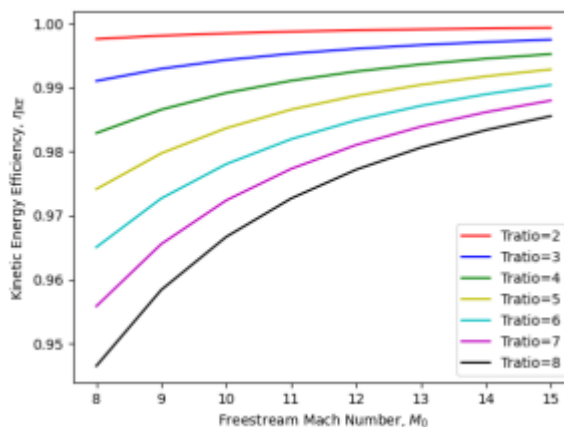


Figure 4.4: Kinetic energy efficiency as function of M_0 [16]

Observing the Figure 4.5, it can be seen how the Mach number at the burner input varies with the freestream Mach number, for different values of T_{ratio} . As the T_{ratio} increases the M_3 decreases.

This chart allows us to choose the value of T_{ratio} , which allows a supersonic flow at the burner inlet, although low Mach numbers at the burner inlet, have some advantages, namely in terms of combustion. In this case, low Mach numbers mean high values of T_{ratio} , this implies high temperatures inside the engine, implying specific materials in this region, increasing the cost of the developed engine.

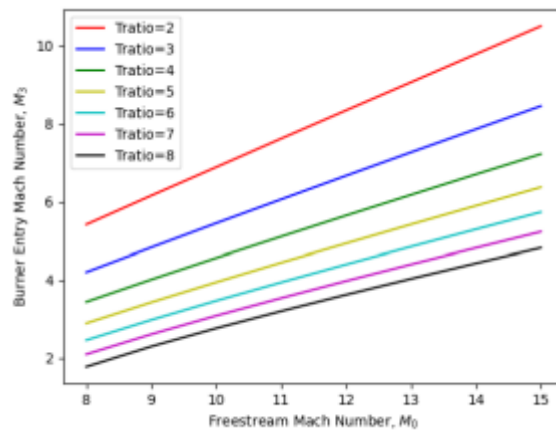


Figure 4.5: M_3 as a function of M_0 [16]

4.2.2 Oblique Detonation Wave

The next study to be carried out was that of oblique detonation waves. This study seeks to understand how $\tilde{Q}$ influences the wedge and detonation angles of a Chapman-Jouguet detonation. Both influences can be seen in Figure 4.6 and Figure 4.7, respectively.

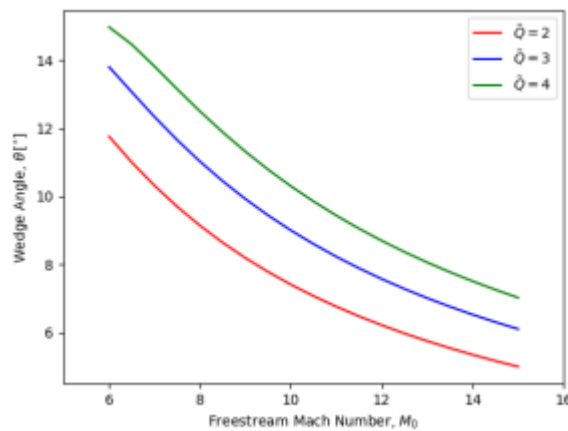


Figure 4.6: Wedge angle as a function of freestream Mach number [16]

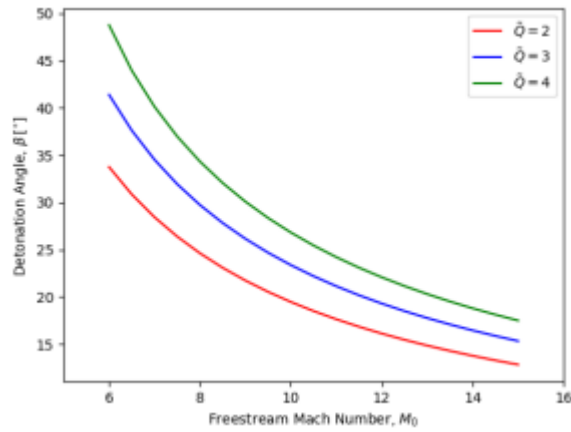


Figure 4.7: Detonation angle as a function of freestream Mach number [16]

As the value of $\tilde{Q}$ increases, the wedge angle required to generate a Chapman-Jouguet detonation will also increase, also leading to an increase in the detonation angle.

Next, it was studied how the detonation pressure and temperature ratio are influenced by the heat flux. This influence can be visualized in the Figure 4.8 and Figure 4.9.

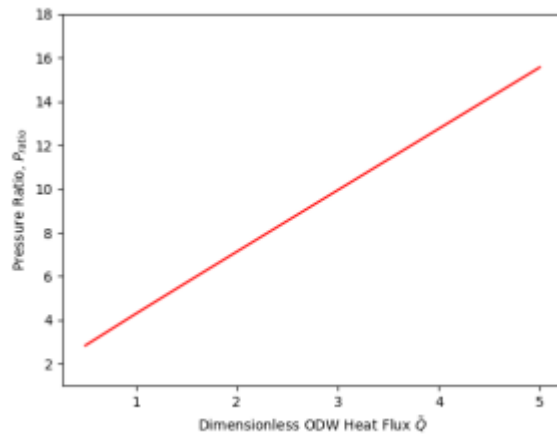


Figure 4.8: Pressure ratio as function of ODW heat flux [16]

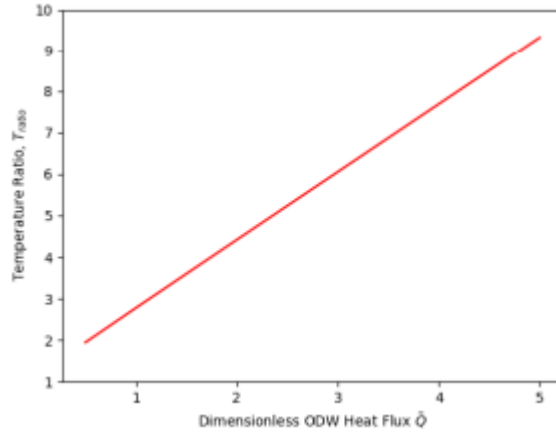


Figure 4.9: Temperature ratio as function of ODW heat flux [16]

It can then be concluded that as the heat flux of ODW increases, there is also a linear increase in the p_{ratio} and T_{ratio} .

4.2.3 Scramjet combustion

In this section, the influence of ϕ for a constant pressure burner will be studied. To carry out this study, the properties visible in Table 4.7.

Table 4.7: Inputs of the study of influence of ϕ [16]

Property	Assigned Value
$On - design M_0$	8
$On - design T_{ratio}$	2
$\frac{V_{fx}}{V_3}$	0.5
$\frac{V_f}{V_3}$	0.5
$C_f \frac{A_w}{A_3}$	0
C_{PB}	1510
T°	J/kgK
h_{pr}	222 K
	119.96
	MJ/kg
η_b	1

The results obtained for the burner at constant pressure can be seen in the Figure 4.10, Figure 4.11 and Figure 4.12.

As expected, as the equivalence ratio increases, there is an increase in the temperature at the output of the burner.

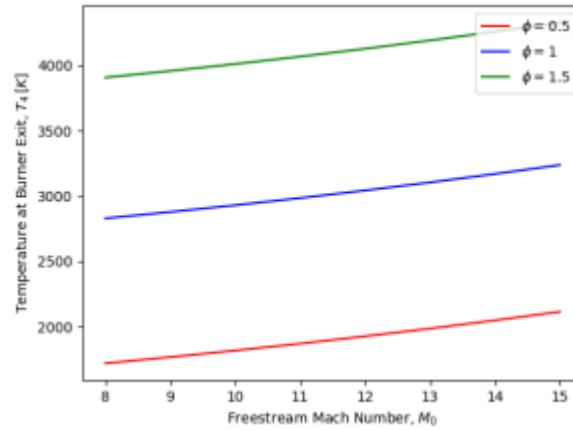


Figure 4.10: Temperature at the exit of the burner as a function of M_0 [16]

A different behaviour is observed at the burner output speed, where low equivalence ratio values lead to higher burner exit speed values.

At the level of the area along the burner, it is necessary that it increases, so that the pressure is maintained. Therefore, an increase in the equivalence ratio leads to higher variations in the area along the burner, making it easier to maintain pressure.

For the engine in general, by increasing the cross-section area, you are increasing the drag generated by the engine, which is a negative aspect.

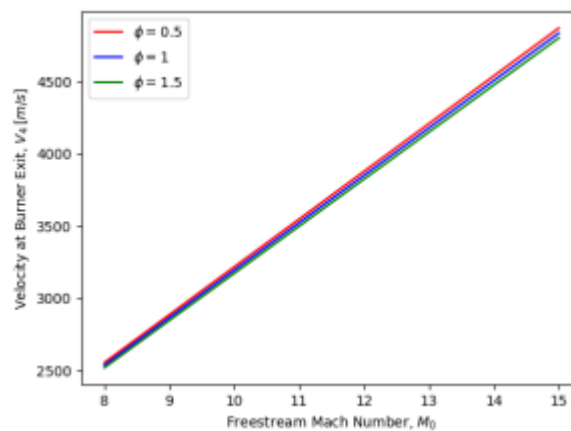


Figure 4.11: Velocity at the exit of the burner as a function of M_0 [16]

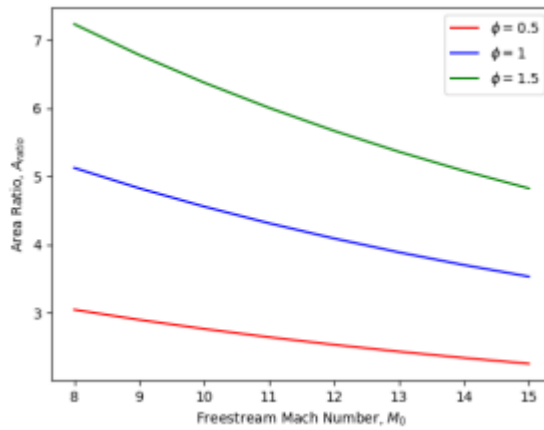


Figure 4.12: Area ratio as a function of M_0 [16]

4.3 Case Study

After the explanation, of the parametric studies obtained by Pereirinha et al. [16], and after carrying them out, to prove the results obtained by Pereirinha et al. [16], the case study intended in this work, can be developed.

Based on the parametric studies, the inputs for the case study have been carefully selected, so that the case study is as accurate and real as possible.

4.3.1 Input

Some considerations were made during the choice of inputs, namely the fact that the freestream number of Mach is 8, although in this work an isolator is implemented. The range of the Mach number implemented in the study should be small since some properties are considered constant along this range.

Other points to be considered are the choice of the T_{ratio} , as seen in the chapter on parametric studies, in addition to the temperature before detonation being at least 1000 K, to prevent pre-ignition. If these conditions are not registered, the code will warn the user.

In the case of the values, especially the specific heats, these must be different in the scramjet and in the ODWE, since the temperature and pressure recorded in the burner of the ODWE's is much higher when compared to those of the scramjet.

The values used as input for the case study can be visualized in Table 4.8.

Table 4.8: Case study inputs

Property	Assigned Value
h [m]	0
M_{0D}	10
M_{ub}	15
T_{ratio}	2
γ_c	1.362
Scramjet γ_b	1.238
ODWE γ_b	1.170
Scramjet γ_c	1.238
ODWE γ_c	1.170
C_{p0}	1005
C_{pc}	1090
Scramjet C_{pb}	1510
ODWE C_{pb}	2000
Scramjet C_{pe}	1510
ODWE C_{pe}	2000
$\frac{V_{fx}}{V_3}$	0.5
$\frac{V_f}{V_3}$	0.5
Scramjet η_b	0.9
ODWE η_b	0.9
Scramjet η_e	0.9
ODWE η_e	0.9
x of CxHy	0
y of CxHy	2
h_{pr} [MJ/kg]	119.96

4.3.2 Performance

In this section, the results obtained will be analysed, after the implementation of the case study in the numerical tool developed.

Primarily, the analysis of the specific thrust will be made as a function of the Mach number at the inlet, as shown in Figure 4.13. As expected, for all three cases, the maximum specific thrust is recorded for a Mach number of 10, and as the Mach number increases, there is a decrease

in specific thrust, due to the more expressive increase in the velocity of the flow outside in relation to the escape velocity. For scramjet engines, we found that the variable area configuration has a specific thrust higher than the constant pressure configuration. Even so, this value is much lower when compared to the specific thrust for the case of ODWE's. Another important point is the fact that the slope of the curve of the variable area configuration is quite pronounced, being in ODWE, lighter, and the constant pressure almost non-existent.

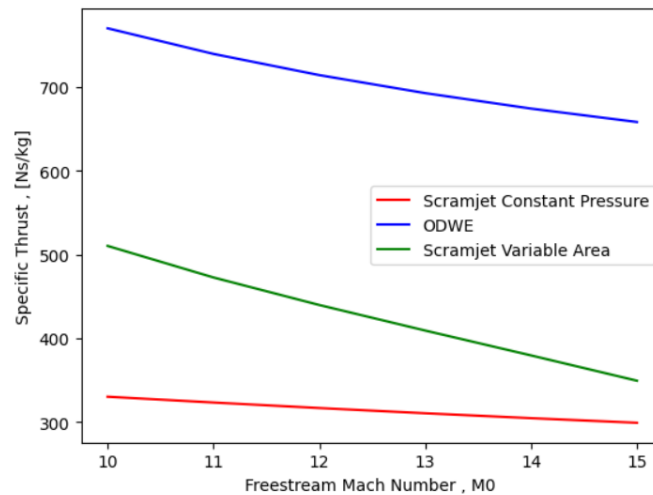


Figure 4.13: Specific Thrust as function of freestream Mach number

According to these results, it is expected that the engine with the highest specific thrust will have the lowest specific fuel consumption. This is proven by the Figure 4.14. It is also noted that the lowest specific fuel consumption is recorded for lower Mach numbers, and as the Mach number increases, so does the specific fuel consumption. Once again, the ODWE is the engine that has the lowest specific fuel consumption, being inferior to the variable area engine, and even more so to the constant pressure engine.

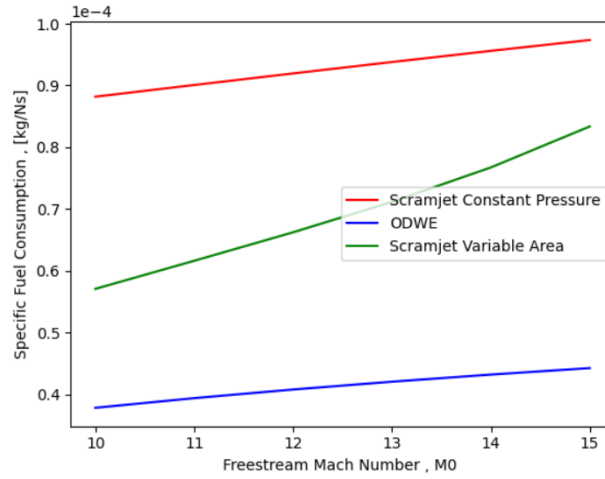


Figure 4.14: Specific fuel consumption as function of freestream Mach number

Next, the specific impulse will be analysed, which can be observed in Figure 4.15. The motor that has the highest specific thrust is the ODWE, followed by the variable area and finally the constant pressure motor. It is also noted that the highest specific impulse values recorded are for lower Mach numbers.

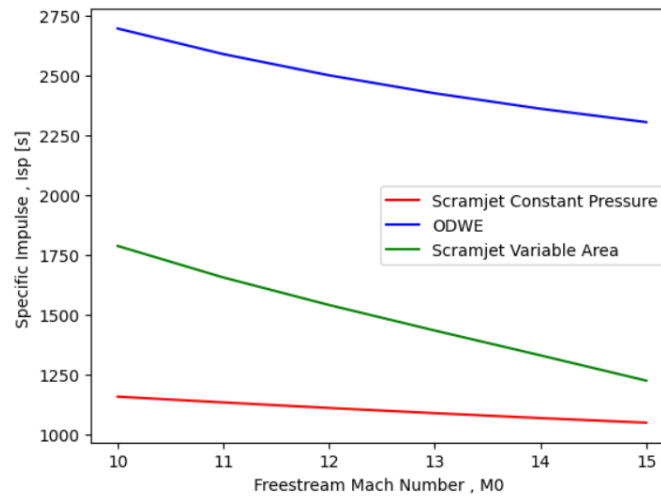


Figure 4.15: Specific impulse as function of freestream Mach number

Observing the Figure 4.16, we realized that the next factor to be studied was propulsive efficiency. We realized that in this case the ODWE has the lowest propulsive efficiency, since it has the highest exhaust velocity, which can be proven by the equation (3.55). On the contrary, the constant pressure motor has the highest propulsive efficiency, since it has the lowest exhaust speed. An important point is the fact that the constant-pressure motor is the only motor that converts power, mechanical energy into thrust power more efficiently.

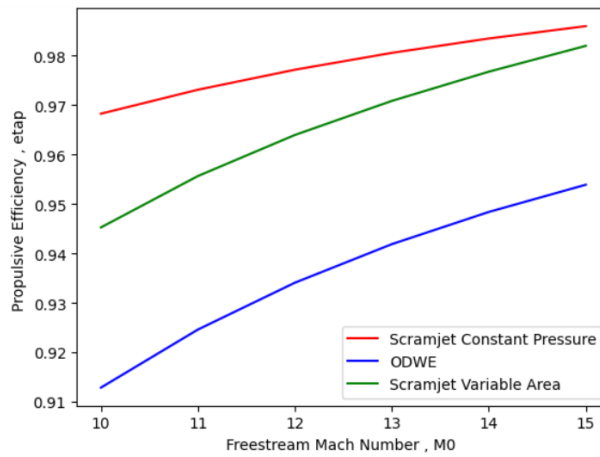


Figure 4.16: Propulsive efficiency as function of freestream Mach number

In terms of thermal efficiency and observing the Figure 4.17, we realize that ODWE has the highest thermal efficiency compared to scramjet engines, since ordinary thermal machines do not have shock waves, other than burning. The shock waves allow an increase in pressure and temperature, without using more fuel, improving thermal efficiency. This value increases as the number of Mach in the inlet also increases. In the case of scramjet engines, the variable area engine has better thermal efficiency when compared to the constant pressure engine, although it decreases as the Mach number increases.

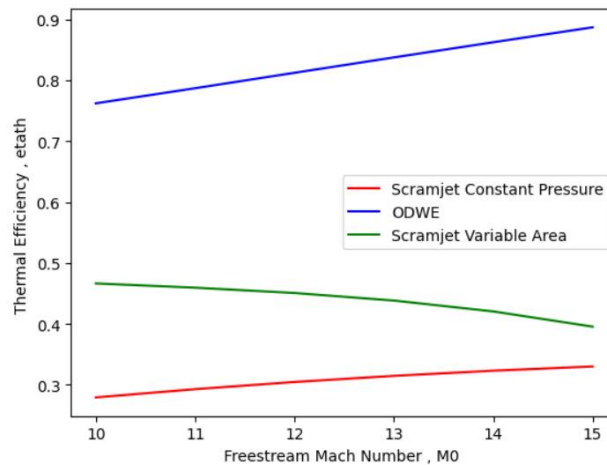


Figure 4.17: Thermal efficiency as function of freestream Mach number

The next study to be carried out is the overall efficiency study, as shown in Figure 4.18. This parameter demonstrates how efficiently the engine uses the energy that was initially stored

in the fuel tanks. Once again it is verified that the ODWE is the most efficient motor, followed by the variable area and finally the constant pressure motor.

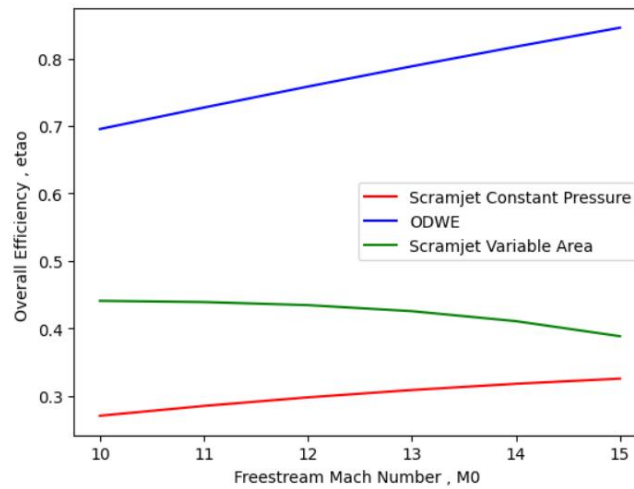


Figure 4.18: Overall efficiency as function of freestream Mach number

After analyzing the performance graphs obtained, we can conclude that the ODWE is a very promising engine, with a performance to highlight, when compared to both configurations of scramjet engines studied. Although not the main objective of this work, we can also conclude that the variable area scramjet engine has a higher performance than the constant pressure scramjet engine.

Although it is not a performance parameter, it was decided to generate the graph of the ratio between the engine inlet and outlet area, which can be observed in Figure 4.19. This is an indicator of the amount of drag generated by the geometry of the engine. We noticed that the greatest variation in areas between the entrance and the exit is verified in the variable area scramjet engine.

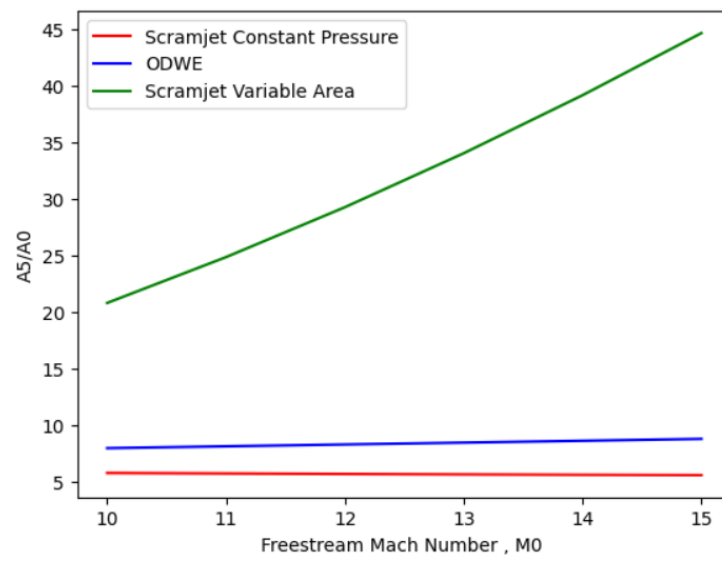


Figure 4.19: Area ratio as function of freestream Mach number

5. CONCLUSION

The main objective of this dissertation was to compare the performance of ODWE with the performance of different configurations of scramjet engines. With this objective, it was necessary to create a numerical tool using the Python language, joining the various studies that have been carried out in the context of new engine configurations that reach supersonic speeds, given the advantages they have over the engines traditionally used, both in aviation and at a special level.

First, research was carried out on the concepts that served as the basis for this work, and later it was decided that the study would be one-dimensional. The method for the configurations of the scramjet engines was chosen, either constant pressure or variable area. Next, the ODWE detonation process was defined since it does not occur in the configurations of the scramjet engines. It was also necessary to define the different values for the specific heats in the different stages of the engine, considering that these were constant for the range of the Mach number considered. This seeks to simplify the analysis without compromising the accuracy of the results.

To ensure that the results obtained were valid, several tests were carried out, where their results were compared with those obtained in similar existing studies. The results obtained in these tests were quite satisfactory, and it was possible to validate the implemented model, since the relative error obtained in the different properties analysed was less than 1%.

Then, parametric studies were carried out, with the objective of understanding the influence of certain parameters on the functioning of the engine, but also to serve as a basis for the values chosen as inputs in the case study developed.

Finally, the case study was implemented, and its results analysed. Thus, it was concluded that ODWE has a superior performance in relation to scramjet configurations, and this difference is mainly due to the combustion process. The ODWE has been found to have a higher specific thrust than scramjet engines, as well as a higher thermal efficiency and lower specific fuel consumption. Among the scramjet engines, the variable area engine showed a higher performance, in practically all the factors analysed, than the constant pressure scramjet engine.

Although ODWE has a higher performance than scramjet engines, its implementation in the real world is still far away, due to obstacles to its use, especially in the stability of ODW, in

addition to materials that can withstand the existing conditions after detonation. Although oblique detonation engines, due to the fact that they use shock waves, increase the pressure and temperature before combustion, resulting in a higher thermal efficiency, unlike common thermal machines, which only have the burning phase, they are not as efficient.

To conclude, the main objective of this dissertation was achieved, and demonstrates, the increasing interest in supersonic flights, namely ODWE's.

5.1 Future Works

As far as future work is concerned, several improvements can be implemented in the code. These improvements can go through different configurations of scramjet engines, namely in terms of the combustion process, such as constant volume. Another way would be to implement an existing engine geometry, to be implemented in the code, to estimate the dimensions of the engine, allowing for a more complete analysis and more realistic comparison. Finally, a study with different fuels, in addition to hydrogen, would also be interesting, to understand the behaviour of the engine for other fuels, and its influence.

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ANNEXES

Annex 1 – Code input

```
Master Thesis Project by Tomás Ferreira, University of Minho, 2023/2024
-----Input-----
Altitude (m) = 0
Design Mach Number = 10
Lower Mach Number = 10
Upper Mach Number = 15
Number of shocks for compression system = 4
Desired compression ratio = 2
Ratio of specific heats for compression = 1.362
Ratio of specific heats for combustion - SCRAMJET = 1.238
Ratio of specific heats for combustion - ODWE = 1.17
Ratio of specific heats for expansion - SCRAMJET = 1.238
Ratio of specific heats for expansion - ODWE = 1.17
Specific heat at constant pressure - free-stream (J/kgK) = 1005
Specific heat at constant pressure - compression (J/kgK) = 1090
Specific heat at constant pressure - burner Scramjet (J/kgK) = 1510
Specific heat at constant pressure - burner ODWE (J/kgK) = 2000
Specific heat at constant pressure - expansion Scramjet (J/kgK) = 1510
Specific heat at constant pressure - expansion ODWE (J/kgK) = 2000
Ratio of fuel injection axial velocity to velocity at burner entry = 0.5
Combustion efficiency SCRAMJET = 0.9
Combustion efficiency ODWE = 0.9
Expansion efficiency SCRAMJET = 0.9
Expansion efficiency ODWE = 0.9
Fuel definition (considering CxHy, insert x) = 0
Fuel definition (considering CxHy, insert y) = 2
Fuel Heating Value (MJ/kg) = 119.96
```

Annex 2 – Code

```
from cgi import print_arguments
from cmath import pi, sin, sqrt
from curses.ascii import SI
from getopt import gnu_getopt
from gettext import translation
import math
from posixpath import abspath
from re import X
import string
import sys
from tkinter.messagebox import YES
from tracemalloc import stop
from turtle import xcor
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.backends.backend_pdf import PdfFile , PdfPages
import random
from scipy.optimize import fsolve
import warnings
warnings.simplefilter("ignore", np.ComplexWarning)

#Reading Input from input.txt file
rdm=[]
file = open("/content/input.txt","r")
lines=file.readlines()

for line in lines:
    s=''
    for c in line:
        if c.isdigit()==True:
            s1=str(c)
            s=s+s1
```

```

        if c==" ":
            s1=" "
            s=s+s1
    rdm.append(s)
file.close()

h=float(rdm[2])
M0=float(rdm[3])
A=int(rdm[4])
B=int(rdm[5])
x=int(rdm[6])
Tratio=float(rdm[7])
gamma_c=float(rdm[8])
gamma_b1=float(rdm[9])
gamma_b2=float(rdm[10])
gamma_e1=float(rdm[11])
gamma_e2=float(rdm[12])
Cp0=float(rdm[13])
Cpc=float(rdm[14])
Cpb1=float(rdm[15])
Cpb2=float(rdm[16])
Cpe1=float(rdm[17])
Cpe2=float(rdm[18])
VfxV4=float(rdm[19])
eta_b1=float(rdm[20])
eta_b2=float(rdm[21])
eta_e1=float(rdm[22])
eta_e2=float(rdm[23])
n_of_C=float(rdm[24])
n_of_H=float(rdm[25])
h_pr=float(rdm[26])
gamma_0=1.4
R0=- (Cp0/gamma_0)+Cp0

```

```

Rc=-(Cpc/gamma_c)+Cpc
Rb1=-(Cpb1/gamma_b1)+Cpb1
Rb2=-(Cpb2/gamma_b2)+Cpb2
Re1=-(Cpe1/gamma_e1)+Cpe1
Re2=-(Cpe2/gamma_e2)+Cpe2

if M0<8:
    print("Calculations might not be accurate , as an isolator might be needed for Mach < 8!")

if A<M0:
    print("This input will result in an underdriven detonation , therefore stable detonation cannot be assured!")
    quit()

#ISA Model
def atm(h):
    from cmath import exp, sqrt
    T_i=[288.15,216.65] #K
    P_i=[101325 ,22632.0637315355 ,5474.88855574362] #Pa
    g0=9.80665 #m/s^2
    R=287.05307 #J/kgK
    gamma=1.4
    lam=[-6.5*10**-3,1*10**-3] #K/m

    #Calculation of Temperature
    if h<11000:
        T=T_i[0]+lam[0]*h
    elif h<20000:
        T=T_i[1]
    else:
        T=T_i[1]+lam[1]*(h-20000)

    #Calculation of Pressure
    if h<11000:

```

```

        P=P_i[0]*(T/T_i[0])**(-g0/lam[0]/R)
    elif h<20000:
        P=P_i[1]*exp(-g0*(h-11000)/R/T_i[1])
    else:
        P=P_i[2]*(T/T_i[1])**(-g0/lam[1]/R)

    #Calcualtion of Density
    rho=P/(R*T)

    #Calculation of Speed of Sound
    a=sqrt(gamma*R*T)

    #Calculation of g with altitude
    g=g0*(6371000/(6371000+h))**2

    return T,P,rho,a,g

#Function to determine properties after an oblique shock wave

def oblique(M1,theta ,gamma_c ,T1,P1):
    beta=math.asin(1/M1)
    i1=1
    i2=0.5
    while i1>i2:
        a1=(2*(1/math.tan(beta)))*(M1**2*math.sin(beta)**2-1)
        a2=(M1**2*(gamma_c+math.cos(2*beta)))+2
        i1=abs(math.tan(theta)-a1/a2)
        beta+=0.00001
        a3=(2*(1/math.tan(beta)))*(M1**2*math.sin(beta)**2-1)
        a4=(M1**2*(gamma_c+math.cos(2*beta)))+2
        i2=abs(math.tan(theta)-a3/a4)
    beta=beta -0.00001
    i=M1*math.sin(beta)

```

```

if abs(i) <1.0:
    print("At this wave angle , no oblique shock wave is possible")
    quit()
rhoratio=((gamma_c+1)*(M1**2*math.sin(beta)**2))/(2+(gamma_c -1)*M1**2*
math.sin(beta)**2)
Pratio=1+(2*gamma_c)/(gamma_c+1)*(M1**2*math.sin(beta)**2-1)
Tratio=Pratio/rhoratio
T2=Tratio*T1
P2=Pratio*P1
M2=2/(gamma_c -1)
M2=M2*((1/Tratio)*(1+((gamma_c -1)/2)*M1**2) -1)
M2=sqrt(M2)
return abs(M2),T2,P2

#Inlet Design
T0,P0,rho,a,g0=atm(h)
theta=math.radians(1)
while theta <1:
    #1st compression shock
    beta=math.asin(1/M0)
    i1=(2*(M0**2*math.sin(beta)**2-1))
    i2=(math.tan(beta)*((M0**2*(gamma_c+math.cos(2*beta))+2)))
    while abs(math.tan(theta)-(i1/i2)) >0.0001:
        beta=beta+0.0001
        i1=(2*(M0**2*math.sin(beta)**2-1))
        i2=(math.tan(beta)*((M0**2*(gamma_c+math.cos(2*beta))+2)))
    pratio=abs(1+(2*gamma_c/(gamma_c+1))*(abs(M0**2*math.sin(beta)**2-1)))
    e=pratio
    rhoratio=abs((M0**2*math.sin(beta)**2*(gamma_c+1))/(2+(gamma_c -1)*M0**2*math.sin(beta)**2))
    M1=abs(sqrt((2/(gamma_c -1))*((rhoratio/pratio)*(1+((gamma_c -1)/2)*M0**2) -1)))
    a=abs(pratio/rhoratio)

```

```

#2nd compression shock
beta=math.asin(1/M1)
i1=(2*(M1**2*math.sin(beta)**2-1))
i2=(math.tan(beta)*((M1**2*(gamma_c+math.cos(2*beta))+2)))
while abs(math.tan(theta)-(i1/i2)) >0.0001:
    beta=beta+0.0001
    i1=(2*(M1**2*math.sin(beta)**2-1))
    i2=(math.tan(beta)*((M1**2*(gamma_c+math.cos(2*beta))+2)))
pratio=1+(2*gamma_c/(gamma_c+1))*(abs(M1**2*math.sin(beta)**2-1))
f=pratio
rhoratio=(M1**2*math.sin(beta)**2*(gamma_c+1))/(2+(gamma_c-1)*M1**2*math.sin(beta)**2)
M2=abs(sqrt((2/(gamma_c-1))*((rhoratio/pratio)*(1+((gamma_c-1)/2)*M1**2-1))))
b=abs(pratio/rhoratio)

#3rd compression shock
beta=math.asin(1/M2)
i1=(2*(M2**2*math.sin(beta)**2-1))
i2=(math.tan(beta)*((M2**2*(gamma_c+math.cos(2*beta))+2)))
while abs(math.tan(theta)-(i1/i2)) >0.0001:
    beta=beta+0.0001
    i1=(2*(M2**2*math.sin(beta)**2-1))
    i2=(math.tan(beta)*((M2**2*(gamma_c+math.cos(2*beta))+2)))
pratio=1+(2*gamma_c/(gamma_c+1))*(abs(M2**2*math.sin(beta)**2-1))
g=pratio
rhoratio=(M2**2*math.sin(beta)**2*(gamma_c+1))/(2+(gamma_c-1)*M2**2*math.sin(beta)**2)
M3=abs(sqrt((2/(gamma_c-1))*((rhoratio/pratio)*(1+((gamma_c-1)/2)*M2**2-1))))
c=pratio/rhoratio

#4th compression shock
beta=math.asin(1/M3)
i1=(2*(M3**2*math.sin(beta)**2-1))
i2=(math.tan(beta)*((M3**2*(gamma_c+math.cos(2*beta))+2)))
while abs(math.tan(theta)-(i1/i2)) >0.0001:

```

```

        beta=beta+0.00001
        i1=(2*(M3**2*math.sin(beta)**2-1))
        i2=(math.tan(beta)*((M3**2*(gamma_c+math.cos(2*beta))+2)))
        pratio=1+(2*gamma_c/(gamma_c+1))*(abs(M3**2*math.sin(beta)**2-1))
        h=pratio
        rhoratio=(M3**2*math.sin(beta)**2*(gamma_c+1))/(2+(gamma_c-1)*M3**2*math.sin(beta)**2)
        M4=abs(sqrt((2/(gamma_c-1))*((rhoratio/pratio)*(1+((gamma_c-1)/2)*M3**2-1))))
        d=pratio/rhoratio

        attempt=abs(d*c*b*a-Tratio)
        if abs(attempt) < 0.01:
            tpratio=e*f*g*h*((1/(a*b*c*d)))**((gamma_c/(gamma_c-1)))
            eta_c=a*b*c*d-(1/tpratio)**((gamma_c-1)/gamma_c)
            eta_c=eta_c/(a*b*c*d-1)
            eta_ke=1-(2/((gamma_c-1)*M0**2))*(-1+(1/tpratio)**((gamma_c-1)/gamma_c))
            break
        theta=theta+0.001

#Determination of ODWE wedge angle for Ch.J detonation
f=(36*n_of_C+3*n_of_H)/(412*n_of_C+103*n_of_H)
Q=((h_pr*10**6)*f)/(f+1)*eta_b2
T4=a*b*c*d*T0
Q_bar=Q/(Cpc*T4)
V4=abs(M4*sqrt(gamma_c*(Rc)*T4))
V4=V4*((1+f*Vf*xV4)/(1+f))
M4=M4*((1+f*Vf*xV4)/(1+f))

def ChJ(var):
    x=var
    eq1 = ((M4*math.sin(x))**2-1)**2-2*(gamma_c+1)*((M4*math.sin(x))**2)*Q_bar
    return eq1
beta= fsolve(ChJ ,((math.asin(1/M4)+1)))
if math.asin(1/M4)>beta >(pi/2):

```

```

    print("ERROR with detonation angle! Angle must be between",math.degrees(math.asin(1/M4)), "and 90 degrees")
i=(M4*math.sin(beta))**2
wedge_angle=beta-math.atan(((1+gamma_c*i)/((gamma_c+1)*i*abs(sqrt((M4/sqrt(i))**2-1))))))

#Determination of Properties after the compression system
i=0
matrix = [ [ 0 for y in range( 4 ) ] for x in range( (B-A)+1 ) ]
for M in range(A,B+1,1):
    T1=T0
    P1=P0
    M1=M
    for n_of_shocks in range (1,5,1):
        M2,T2,P2=oblique(M1,theta ,gamma_c ,T1,P1)
        if abs(M2)<1:
            print("ERROR: Supersonic combustion is not possible!")
            quit()
        if T2>1000:
            print("Preignition can occur!")
        M1=M2
        T1=T2
        P1=P2
        matrix[i][0]=M2
        matrix[i][1]=T2
        matrix[i][2]=P2
        matrix[i][3]=M
    i=i+1

#Scramjet Combustion
#Constant Pressure
def combustion_scramjet(T1,Cpb,f,h_pr,M1):
    V1=abs(M1*sqrt(gamma_c*(Rc)*T1))
    V2=V1*((1+f*VfxV4)/(1+f))
    T2=(T1/(1+f))*(1+(1/(Cpb*T1))*(eta_b1*f*h_pr*10**6+f*Cpb*222+(1+f*(VfxV4)**2)*((V1**2)/2)))-((V2**2)/(2*Cpb))

```

```

return T2,V2

T_pb1 = [ [ 0 for y in range( 2 ) ] for x in range( (B-A)+1 ) ]
for i in range(0,(B-A)+1,1):
    T_pb1[i][0],T_pb1[i][1]=combustion_scamjet(matrix[i][1],Cpb1,f,h_pr,matrix[i][0])

#Variable area
def combustion_scamjetca(T1,Cpb,f,h_pr,M1,P1):
    Pi2=3
    C=(M1 * np.sqrt(gamma_0 * 287 * T1) + (1 - Pi2) * 287 * T1 / (M1 * np.sqrt(gamma_0 * 287 * T1)))/(np.sqrt((1.3* 287 * T1) * (1 + ((1.3 - 1) / 2) * M1**2)))
    ML=sqrt((C**2)/(1-((gamma_0 - 1)/2)*C**2))
    TI=T1*((1+((1.4- 1)/2)*M1**2)/(1+((1.3-1)/2)*abs(ML)**2))
    A1=3.14*(8*10**-3)
    tau=((0.2 * 119960000) / (34.32 * Cpc * TI))+1
    T2=TI*tau
    A2=A1/(gamma_0*abs(ML)**2)*((1/Pi2)*(1+gamma_0*M1**2)-1)
    PL=P1*(Pi2*((1 + ((1.3 - 1) / 2) * abs(ML)**2) ** (1.3 / (1.3 - 1)))/((1 + (gamma_0 - 1) / 2) * M1**2) ** (gamma_0 / (gamma_0 - 1)))
    M2=abs(ML)/(np.sqrt(tau*((1+1.286-1)/2*abs(ML)**2)-((1.286-1)/2*abs(ML)**2)))
    P2=PL / ((1 + (1.286 - 1)/2 * (abs(ML)**2) * (1 - 1/tau))**(1.286/(1.286 - 1)))
    V2=340*M2
    print(f"C: {C}")
    print(f"tau: {tau}")
    print(f"ML: {ML}")
    print(f"TI: {TI}")
    print(f"M2: {M2}")
    print(f"T2: {T2}")
    print(f"P2: {P2}")
    print(f"A2: {A2}")

    return T2,V2,P2

T_pb1ca = [ [ 0 for y in range( 3 ) ] for x in range( (B-A)+1 ) ]
for i in range(0,(B-A)+1,1):

```

```

T_pb1ca[i][0],T_pb1ca[i][1],T_pb1ca[i][2]=combustion_scramjetca(matrix[i][1],Cpb1,f,h_pr,matrix[i][0],matrix[i][2])

#Expansion Scramjet
def expansion(T1,P1,V1,eta_e ,R,Cp,f,M0,T0,P0):
    #1 is 4 and 2 is to 10
    V0=abs(M0*sqrt(gamma_0*(R0)*T0))
    T2=T1*(1-eta_e*(1-(P0/P1)**(R/Cp)))
    V2=abs(sqrt(V1**2+2*Cp*(T1-T2)))
    P2=P0
    Aratio=(1+f)*(T2/T0)*(V0/V2)
    return Aratio ,T2,P2,V2

Station10_1 = [ [ 0 for y in range( 4 ) ] for x in range( (B-A)+1 ) ]
for i in range(0,(B-A)+1,1):
    Station10_1[i][0],Station10_1[i][1],Station10_1[i][2],Station10_1[i][3]=expansion(T_pb1[i][0],matrix[i][2],T_pb1[i][1],eta_e1 ,Re1,Cpe1,f,matrix[i][3],T0,P0)

Station10_1ca = [ [ 0 for y in range( 4 ) ] for x in range( (B-A)+1 ) ]
for i in range(0,(B-A)+1,1):
    Station10_1ca[i][0],Station10_1ca[i][1],Station10_1ca[i][2],Station10_1ca[i][3]=expansion(T_pb1ca[i][0],T_pb1ca[i][2],T_pb1ca[i][1],eta_e1 ,Re1,Cpe1,f,matrix[i][3],T0,P0)

#ODWE Combustion
def post_detonation(P1,T1,rho1,V1,theta ,Cp,M1,Q):
    M1=M1*((1+f*VfxV4)/((1+f)))
    def first_root(var):
        x=var
        eq1=(-(gamma_c+1)/2)*(math.tan(x-theta)/math.tan(x))**2*(M1**2)*(math.sin(x)**2)+(math.tan(x-theta)/math.tan(x))*(1+gamma_c*(M1**2)*(math.sin(x)**2)) - ((1+((gamma_c -1)/2)
        *(M1**2)*(math.sin(x)**2))-Q
    return eq1

beta=fsolve(first_root ,(math.asin(1/M1)))
X=math.tan(beta-theta)/math.tan(beta)
V1=abs(M1*sqrt(Rc*T1*gamma_c))

```

```

V1=V1*math.sin(beta)
P2=P1+rho1*(V1**2)*(1-X)
Tratio=1+((V1**2)/(2*Cp*T1))*(1-X**2)+(Q)
T2=Tratio*T1
V2=(V1*X)/(math.sin(beta-theta))
M2=abs(V2/sqrt(gamma_b2*Rb2*T2))
if M2<1:
    print("Warning - Detonation can be unstable!")
M2n=abs((V1*X)/sqrt(gamma_b2*Rb2*T2))
return T2,P2,V2

T_pb2 = [ [ 0 for y in range( 3 ) ] for x in range( (B-A)+1 ) ]
for i in range(0,(B-A)+1,1):
    rho=(matrix[i][2])/(Rc*matrix[i][1])
    Q=((h_pr*10**6)*f)/(f+1)/(Cpc*matrix[i][1])*eta_b2
    T_pb2[i][0],T_pb2[i][1],T_pb2[i][2]=post_detonation(matrix[i][2],matrix[i][1],rho,matrix[i][0],wedge_angle ,Cpc,matrix[i][0],Q)

#Expansion ODWE
Station10_2 = [ [ 0 for y in range( 4 ) ] for x in range( (B-A)+1 ) ]
for i in range(0,(B-A)+1,1):
    Station10_2[i][0],Station10_2[i][1],Station10_2[i][2],Station10_2[i][3]=expansion(T_pb2[i][0],T_pb2[i][1],T_pb2[i][2],eta_e2 ,Re2,Cpe2,f,matrix[i][3],T0,P0)

#Performance
def performance(R1,R2,V10,T10,V0,Aratio ,f,g):
    Ve=V10
    Sa10=V10+((R2*T10)/V10)
    Sa0=V0+((R1*T0)/V0)
    st=(1+f)*Sa10-Sa0 -((R1*T0)/V0)*(Aratio -1)
    sfc=f/st
    si=1/(g0*sfc)
    Vratio=Ve/V0
    eta_p=2/(Vratio+1)
    eta_th=((((1+f)*(Ve**2)/2) -((V0**2)/2)))/(f*h_pr*1000000)

```

```

eta_o=eta_th*eta_p
return st,sfc,si,eta_p ,eta_th ,eta_o

per_scramjet = [ [ 0 for y in range( 6 ) ] for x in range( (B-A)+1 ) ]
per_scramjetca = [ [ 0 for y in range( 6 ) ] for x in range( (B-A)+1 ) ]
per_ODWE = [ [ 0 for y in range( 6 ) ] for x in range( (B-A)+1 ) ]

for i in range(0,(B-A)+1,1):
    V0=abs(matrix[i][3]*sqrt(T0*gamma_0*R0))
    per_scramjet[i][0],per_scramjet[i][1],per_scramjet[i][2],per_scramjet[i][3],per_scramjet[i][4],per_scramjet[i][5]=
    performance(R0,Re1,Station10_1[i][3],Station10_1[i][1],V0,Station10_1[i][0],f,g0)
    per_scramjetca[i][0],per_scramjetca[i][1],per_scramjetca[i][2],per_scramjetca[i][3],per_scramjetca[i][4],per_scramjetca[i][5]=
    performance(R0,Re1,Station10_1ca[i][3],Station10_1ca[i][1],V0,Station10_1ca[i][0],f,g0)
    per_ODWE[i][0],per_ODWE[i][1],per_ODWE[i][2],per_ODWE[i][3],per_ODWE[i][4],per_ODWE[i][5]=
    performance(R0,Re2,Station10_2[i][3],Station10_2[i][1],V0,Station10_2[i][0],f,g0)

#Plotting Results
Mach=[]
Mach=[0 for i in range((B-A)+1)]
ST_SCRAMJET=[]
ST_SCRAMJET=[0 for i in range((B-A)+1)]
ST_SCRAMJETca=[]
ST_SCRAMJETca=[0 for i in range((B-A)+1)]
ST_ODWE=[]
ST_ODWE=[0 for i in range((B-A)+1)]
SFC_SCRAMJET=[]
SFC_SCRAMJET=[0 for i in range((B-A)+1)]
SFC_SCRAMJETca=[]
SFC_SCRAMJETca=[0 for i in range((B-A)+1)]
SFC_ODWE=[]
SFC_ODWE=[0 for i in range((B-A)+1)]
SI_SCRAMJET=[]
SI_SCRAMJET=[0 for i in range((B-A)+1)]

```

```

SI_SCRAMJETca=[]
SI_SCRAMJETca=[0 for i in range((B-A)+1)]
SI_ODWE=[]
SI_ODWE=[0 for i in range((B-A)+1)]
ETA_p_SCRAMJET=[]
ETA_p_SCRAMJET=[0 for i in range((B-A)+1)]
ETA_p_SCRAMJETca=[]
ETA_p_SCRAMJETca=[0 for i in range((B-A)+1)]
ETA_p_ODWE=[]
ETA_p_ODWE=[0 for i in range((B-A)+1)]
ETA_th_SCRAMJET=[]
ETA_th_SCRAMJET=[0 for i in range((B-A)+1)]
ETA_th_SCRAMJETca=[]
ETA_th_SCRAMJETca=[0 for i in range((B-A)+1)]
ETA_th_ODWE=[]
ETA_th_ODWE=[0 for i in range((B-A)+1)]
ETA_o_SCRAMJET=[]
ETA_o_SCRAMJET=[0 for i in range((B-A)+1)]
ETA_o_SCRAMJETca=[]
ETA_o_SCRAMJETca=[0 for i in range((B-A)+1)]
ETA_o_ODWE=[]
ETA_o_ODWE=[0 for i in range((B-A)+1)]
Aratiocp=[]
Aratiocp=[0 for i in range((B-A)+1)]
Aratioca=[]
Aratioca=[0 for i in range((B-A)+1)]
AratioODWE=[]
AratioODWE=[0 for i in range((B-A)+1)]

for i in range(0,(B-A)+1,1):
    Mach[i]=matrix[i][3]
    ST_SCRAMJET[i]=per_scramjet[i][0]
    SFC_SCRAMJET[i]=per_scramjet[i][1]

```

```

SI_SCRAMJET[i]=per_scramjet[i][2]
ETA_p_SCRAMJET[i]=per_scramjet[i][3]
ETA_th_SCRAMJET[i]=per_scramjet[i][4]
ETA_o_SCRAMJET[i]=per_scramjet[i][5]
SI_SCRAMJETca[i]=per_scramjetca[i][2]
ETA_p_SCRAMJETca[i]=per_scramjetca[i][3]
ETA_th_SCRAMJETca[i]=per_scramjetca[i][4]
ETA_o_SCRAMJETca[i]=per_scramjetca[i][5]
ST_ODWE[i]=per_ODWE[i][0]
SFC_ODWE[i]=per_ODWE[i][1]
SI_ODWE[i]=per_ODWE[i][2]
ETA_p_ODWE[i]=per_ODWE[i][3]
ETA_th_ODWE[i]=per_ODWE[i][4]
ETA_o_ODWE[i]=per_ODWE[i][5]
Aratiocp[i]=Station10_1[i][0]
Aratioca[i]=Station10_1ca[i][0]
AratioODWE[i]=Station10_2[i][0]

```

#PLOTING SPECIFIC THRUST VS. FREESTREAM MACH NUMBER

```

with PdfPages("/content/Output.pdf") as pdf:
    x = Mach
    y=ST_SCRAMJET
    z = ST_ODWE
    w=ST_SCRAMJETca
    plt.plot(x, y,'r',label="Scramjet Constant Pressure")
    plt.plot(x, z,'b',label="ODWE")
    plt.plot(x, w,'g',label="Scramjet Variable Area")
    plt.xlabel('Freestream Mach Number , M0 ')
    plt.ylabel('Specific Thrust , [Ns/kg]')
    plt.legend()

```

```
pdf.savefig()
plt.show()
plt.close()
```

#PLOTING SPECIFIC FUEL CONSUMPTION VS. FREESTREAM MACH NUMBER

```
x = Mach
y=SFC_SCRAMJET
z = SFC_ODWE
w=SFC_SCRAMJETca
plt.plot(x, y, 'r', label="Scramjet Constant Pressure")
plt.plot(x, z, 'b', label="ODWE")
plt.plot(x, w, 'g', label="Scramjet Variable Area")
plt.xlabel('Freestream Mach Number , M0 ')
plt.ylabel('Specific Fuel Consumption , [kg/Ns]')
plt.ticklabel_format(axis="y", style='sci', scilimits=(0,0))
plt.legend()
pdf.savefig()
plt.show()
plt.close()
```

#PLOTING SPECIFIC IMPULSE VS. FREESTREAM MACH NUMBER

```
x = Mach
y=SI_SCRAMJET
z = SI_ODWE
w=SI_SCRAMJETca
plt.plot(x, y, 'r', label="Scramjet Constant Pressure")
plt.plot(x, z, 'b', label="ODWE")
plt.plot(x, w, 'g', label="Scramjet Variable Area")
plt.xlabel('Freestream Mach Number , M0 ')
plt.ylabel('Specific Impulse , Isp [s] ')
plt.legend()
```

```
pdf.savefig()
plt.show()
plt.close()
```

#PLOTING PROPULSIVE EFFICIENCY VS. FREESTREAM MACH NUMBER

```
x = Mach
y = ETA_p_SCRAMJET
z=ETA_p_ODWE
w=ETA_p_SCRAMJETca
plt.plot(x, y, 'r', label="Scramjet Constant Pressure")
plt.plot(x, z, 'b', label="ODWE")
plt.plot(x, w, 'g', label="Scramjet Variable Area")
plt.xlabel('Freestream Mach Number , M0 ')
plt.ylabel('Propulsive Efficiency , etap ')
plt.legend()
pdf.savefig()
plt.show()
plt.close()
```

#PLOTING THERMAL EFFICIENCY VS. FREESTREAM MACH NUMBER

```
x = Mach
y = ETA_th_SCRAMJET
z= ETA_th_ODWE
w=ETA_th_SCRAMJETca
plt.plot(x, y, 'r', label="Scramjet Constant Pressure")
plt.plot(x, z, 'b', label="ODWE")
plt.plot(x, w, 'g', label="Scramjet Variable Area")
plt.xlabel('Freestream Mach Number , M0 ')
plt.ylabel('Thermal Efficiency , etath ')
plt.legend()
pdf.savefig()
```

```
plt.show()
plt.close()
```

#PLOTING GLOBAL EFFICIENCY VS. FREESTREAM MACH NUMBER

```
x = Mach
y = ETA_o_SCRAMJET
z=ETA_o_ODWE
w=ETA_o_SCRAMJETca
plt.plot(x, y, 'r',label="Scramjet Constant Pressure")
plt.plot(x, z, 'b',label="ODWE")
plt.plot(x, w, 'g',label="Scramjet Variable Area")
plt.xlabel('Freestream Mach Number , M0 ')
plt.ylabel('Overall Efficiency , etao ')
plt.legend()
pdf.savefig()
plt.show()
plt.close()
```

#PLOTING $A5/A0$ VS. FREESTREAM MACH NUMBER

```
x = Mach
y = Aratiocp
z=Aratioca
w=AratioODWE
plt.plot(x, y, 'r',label="Scramjet Constant Pressure")
plt.plot(x, w, 'b',label="ODWE")
plt.plot(x, z, 'g',label="Scramjet Variable Area")
plt.xlabel('Freestream Mach Number , M0 ')
plt.ylabel('A5/A0')
plt.legend()
pdf.savefig()
plt.show()
plt.close()
```